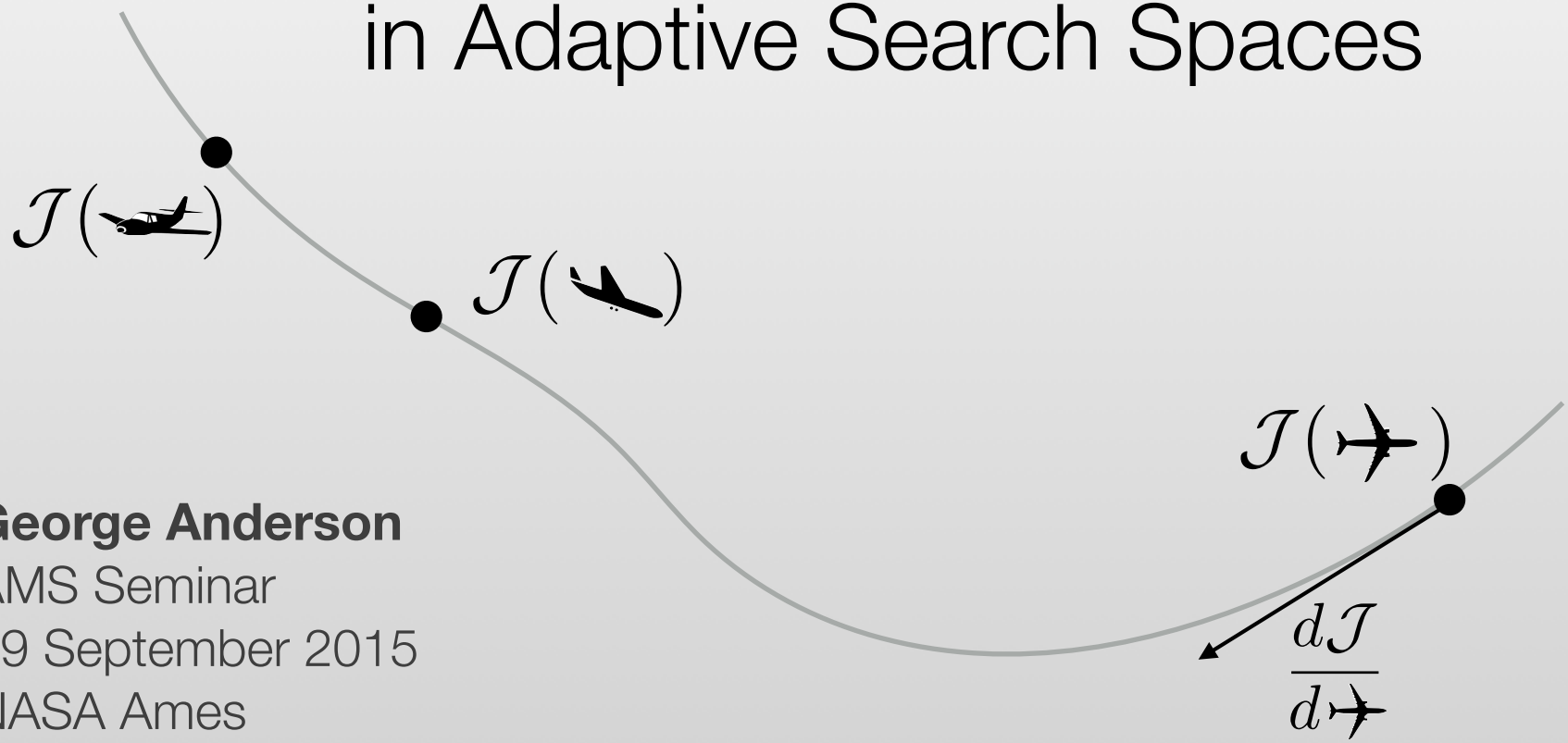
Shape Optimization in Adaptive Search Spaces

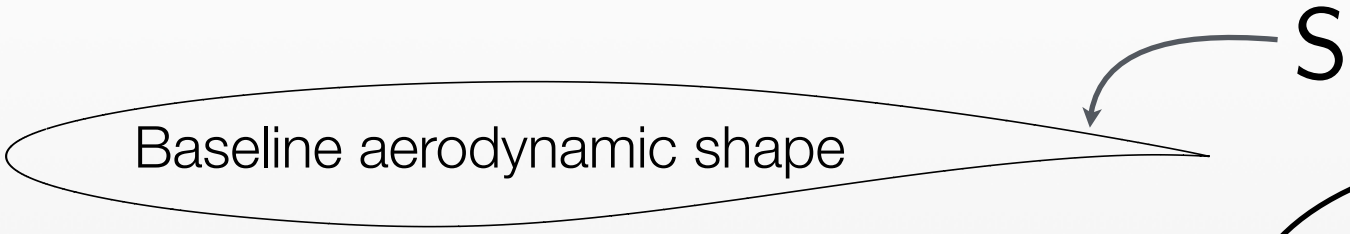


George Anderson
AMS Seminar
29 September 2015
NASA Ames

Outline

1. Parametric Shape Optimization
2. Adaptive Parameterization
3. Verification Studies
4. Design Examples

Aerodynamic Shape Optimization



Designer-driven

1. Define **goals**:
   Minimize objective
   Subject to constraints

$$\min_{\mathsf{S}} \mathcal{J}(\mathsf{S})$$
$$\mathcal{C}_j(\mathsf{S}) \leq 0$$

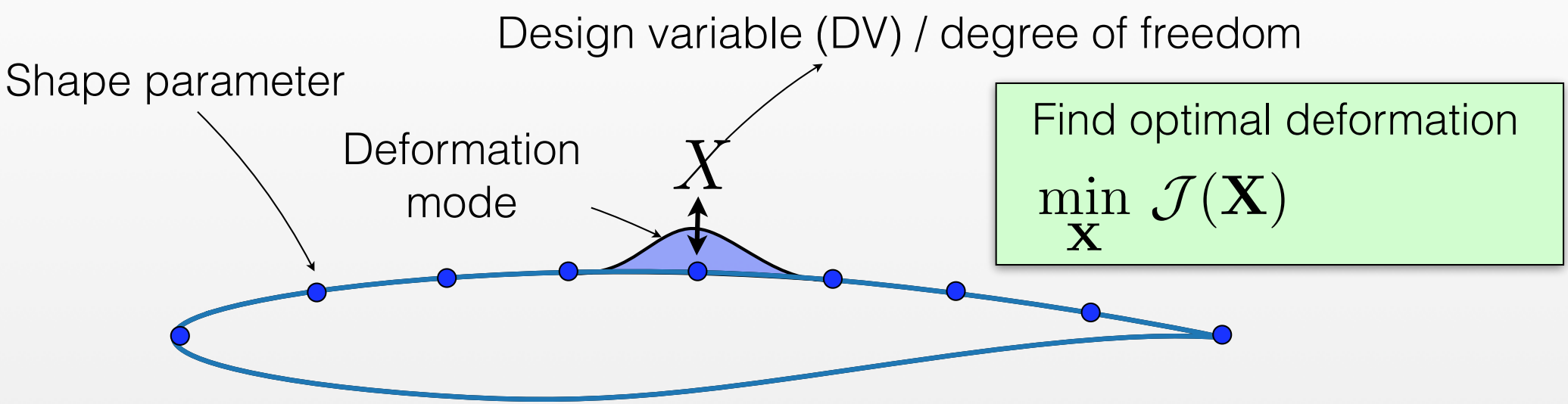
2. Select **design variables**
   (shape parameterization)

High-fidelity (expensive) **analysis**

Automated

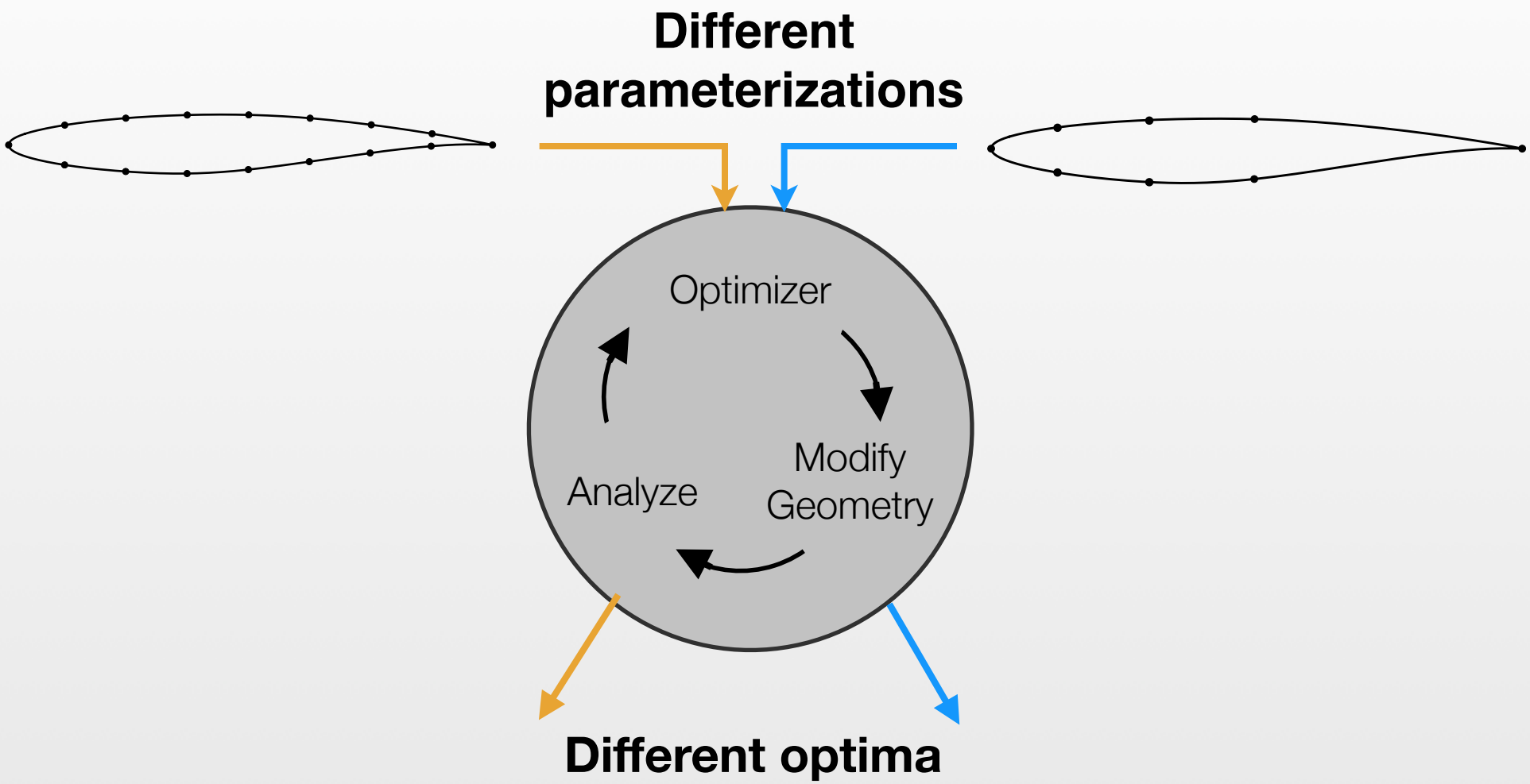
3. Numerical **optimizer** iteratively modifies shape to improve performance

Shape Parameterization



- Shape parameterization reduces continuous design space into finite search space
- Reduces range of reachable shapes

Static Parameterization



Motivation

- Choice of shape parameters impacts:
  - **Coverage** (want *more* DOF).
  - **Computational cost** (want *fewer* DOF) — especially important for optimization driven by high-fidelity simulations.
- For design of complex vehicles in unfamiliar settings, would like ability to
  - **Efficiently** approach the **continuous** optimal design.
  - Quantify optimality in each search space.

Objective

Research Goal:

Develop optimization system with automatic, **adaptive shape parameterization** refinement

Requirements:

- Gradually approach continuous optimum (**convergent**)
- Without *a priori* knowledge (**automated**)
- Using as few design variables as possible (**adaptive**)

Previous Work

Progressive
(uniform "h"-refinement)

- Gradually **increase resolution**
- **(1991)** *Kohli and Carey* — Multi-fidelity shape representation for structural optimization
- **(1993)** *Marco et al.* — Aerodynamic optimization with nested parameters

Redistribution
(*"r"-refinement*)

- Improve **distribution** of shape control
- **(2004, 2006)** *Desideri and El Majd, Duvigneau* — Minimize total variation of Bezier/FFD control points
- **(2012)** *Hwang and Martins* — Equally distribute arc-length of curve between B-spline control points

These approaches are **insensitive to the goals of aerodynamic optimization.**

Previous Work

Towards **goal-oriented** adaptation:

- **(2011)** *Han and Zingg* — **Discrete** refinement approach
  - **Restrictions:** Single-component design, only localized constraints, can only add one new variable at a time
- **(2014)** *Poole and Allen* — **Redistribution** approach
  - **Restrictions:** Only geometric matching of airfoils
- **(2015)** *Anderson* — **Discrete adaptation** approach appropriate for general aerodynamic design problems

This Work

- Complete **system** for automatic, adaptive parameterization
  - Novel **refinement indicator** that enables adaptive parameterization for general problems:
    - Multiple components
    - Multiple classes of shape control
    - High curvature variation in design space
    - General constraints
  - Support for (nearly) **arbitrary geometry modelers**
  - More efficient adaptation strategies and algorithms

Outline

- ✓ Parametric Shape Optimization
- **Adaptive Parameterization**
  - **Discrete Adaptation (How?)**
  - Refinement Indicator (Where?)
  - Adaptation Strategy
- Verification
- Design Examples

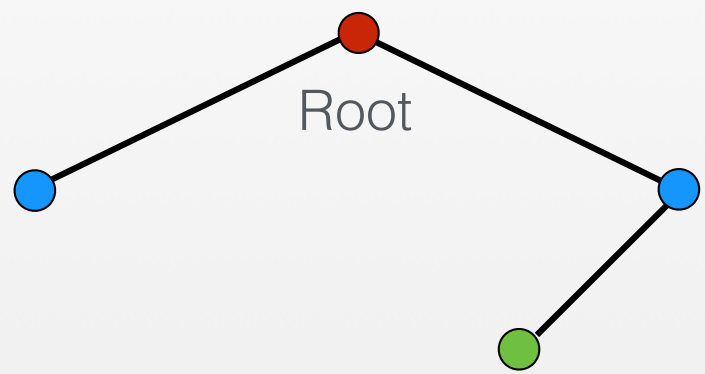
Shape Control Refinement

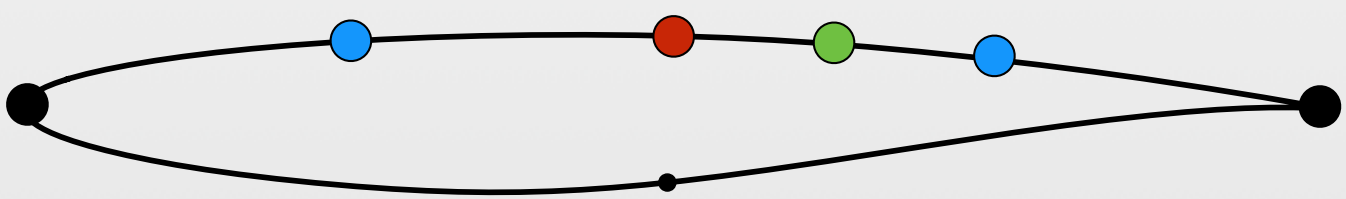
View shape parameterization as **binary tree:**

Level 0

Level 1

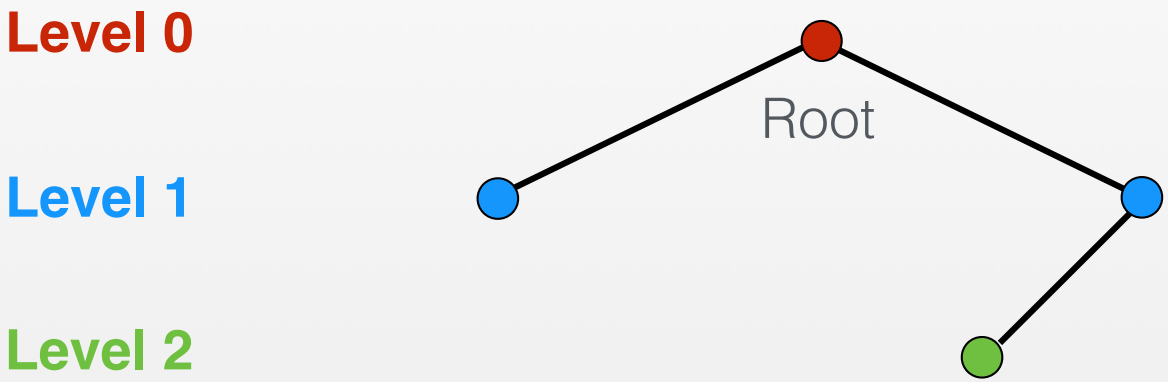
Level 2

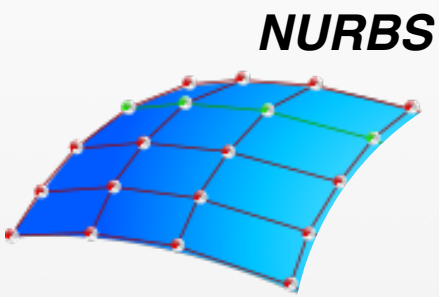


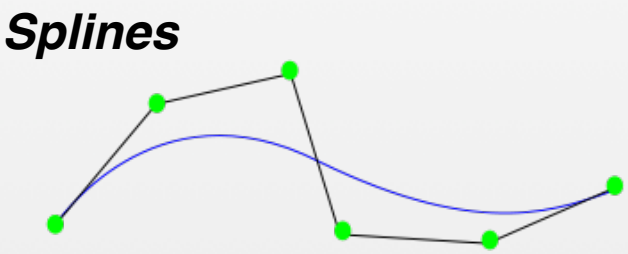


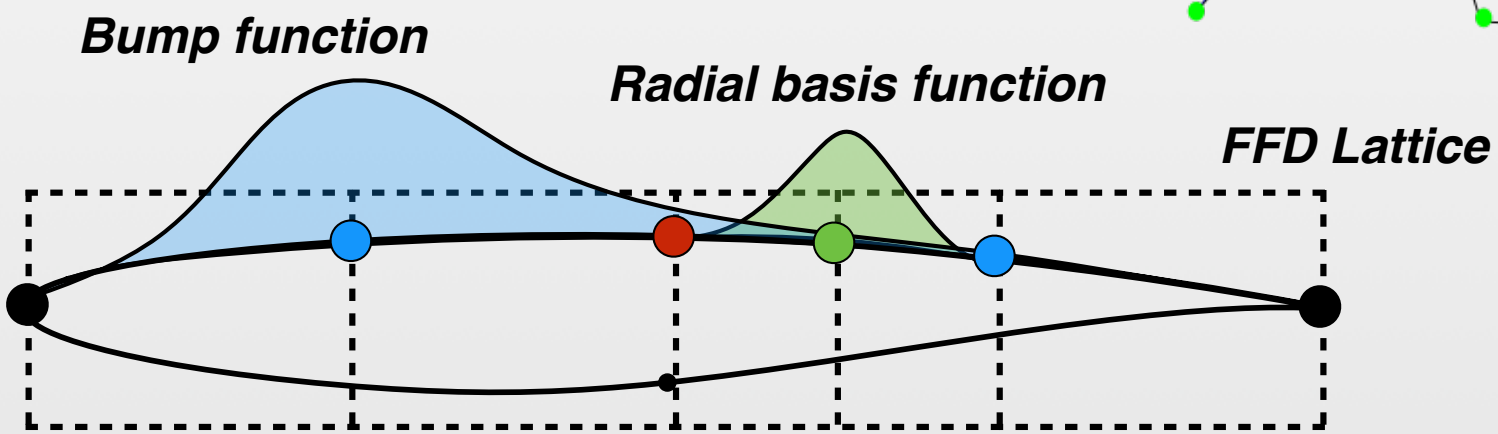
Shape Control Refinement

View shape parameterization as **binary tree:**



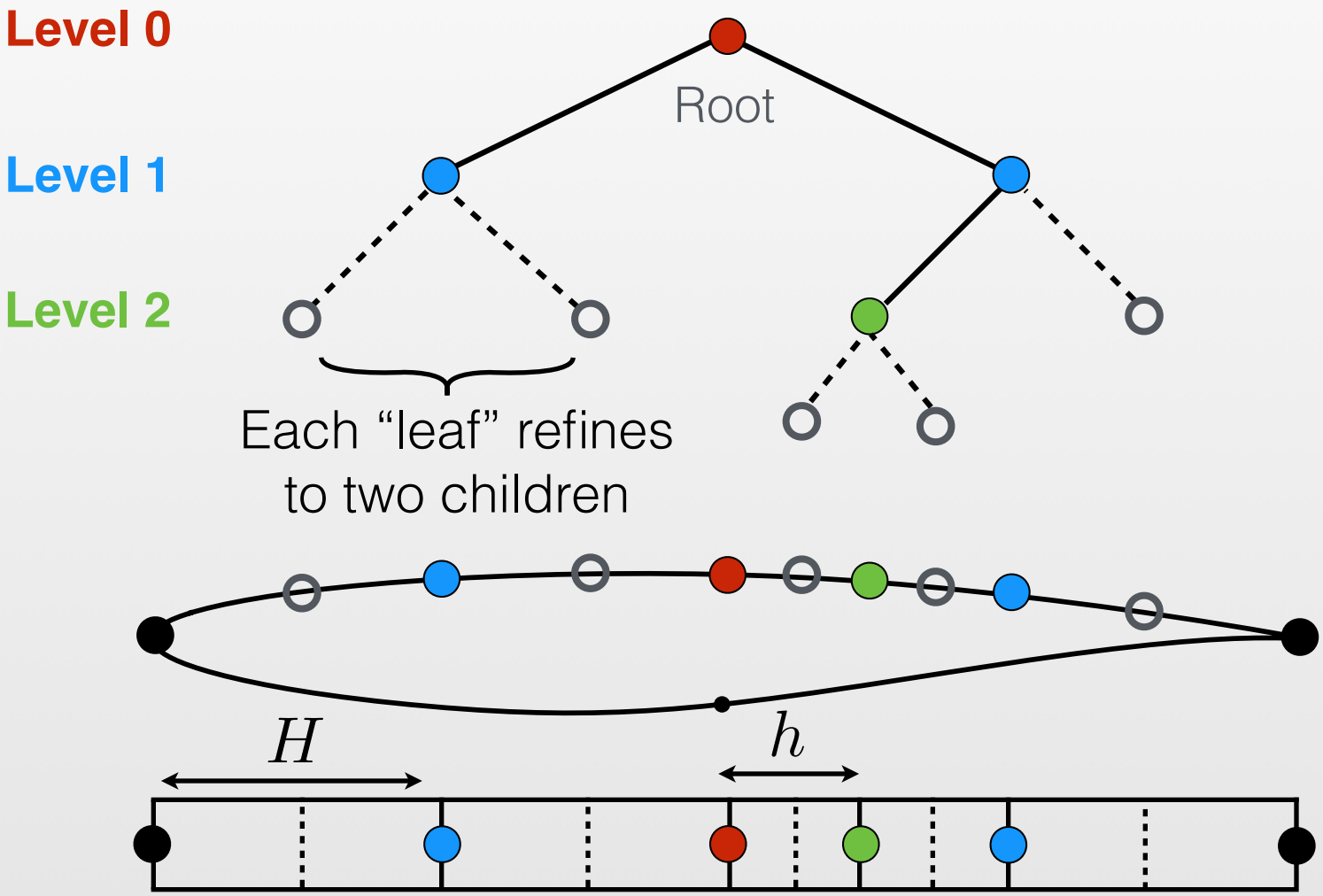




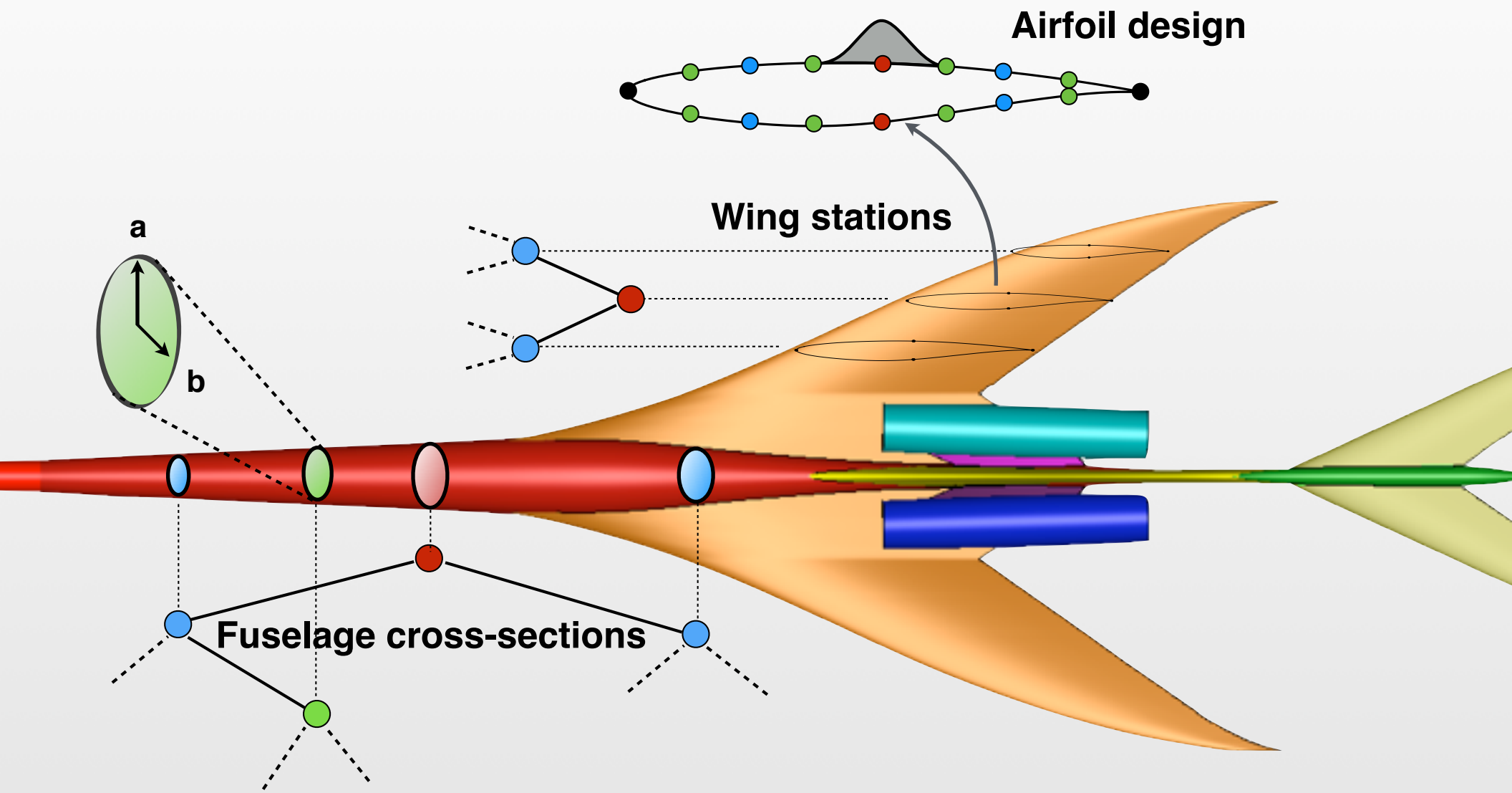


Shape Control Refinement

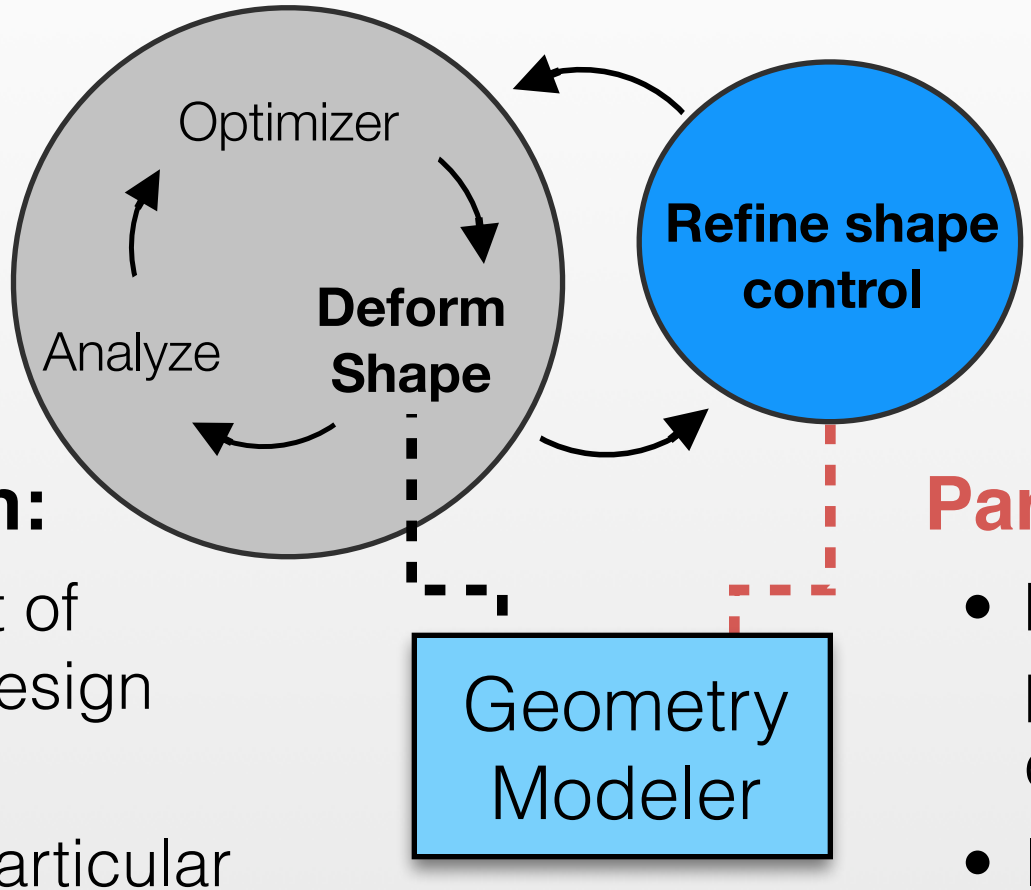
View shape parameterization as **binary tree:**



Configuration Design



Interface to Arbitrary Modelers



Deformation:

- Provide list of **existing** design variables
- Invoke a particular **deformation**

Parameterization:

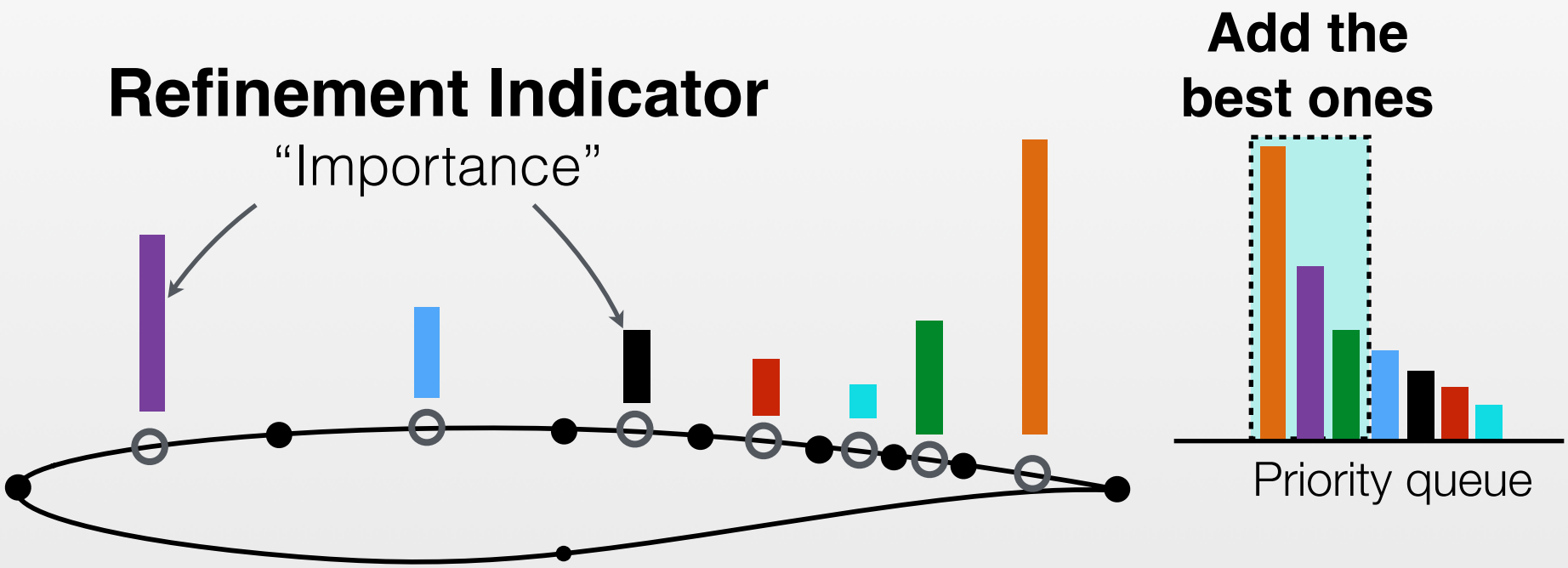
- Provide list of possible **new** design variables
- Invoke a particular **refinement**

Outline

- ✓ Parametric Shape Optimization
- **Adaptive Parameterization**
  - ✓ Discrete Adaptation
  - **Refinement Indicator**
  - Adaptation Strategy
- Verification
- Design Examples

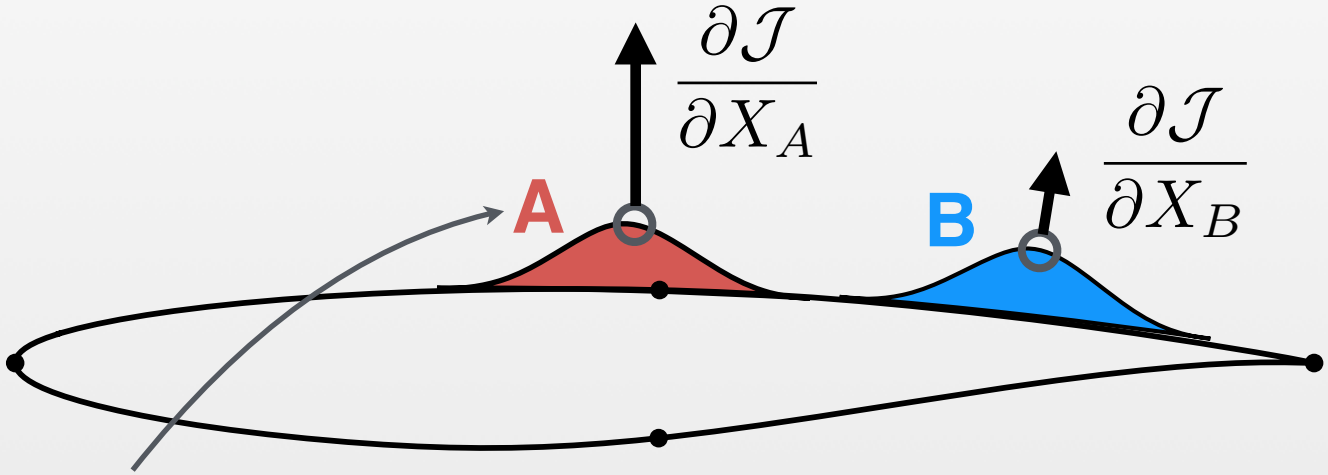
Adaptive Refinement

Goal: Determine **most important** candidate parameters



Previous Approach

- (2011) Han and Zingg rank parameters by magnitude of **objective gradient** with respect to **candidate** design variables.†



Prefer A, because objective is more **sensitive** to it**.**

† (2011) X. Han, D. Zingg. *"An Evolutionary Geometry Parametrization for Aerodynamic Shape Optimization." AIAA 2011-3536*

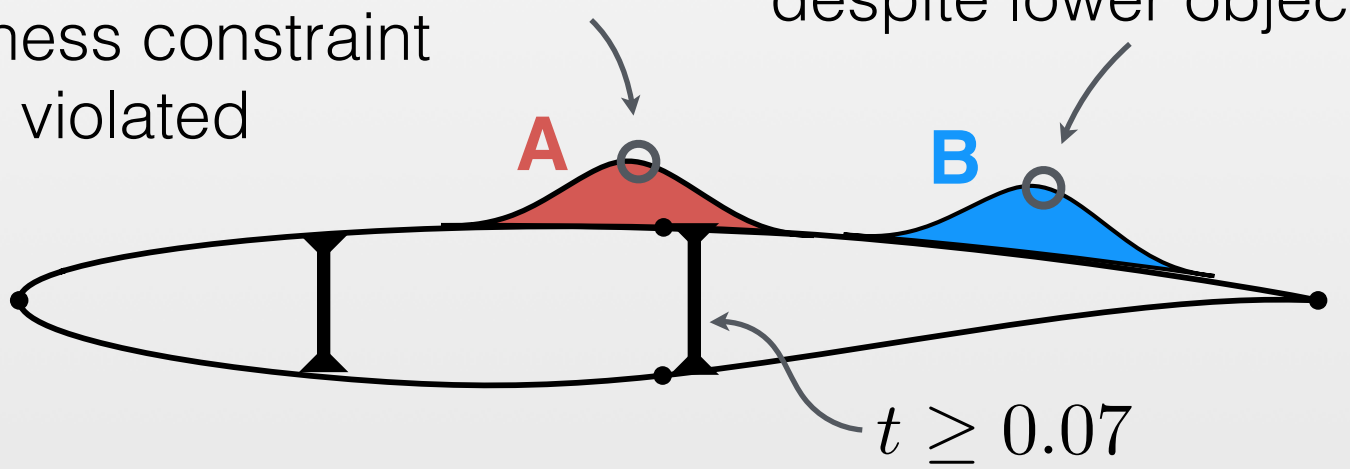
Limitations of Previous Approach

- **Ignores constraints**

Inconsistent units
Ignores curvature variation
Insensitive to redundancy

Drag is **more sensitive** to **A**, but thickness constraint would be violated

B offers **more real potential**, despite lower objective gradient



Limitations of Previous Approach

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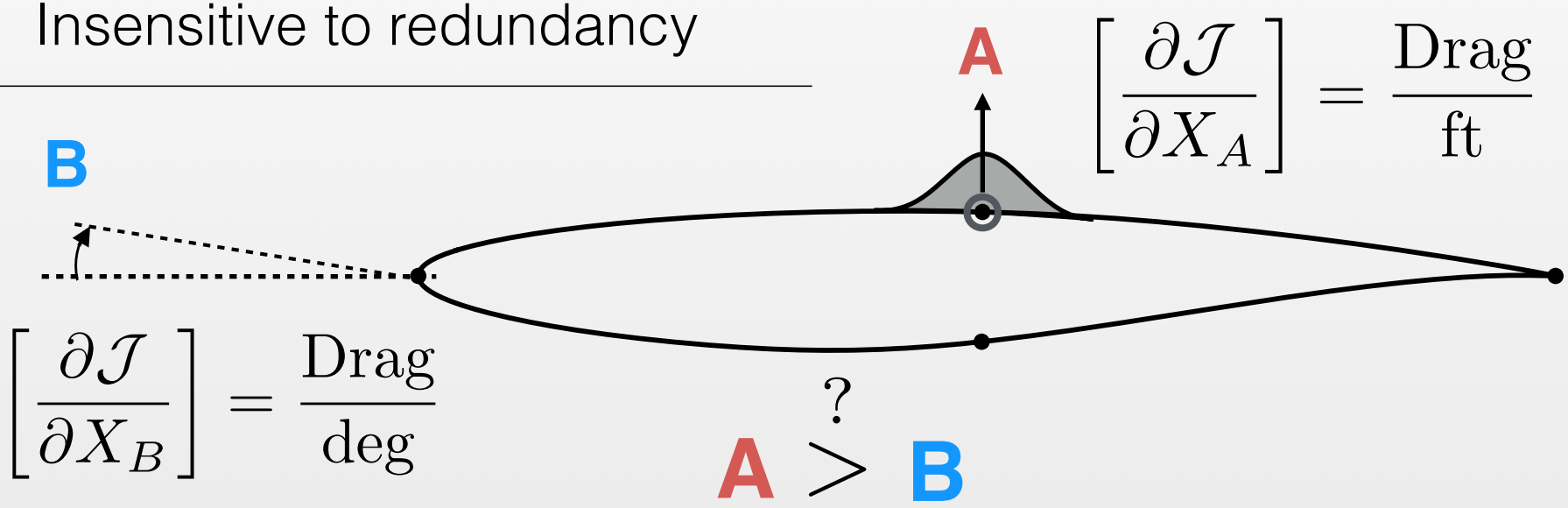
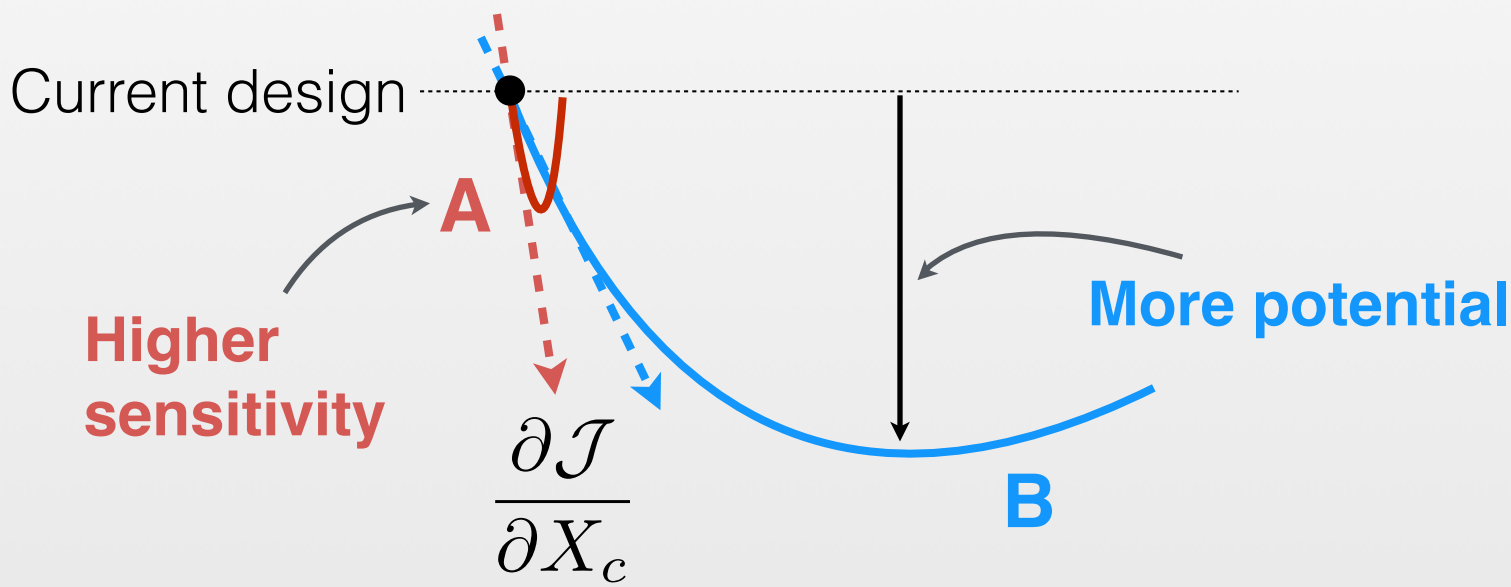


Limitations of Previous Approach

Ignores constraints
Inconsistent units
- **Ignores curvature variation** $\frac{\partial^2 \mathcal{J}}{\partial X_c^2}$

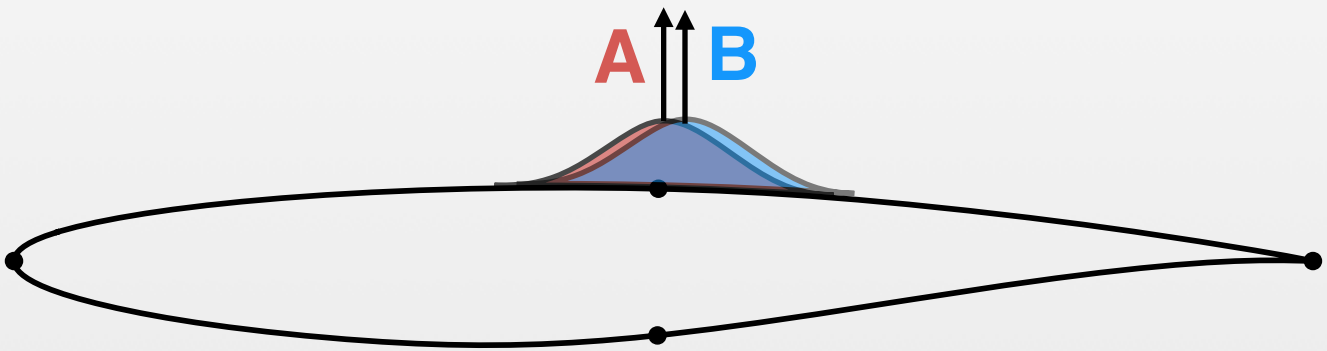
Insensitive to redundancy



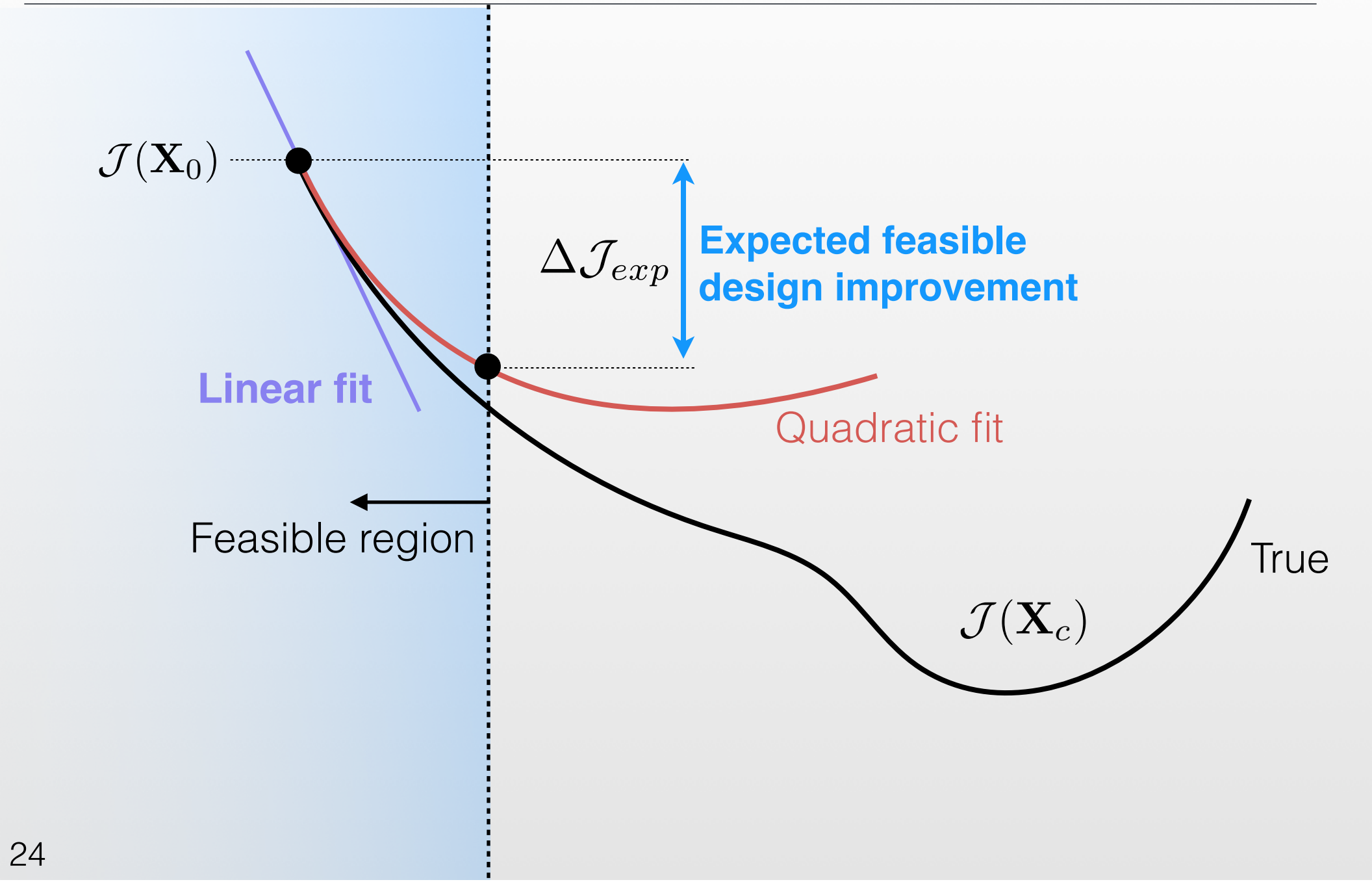
Limitations of Previous Approach

Ignores constraints
Inconsistent units
Ignores curvature variation
- **Insensitive to redundancy**

Either one would be useful, but **not both**



New Refinement Indicator



Expected Feasible Design Improvement

KKT system

Gradients of active constraints

Objective gradients

Hessian

$$\begin{bmatrix} \mathcal{H} & \frac{\partial \boldsymbol{\mathcal{C}}^a}{\partial \mathsf{S}} \\ \left(\frac{\partial \boldsymbol{\mathcal{C}}^a}{\partial \mathsf{S}}\right)^{\mathsf{T}} & \overline{\mathbf{0}} \end{bmatrix} \begin{pmatrix} \delta \mathsf{S}^\star \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -\frac{\partial \mathcal{J}}{\partial \mathsf{S}} \\ \mathbf{0} \end{pmatrix}$$

Lagrange multipliers

Solve for **Newton step** to predicted optimum

$$\delta \mathsf{S}^\star = -\mathcal{H}^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathsf{S}} + \boldsymbol{\lambda}\frac{\partial \boldsymbol{\mathcal{C}}^a}{\partial \mathsf{S}}\right)$$

Gradient

Hessian

Quadratic Taylor expansion

$$\mathcal{J}(\mathsf{S}_0 + \delta \mathsf{S}) \approx \mathcal{J}(\mathsf{S}_0) + \left\langle \frac{\partial \mathcal{J}}{\partial \mathsf{S}}, \delta \mathsf{S} \right\rangle + \frac{1}{2}\left\langle \mathcal{H}\delta \mathsf{S}, \delta \mathsf{S} \right\rangle + \ldots$$

$$\Delta \mathcal{J}^{\infty}_{exp} = \frac{1}{2}\left\langle \left(\frac{\partial \mathcal{J}}{\partial \mathsf{S}} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathsf{S}}\right), \mathcal{H}^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathsf{S}} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathsf{S}}\right) \right\rangle$$

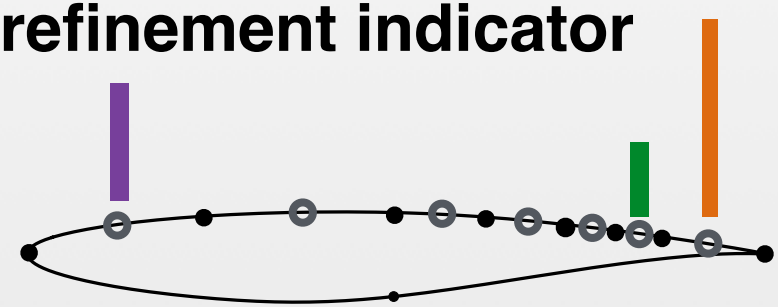
Refinement Indicator

Expected feasible objective reduction in **candidate** search space:

KKT stationarity
0 at optimum

$$I \equiv \Delta\mathcal{J}^{\infty}_{exp} = \frac{1}{2}\left\langle \left(\frac{\partial\mathcal{J}}{\partial\mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial\mathcal{C}^a}{\partial\mathbf{X}_c}\right), (\mathcal{M}\overline{\mathbf{H}}_c)^{-1}\left(\frac{\partial\mathcal{J}}{\partial\mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial\mathcal{C}^a}{\partial\mathbf{X}_\mathbf{c}}\right)\right\rangle$$

Use as **refinement indicator**



Has sensible **units**

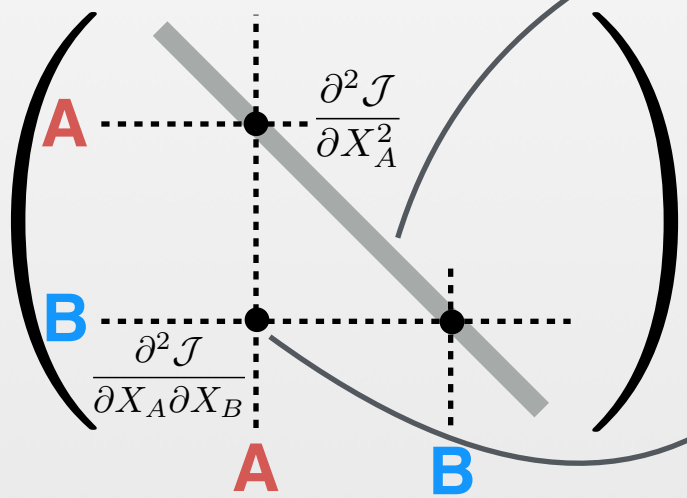
$$[I] = \frac{\text{Drag}}{\text{ft}}\left(\frac{\text{ft}^2}{\text{Drag}}\right)\frac{\text{Drag}}{\text{ft}} = \text{Drag}$$

"expected drag reduction"

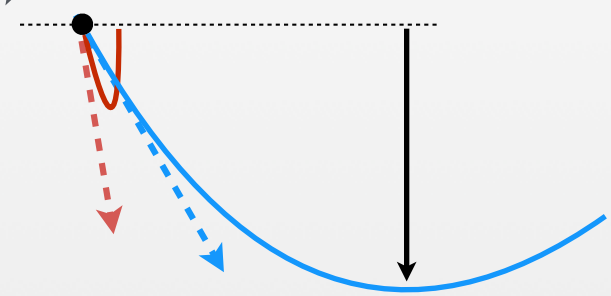
Refinement Indicator

$$I = \frac{1}{2}\left\langle \underbrace{\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_c}\right)}_{\text{Explicitly accounts for constraints}}, (\mathcal{M}\overline{\mathbf{H}}_c)^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_\mathbf{c}}\right)\right\rangle$$

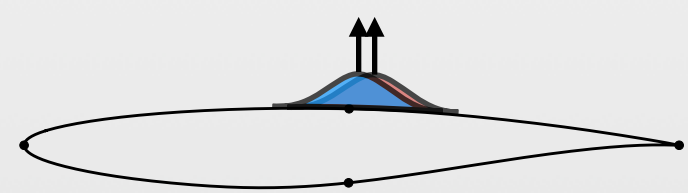
Hessian matrix



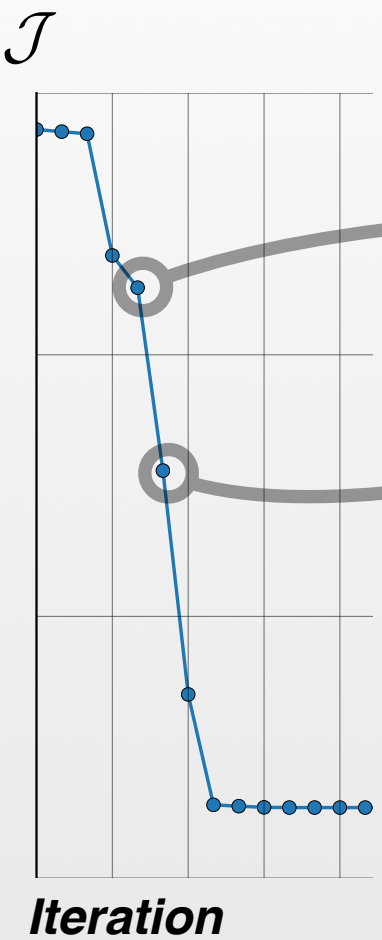
Accounts for **curvature** variation



Detects **redundancy**



Candidate Gradient Computation



Design variables

$$\left\{ \begin{array}{ccc} X_1 & \otimes & \otimes \\ X_2 & \otimes & \otimes \\ \vdots & \vdots & \vdots \\ X_n & \otimes & \otimes \end{array} \right\} \quad \frac{d\mathcal{J}}{dX} = \left(\frac{\partial \mathcal{J}}{\partial S} + \psi \frac{\partial R}{\partial S} \right) \frac{\partial S}{\partial X}$$

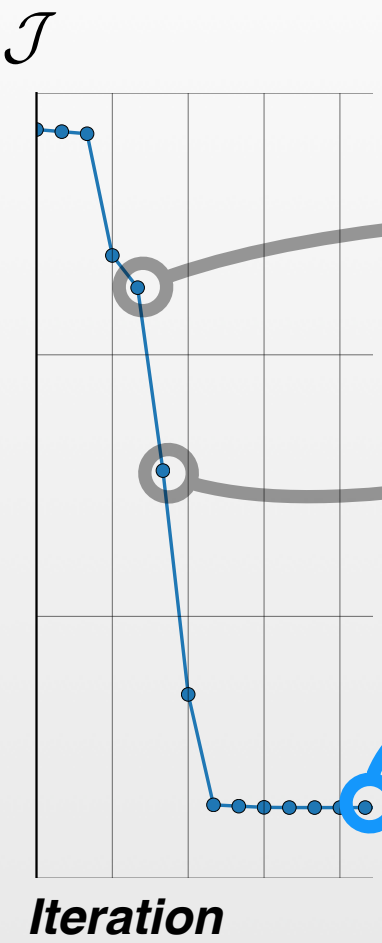
Matrix-vector products

Flow Solve

$$\psi \frac{\partial R}{\partial Q} = -\frac{\partial \mathcal{J}}{\partial Q} \qquad \psi \frac{\partial R}{\partial Q} = -\frac{\partial \mathcal{C}}{\partial Q}$$

Adjoint solutions

Candidate Gradient Computation



Design variables

$$\left\{ \begin{array}{c} X_1 \\ X_2 \\ \vdots \\ X_n \end{array} \right\} \quad \begin{array}{c} \otimes \\ \otimes \\ \vdots \\ \otimes \end{array} \quad \begin{array}{c} \otimes \\ \otimes \\ \vdots \\ \otimes \end{array}$$

$$\frac{d\mathcal{J}}{dX} = \left(\frac{\partial \mathcal{J}}{\partial S} + \psi \frac{\partial R}{\partial S} \right) \frac{\partial S}{\partial X}$$

Matrix-vector products

Flow Solve

$$\psi \frac{\partial R}{\partial Q} = -\frac{\partial \mathcal{J}}{\partial Q} \qquad \psi \frac{\partial R}{\partial Q} = -\frac{\partial \mathcal{C}}{\partial Q}$$

Adjoint solutions

Candidates

$$\left\{ \begin{array}{c} X_1^c \\ X_2^c \\ \vdots \\ X_n^c \end{array} \right\} \quad \begin{array}{c} \otimes \\ \otimes \\ \vdots \\ \otimes \end{array} \quad \begin{array}{c} \otimes \\ \otimes \\ \vdots \\ \otimes \end{array}$$

Project same adjoint sensitivities into **candidate** search space

Computing the Indicator

$$I = \frac{1}{2}\left\langle \left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_c}\right), (\mathcal{M}\overline{\mathbf{H}}_c)^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_\mathbf{c}}\right)\right\rangle$$

Estimate **Hessian** from **quasi-Newton** approximation in **previous** space

$$\mathbf{B}_0^k = \mathbf{I}$$

BFGS

$$\mathbf{B}_n^k \approx \mathbf{H}_n^k$$

Computing the Indicator

$$I = \frac{1}{2}\left\langle \left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_c}\right), (\mathcal{M}\overline{\mathbf{H}}_c)^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_\mathbf{c}}\right)\right\rangle$$

Estimate **Hessian** from **quasi-Newton** approximation in **previous** space

$$\mathbf{B}_0^k = \mathbf{I} \qquad \mathbf{H}_0^{k+1} \approx \mathcal{P}(\mathbf{B}_n^k)$$

BFGS

$$\mathbf{B}_n^k \approx \mathbf{H}_n^k$$

Prolongation
to finer space
(linear interpolation + scale-dependence)

Computing the Indicator

$$I = \frac{1}{2}\left\langle \left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_c}\right), (\mathcal{M}\overline{\mathbf{H}}_c)^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_\mathbf{c}}\right)\right\rangle$$

Estimate **Hessian** from **quasi-Newton** approximation in **previous** space

BFGS

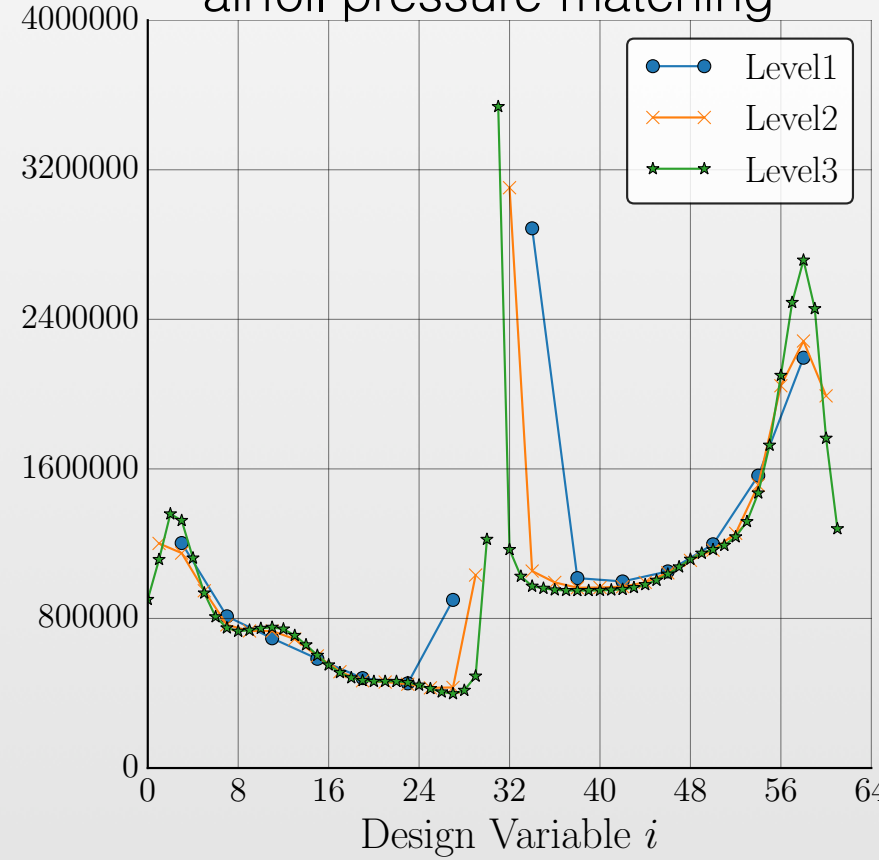
$$\mathbf{B}_0^k = \mathbf{I}$$

$$\mathbf{B}_n^k \approx \mathbf{H}_n^k$$

$$\mathbf{H}_0^{k+1} \approx \mathcal{P}(\mathbf{B}_n^k)$$

Prolongation
to finer space
(linear interpolation + scale-dependence)

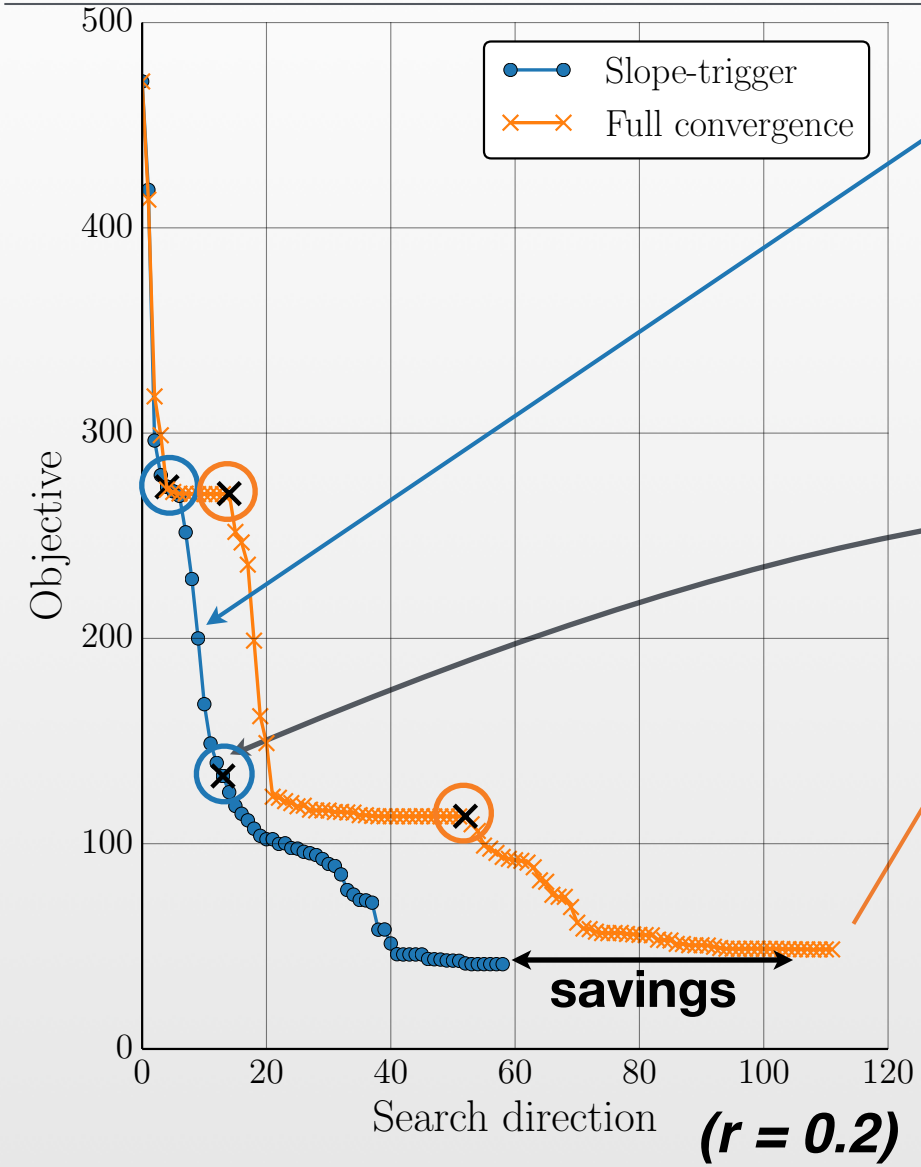
Hessian Diagonal
airfoil pressure-matching



Outline

- ✓ Parametric Shape Optimization
- **Adaptive Parameterization**
  - ✓ Discrete Adaptation
  - ✓ Refinement Indicator
  - **Adaptation Strategy**
- Verification
- Design Examples

When to refine?



Slope Reduction:

Deceleration of design improvement

- **Trigger** detects diminishing returns on **computational** time:

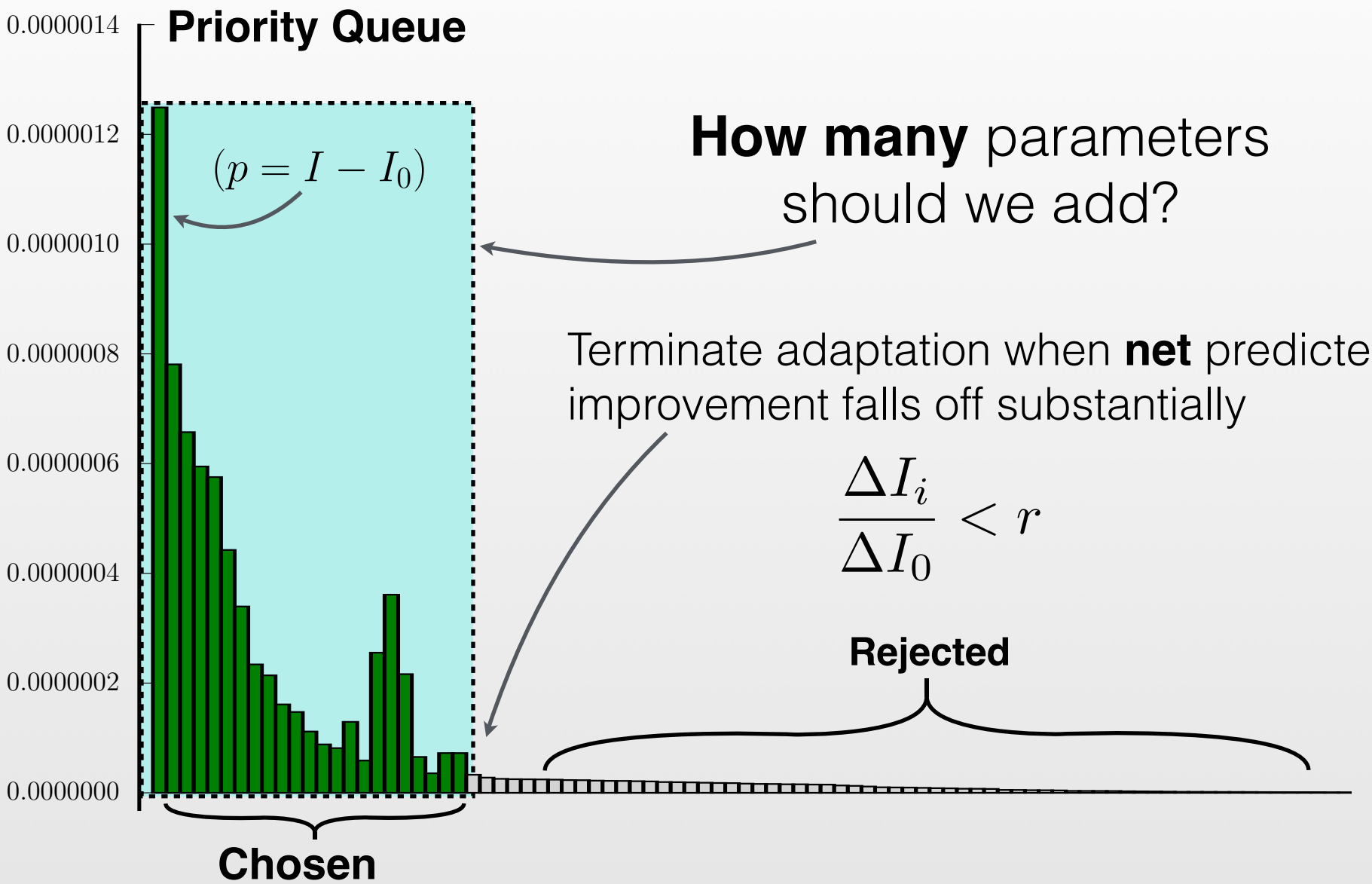
$$\frac{\Delta \mathcal{J}_i}{\max_k(\Delta \mathcal{J}_k)} < r$$

Convergence:

Sufficient optimality (KKT conditions)

- Maximally exploits parameters

Growth rate



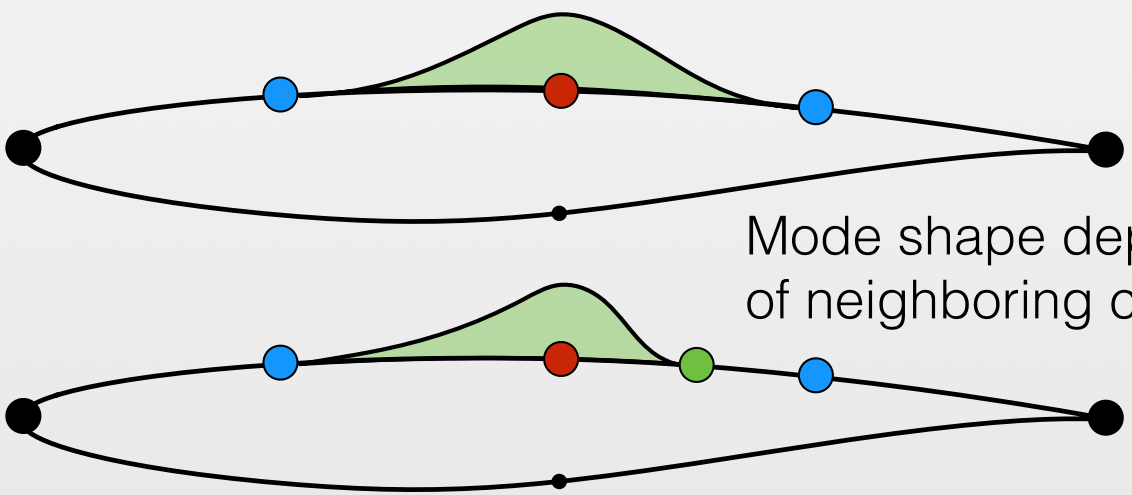
How many parameters should we add?

Terminate adaptation when **net** predicted improvement falls off substantially

$$\frac{\Delta I_i}{\Delta I_0} < r$$

Adding Multiple Parameters

- **Adaptation:** "Find the best *N* out of *M* parameters"
- Properly a **combinatorial optimization** problem
  - Not separable for most deformers
  - But conducive to approximate solutions
- I use an approximate **constructive** (greedy) algorithm†

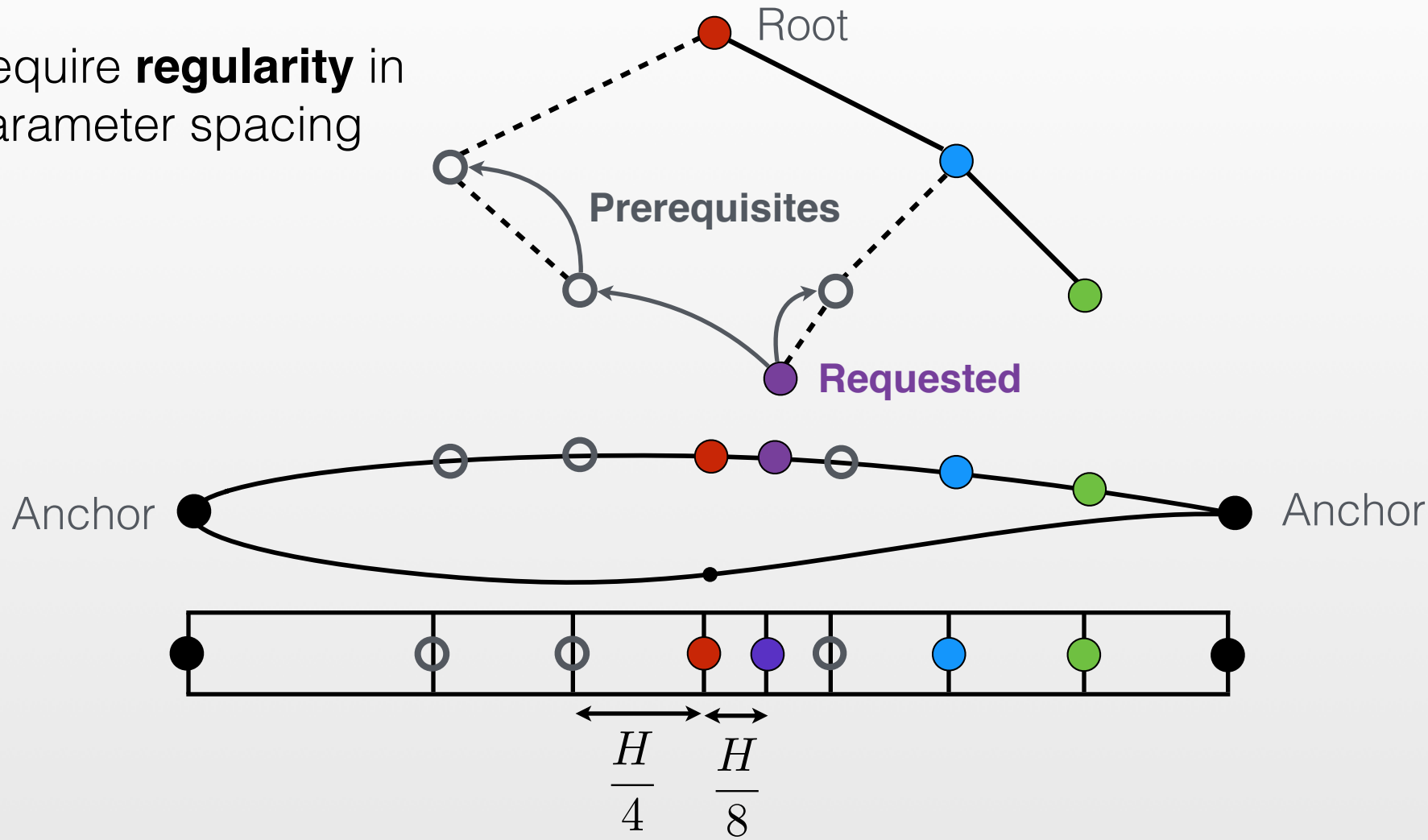


Mode shape depends on locations of neighboring controllers

† (2015) **Anderson**, G.R., Aftosmis, M. J. *"Adaptive Shape Control for Aerodynamic Design."* *AIAA 2015-0398*

Regularity

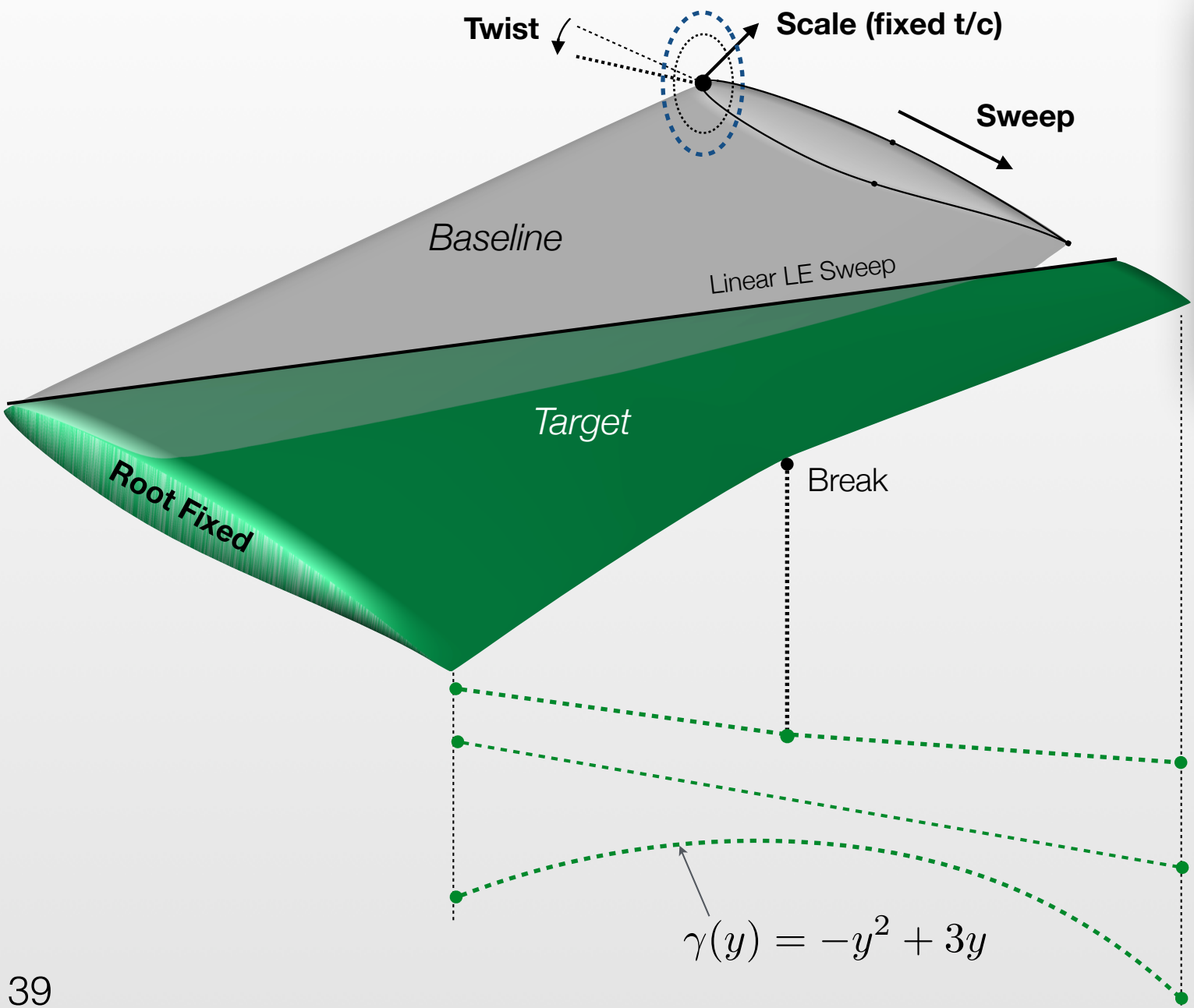
Require **regularity** in parameter spacing



Outline

- ✓ Parametric Shape Optimization
- ✓ Adaptive Parameterization
- **Verification**
  - **Correctness —** Does the indicator predict actual design improvement?
  - **Robustness —** Does the approach always converge to the continuous optimum?
- Design Examples

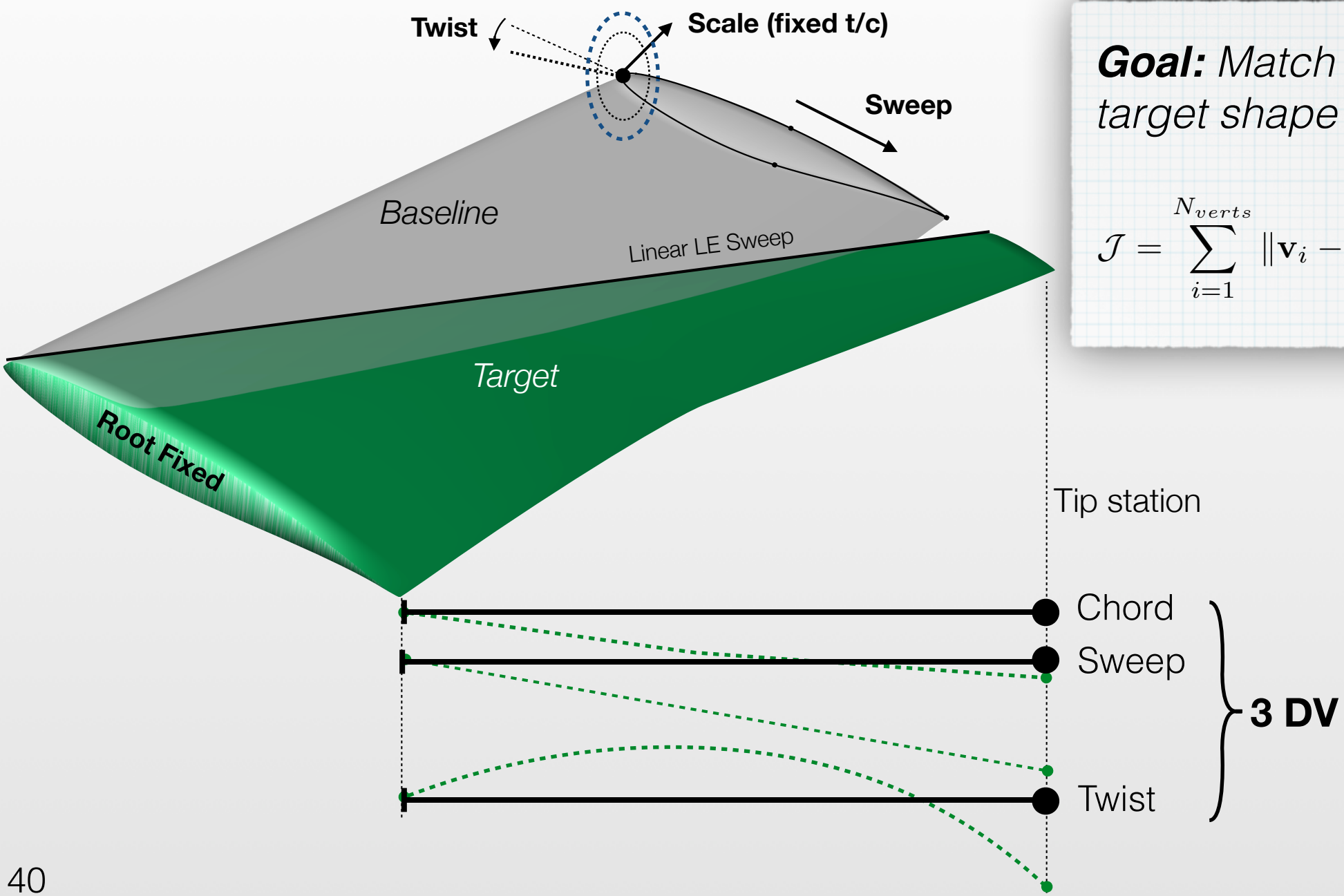
Verification Study 1: Geometric Shape Matching



Goal: *Match target shape*

$$\mathcal{J} = \sum_{i=1}^{N_{verts}} \|\mathbf{v}_i - \mathbf{v}_i^*\|^2$$

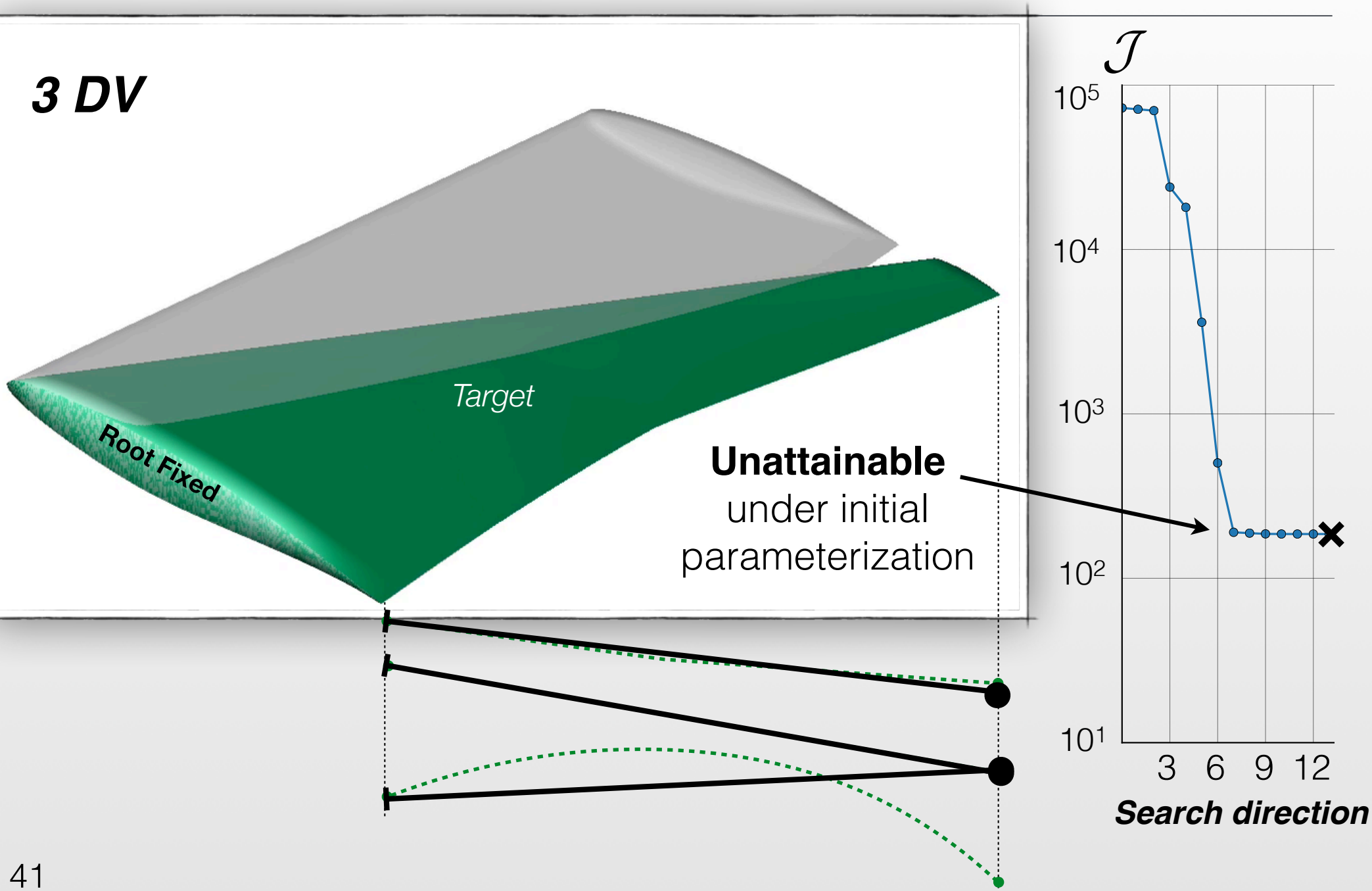
Initial Parameterization



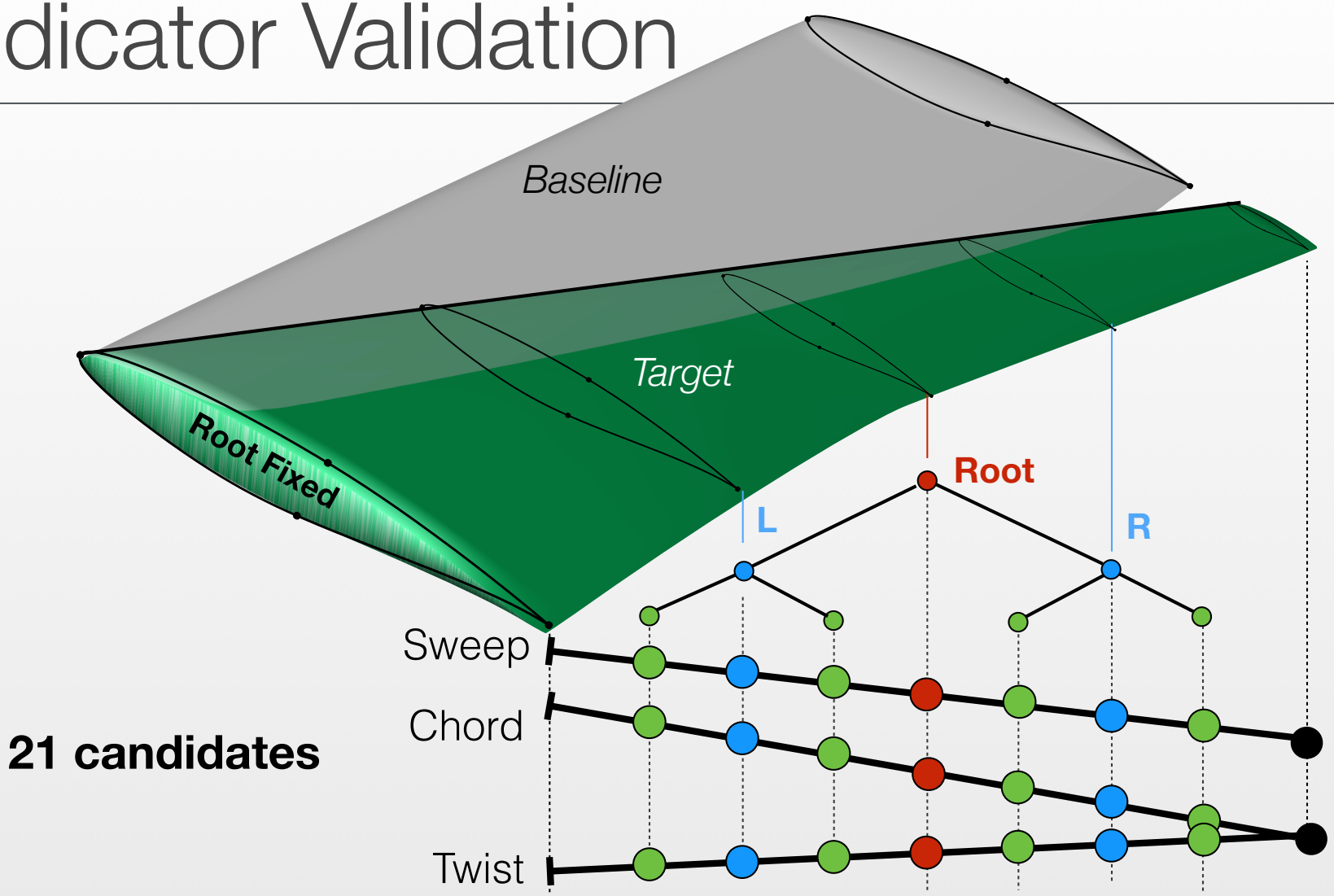
Goal: *Match target shape*

$$\mathcal{J} = \sum_{i=1}^{N_{verts}} \|\mathbf{v}_i - \mathbf{v}_i^*\|^2$$

Shape Matching under Initial Parameterization



Indicator Validation



21 candidates

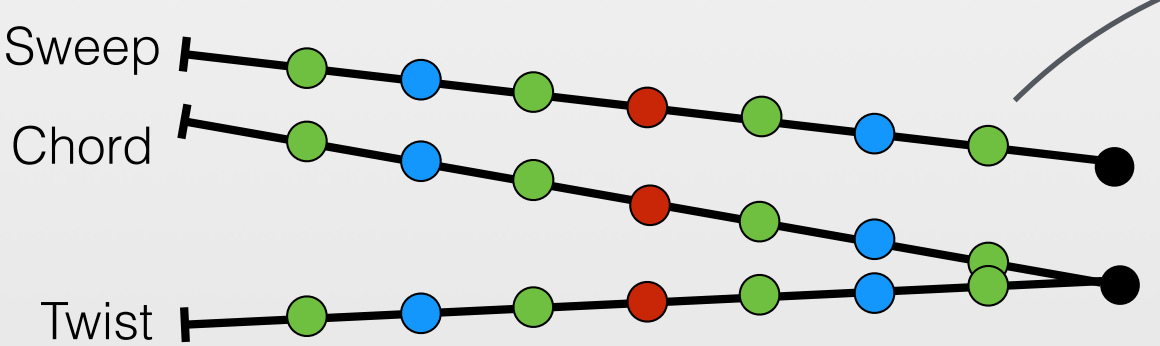
Indicator Validation

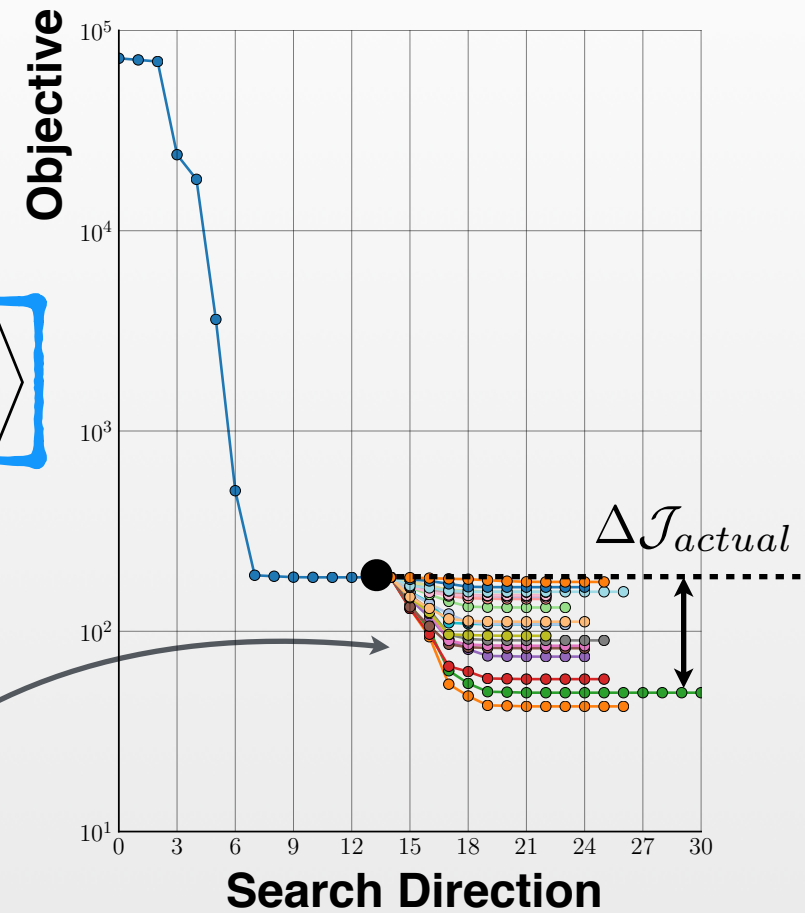
For each candidate:

1. **Predict** design improvement.
   With indicator:

$$I = \frac{1}{2}\left\langle \left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_c}\right), (\mathcal{M}\overline{\mathbf{H}}_c)^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_c} + \boldsymbol{\lambda}\frac{\partial \mathcal{C}^a}{\partial \mathbf{X}_\mathbf{c}}\right)\right\rangle$$

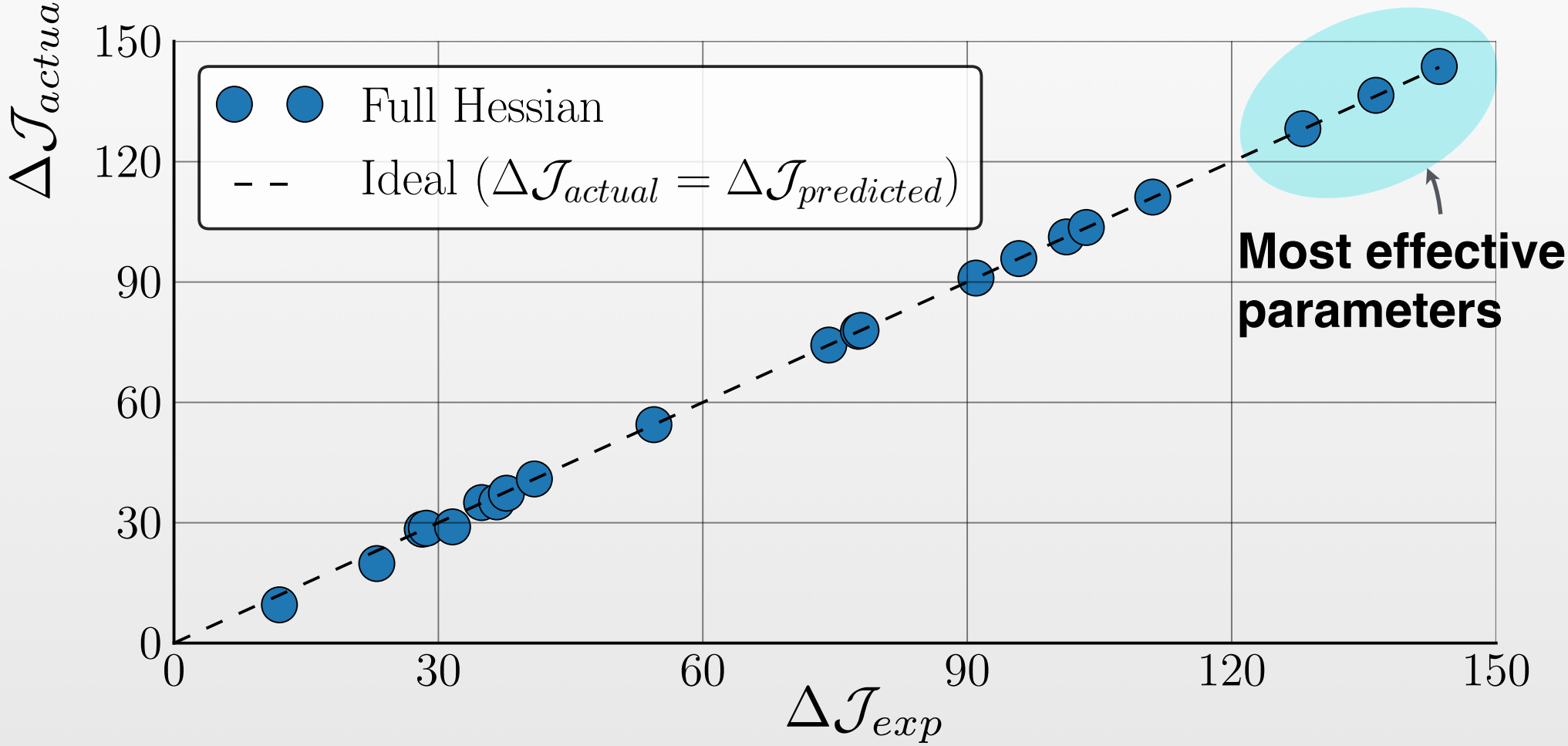
2. Measure **actual** improvement.
   Run optimization for each candidate.





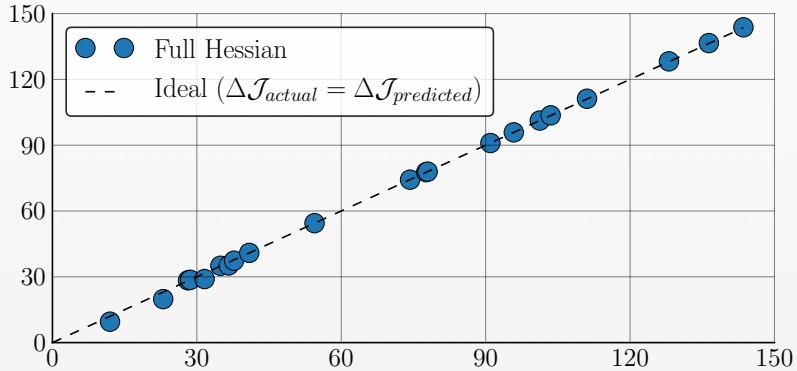
Indicator Validation

Correlate **predicted** and **actual** design improvement



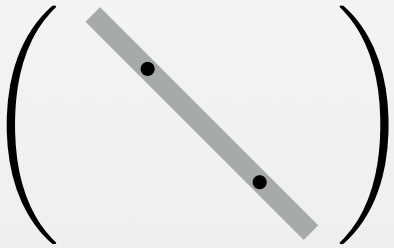
Approximations

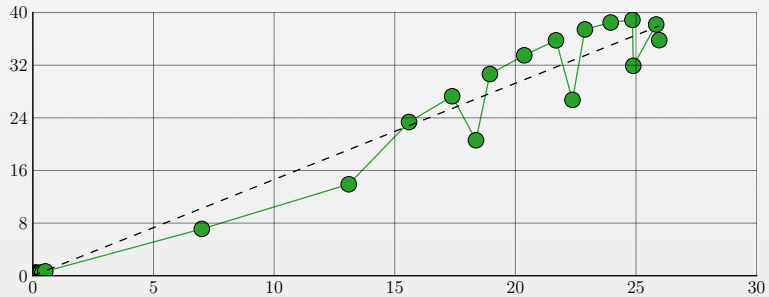
Exact Hessian



Excellent prediction

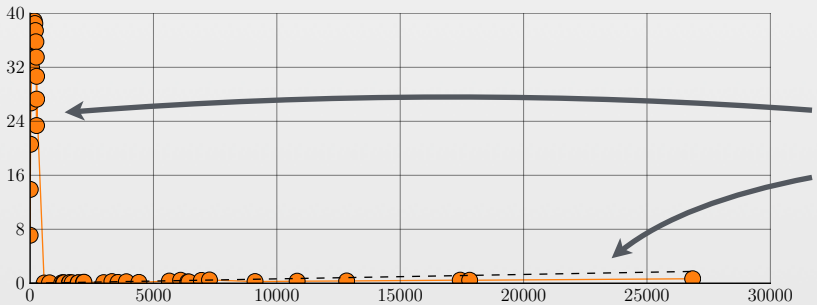
Diagonal





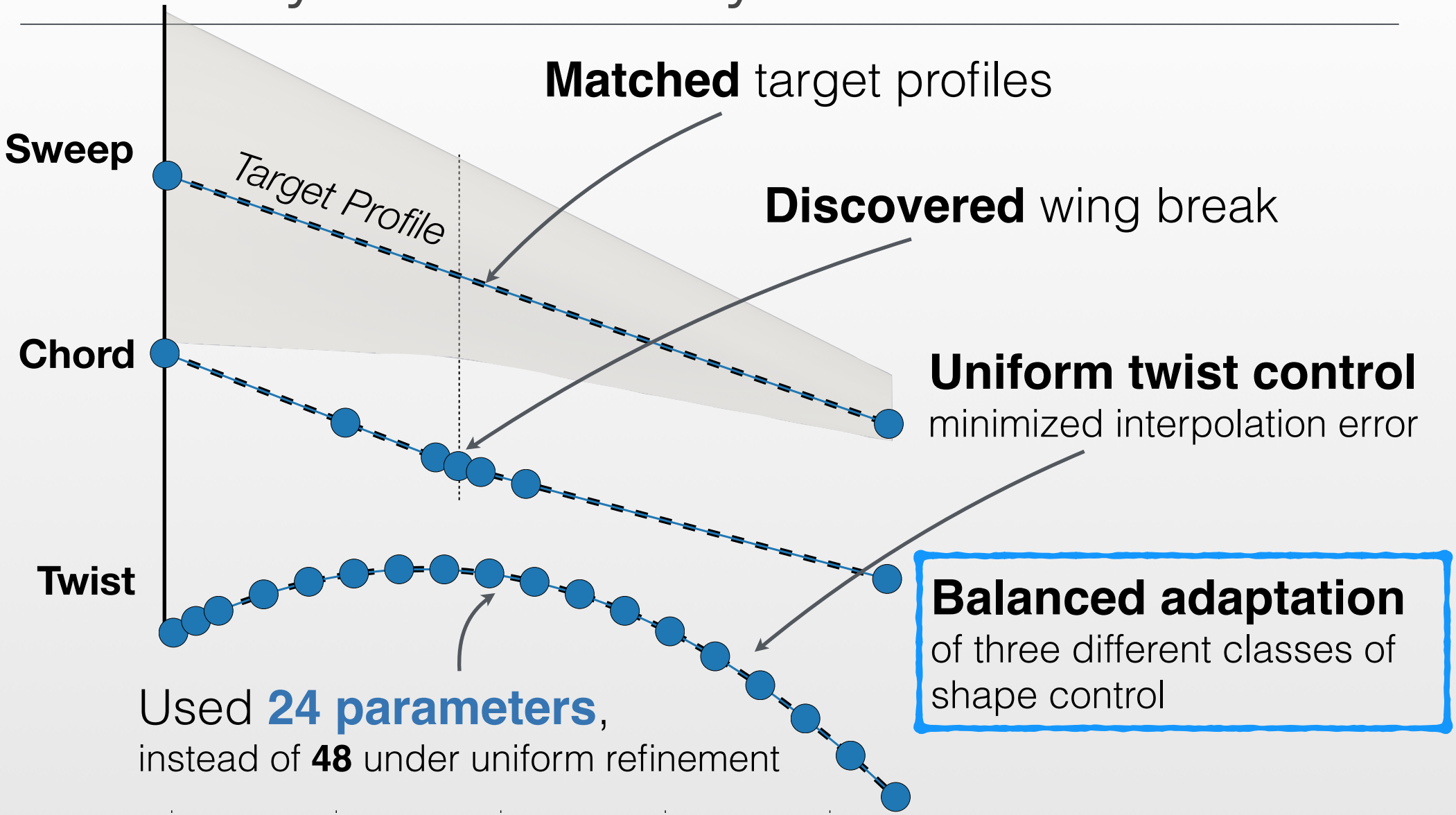
Acceptable — some redundancy

Identity



Poor ranking — systematic difference between classes of shape control

Recovery of Necessary Parameters

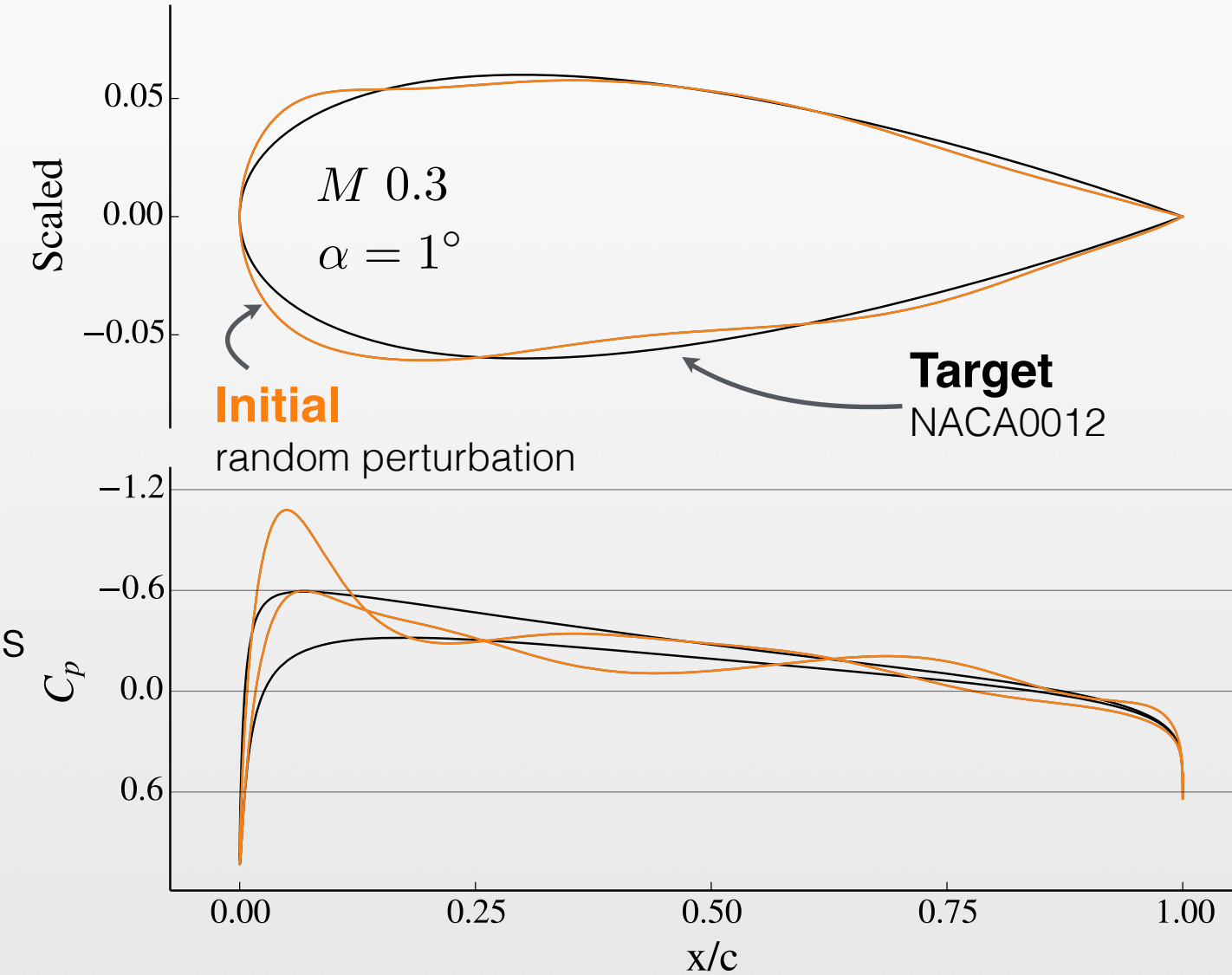


Verification Study 2: Pressure Signature Matching

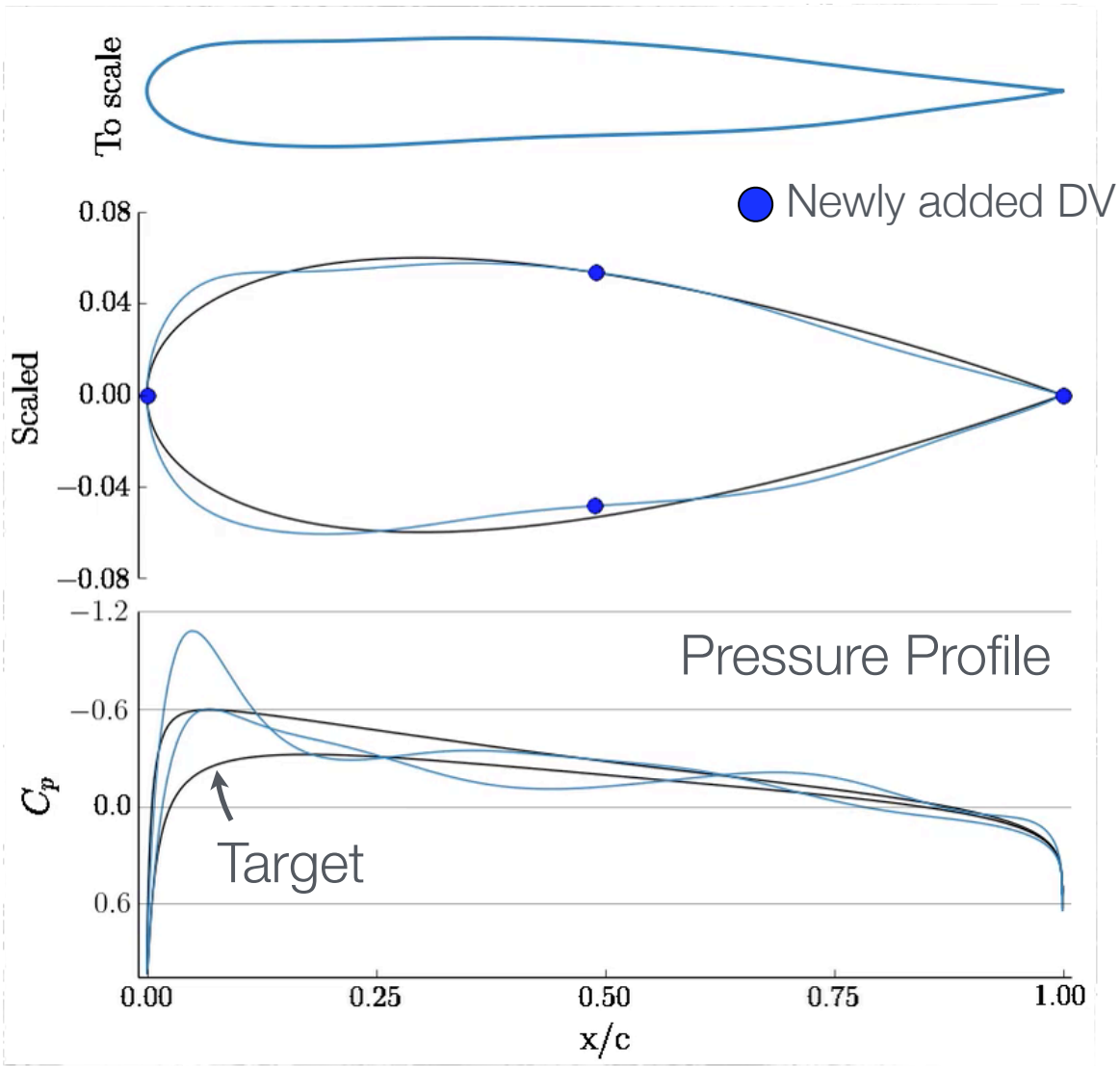
Objective:
Match target pressure profile

$$\mathcal{J} = \frac{1}{2} \sum_{i=1}^{N_{verts}} (p_i - p_i^*)^2$$

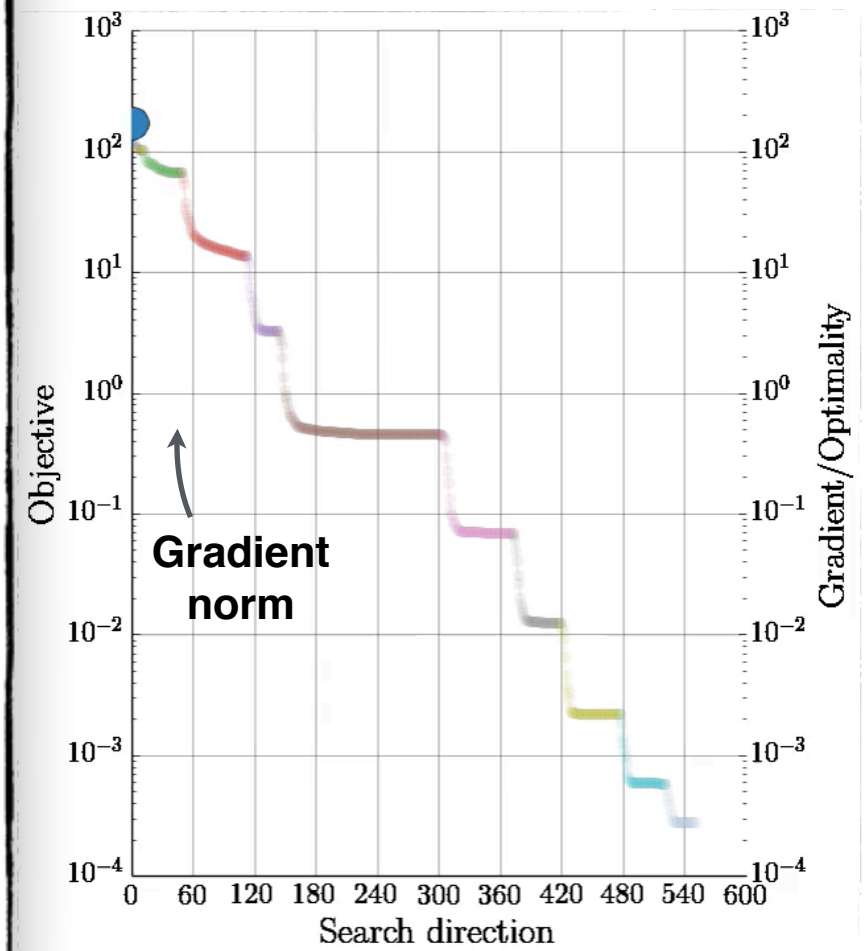
Parameterization:
2D Radial basis functions (localized bumps)
Flow Solver: Cart3D
Optimizer: SNOPT



Adaptive Parameterization



Objective Convergence

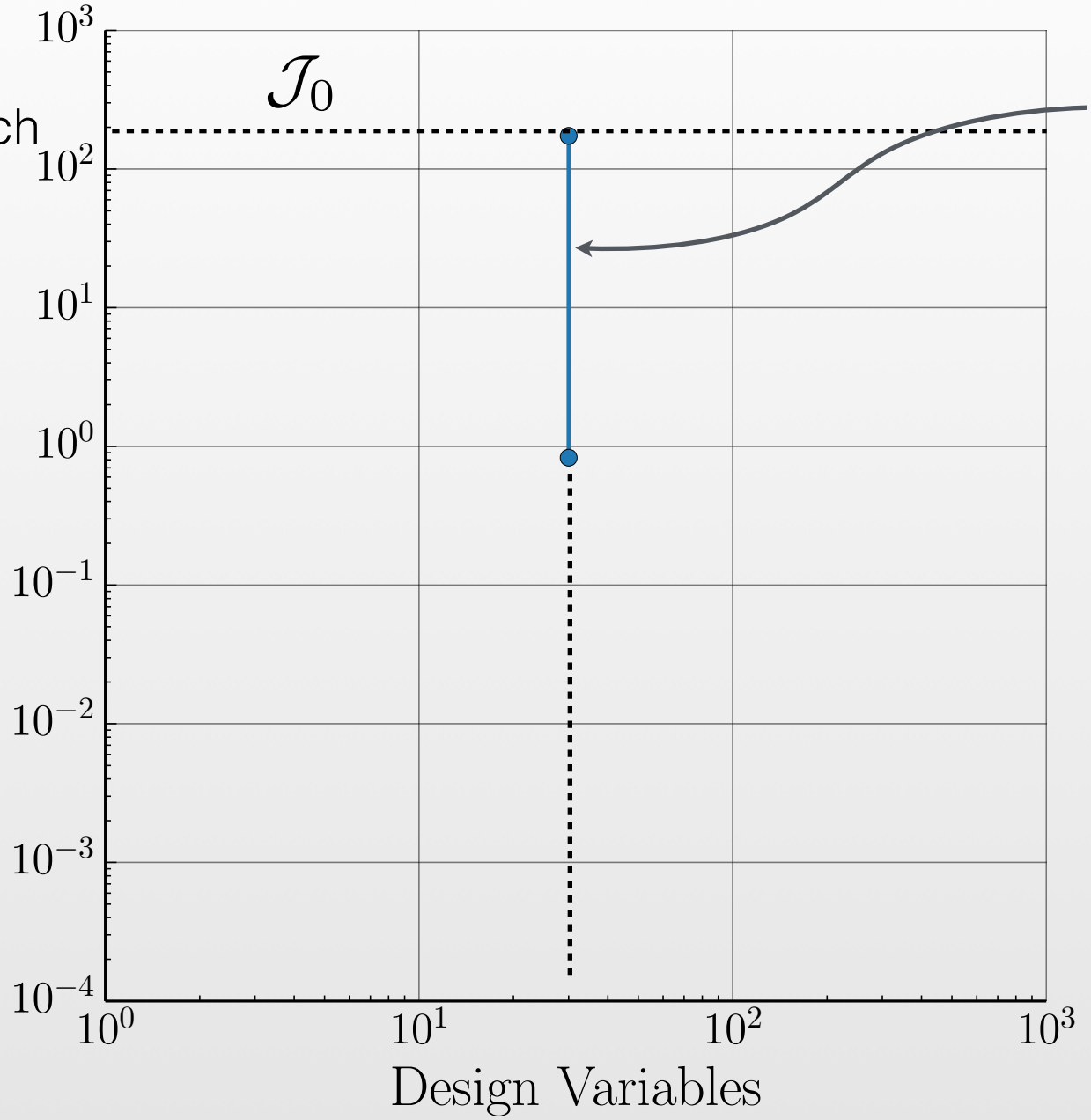


Convergence to Continuous Optimum

Optimum in **current** search space

$$\mathcal{J}_\star^k - \mathcal{J}_\star^\infty$$

Continuous optimum



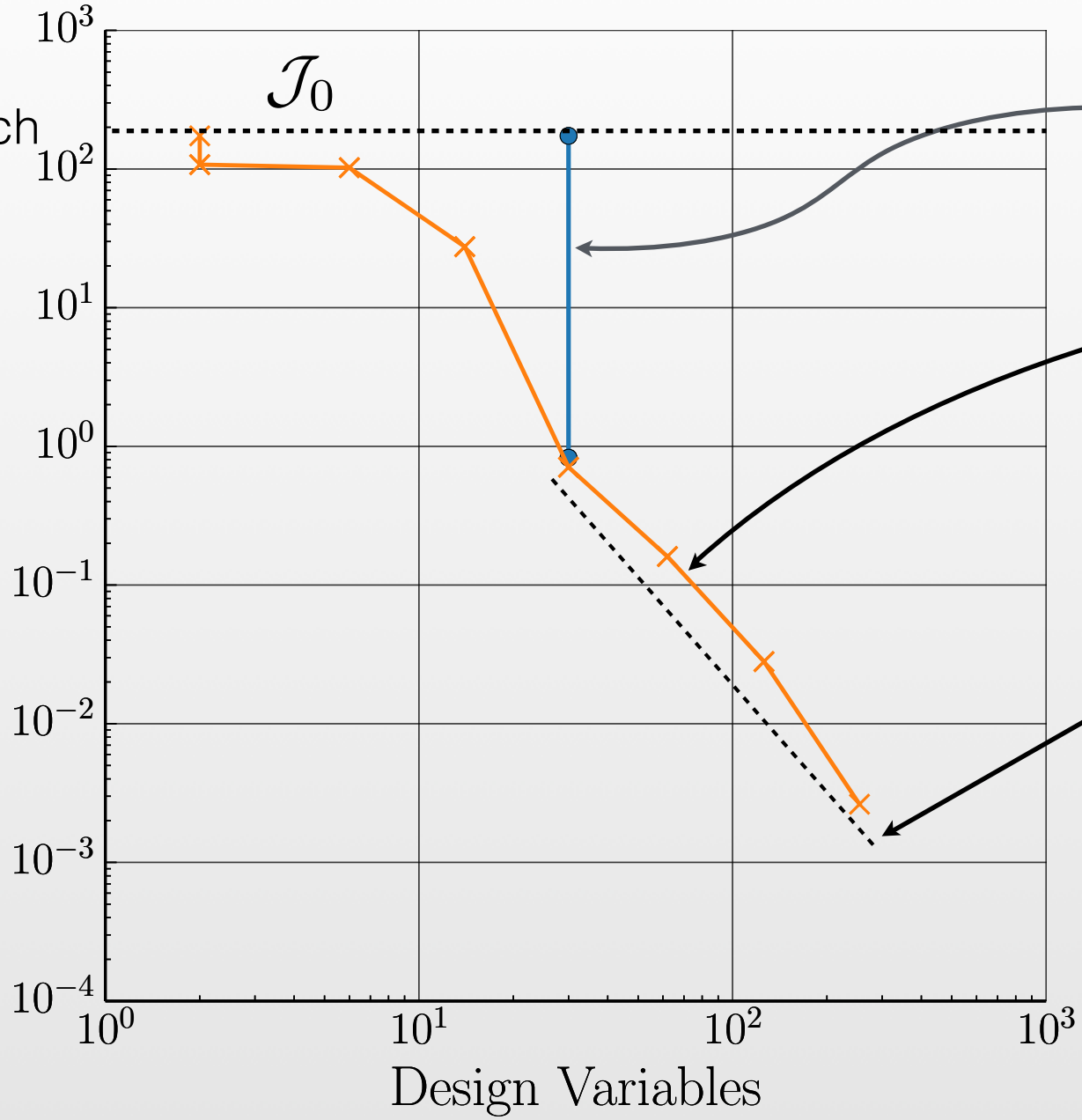
Static
30 DVs

Convergence to Continuous Optimum

Optimum in **current** search space

$$\mathcal{J}_\star^k - \mathcal{J}_\star^\infty$$

Continuous optimum



Static
30 DVs

Uniform
refinement

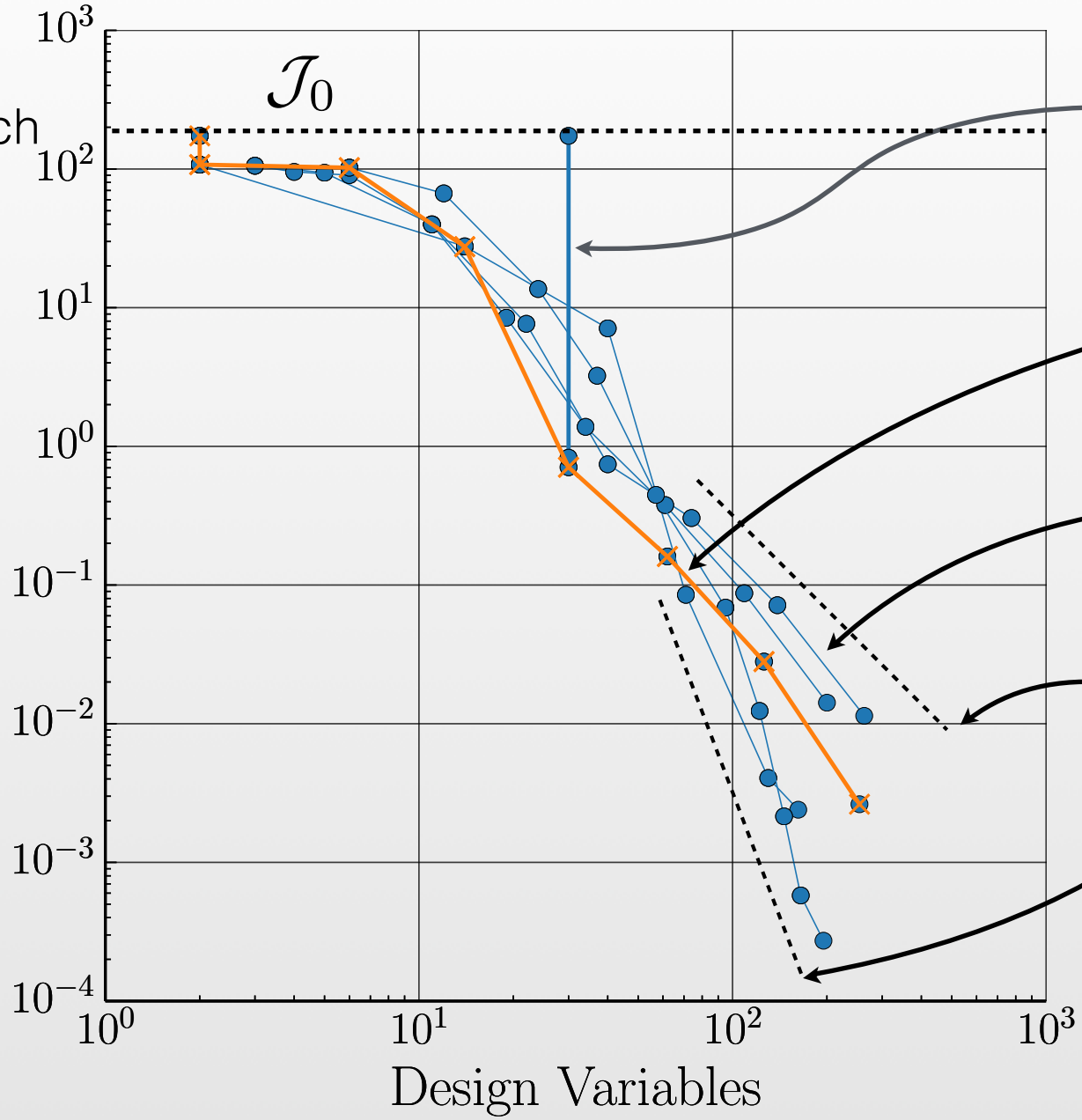
Asymptotic
convergence

Convergence to Continuous Optimum

Optimum in **current** search space

$$\mathcal{J}_\star^k - \mathcal{J}_\star^\infty$$

Continuous optimum



Static 30 DVs

Uniform refinement

4 **adaptive** strategies

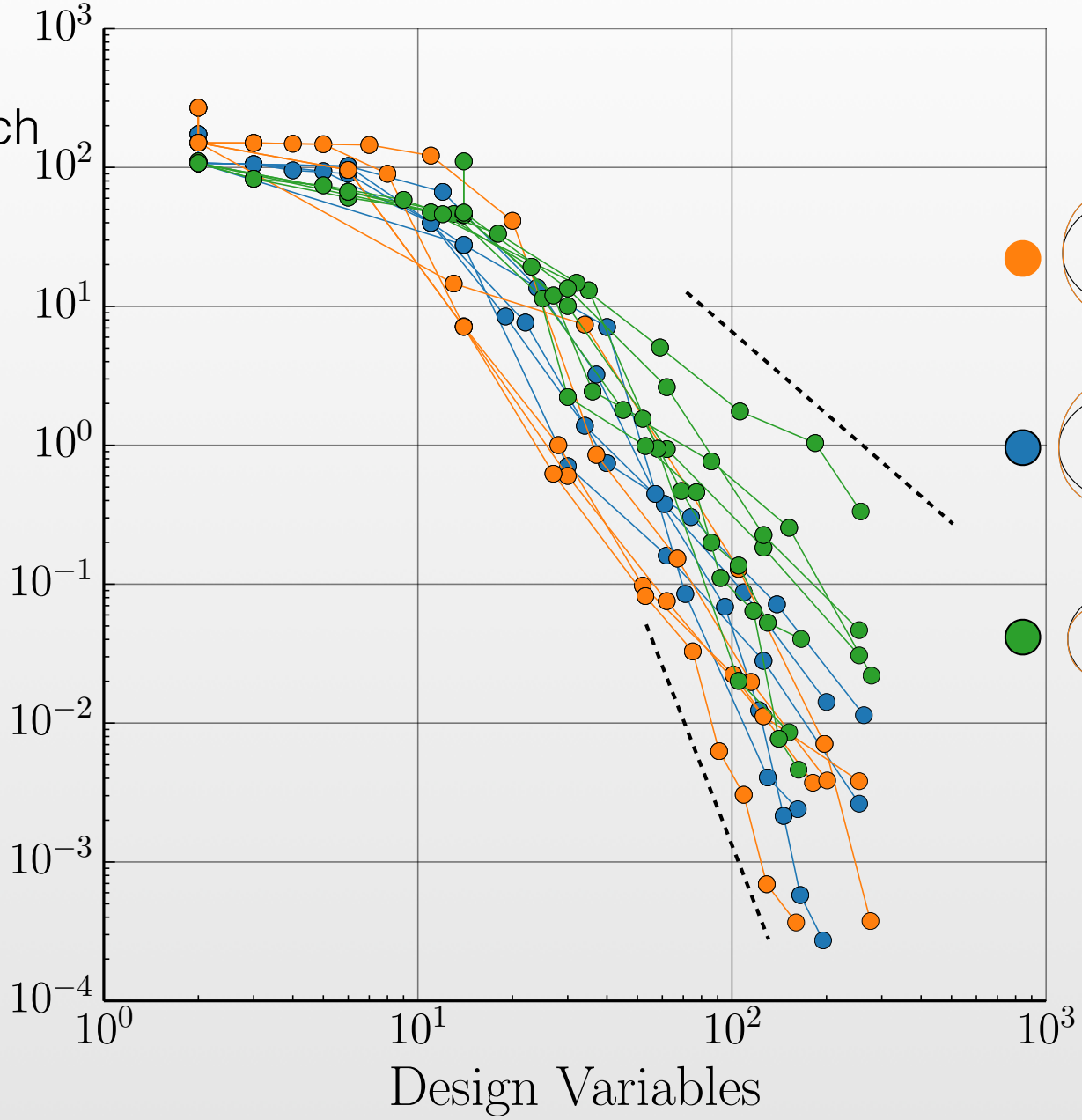
Asymptotic convergence

Convergence to Continuous Optimum

Optimum in **current** search space

$$\mathcal{J}_\star^k - \mathcal{J}_\star^\infty$$

Continuous optimum



Random initial airfoils

Convergence Rate

Efficient in **use of design variables**

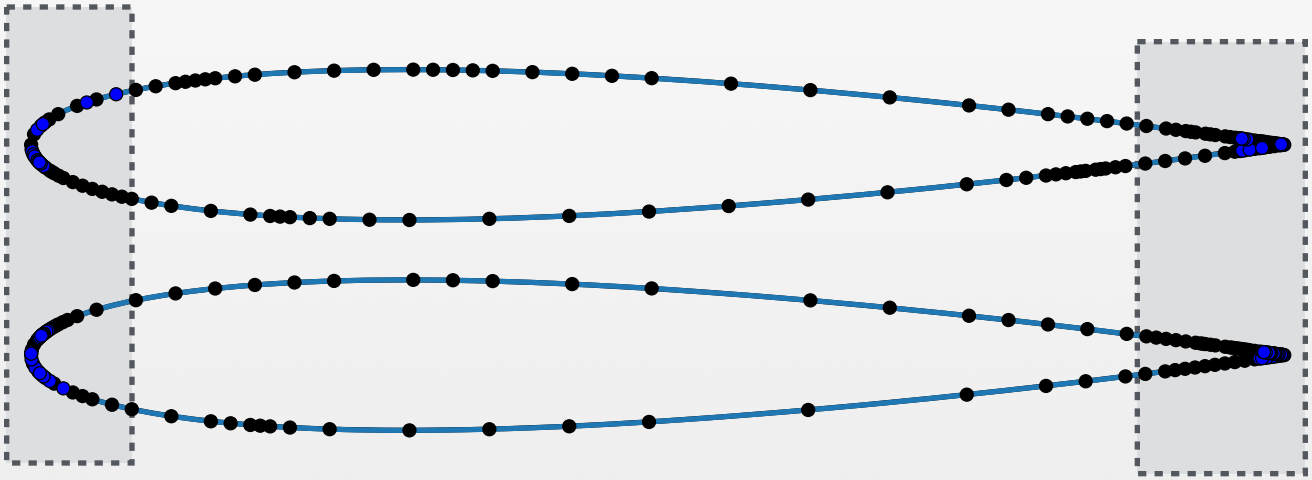
Asymptotic convergence rate of $\mathcal{J}_\star^k - \mathcal{J}_\star^\infty$

Case	Uniform	Adaptive	
		Strategy 1	Strategy 2
1	2.6	8.3	5.0
2	2.4	5.2	5.6
3	2.7	5.7	4.7
mean	**2.6**	**5.75**	
$\frac{\Delta\mathcal{J}}{\Delta N_{DV}}$*	$\sim$ **6**$\times$	$\sim$ **54**$\times$	

* Reduction in objective for $2\times$ increase in N_{DV}

Refinement Patterns

Automatic shape control **clustering**
at leading and trailing edges

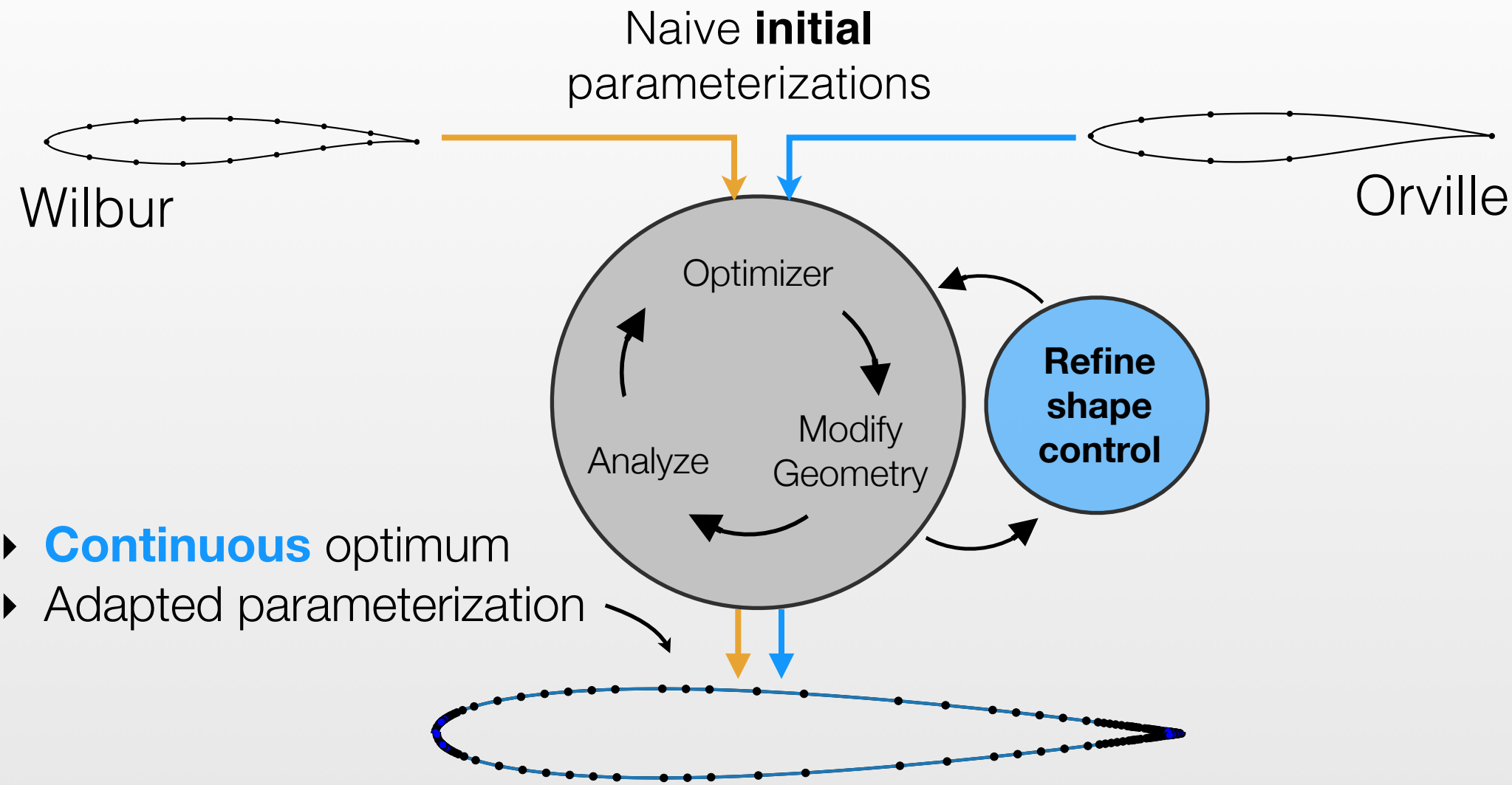


Slower growth

Faster growth

Different adaptation
strategies result in
similar patterns

Adaptive System

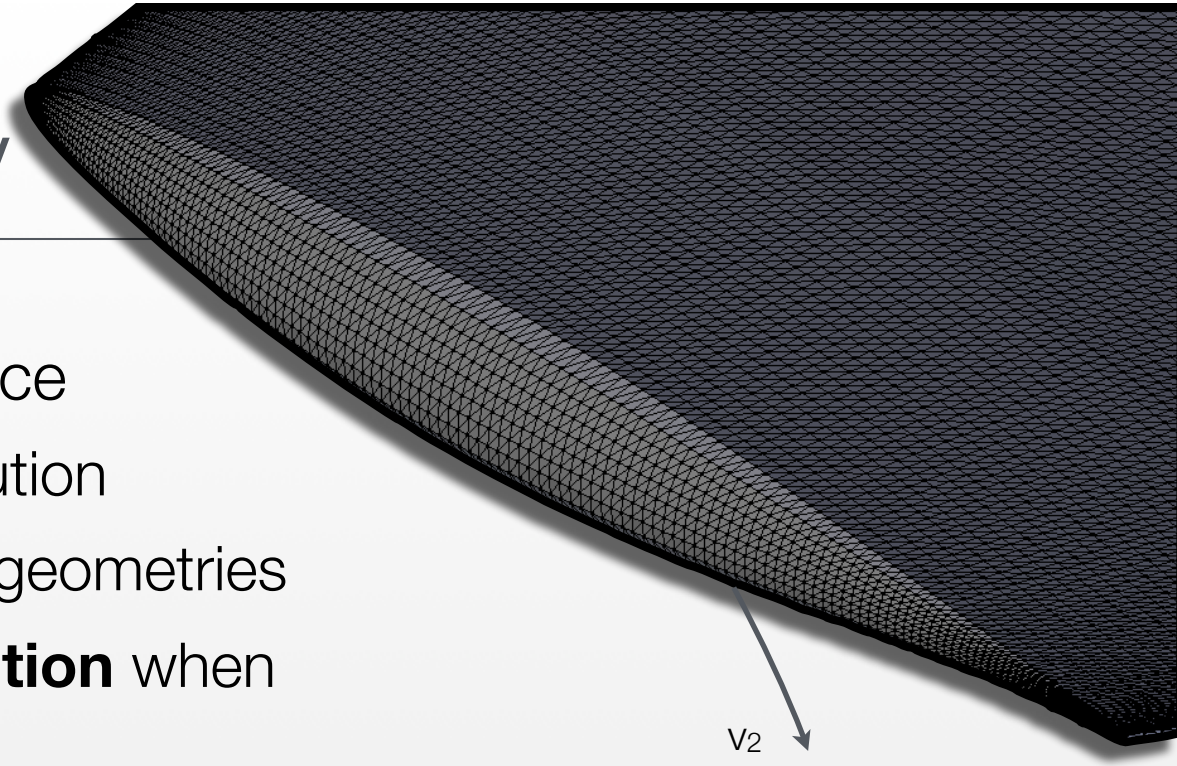


- **Continuous** optimum
- Adapted parameterization

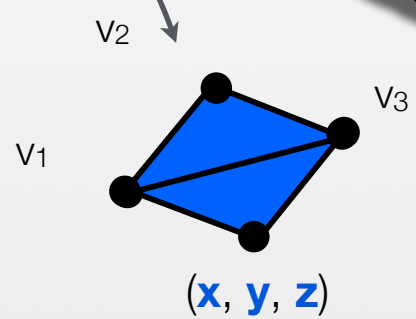
Outline

- ✓ Introduction
- ✓ Theory and Approach
- ✓ Verification
- ‣ **Design Examples**
  - ***Tools:*** *(geometry modeler, flow solver, optimizer)*
  - *Sonic boom signature matching*
  - *Adaptive flaps for Truss-braced wing*

Discrete Geometry



- Direct manipulation of surface
  - Always CFD-ready resolution
  - Optimization of “legacy” geometries
  - Exact **surface preservation** when refining parameterization



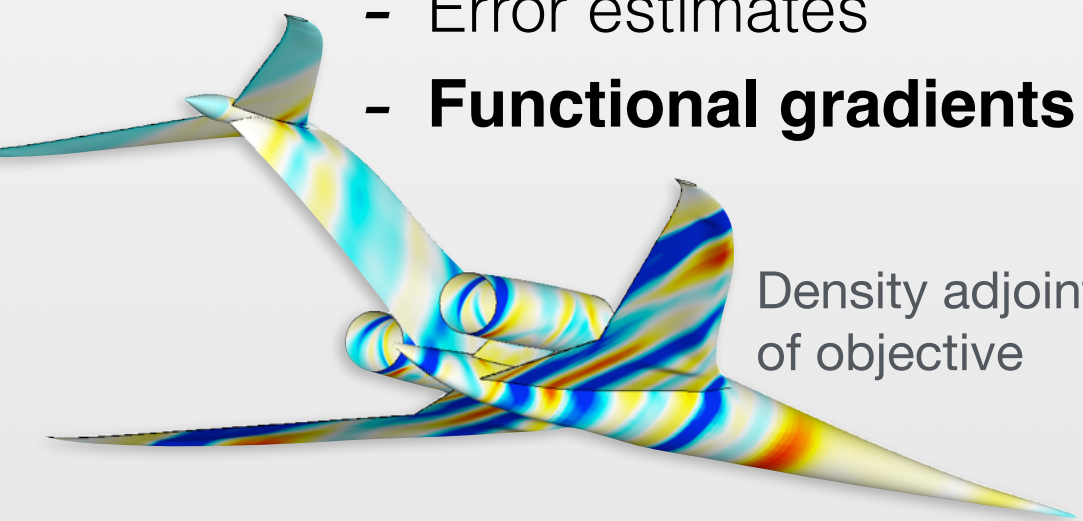
blender

  - Serves as **geometry engine** for optimization
  - Script-driven surface mesh deformation
  - Implemented custom deformation techniques

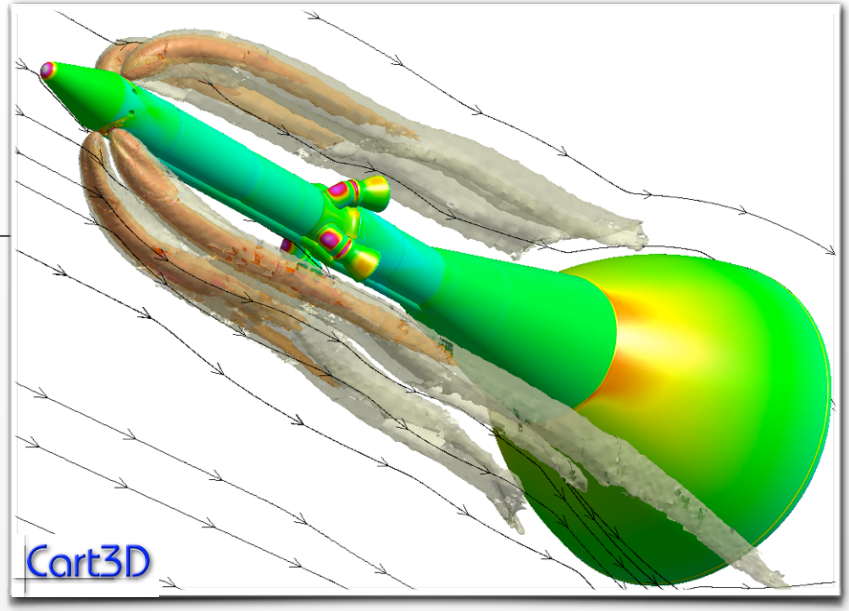
- *(2012)* ***Anderson*** *and Aftosmis,* ***“Parametric Deformation of Discrete Geometry for Aerodynamic Shape Design”****.* *AIAA Paper 2012-0965.*

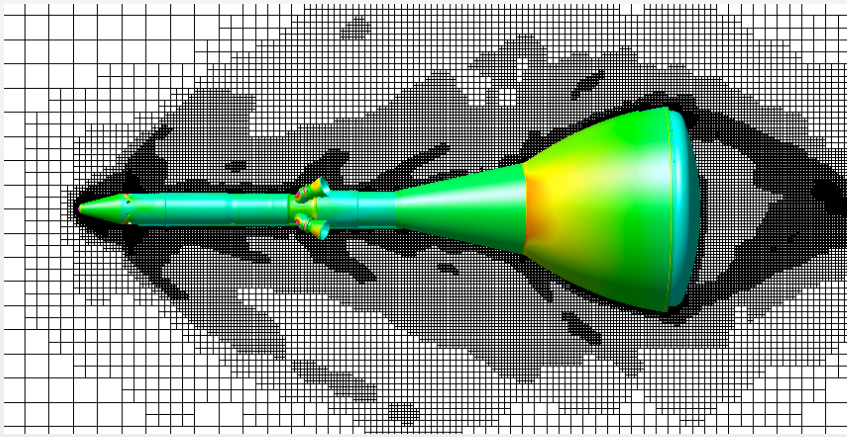
Cart3D

- Cartesian cut-cell method with automated meshing of complex configurations
- Inviscid solver with adjoint-driven
  - Adaptive meshing
  - Error estimates
  - **Functional gradients**



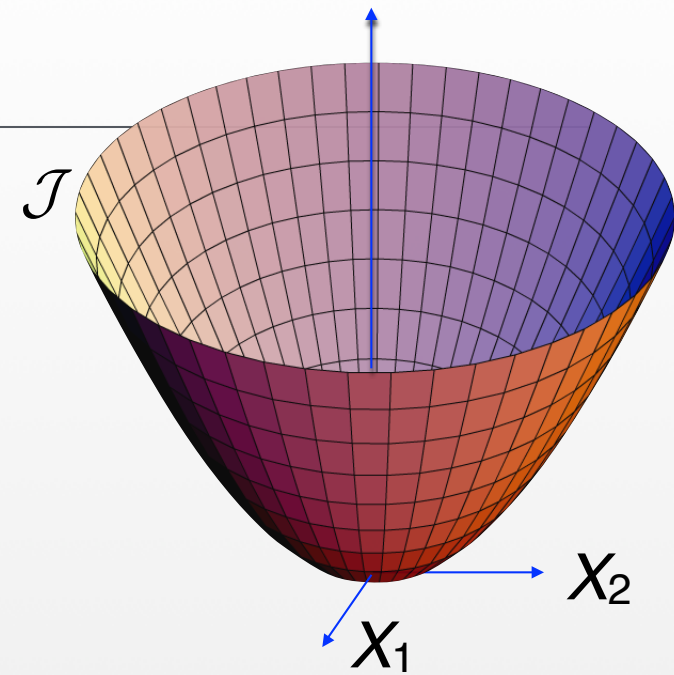
Density adjoint of objective





Output-adaptive meshing

Optimizer



SNOPT — Sparse **N**onlinear **Opt**imizer

- **SQP** method

  Handles nonlinear inequality constraints
- **Quasi-Newton** method

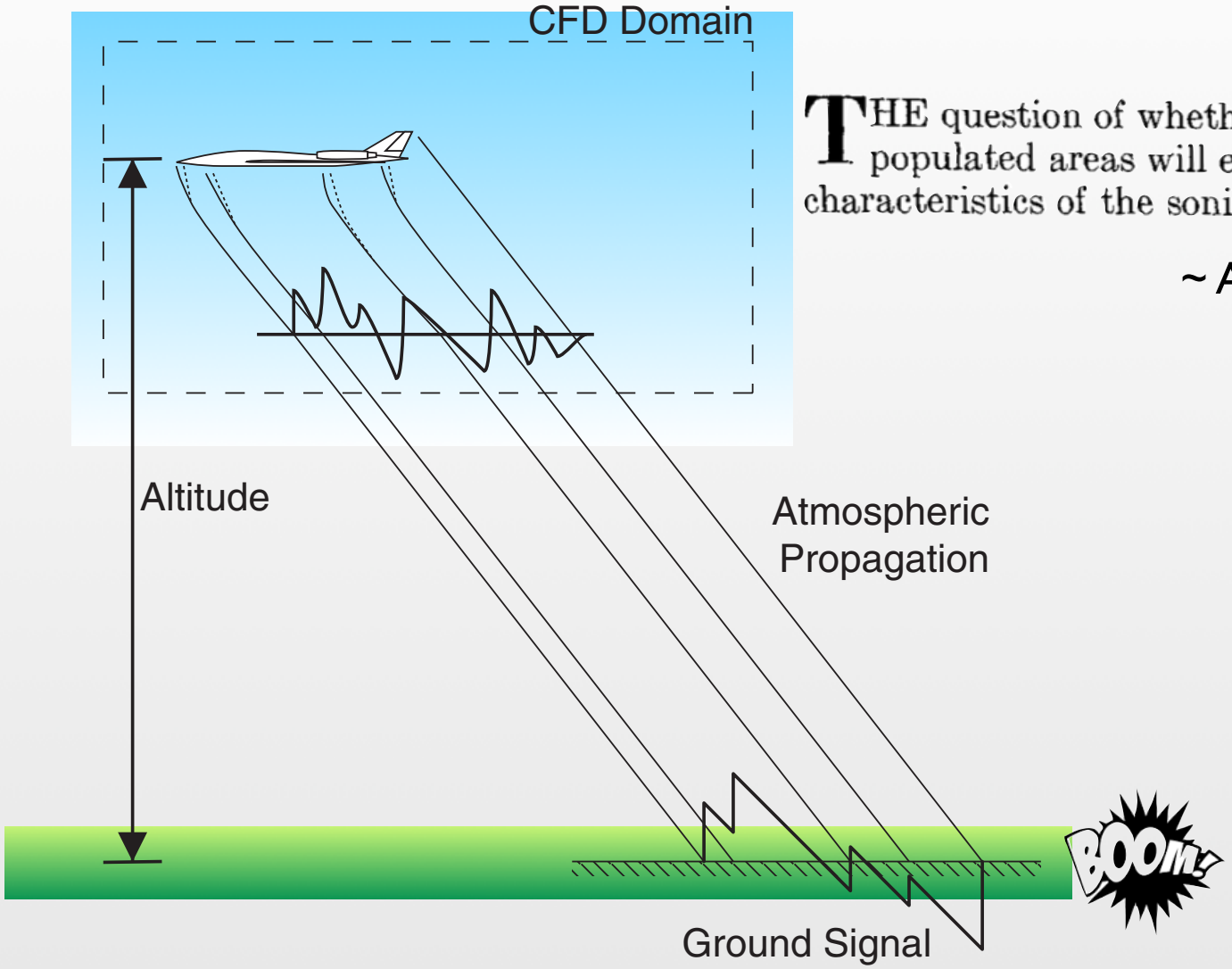
  Gradually builds Hessian approximation
- Use full-memory **BFGS**

  (Test cases involve <1000 DV's)

Can also use any general gradient-based optimizer:

- SLSQP, SciPy, Knitro, pyOpt…

Boom Design



THE question of whether flights of supersonic aircraft over populated areas will ever be acceptable depends upon the characteristics of the sonic booms of these aircraft.

~ A. R. George and Richard Seebass
October 1971

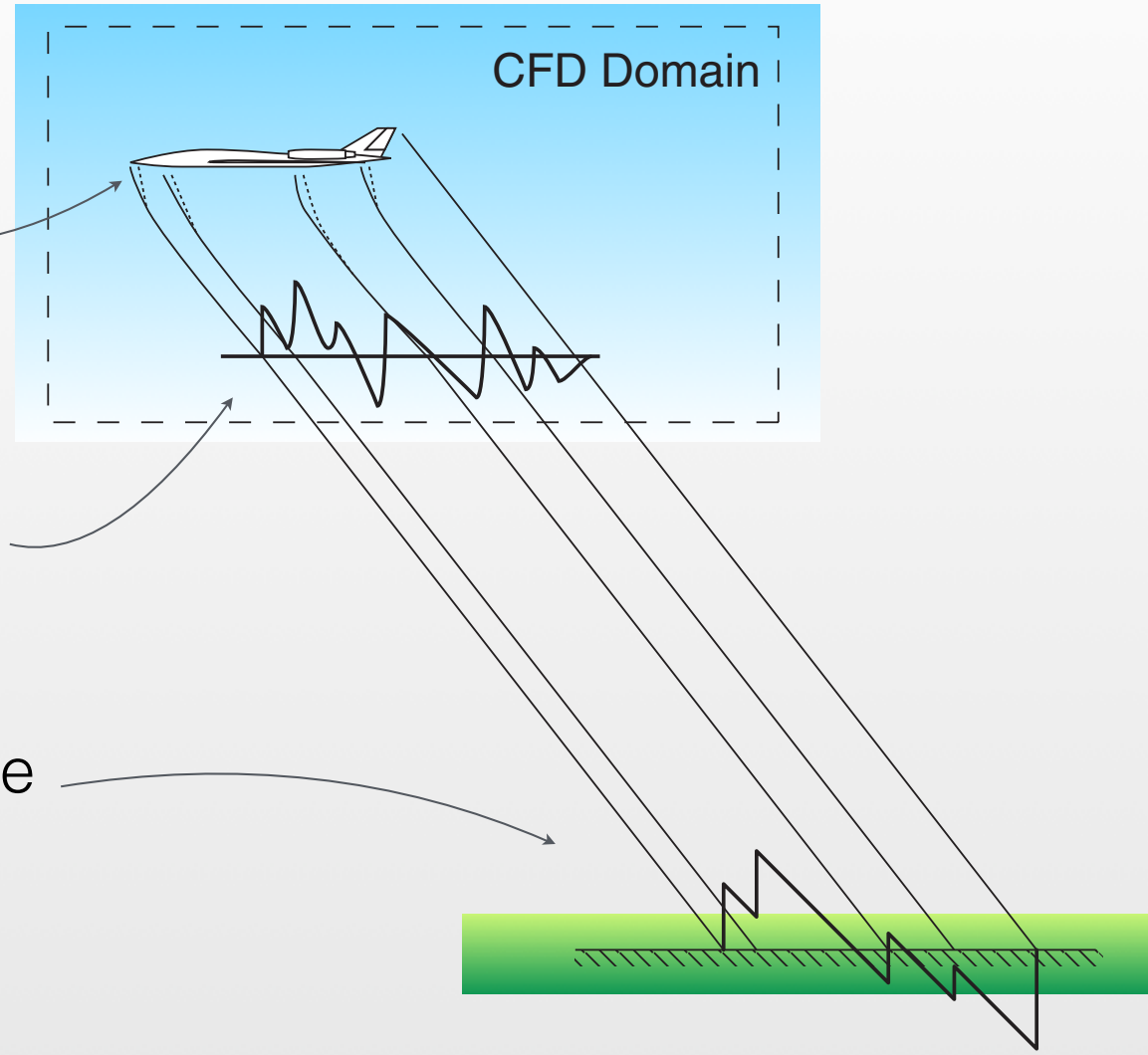
Inverse Design Procedure

3. **Reshape vehicle** to match the near-field signal

$$\mathcal{J} = \frac{1}{p_\infty^2} \int (p - p_{\text{target}})^2 dS$$

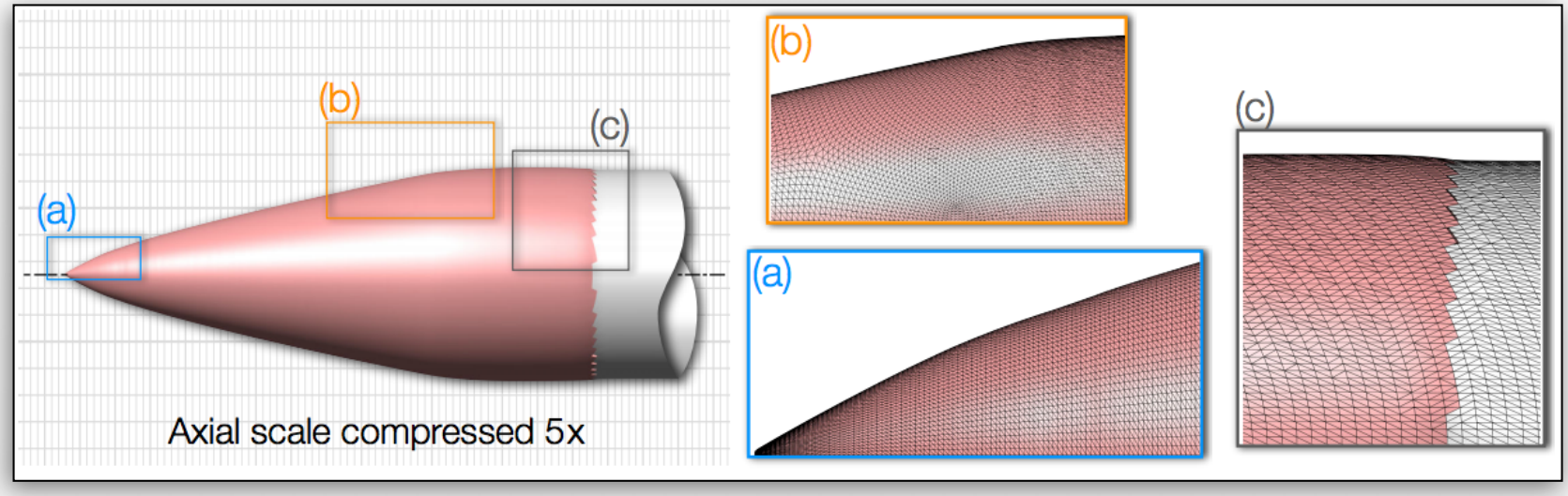
2. Find **near-field signal** that meets these requirements

1. Determine acceptable noise characteristics at **ground**

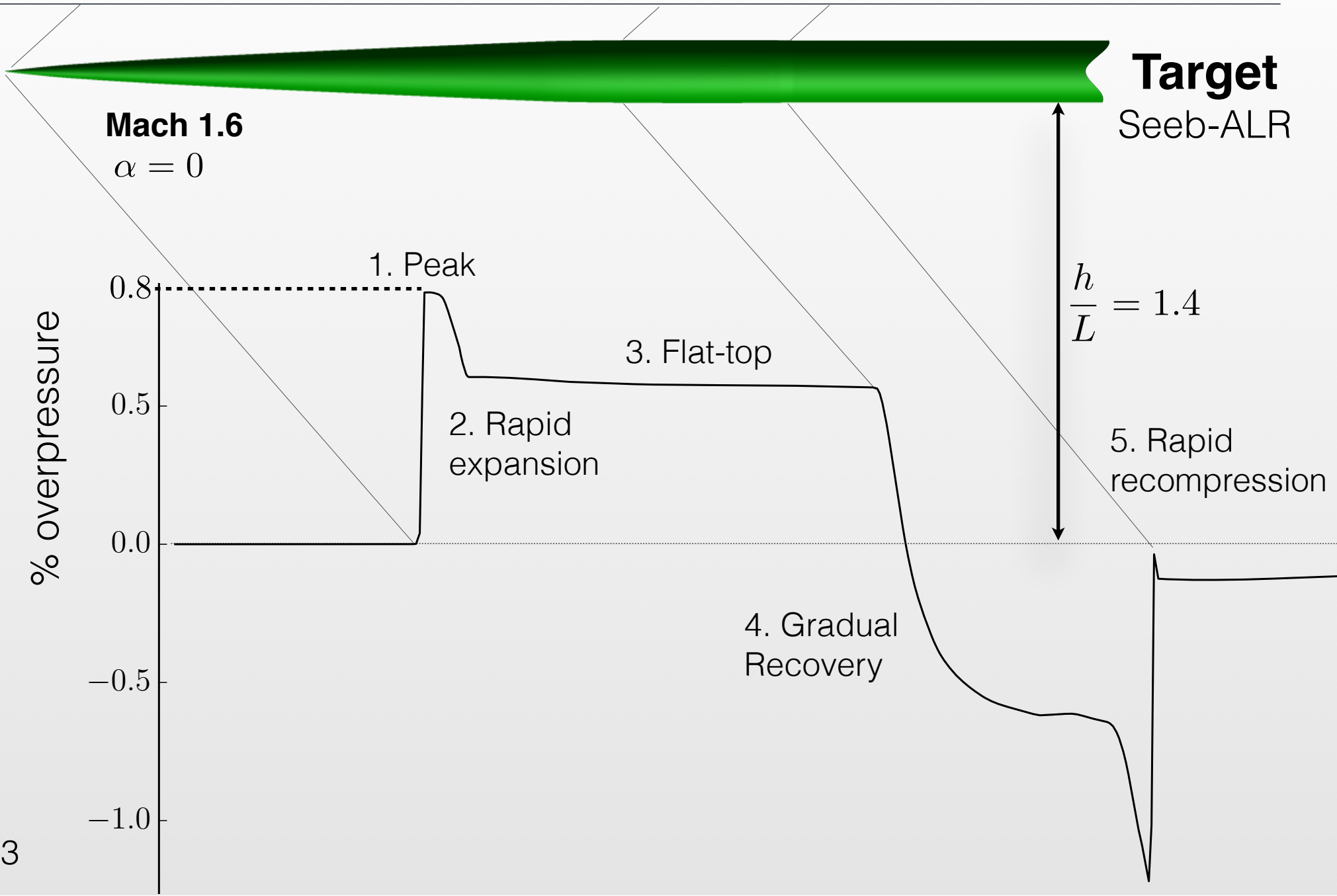


Seeb-ALR

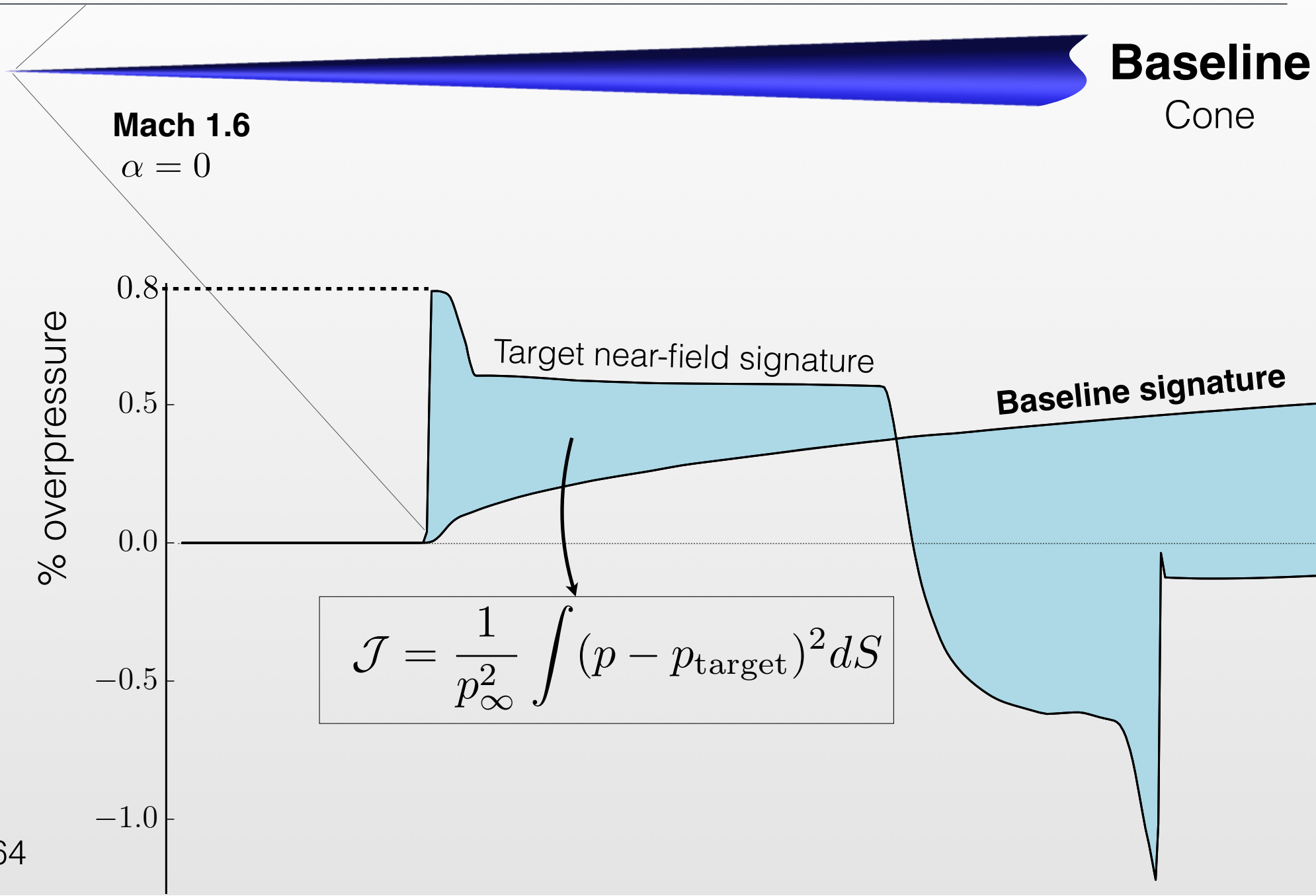
Target
Seeb-ALR



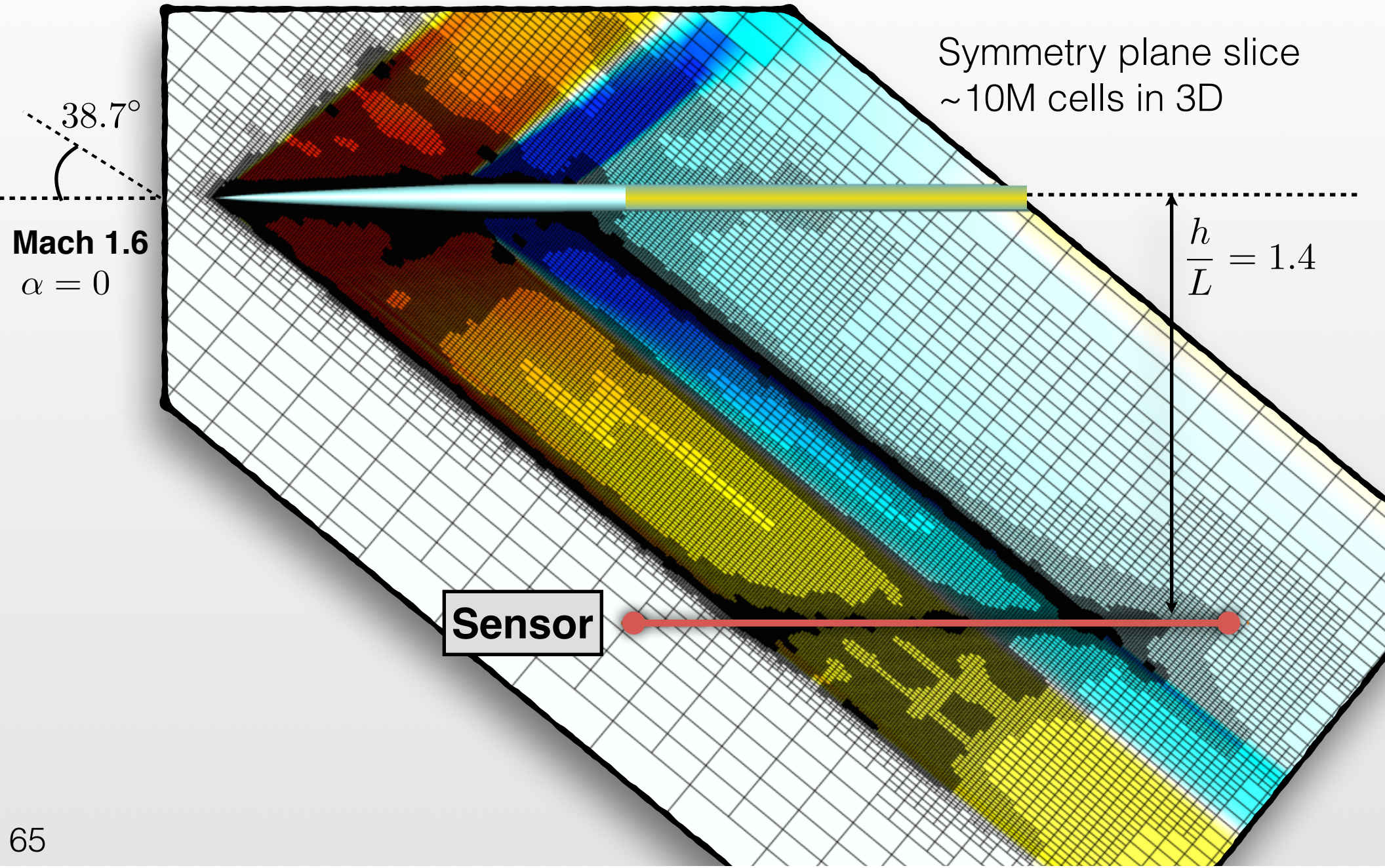
Target Nearfield Signature



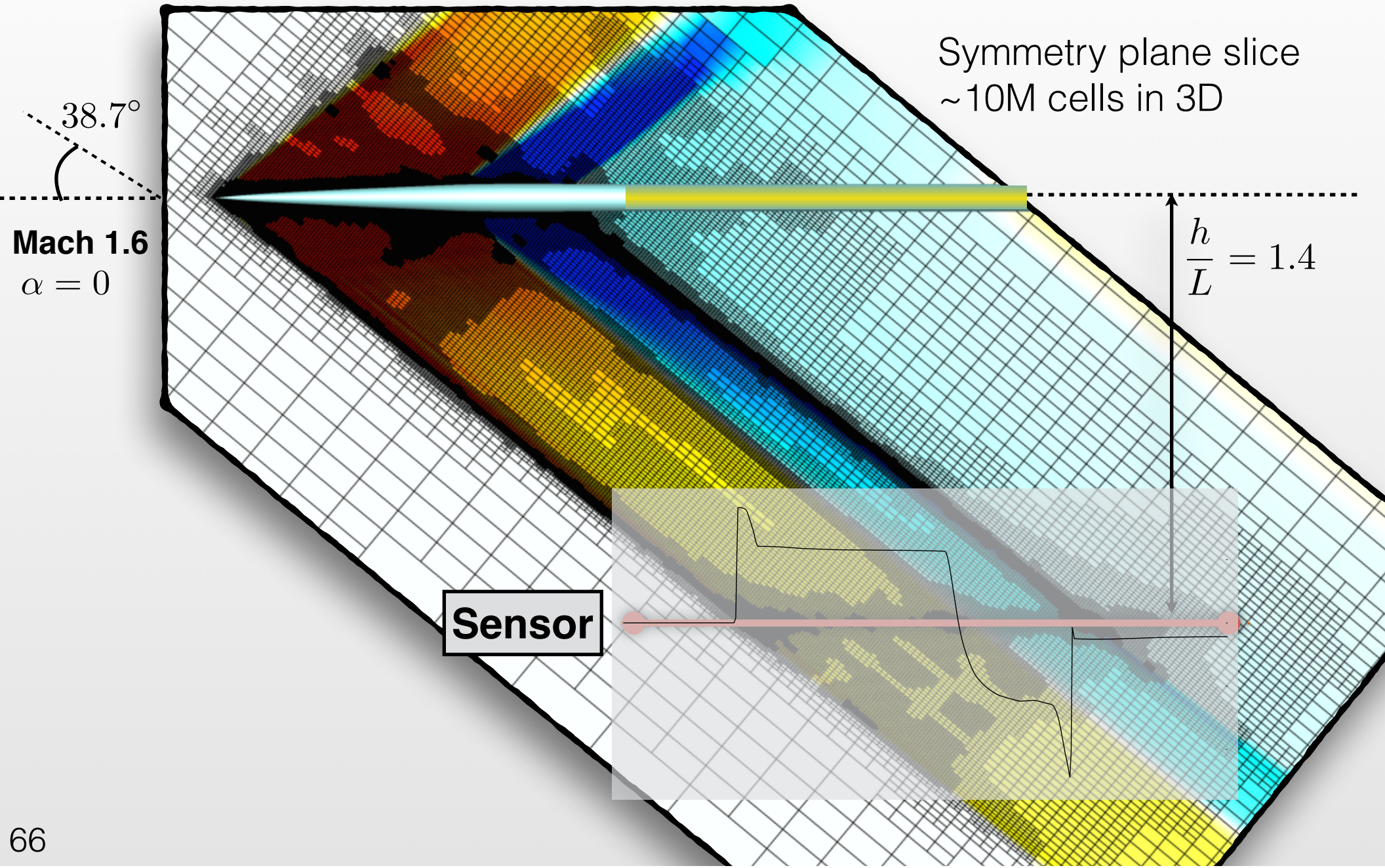
Baseline Geometry



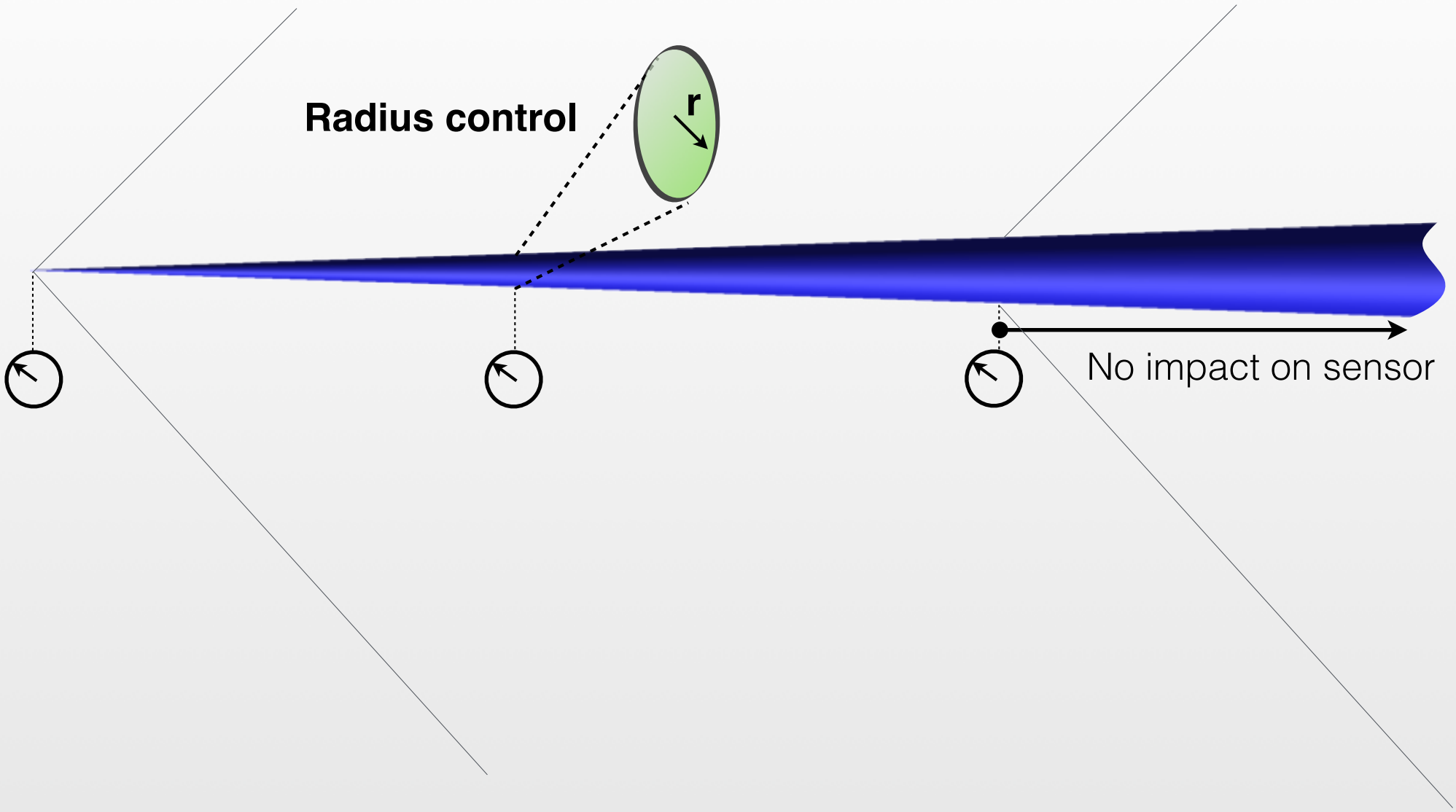
Mesh Adaptation



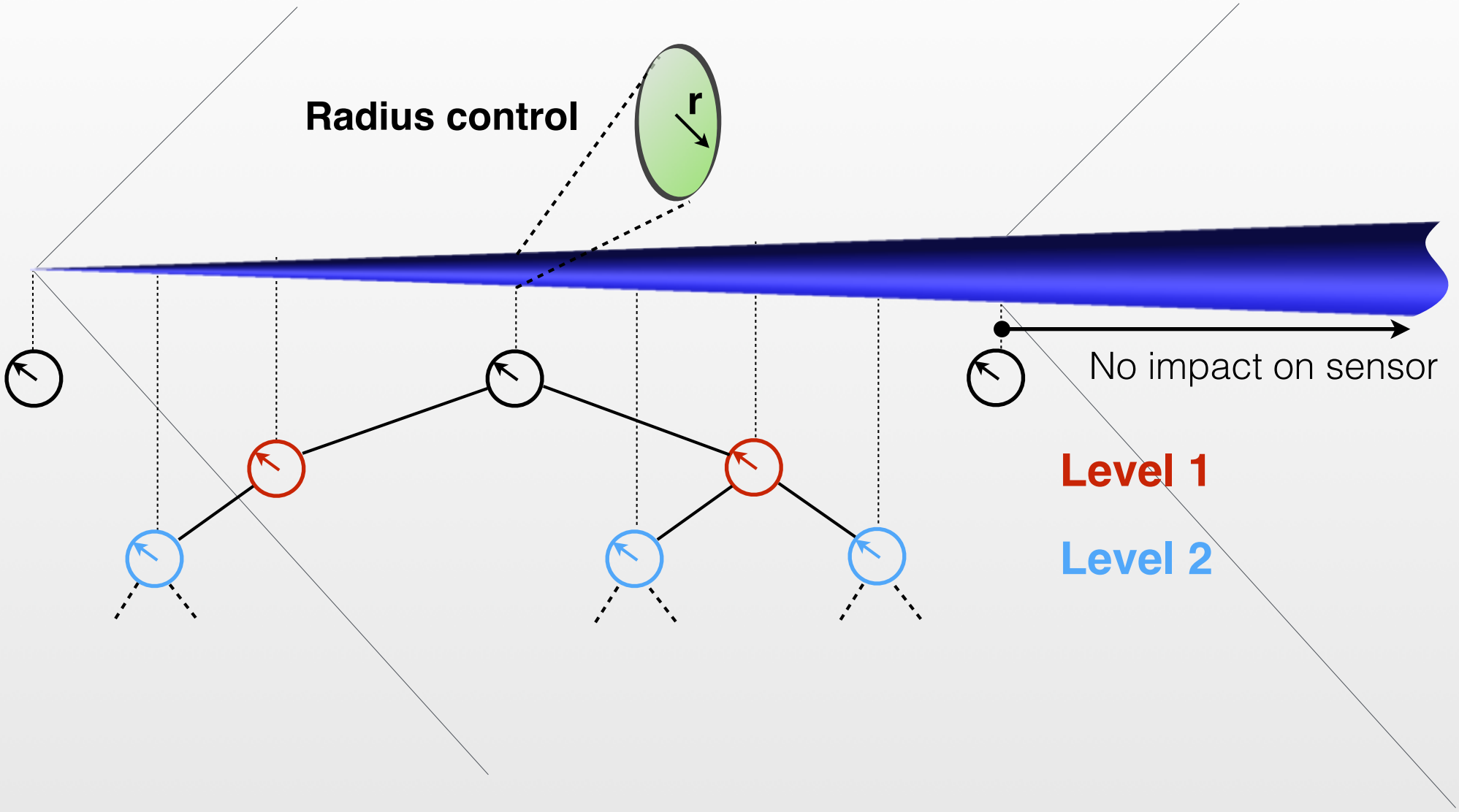
Mesh Adaptation



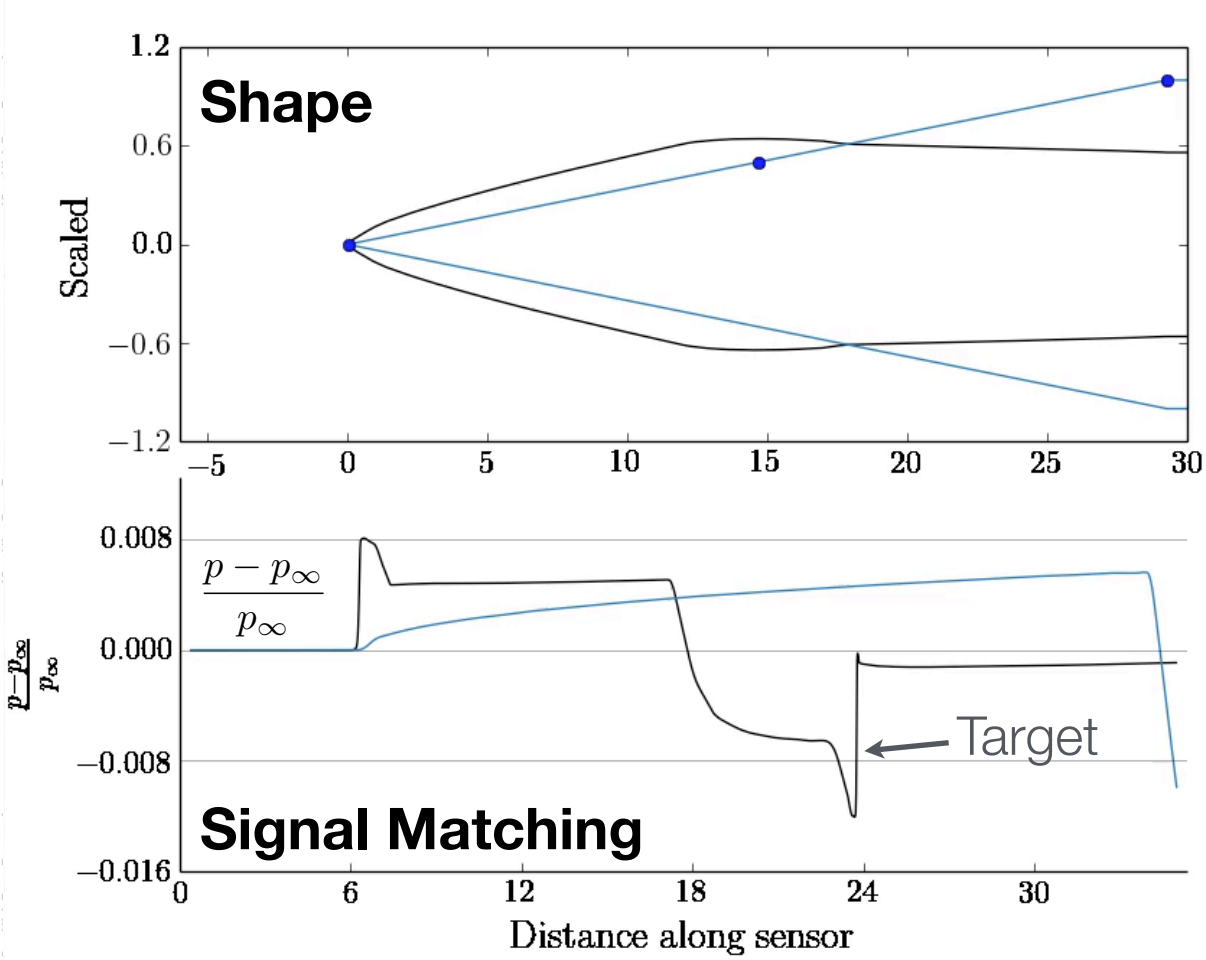
Adaptive Parameterization



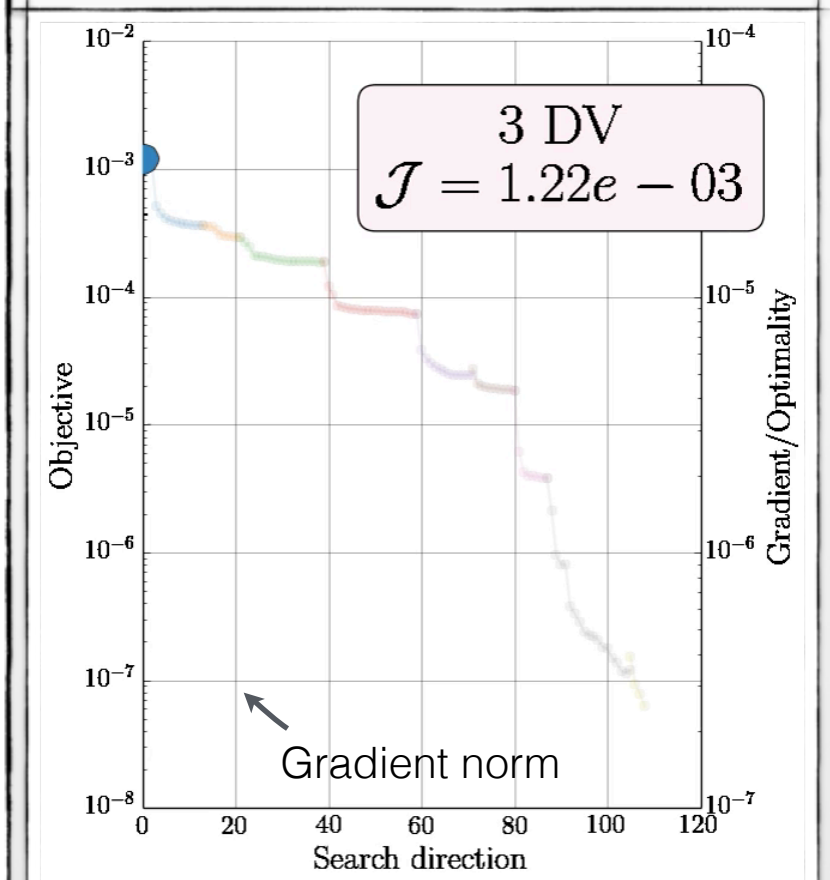
Adaptive Parameterization



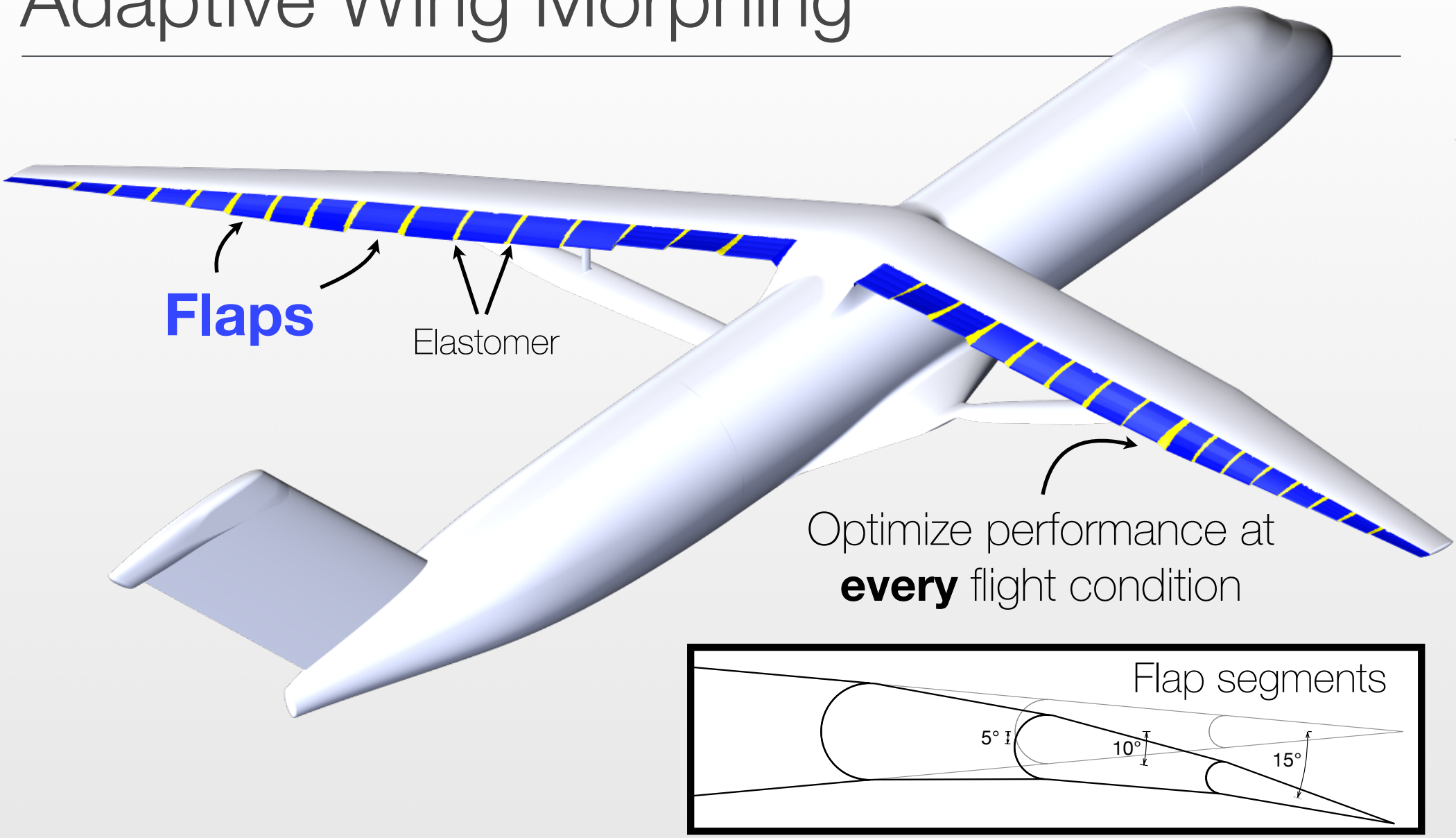
Evolution



Objective Convergence

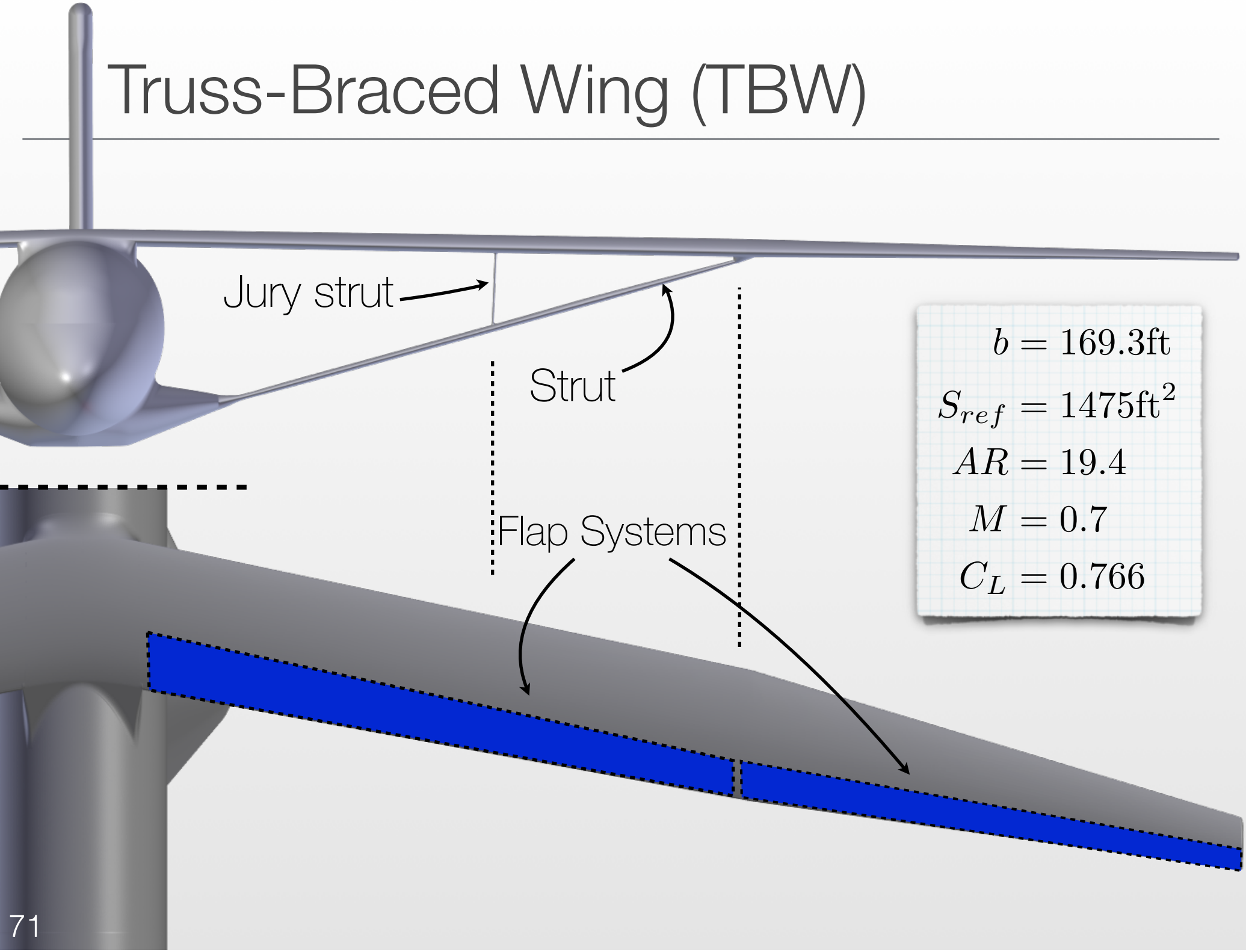


Adaptive Wing Morphing



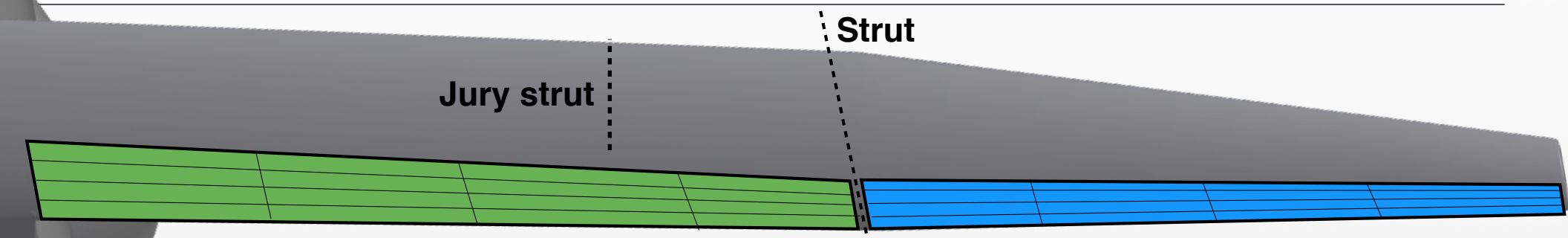
(2015) *Rodriguez, Aftosmis, Nemec,* ***Anderson****, "****Optimized off-design performance of flexible wings with continuous trailing-edge flaps.****" AIAA Paper 2015–1409, AIAA SciTech 2015, Kissimmee, FL.*

Truss-Braced Wing (TBW)



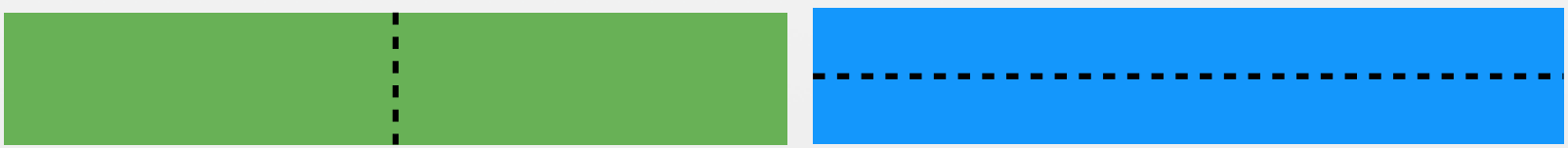
$$b = 169.3\text{ft}$$
$$S_{ref} = 1475\text{ft}^2$$
$$AR = 19.4$$
$$M = 0.7$$
$$C_L = 0.766$$

Flap Refinement



Initially two monolithic flaps, can be subdivided…

span-wise or **stream**-wise

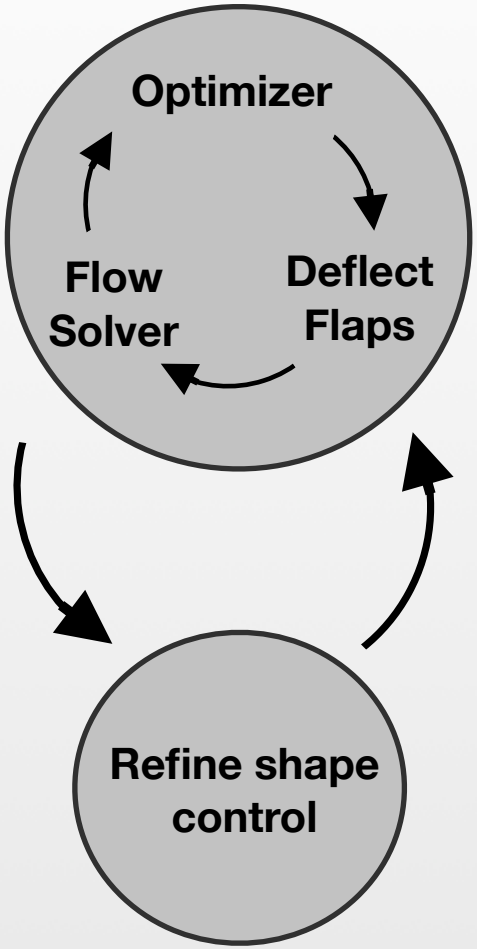


uniformly or **adaptively**

Flap Adaptation Procedure

1) **Morph:** Optimize flap deflections for minimum drag.

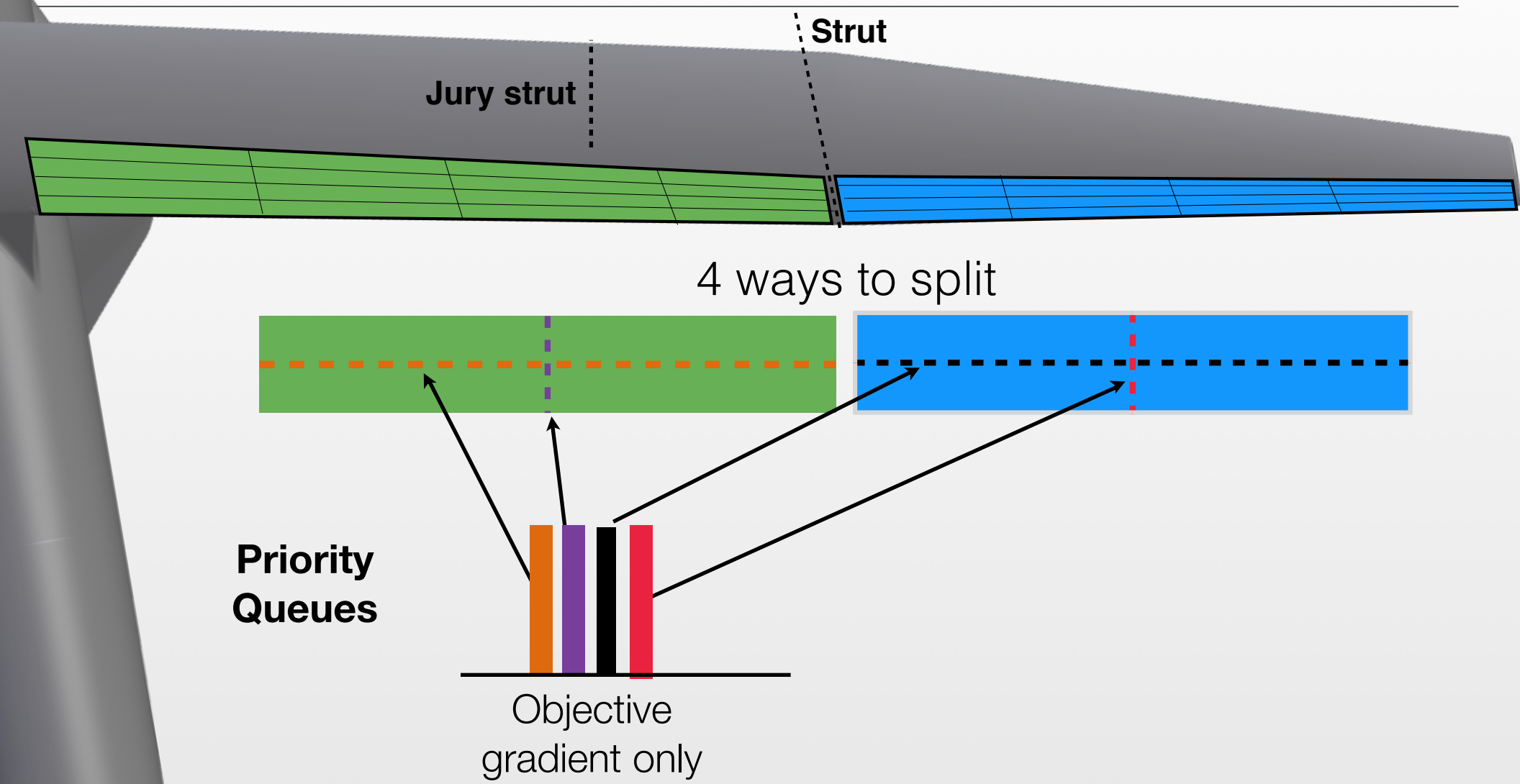
2) **Refine flap topology:** Add the one* additional flap that would best allow the drag to be reduced.



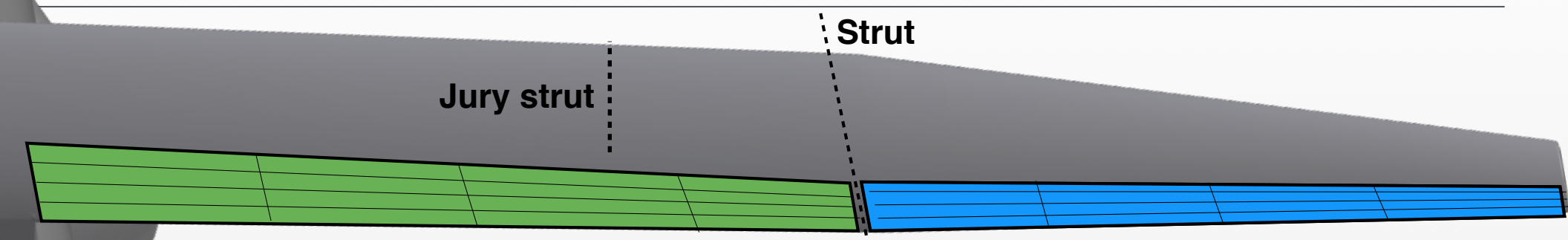
$$\mathcal{J} = C_D$$
$$M = 0.7$$
$$C_L = 0.766$$

*Add flaps **one** at a time, because the cost associated with every flap is real — want to find **minimal** parameterization!

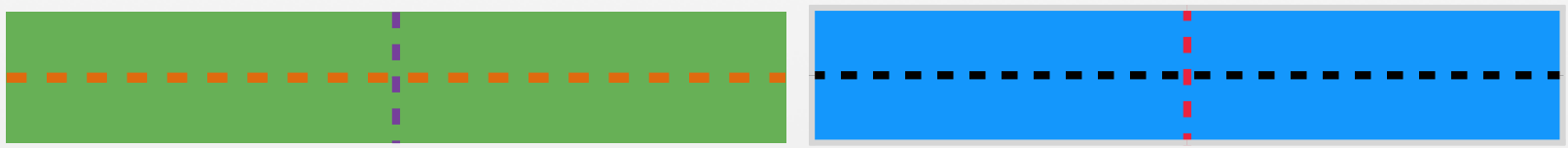
Results


Strut
Jury strut
4 ways to split
Priority Queues
Objective gradient only

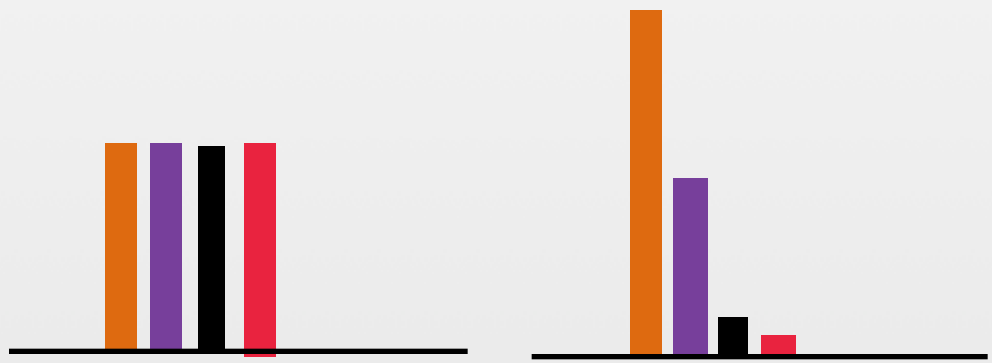
Results



4 ways to split



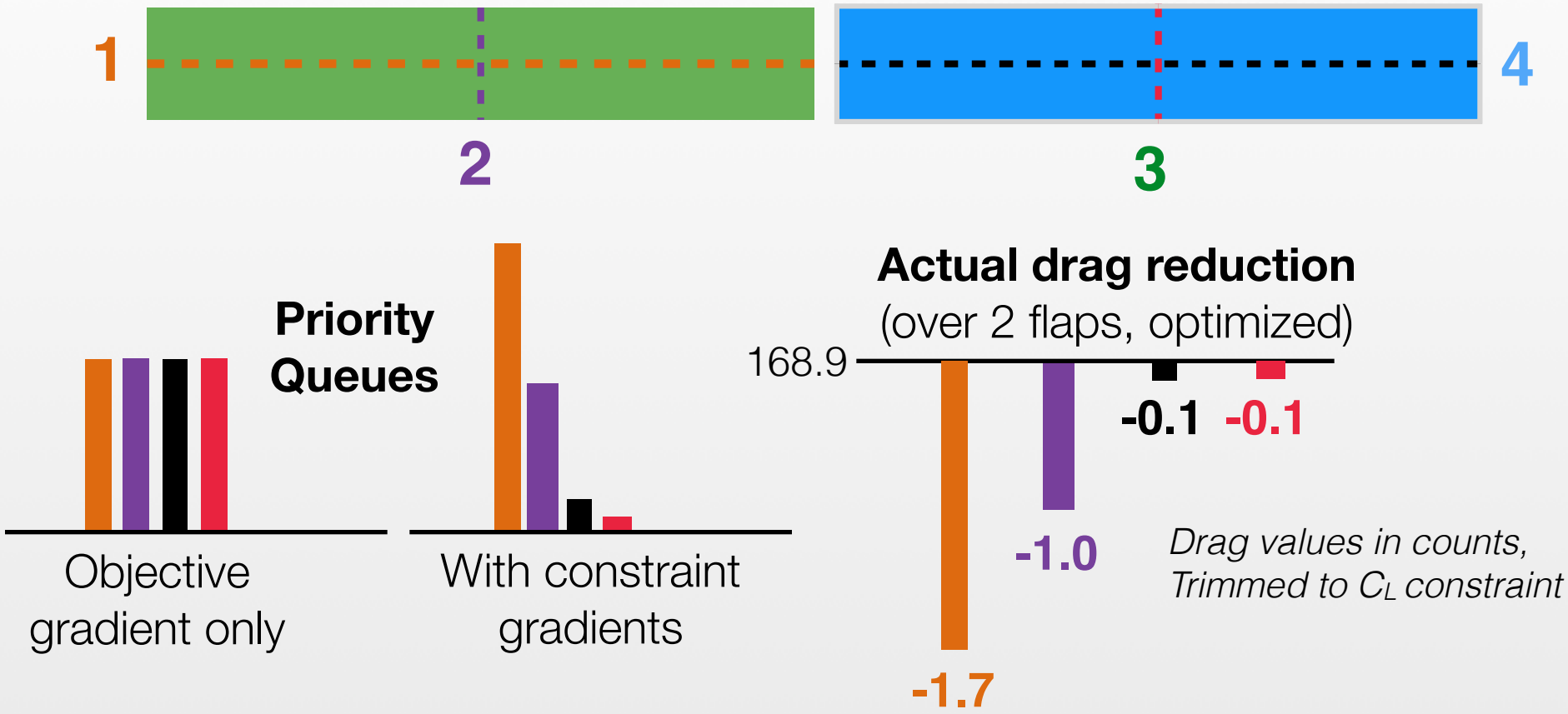
Priority Queues



Objective gradient only

With constraint gradients

Verification of Ranking



Flap Deflection History

Final flap topology

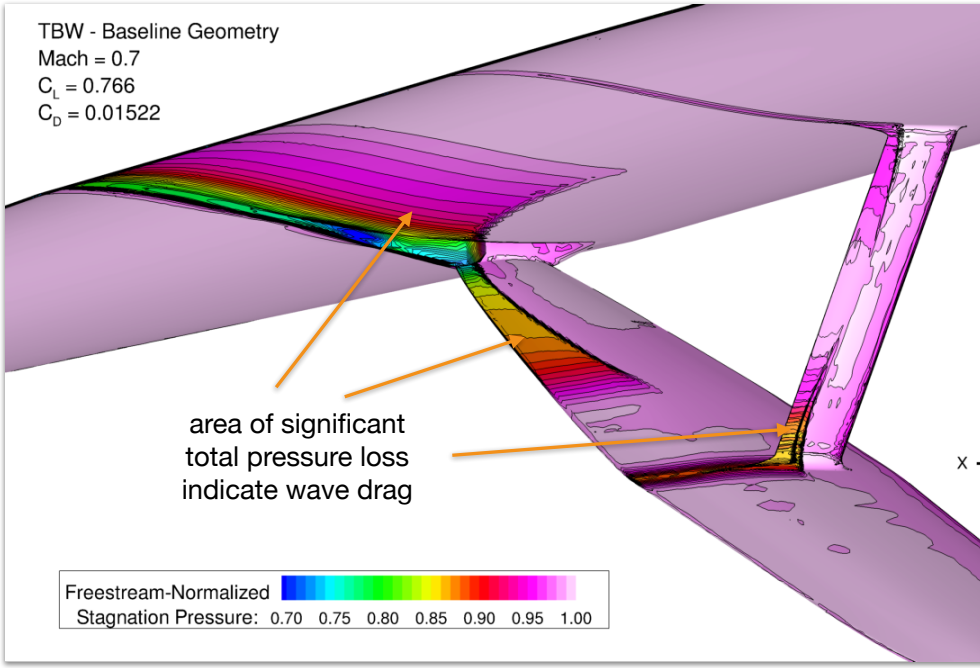
Inboard | Outboard

-1.15° **-1.65°** -0.6° -0.4°

Final deflections

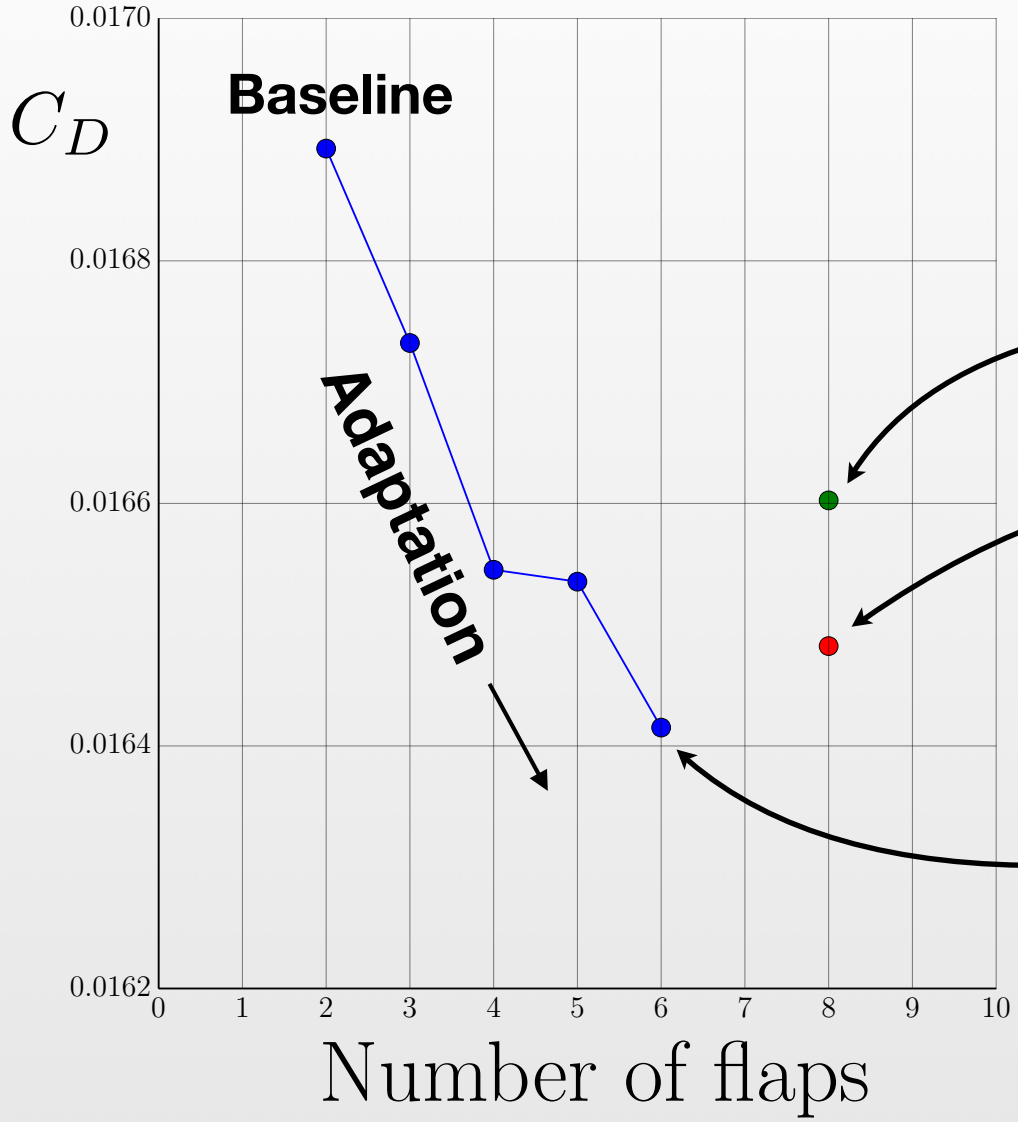
(cumulative deflection at TE)

Negative deflection downward.
Alpha lowered to compensate lift.

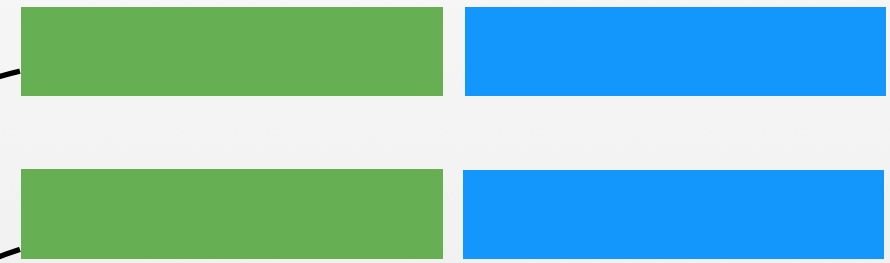


Baseline geometry has substantial **wave drag** through truss

Cost-benefit of Flaps



Two reasonable 8-flap topologies:



Superior drag reduction capability with 6-flap system

Conclusions

- Demonstrated **adaptive shape parameterization** system for automated, high-fidelity aerodynamic optimization.
  - Enables hands-off design **exploration** for unfamiliar problems.
  - Provides **feedback** about the design problem.
- Verification studies confirm that robust **convergence** to continuous optimum is possible.
- A careful adaptive strategy makes the approach substantially **more efficient** both in terms of design variables and computational time.

New Techniques

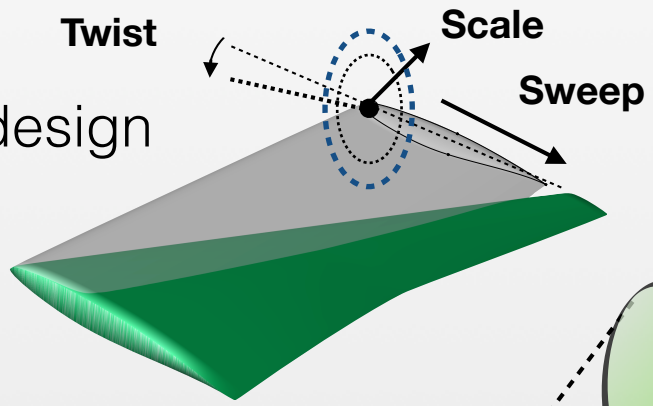
- Goal-oriented **refinement indicator** targeting high potential shape parameters.
  - Substantially improves results over previous best indicator, appropriate for general classes of problems.
  - Leverages information already available during optimization — no *a priori* knowledge required.
- Approximate **Hessian estimation** (prolongation operator)
  - Could also be used to accelerate design in finer design spaces.
- Constructive algorithm to efficiently find an approximate solution to the combinatorial adaptation problem.
- Cost-benefit approaches to automatically determine how many parameters to add and when to trigger refinement.

Future Work

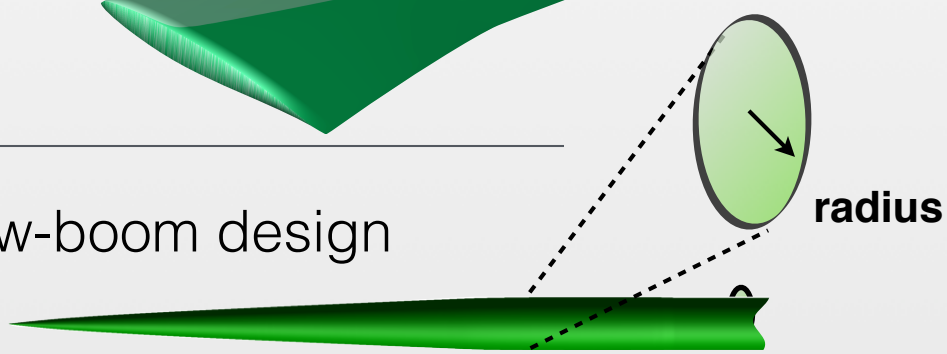
Major outstanding topic:

- Discovering effective **classes** of shape control

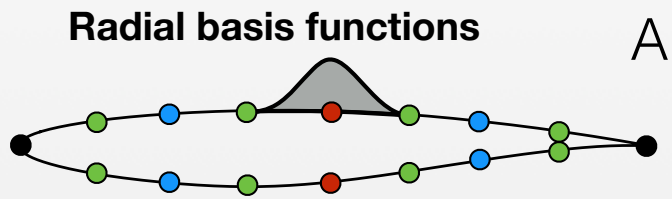
Wing design



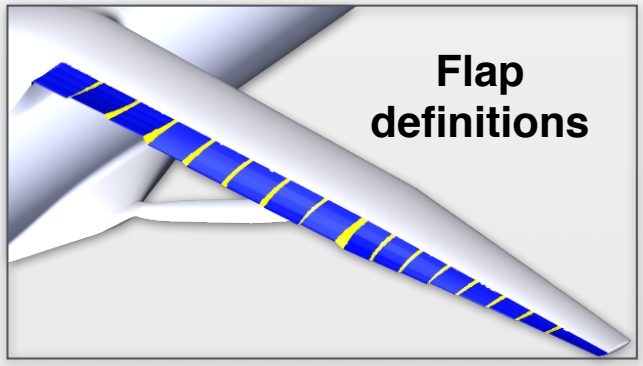
Low-boom design



Airfoil design



Flap system design



Acknowledgements

NASA ARMD Seedling Fund

3 years of outstanding support

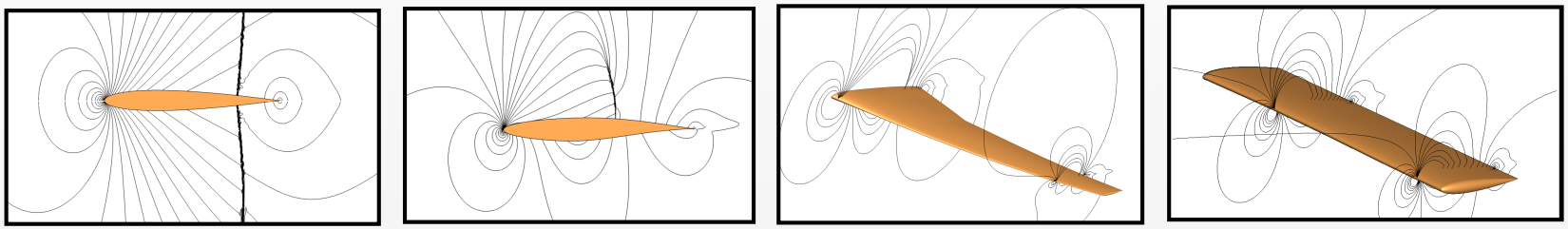
NASA

Michael Aftosmis
Marian Nemec
David Rodriguez
Tom Pulliam

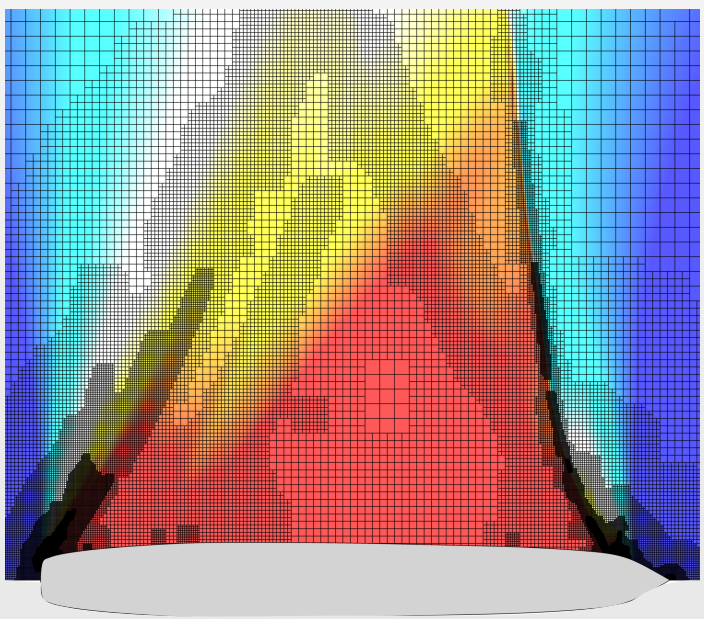
Questions?

Optimization Benchmarks

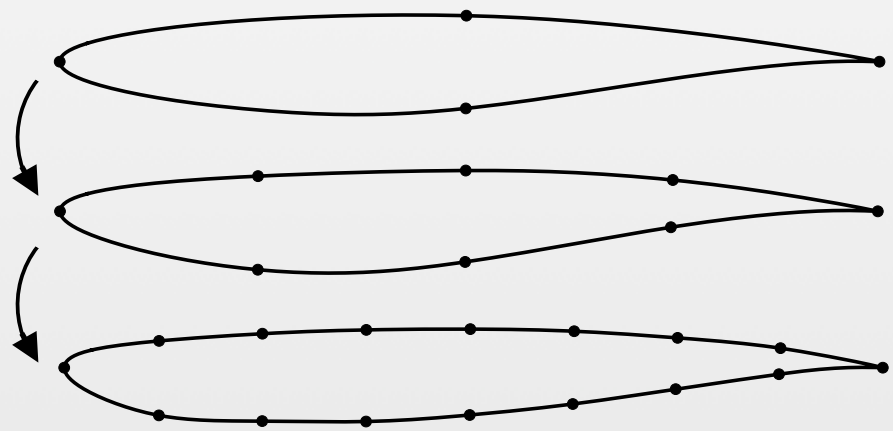
Transonic wing and airfoil design benchmarks



- Combined two **automated, adaptive** elements:



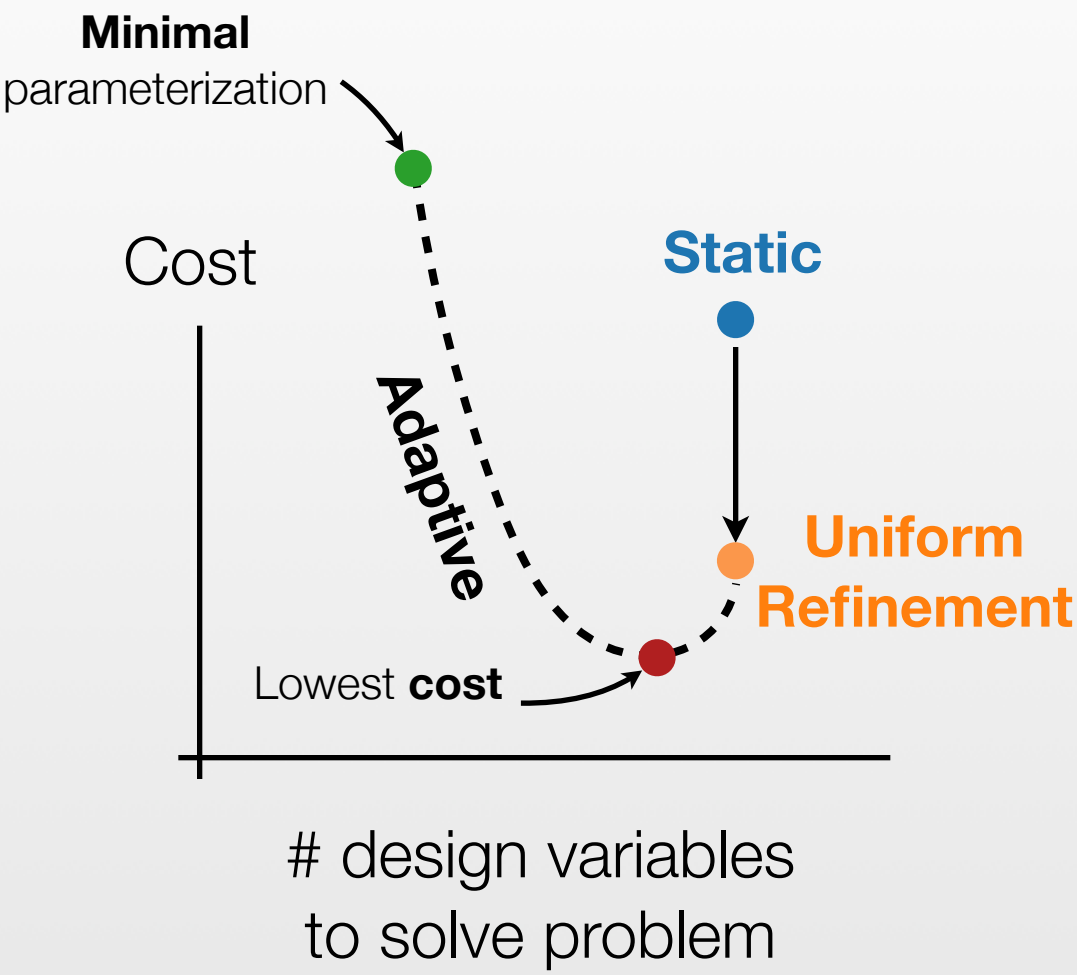
Adaptive mesh refinement



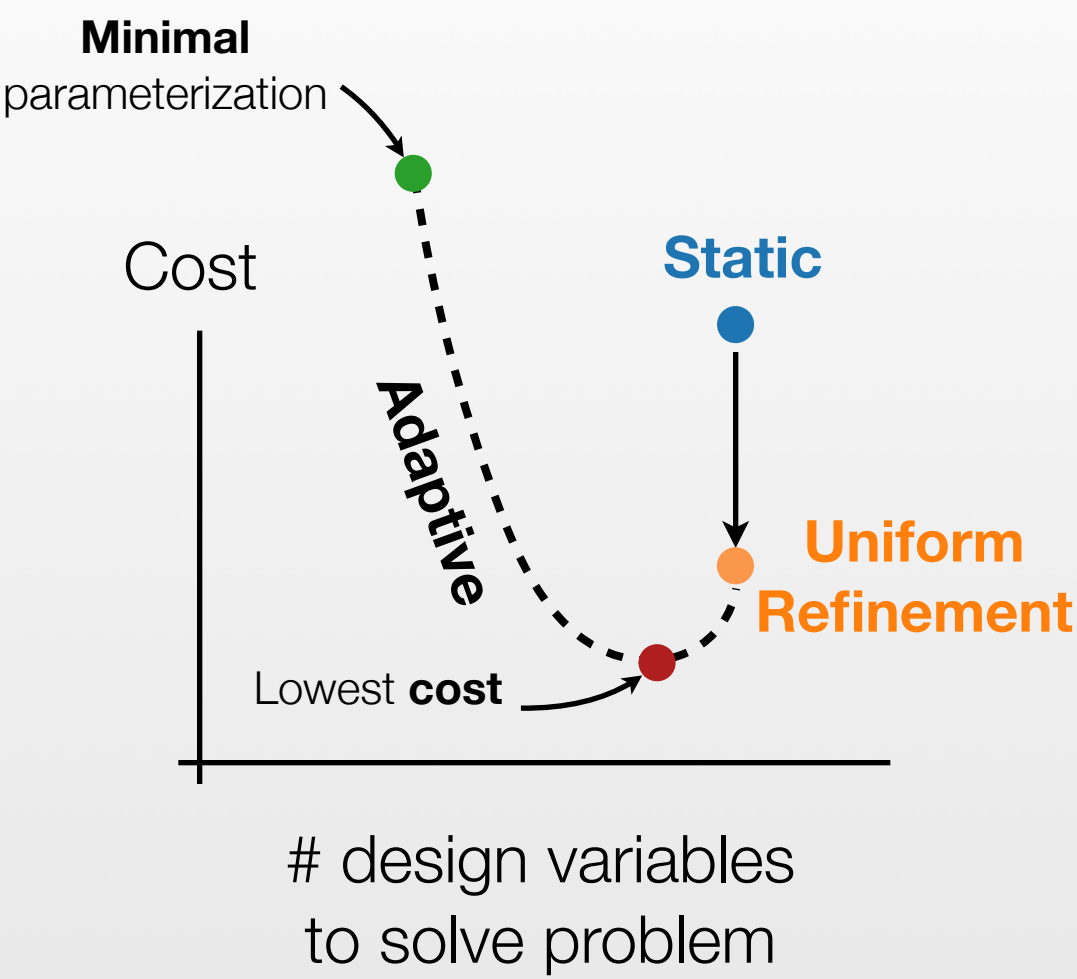
Progressive parameterization

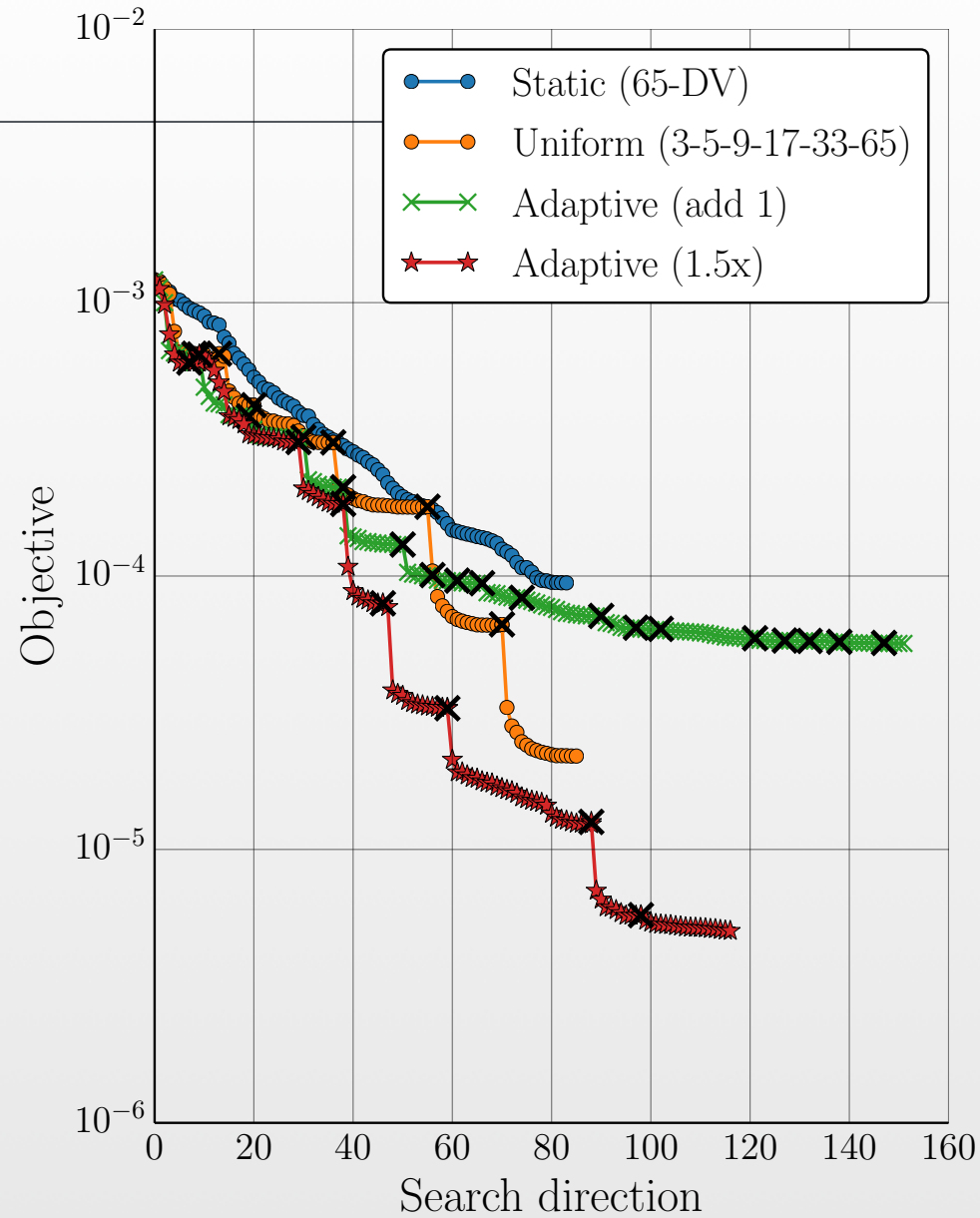
† (2015) **Anderson**, *Nemec, Aftosmis. "Aerodynamic Shape Optimization Benchmarks with Error Control and Automatic Parameterization." AIAA 2015-1719*

How fast is it?



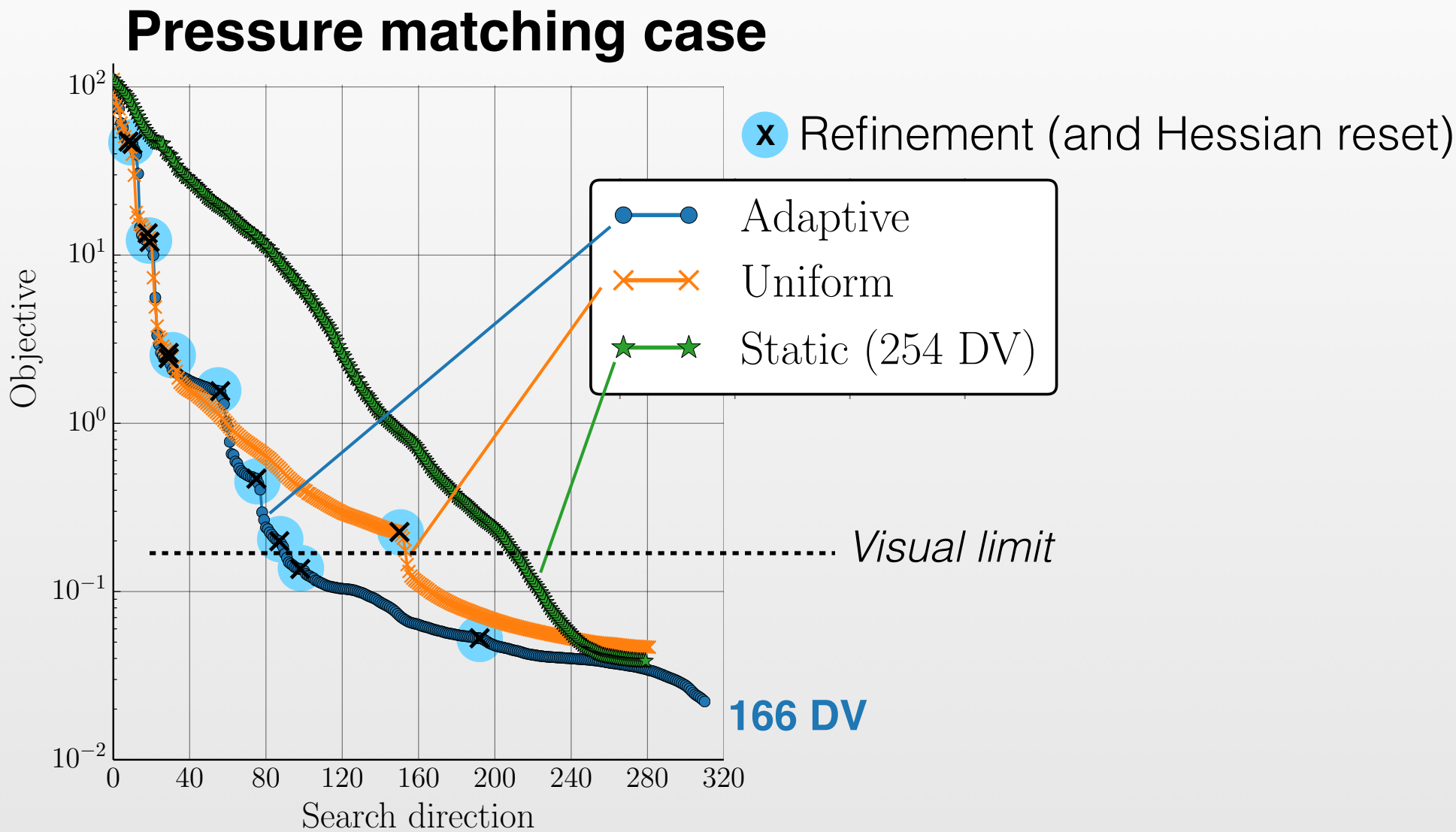
How fast is it?





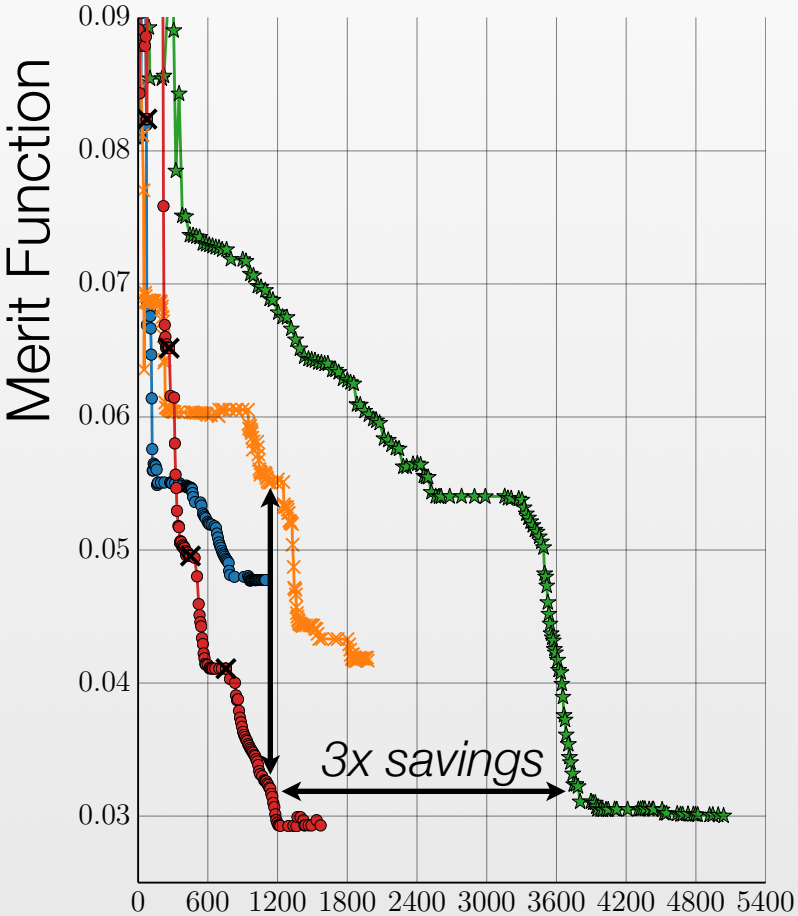
Supersonic low-boom case

Acceleration



How fast is it?

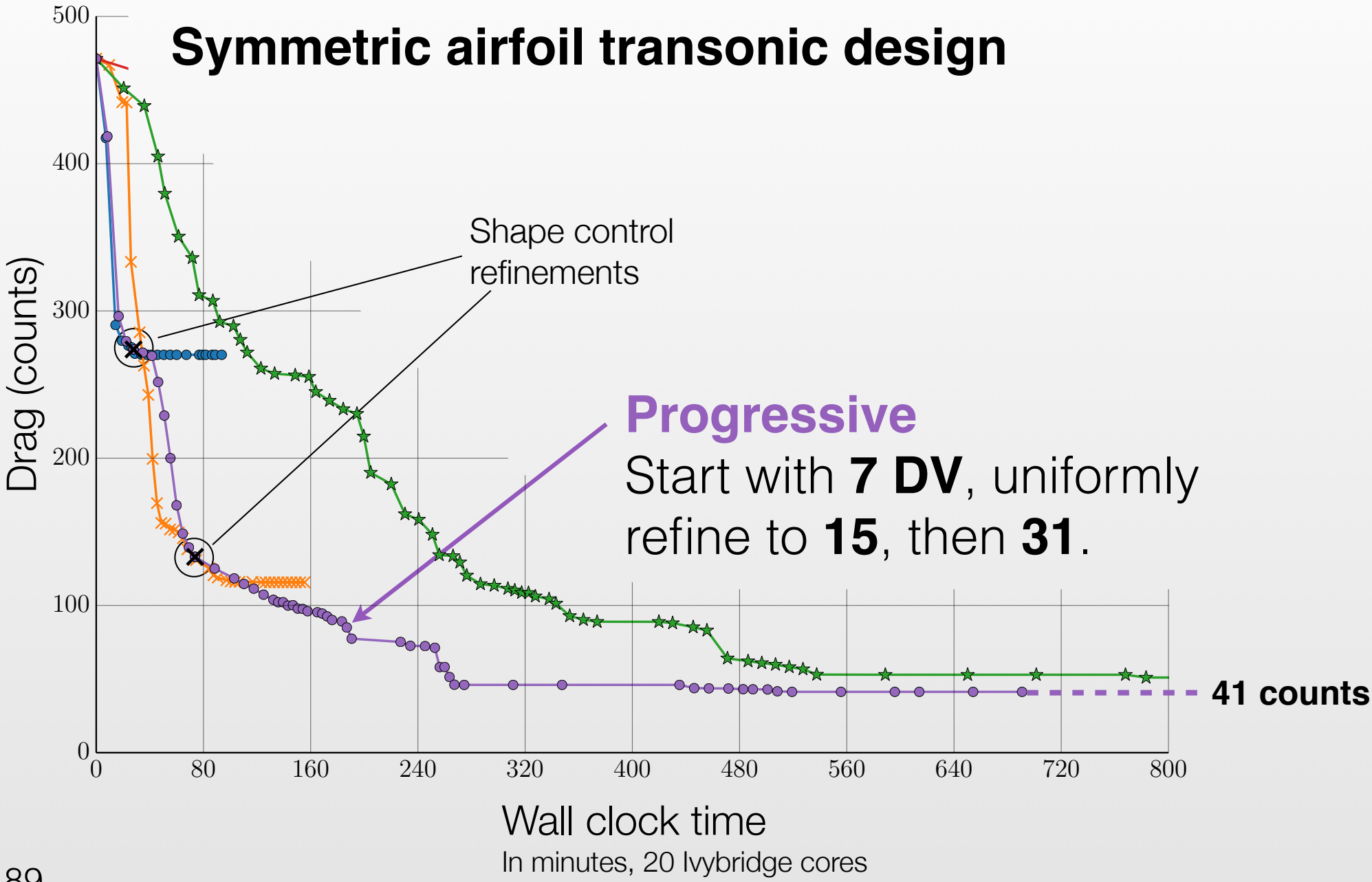
Transonic multipoint design



Wall clock time

In minutes, plotted at major search iterations, on 20 Ivybridge cores

Impact of Parameterization



Why does it work?

Factors contributing to acceleration:

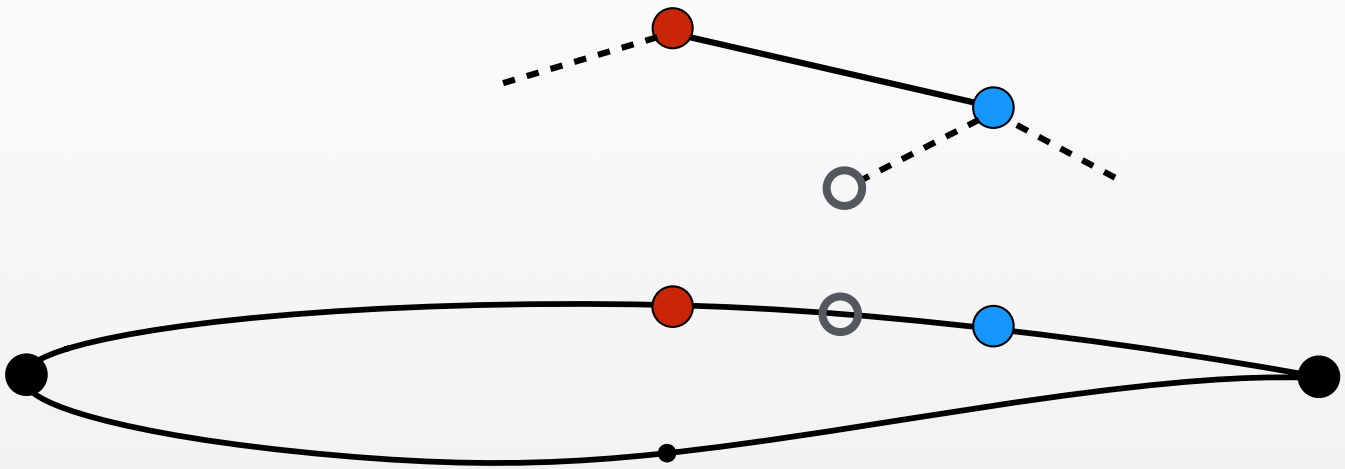
- Early on there are few design variables:
  - Accelerates **BFGS rate of improvement** w.r.t search direction.
  - Reduces # of shape sensitivities and gradient projections.
- Later, more design variables are added, **preventing optimization from stalling.**

Smoothness

(2009) Chaigne and Desideri

- Optimization is intrinsically ***anti***-smoothing
- Shape parameters precondition the optimization
- Multilevel parameterizations have smoothing properties

Hessian Estimation



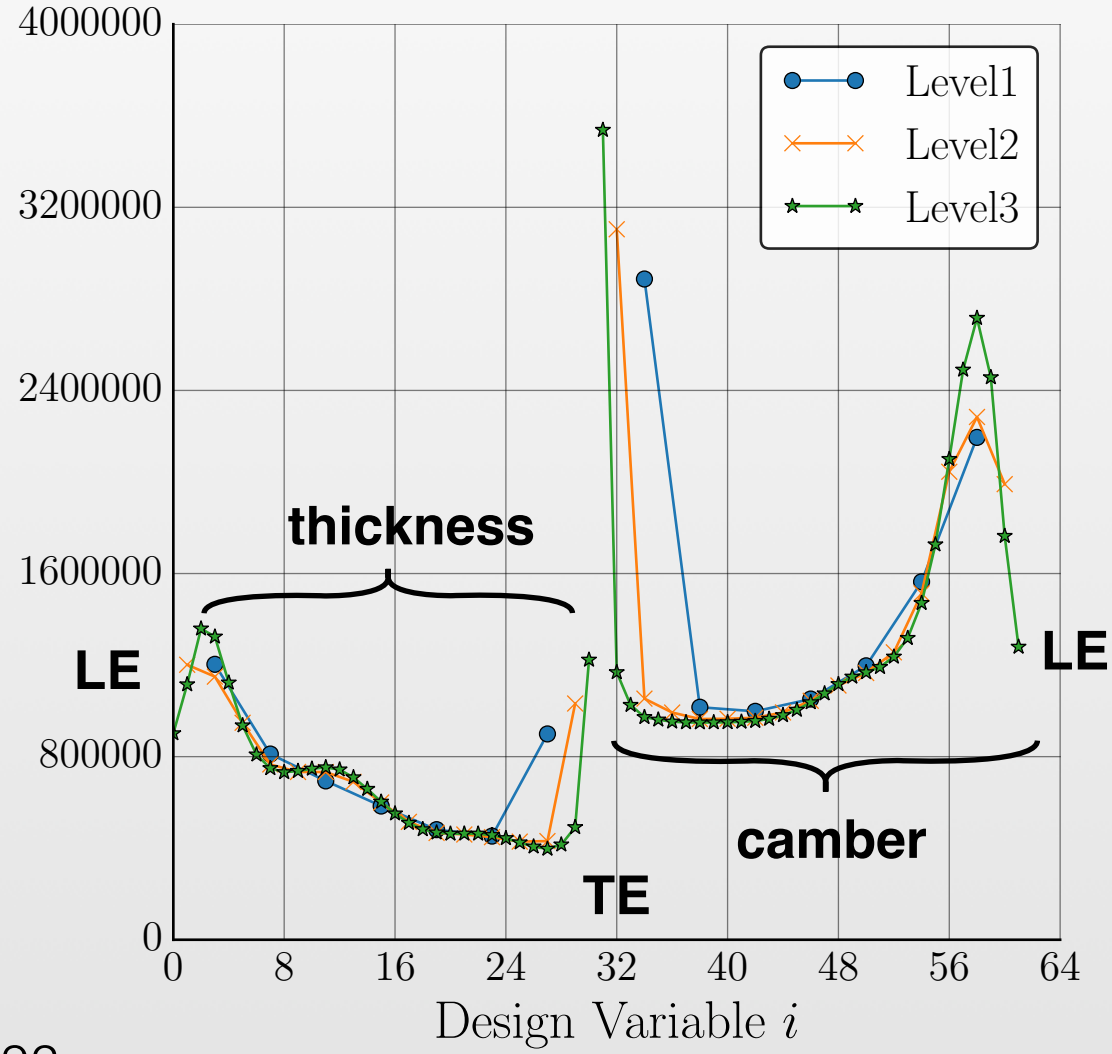
$$B_{\circ}^{k+1} = \underbrace{\frac{1}{2}(B_{\bullet}^{k} + B_{\bullet}^{k})}_{\text{Linear interpolation of Hessian approximation from previous search space}}\mathcal{M}$$

New Hessian estimate

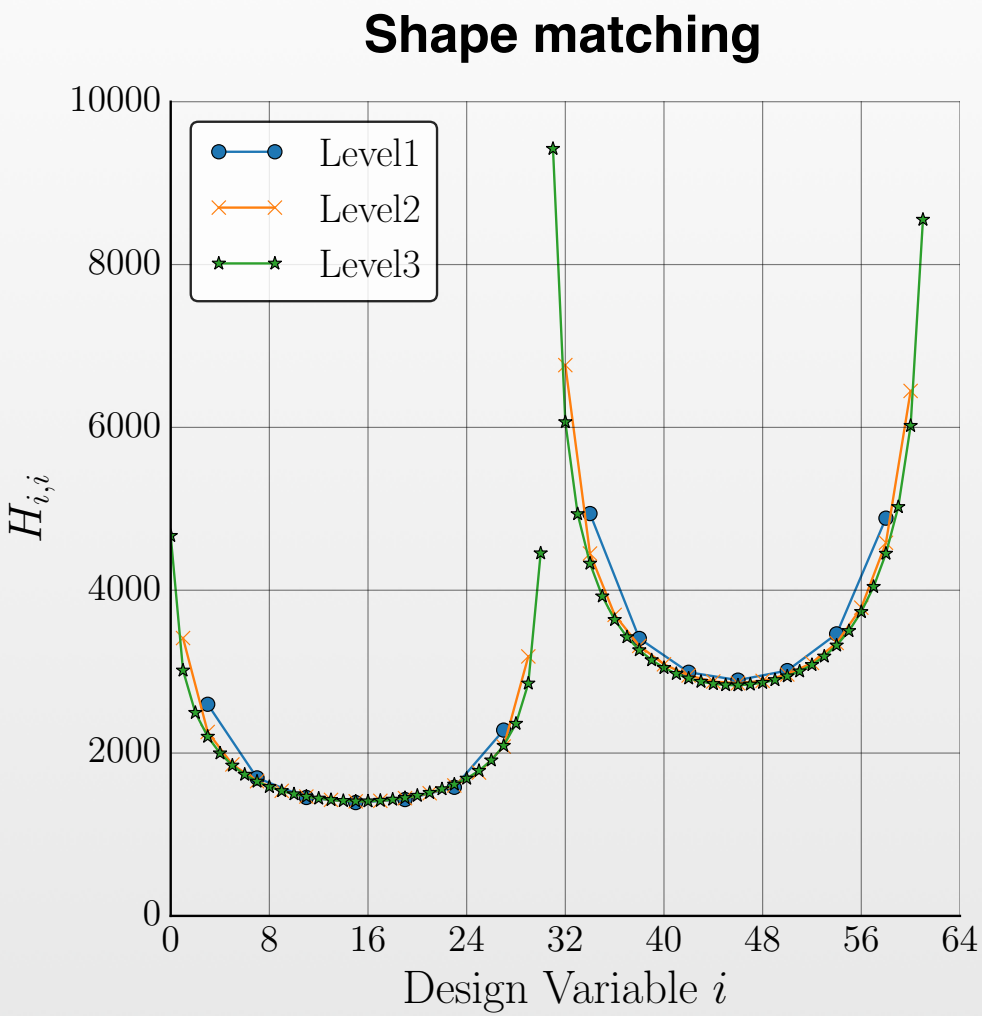
Scale dependence
(see dissertation)

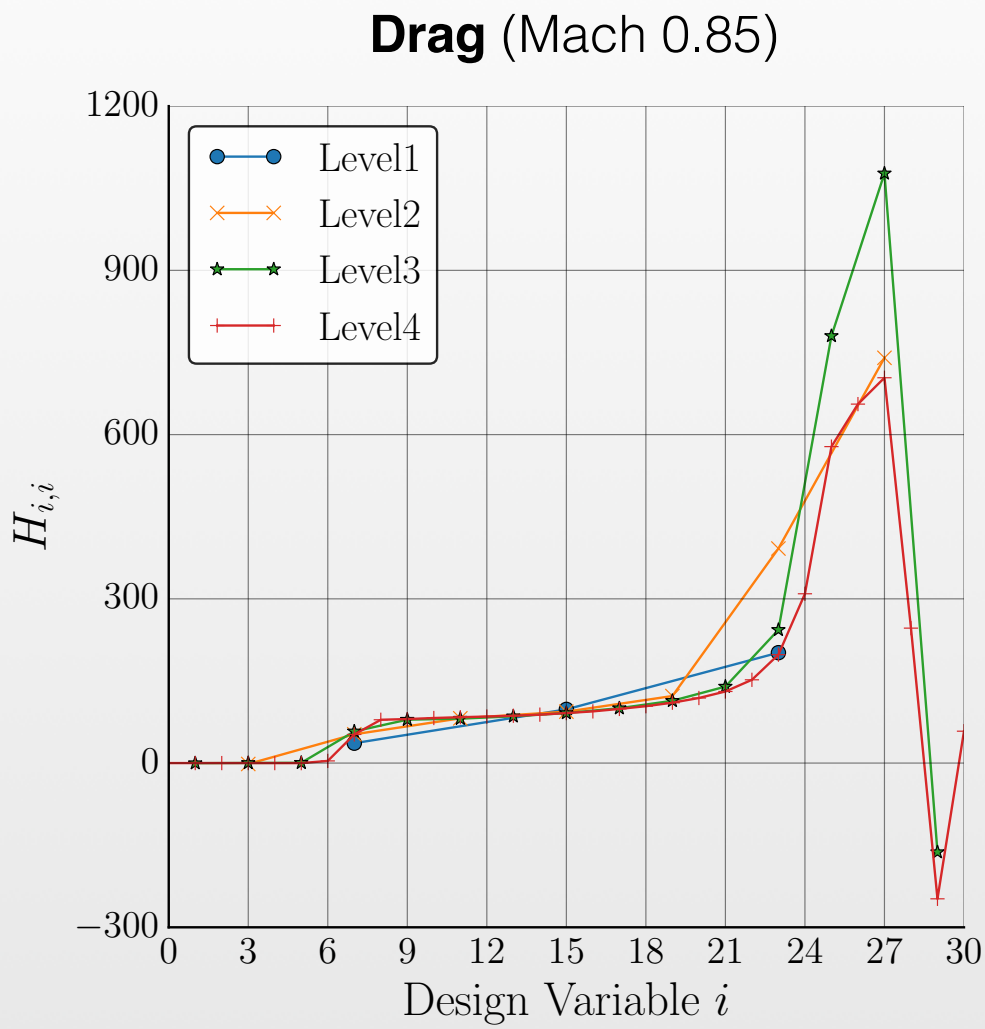
Hessian Estimation — Verification

Hessian diagonal
with scale transformation
for airfoil pressure-matching functional



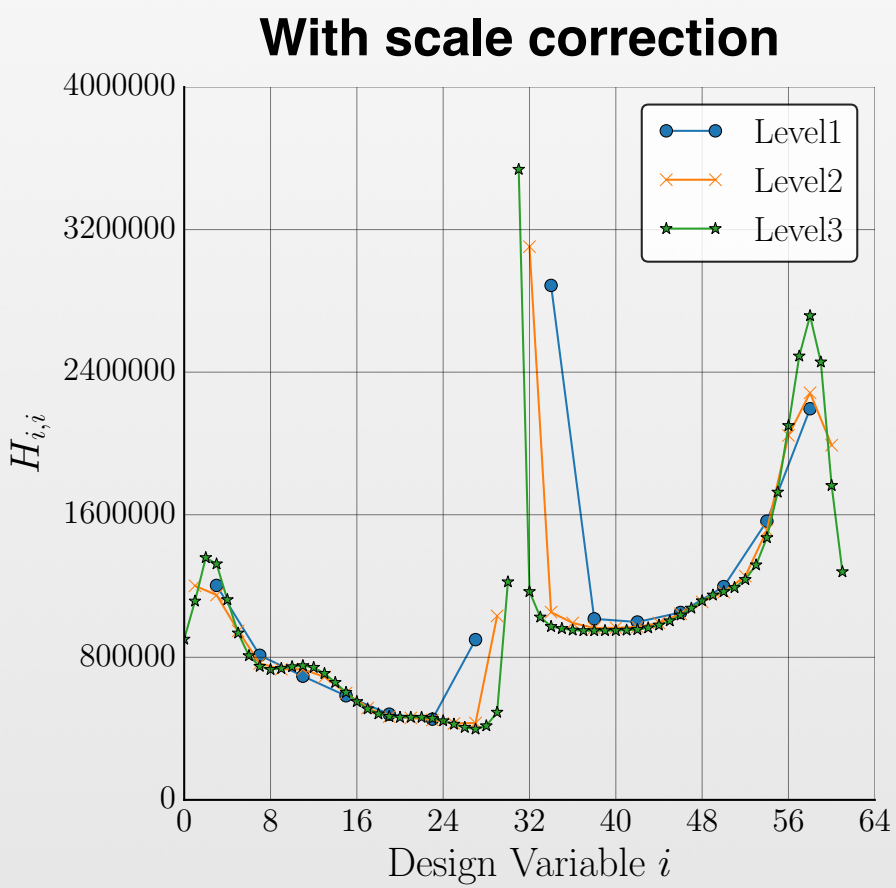
Hessian Estimation — Verification

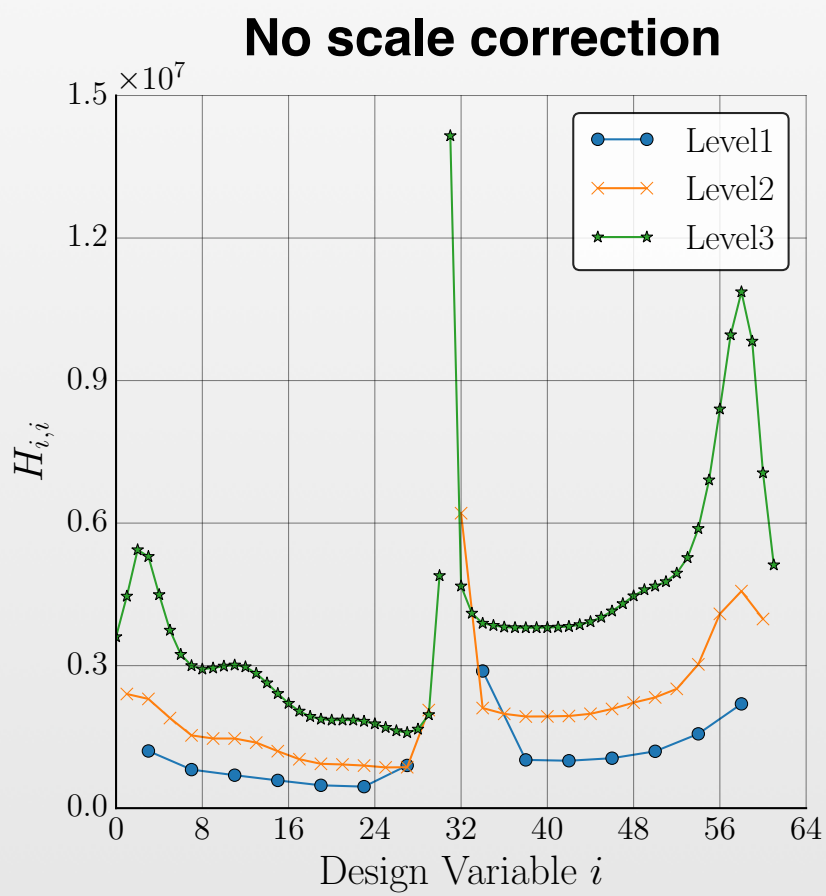




Hessian prolongation

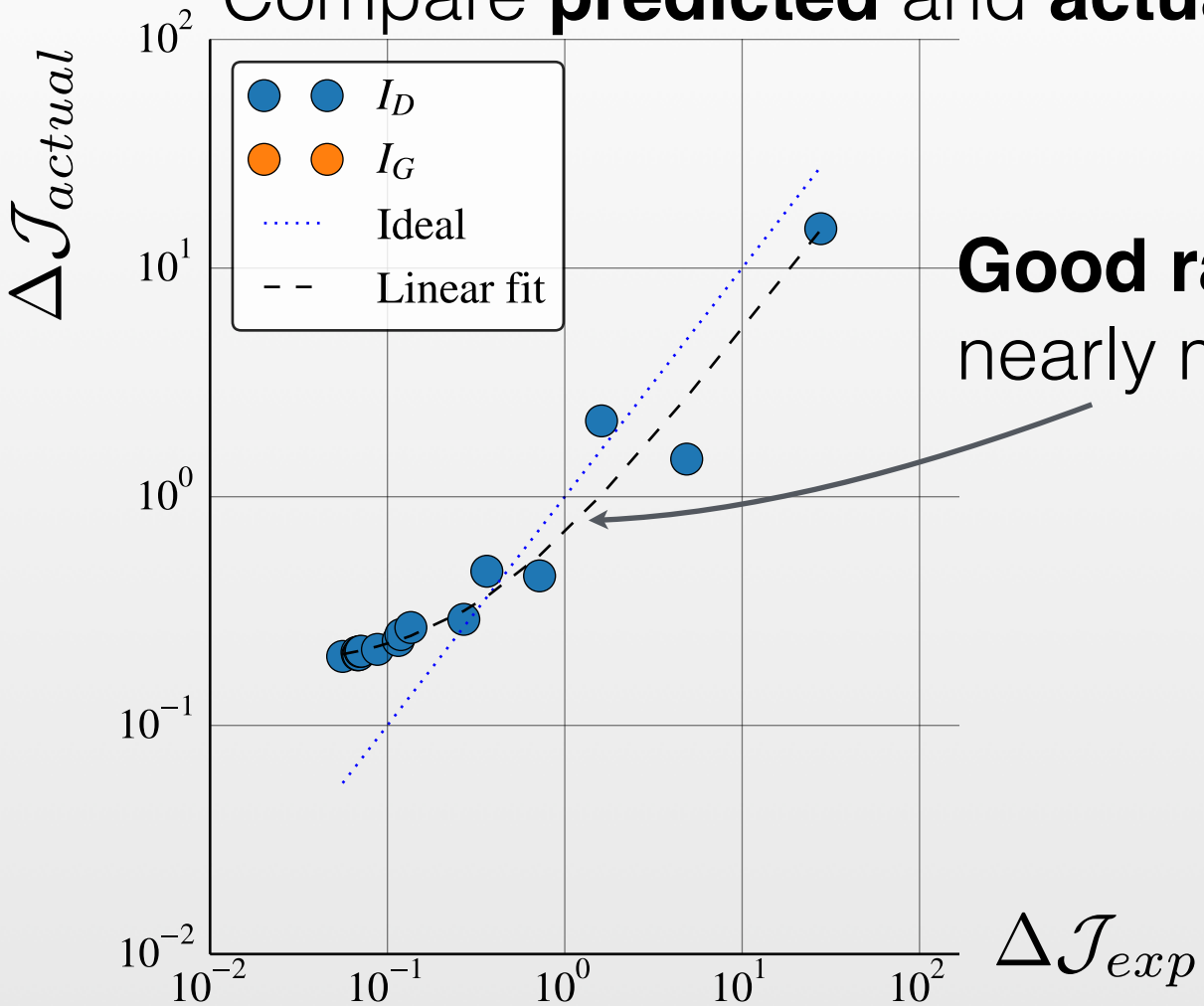
Pressure-matching Hessian diagonal,
across various resolutions





Indicator Validation — Aerodynamic

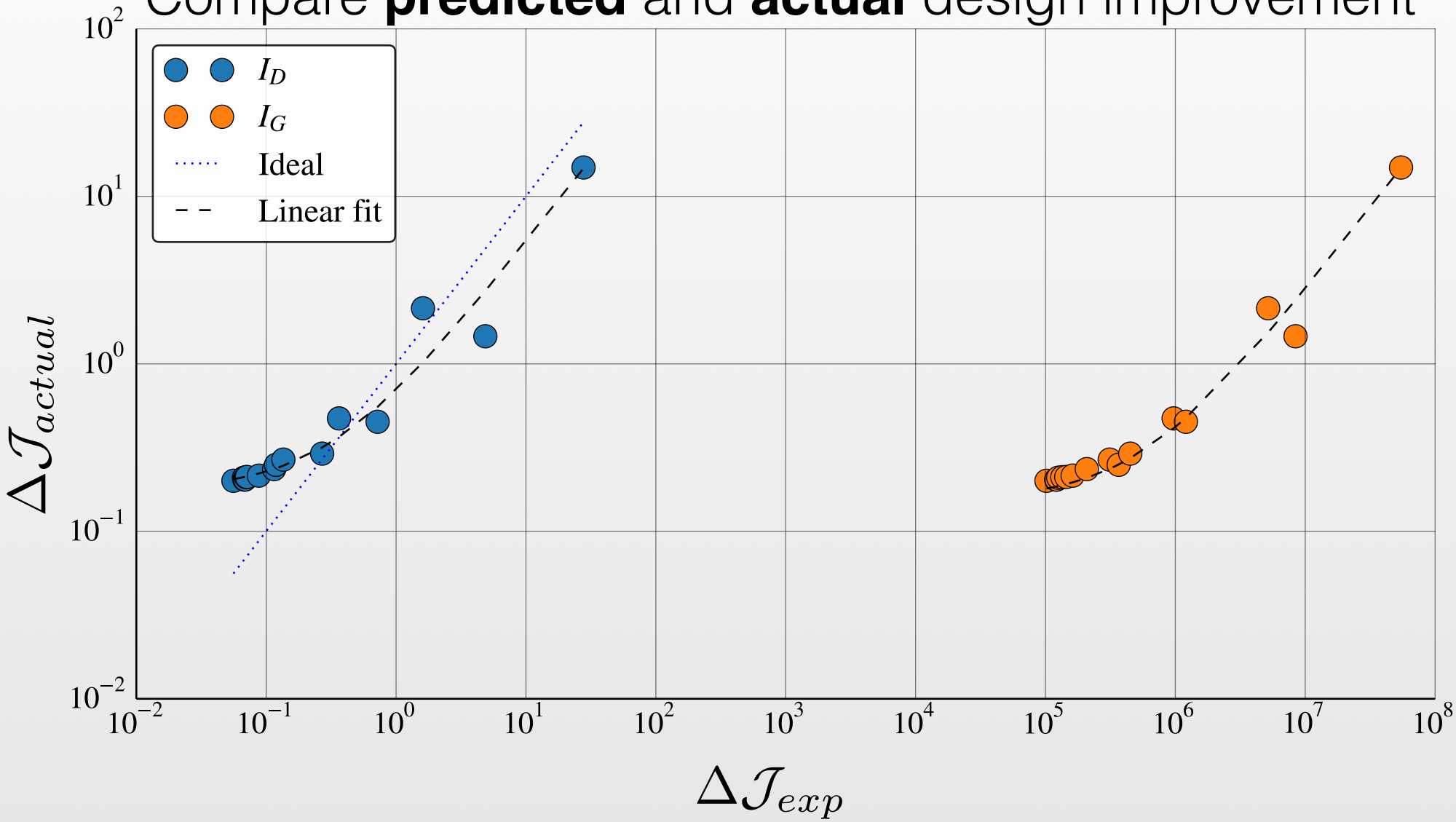
Compare **predicted** and **actual** design improvement



Good ranking —
nearly monotonic

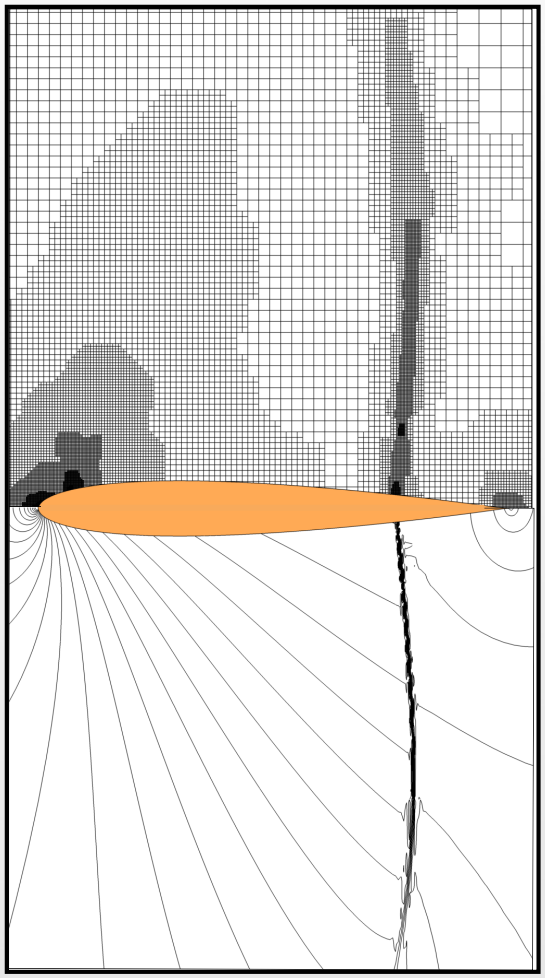
Indicator Verification

Compare **predicted** and **actual** design improvement



Discretization Error Control

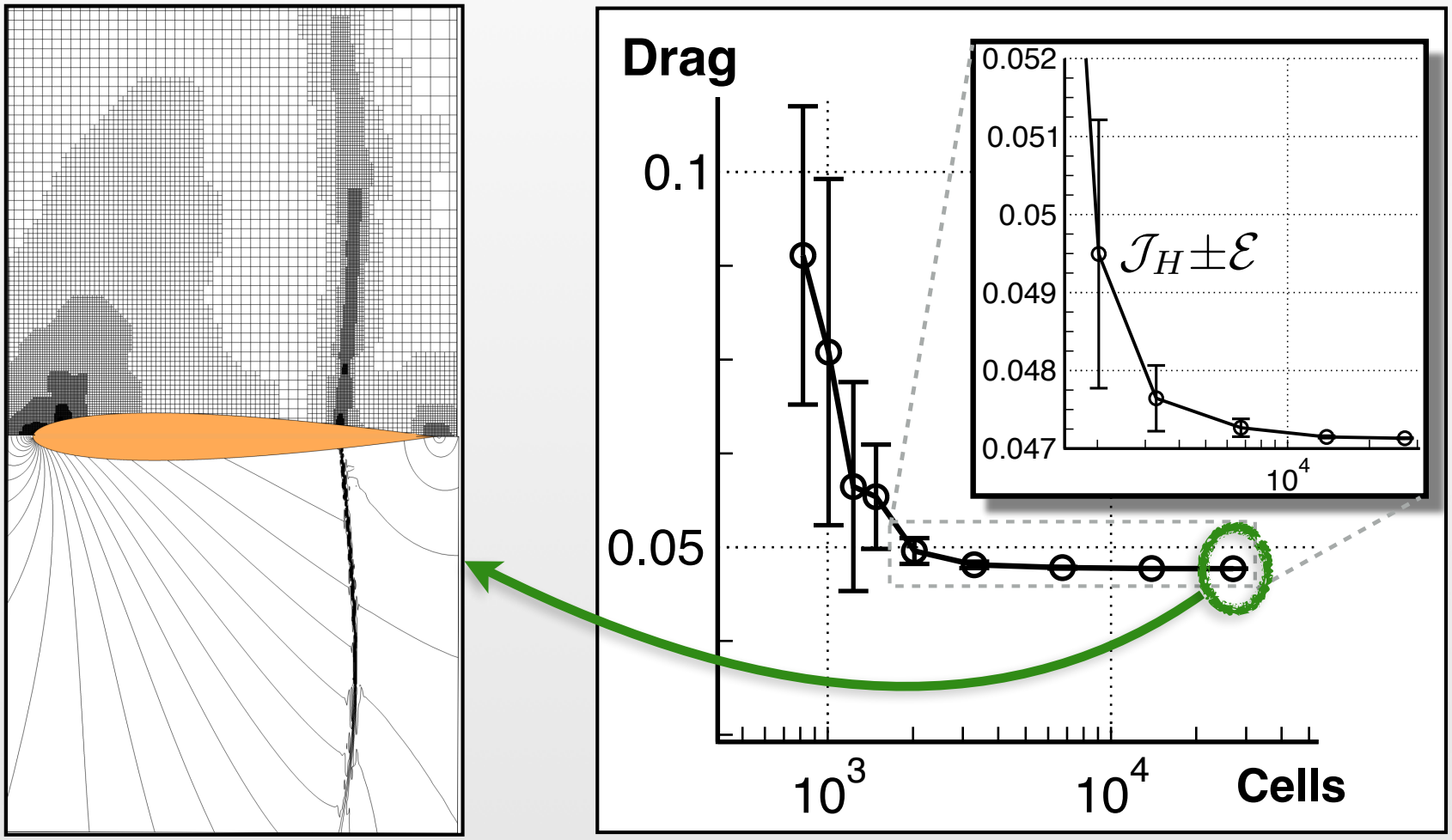
Mesh is adapted to **accurately compute objective**



(2014) Nemec and Aftosmis, ***"Toward Automatic Verification of Goal-Oriented Flow Simulations."*** *NASA TM-2014-218386*

Discretization Error Control

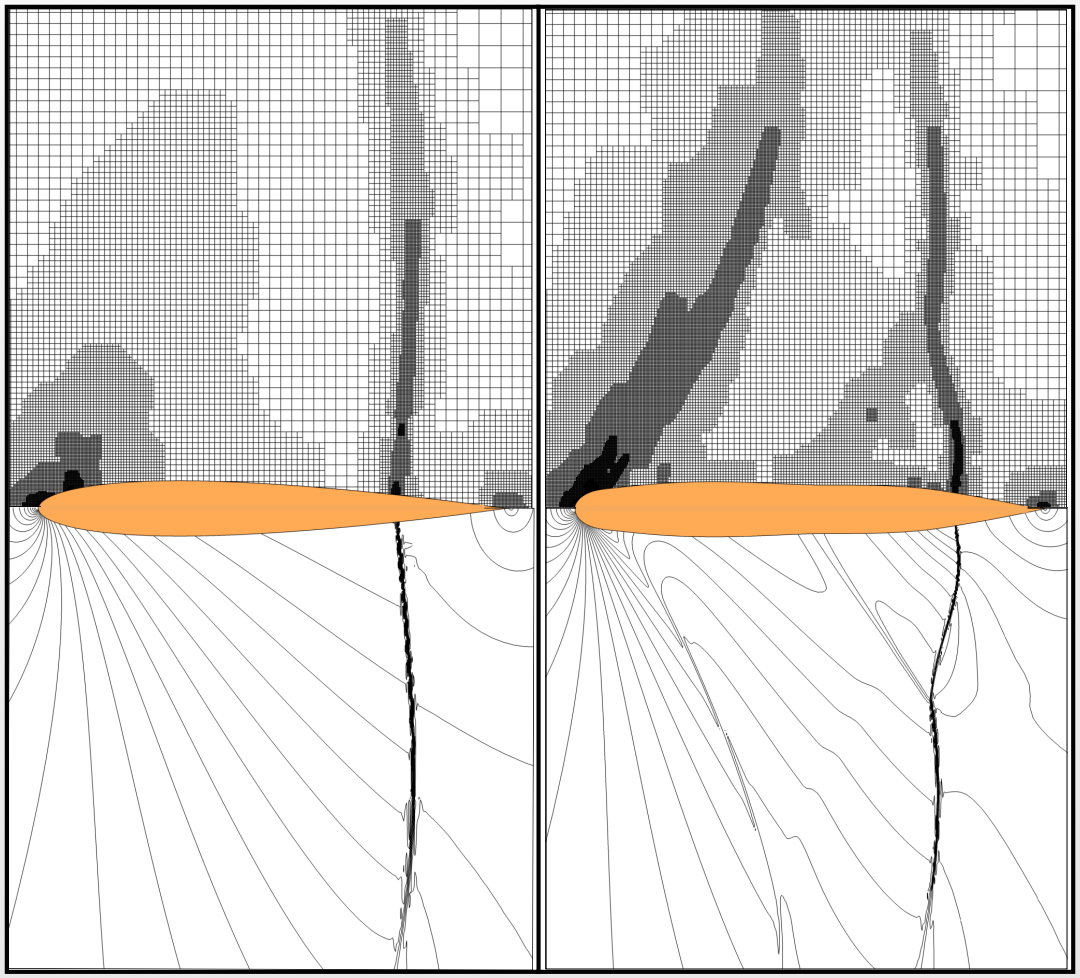
Error tolerance is set low enough to ensure reliable design improvement.



(2013) Nemec and Aftosmis, ***"Output Error Estimates and Mesh Refinement in Aerodynamic Shape Optimization."*** *AIAA 2013-0865*

Discretization Error Control

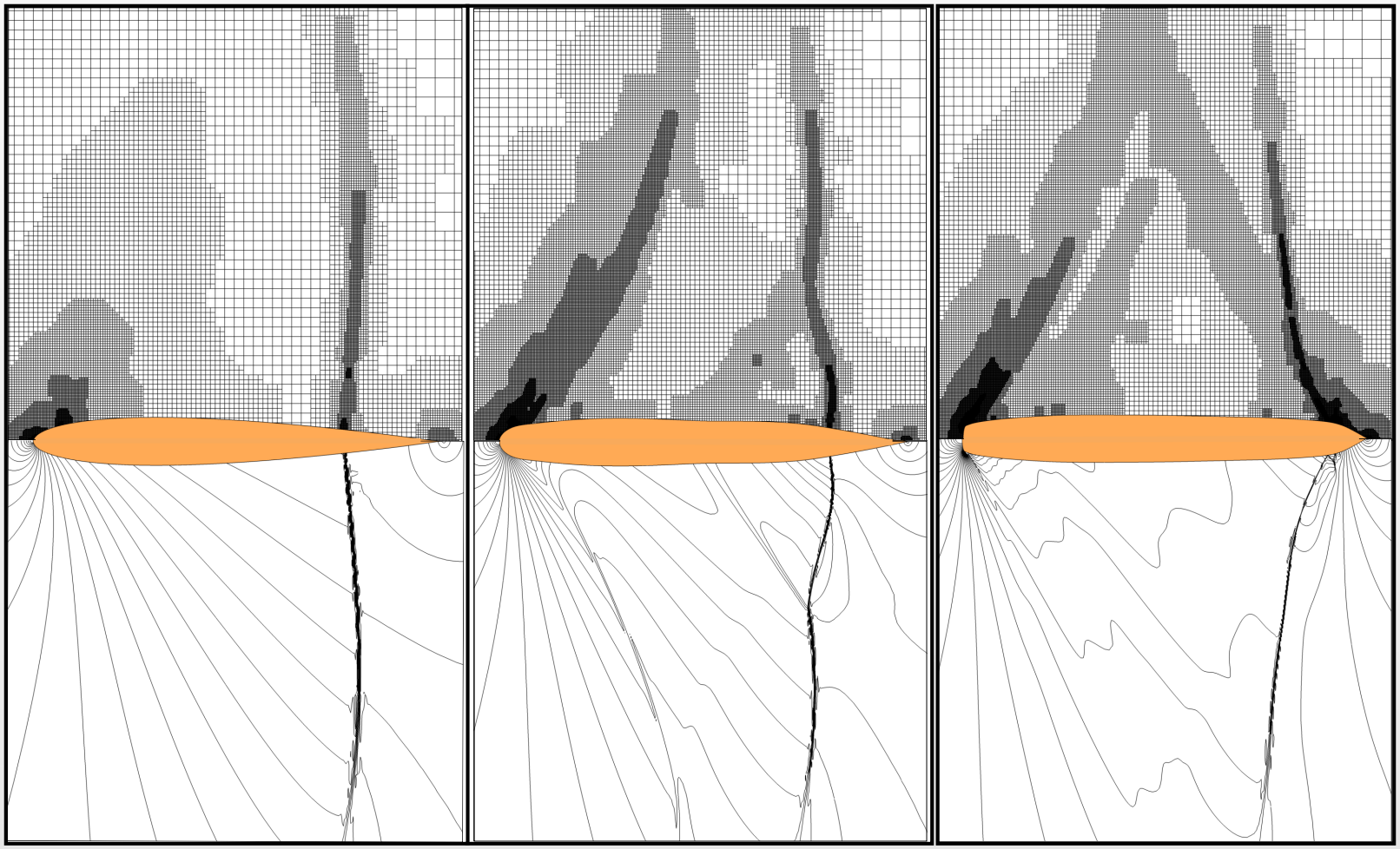
Mesh can be adapted **for each design iteration**.



(2015) Anderson, Nemec, Aftosmis. *"Aerodynamic Shape Optimization Benchmarks with Error Control and Automatic Parameterization." AIAA 2015-1719*

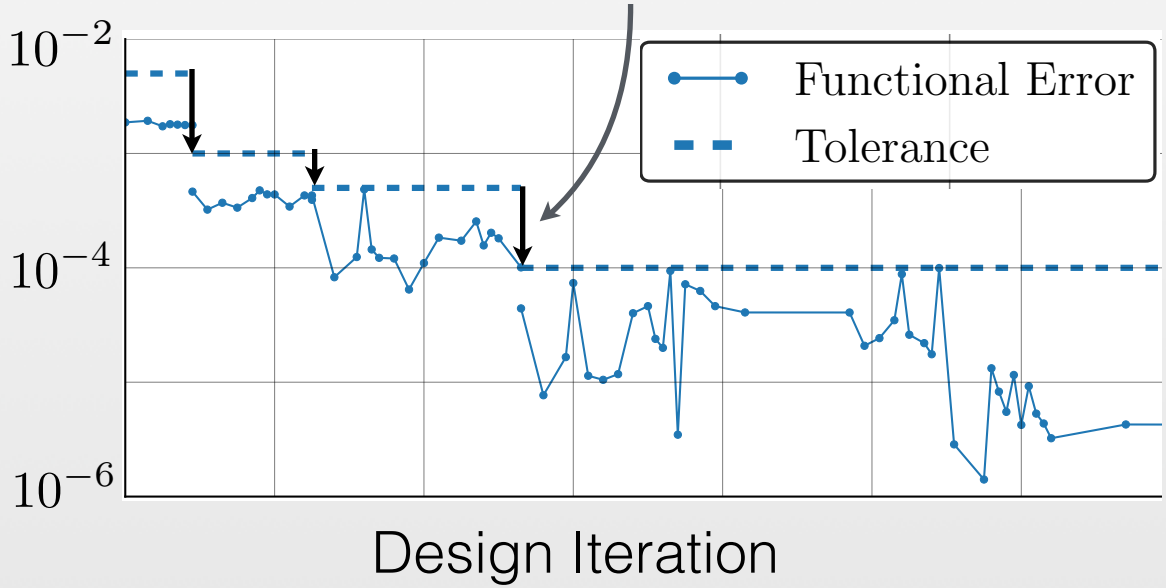
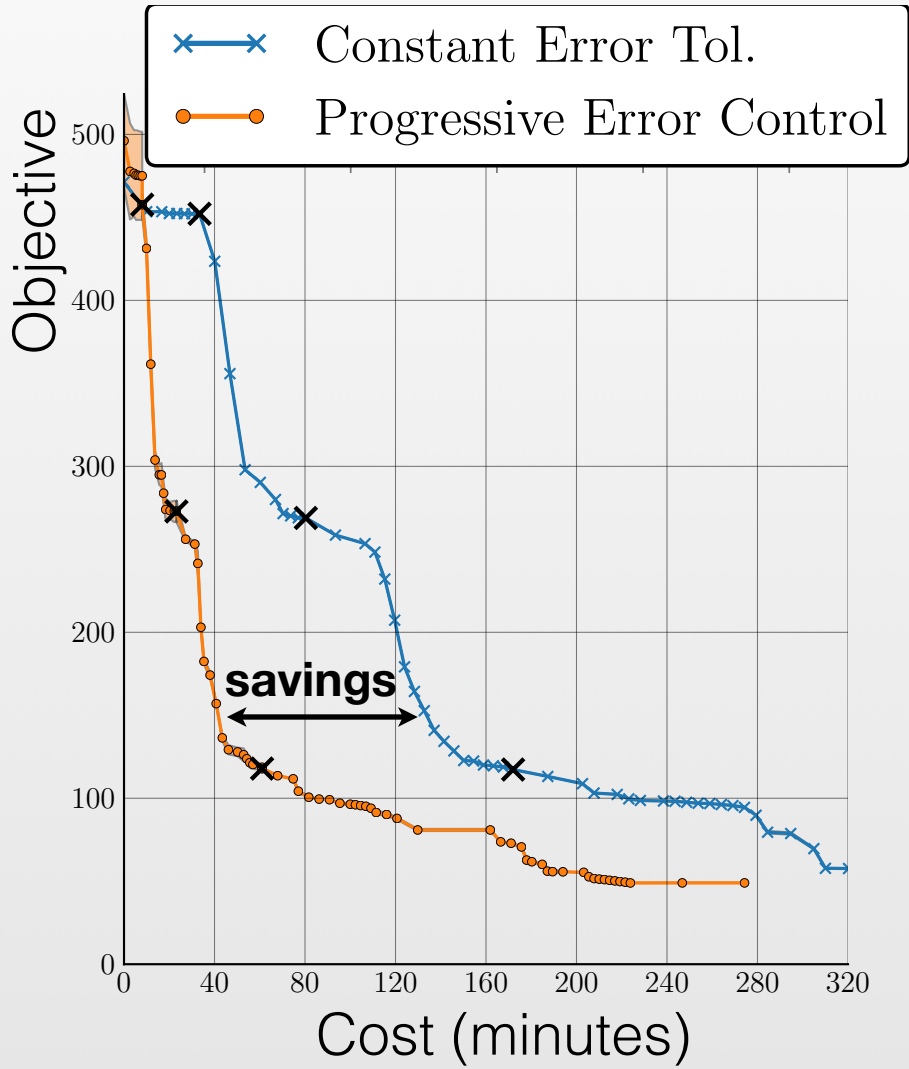
Discretization Error Control

Mesh can be adapted **for each design iteration**.



(2015) Anderson, Nemec, Aftosmis. *"Aerodynamic Shape Optimization Benchmarks with Error Control and Automatic Parameterization." AIAA 2015-1719*

Error Control Scheduling

Use **progressive** error targets to accelerate the optimization.

Coarser meshes used early on, then tolerance is tightened.