

# CAE-based Robust Design optimization

## Challenges and solutions

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President Dynardo GmbH

## Challenges in Virtual Prototyping

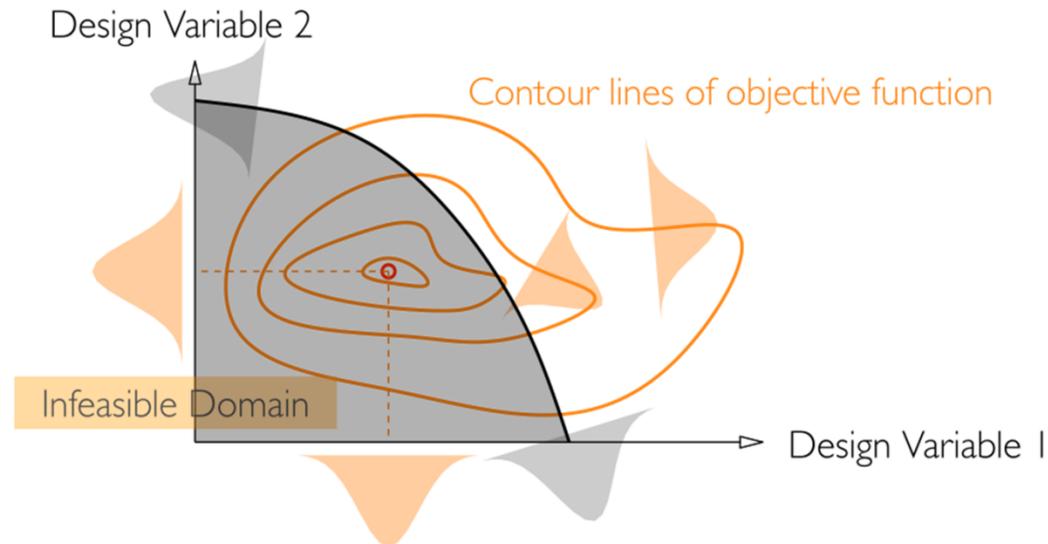
- Virtual prototyping is necessary for cost efficiency
- Test cycles are reduced and placed late in the product development
- CAE-based optimization and CAE-based robustness evaluation becomes more and more important in virtual prototyping
  - Optimization is introduced into virtual prototyping
  - Robustness evaluation (reliability analysis) is the key methodology for safe, reliable and robust products
  - The combination of optimizations and robustness evaluation will lead to robust design optimization strategies



## Robust Design Optimization

Robust Design Optimization (RDO) optimize the design performance with consideration of scatter of design (optimization) variables as well as other tolerances or uncertainties.

As a consequence of uncertainties the location of the optima as well as the contour lines of constraints scatters.



To **proof** Robust Designs safety distances are quantified with variance or probability measurements **using stochastic analysis**.

## Real world CAE-based Robust Design Optimization applications

- CAE-based RDO needs significant compute resources (running 1 Mio. Design realizations is too prohibitive to be done)

- We need to deal with failed designs (design creation, meshing or simulation fails)

- We have much more parameter than just a hand full (at least in the uncertainty domain)

- Problems are coupled over multiple physical domains, are non-linear, high dimensional

- Appropriate result extraction not known a priori (unique result values, decoupling effects, reduce noise, scale/transform responses)

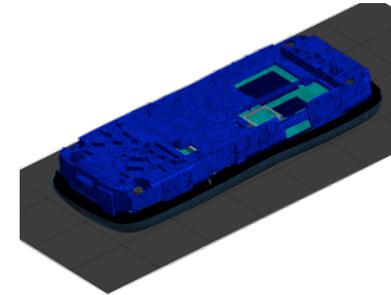
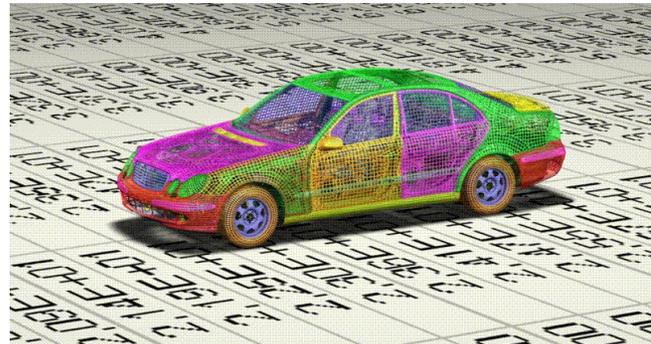
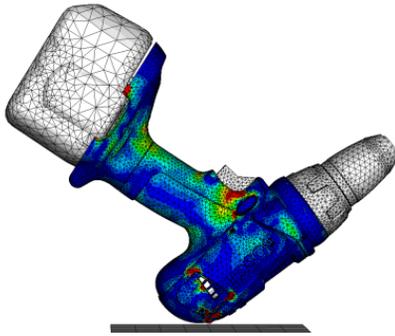
Therefore an iterative approach “understanding your design”, “improve your design” performance and “proof design robustness” will be the method of choice.

## What customer wants do to

### What customer expect

- **Understand your design** using Sensitivity Analysis
  - Easy and safe to use workflow for engineers and designers to get a maximum understanding for the relations of parameterized properties with a minimum number of FE-calculations
- **Improve your Design** using Optimization Analysis
  - Easy and safe to use workflow transfer learning's and suggest optimization strategy
- **Proof Robustness of your Designs** using Stochastic Analysis
  - Easy and safe to use workflow for 2-,3- or even a 6-sigma design

## Premium Consultancy and Software Company for CAE-based Robustness Evaluation, Reliability Analysis and Robust Design Optimization using Stochastic Analysis



Dynardo is the consulting company which successfully introduced stochastic analysis into complex CAE-based virtual product development processes.

Recently, it is applied in the power generation industry, automotive industry and high-level consumer goods industry

Founded: 2001 (Will, Bucher, CADFEM International)

More than 50 employees, offices at Weimar and Vienna

Leading technology companies Daimler, Bosch, Shell, Nokia, Siemens are supported by us



## Software Development



Dynardo is your engineering specialist for CAE-based sensitivity analysis, optimization, robustness evaluation and robust design optimization.

## CAE-Consulting

Our expertise:

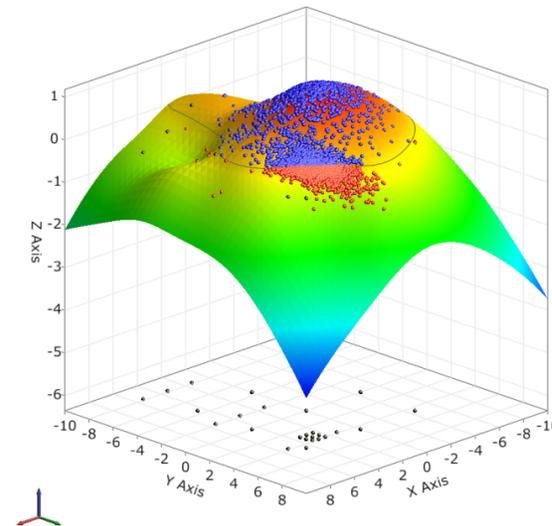
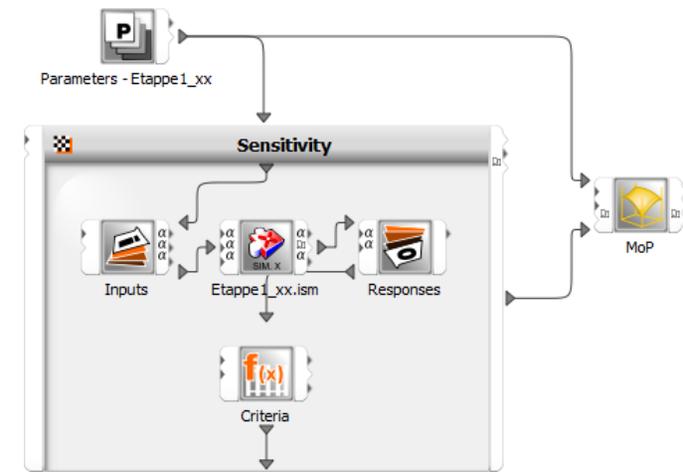
- Mechanical engineering
- Civil engineering & Geomechanics
- Automotive industry
- Consumer goods industry
- Power generation

# OZEN ENGINEERING AT A GLANCE

- ANSYS Channel Partner and Distributor in N. California
- Focused on Mechanical, Fluids and Low Frequency Electromagnetics Products
- Over 25 years expertise in FEA, CFD and Engineering Consulting Services
- Superb Technical Support – “Open Door Policy”
- World Class ANSYS Training and Support
- Distributor of ANSYS Complimentary Solutions
  - Dynardo/optiSLang – Advanced Robust Optimization Optimization
  - PlanetsX - Injection Molding
  - WAON – Advanced Acoustics

## Excellence of optiSLang – the general purpose tool for variation analysis

- optiSLang is an algorithmic toolbox for
  - sensitivity analysis,
  - optimization,
  - robustness evaluation,
  - reliability analysis
  - robust design optimization (RDO)
- functionality of stochastic analysis to run real world industrial applications
- **easy and safe** to use
  - Powerful automation and integration environment
  - predefined workflows
  - algorithmic wizards
  - robust default settings

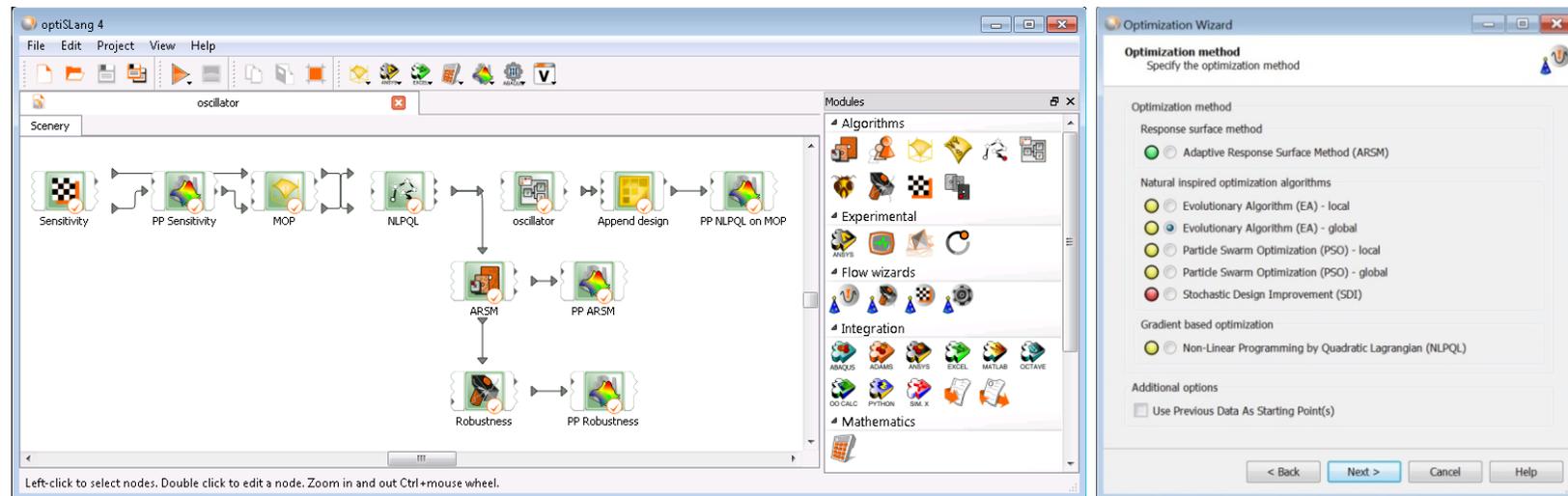


## optiSLang v4

“Robust Design Optimization - easy and flexible to use”



- automated generation of an interactive process chain using the **CAE-based modules of sensitivity analysis, optimization and robustness evaluation**
- minimum of user input required
- automated best practice management for algorithmic defaults
- flexible process integration and post-processing defaults



## optiSLang Integrations

**SIMULATION X<sup>®</sup>** **Pro|ENGINEER<sup>®</sup>**  
Powered by ITI

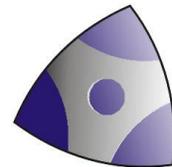
**ABAQUS**



**madymo<sup>®</sup>**



**MSC Software**  
**Adams**



**Midas**  
**Edyson**



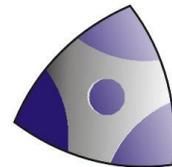
**LS-DYNA**

**ANSYS<sup>®</sup>**

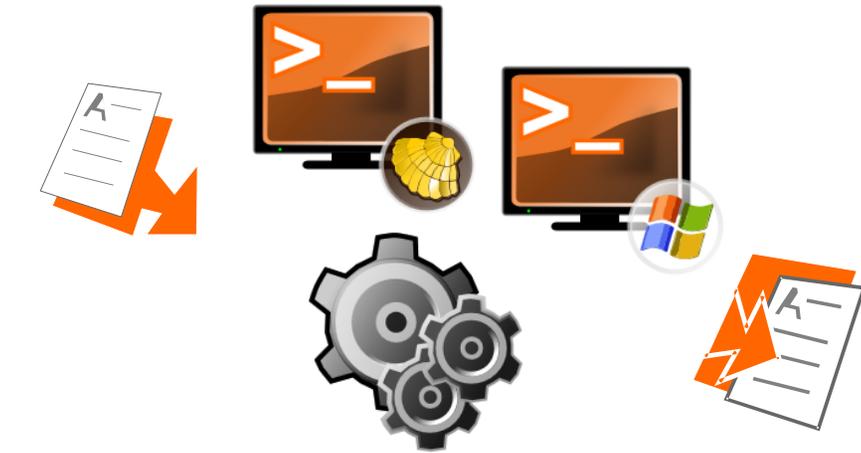


**CONCEPTS NREC**

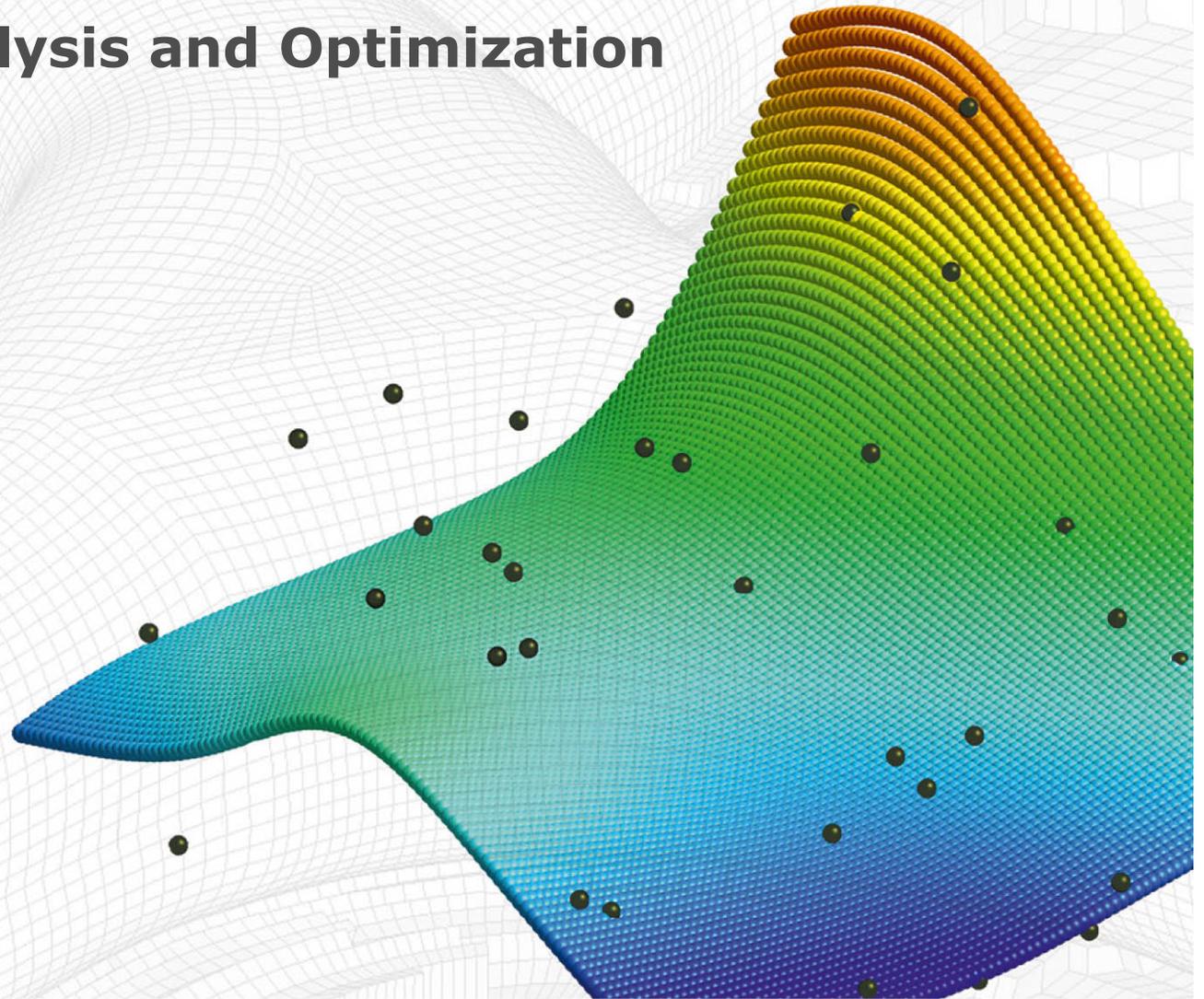
**TurboOpt**

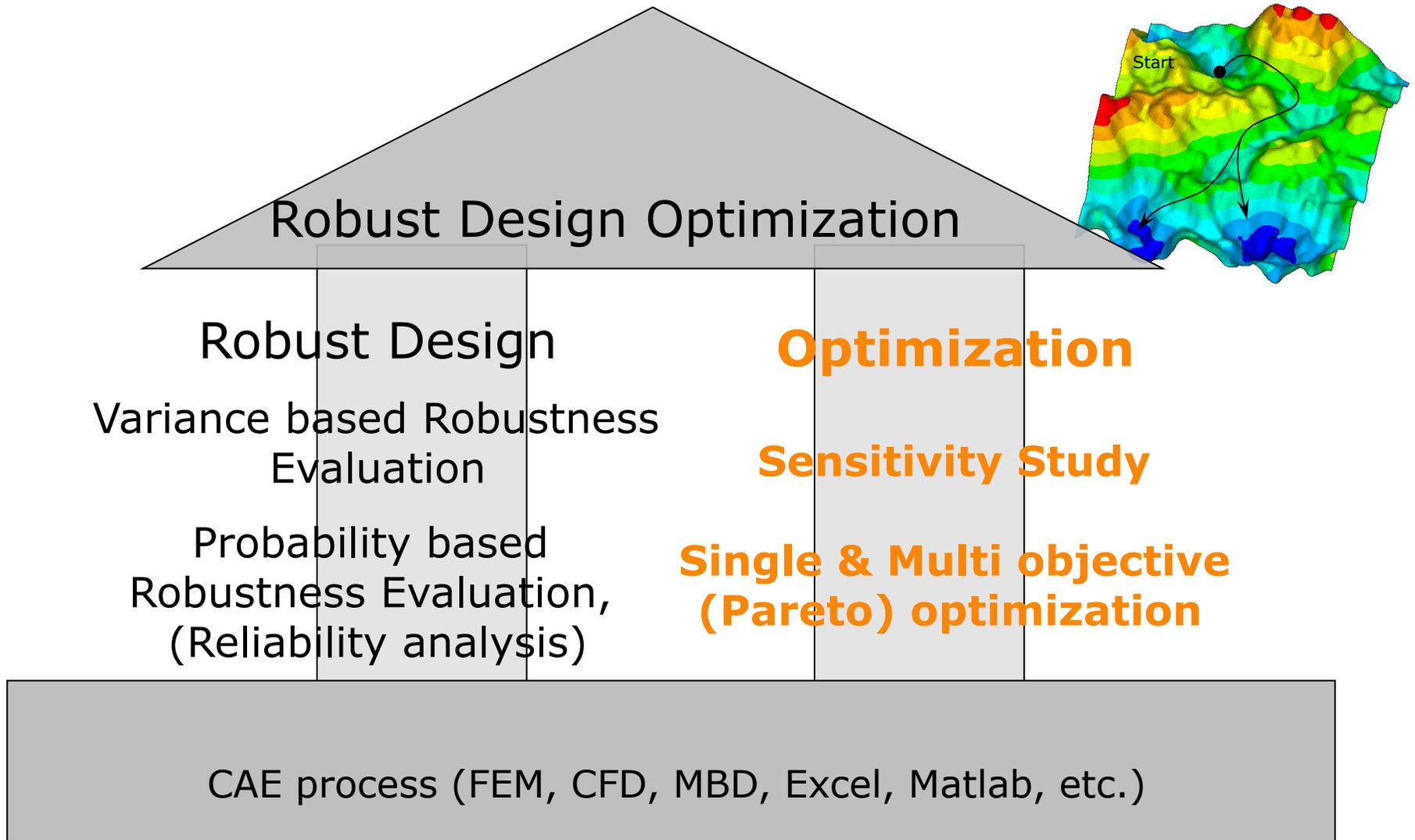


**Mentor Graphics FloEFD<sup>™</sup>**

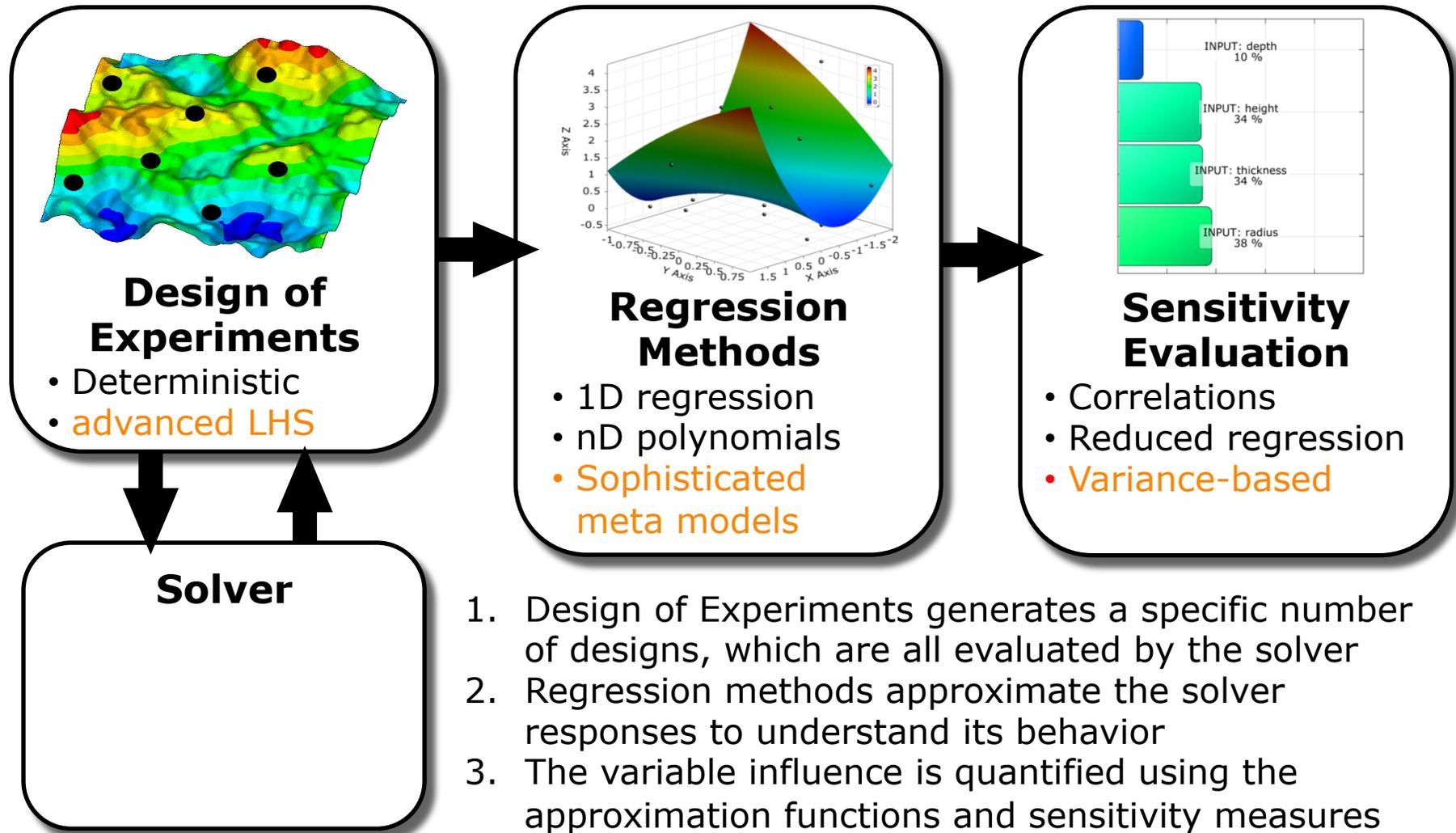


# Sensitivity Analysis and Optimization



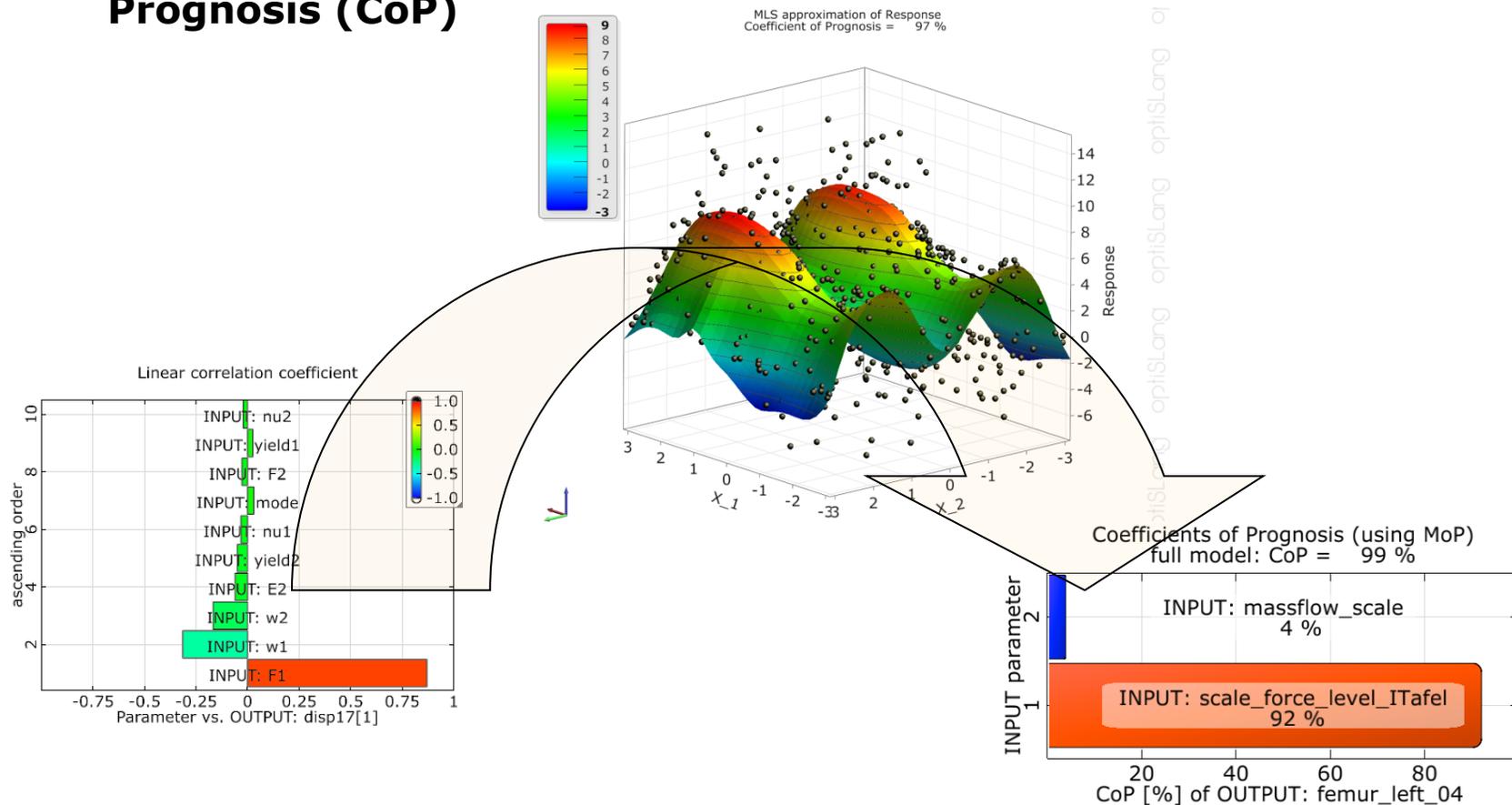


## Flowchart and Methods of Sensitivity Analysis



# Identifying important parameters

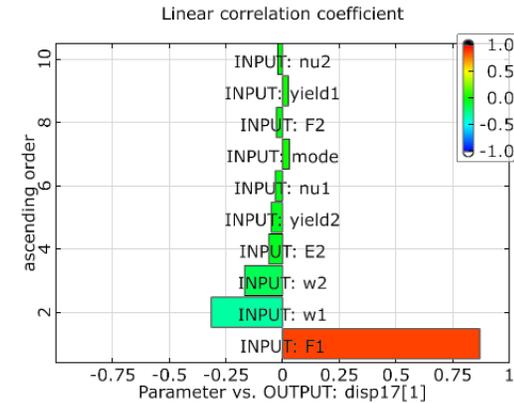
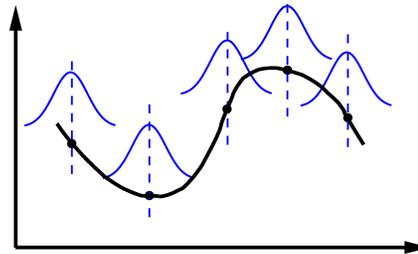
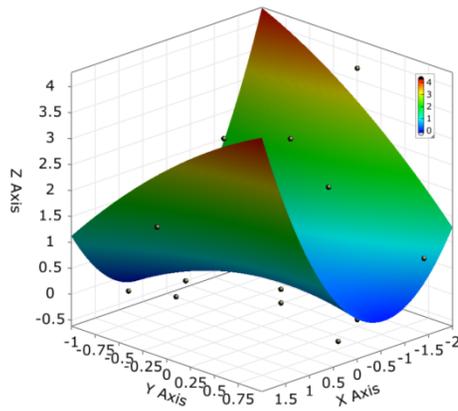
## From tornado chart of linear correlations to the Coefficient of Prognosis (CoP)



# Statistical measurements

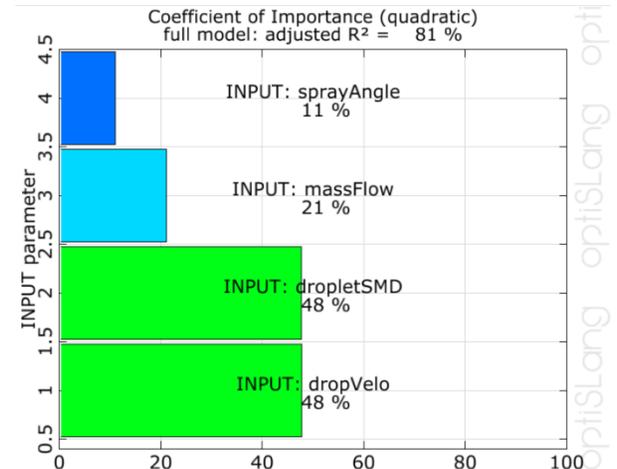
## Correlation Measurements

- Coefficients of pairwise linear/quadratic correlation is the simplest correlation measurement
- Multi-dimensional non-linear correlation can be detected using advanced meta models (Neural networks, Moving least squares,..)



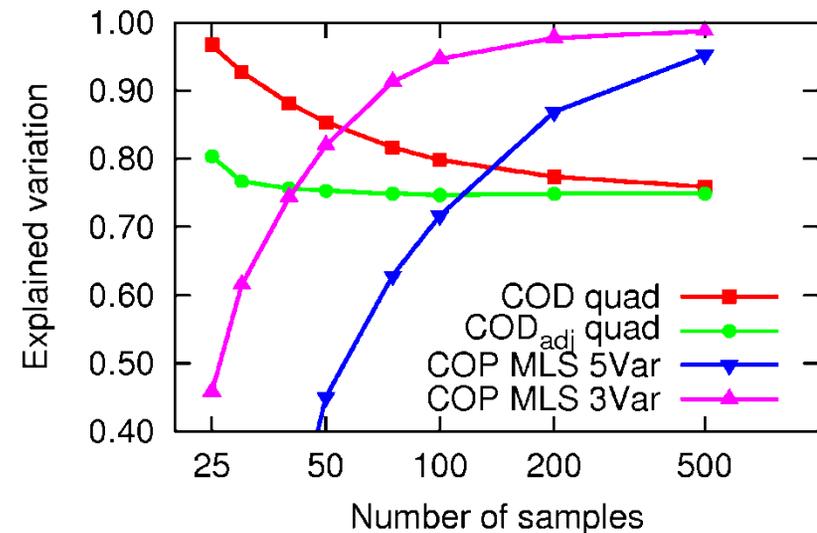
## Goodness of fit Measurements (CoD)

- Goodness of Fit (Coefficient of Determination CoD) summarize correlations on the meta models



## Dynardo's Coefficient of Prognosis (CoP)

- CoD is only based on how good the regression model fits through the sample points, but not on how good the prediction quality is
- Approximation quality is too optimistic for small number of samples
- For interpolation models (MLS, Neural Networks, Radial basis functions,..) with perfect fit, CoD is equal to one
- CoP measures the forecast quality of regression model using an independend test data set

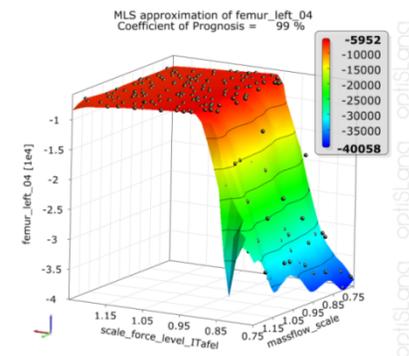


- Prediction quality is better if unimportant variables are removed from the approximation model

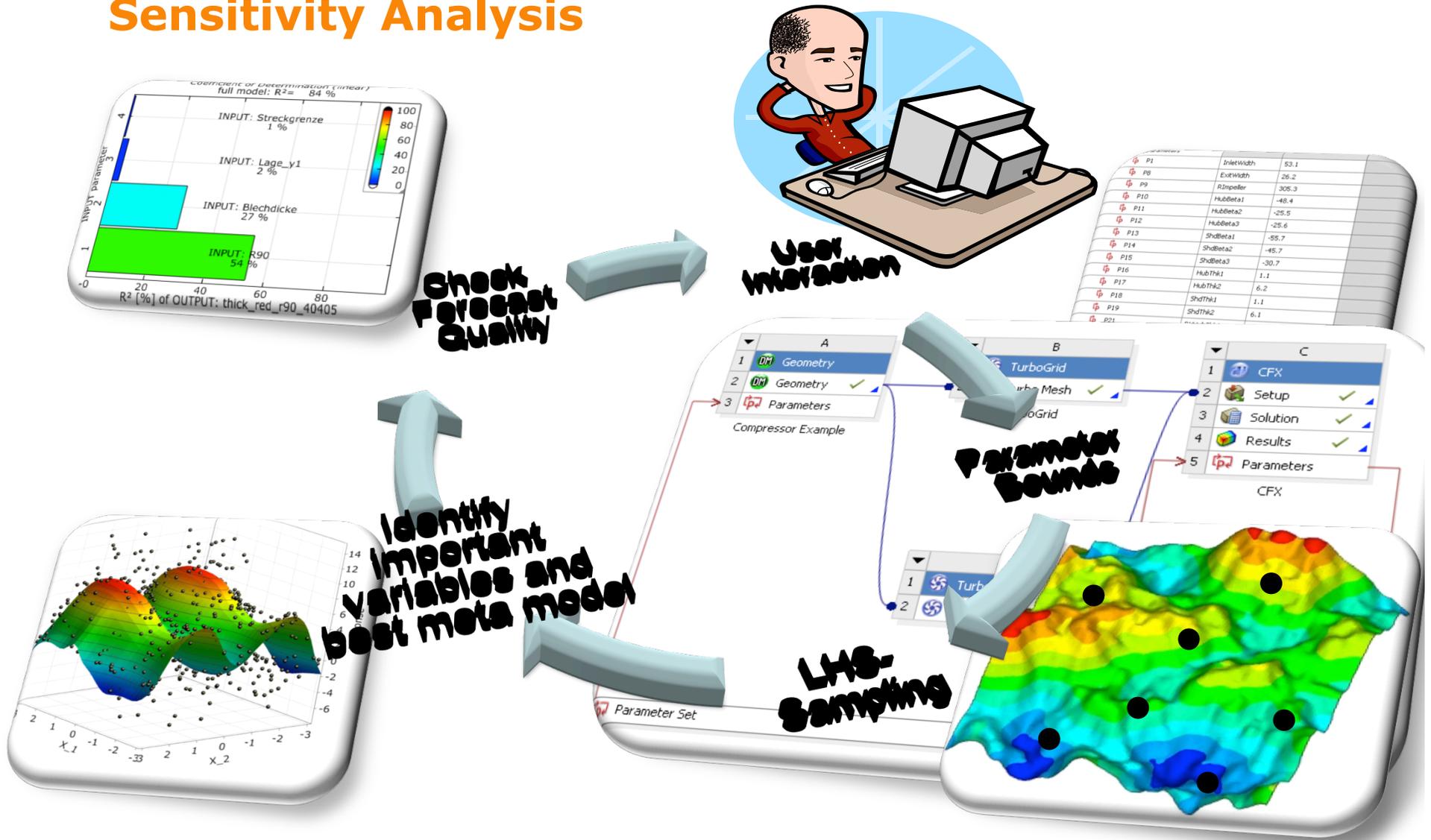
To minimize necessary number of sample optiSLang includes **filter technology** to select significant variables (significance, importance & correlation filter)

## Meta model of optimal Prognosis (MOP)

- optiSLang provides a automatic flow to reduce variables and generate the best possible response surface for every response with a given number of solver calls [Meta model of optimal Prognosis (MOP)] and checks Prognosis quality of the meta model.
- MoP solve following important tasks
  - We reduce the variable space using filter technology= best subspace
  - We check multiple non linear correlations by checking multiple MLS/ Polynomial regression = best Meta Model
  - We check the forecast (prognosis) quality using a test sample set = Coefficient of Prognosis (CoP)
- CoP/MOP allows to minimize the number of solver runs
- Final MOP can be used as approximation function



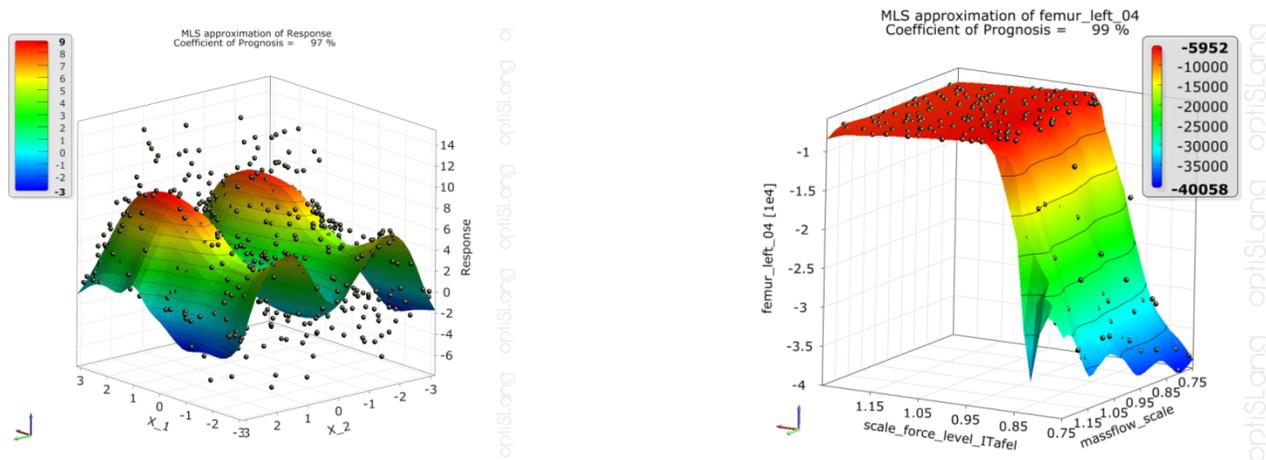
# Sensitivity Analysis



## Easy and safe to use!

What do we mean with that?

- “classic” DOE+RSM technology ask user to reduce number of variables, choose a suitable DOE with a suitable regression function and check the quality of the resulting response surface (RS) and the “optima” on the RS.
- optiSLang provides a automatic flow to reduce variables and generate the best possible response surface for every response with a given number of solver calls [Meta model of optimal Prognosis (MoP)] and checks MoP Prognosis quality and “optima” in real space.

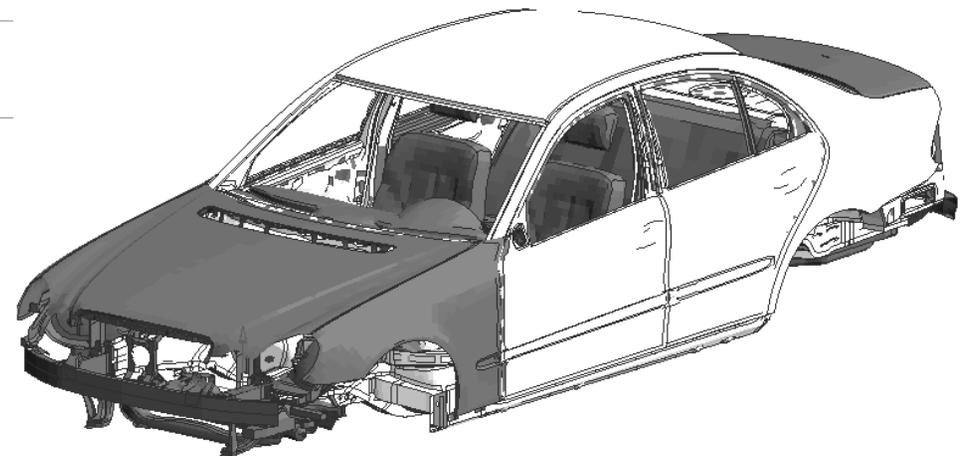
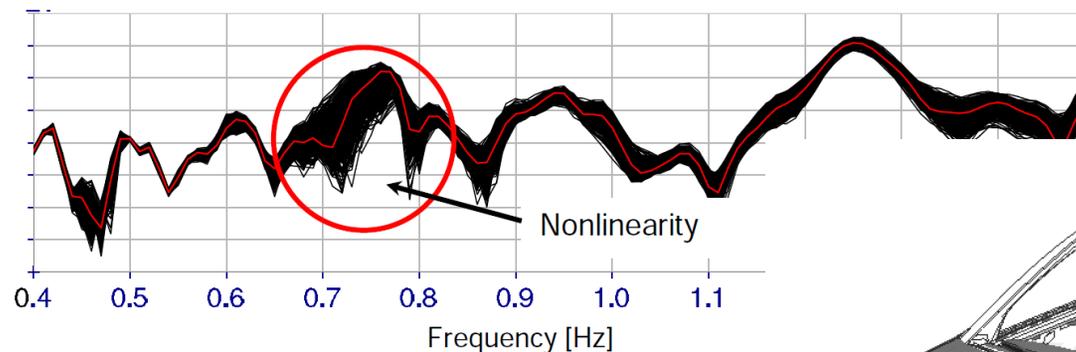


## Application: Noise Vibration Harshness

Comfort point

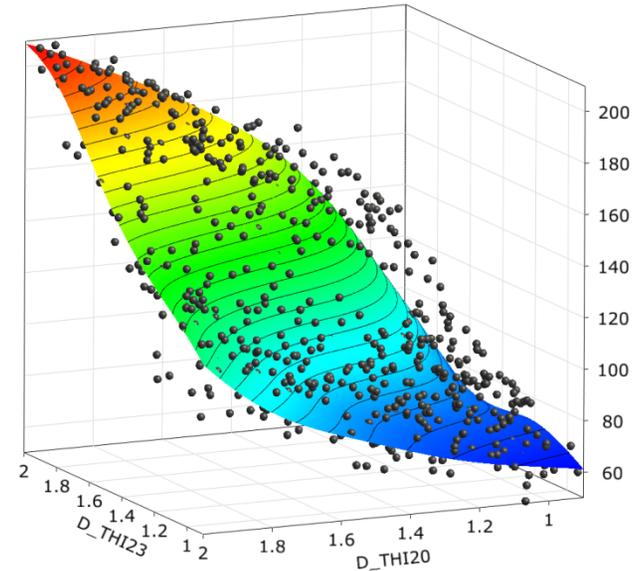
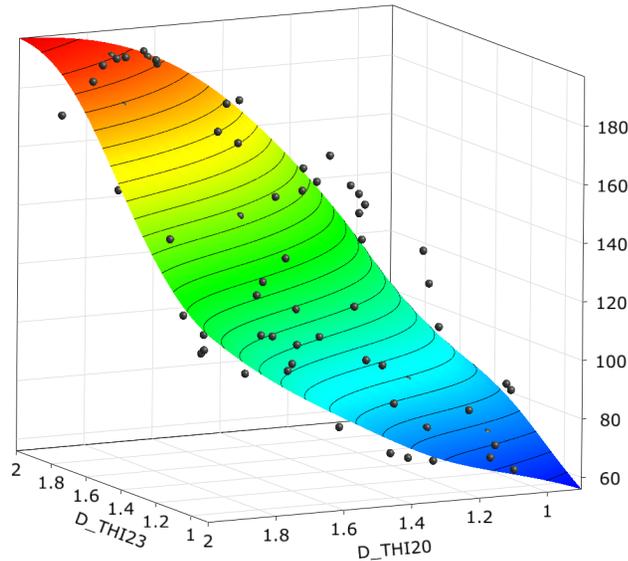
Red  
Black

Reference design  
483 Robustness runs



- Input parameters are 46 sheet thicknesses of a car body
- Variation of inputs within a +/- 20% interval
- Output values are sound pressure levels at certain frequencies
- One single solver run is already very time consuming

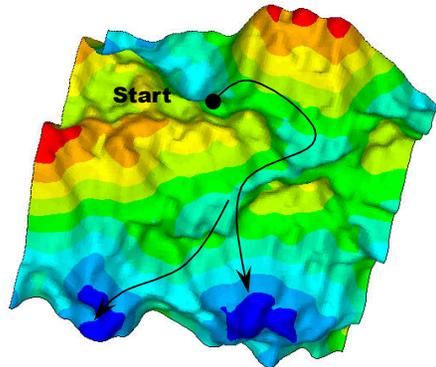
## Application: Noise Vibration Harshness



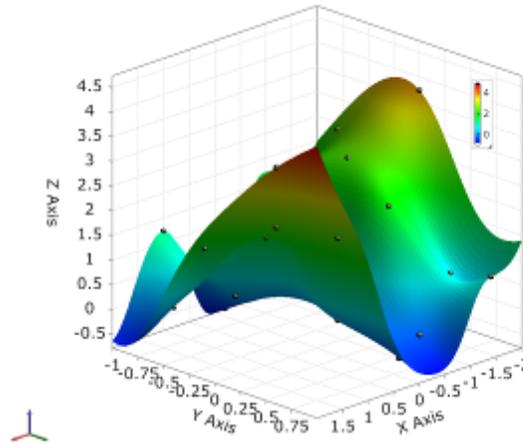
Samples	100	200	400	600	800
Full model CoP	90.9%	91.7%	95.7%	96.3%	96.9%
D_THI5	-	-	2.4%	2.3%	2.7%
D_THI6	6.0%	5.3%	8.2%	8.3%	8.7%
D_THI20	41.3%	42.7%	42.3%	43.4%	42.2%
D_THI23	49.1%	48.0%	50.7%	51.0%	53.8%

# Optimization Algorithms

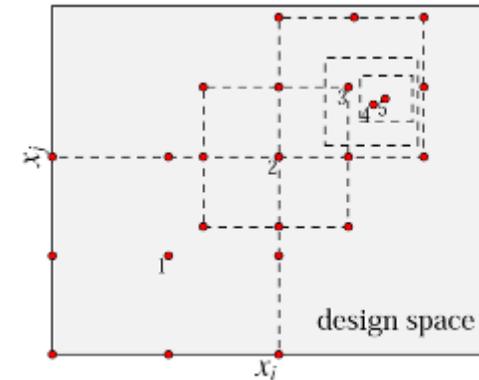
## Gradient-based



## Response surface method

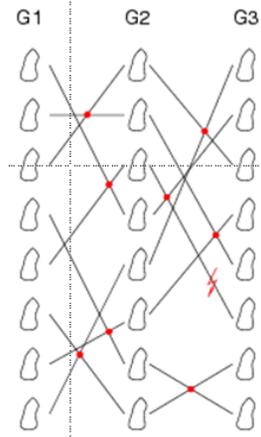
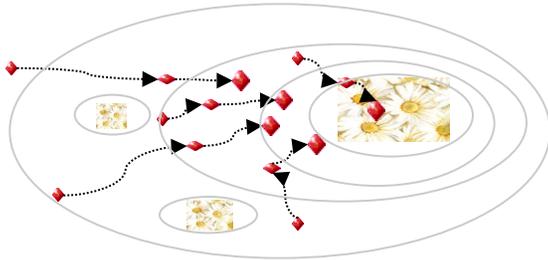


## Adaptive RSM

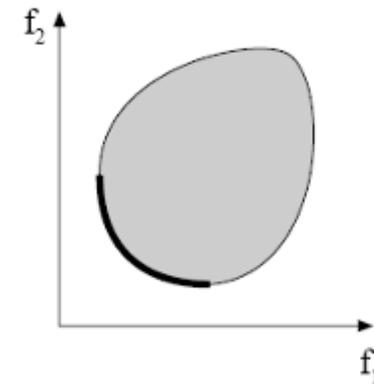


## Biological Algorithms:

- Genetic algorithms,
- Evolutionary strategies
- Particle Swarm Optimization

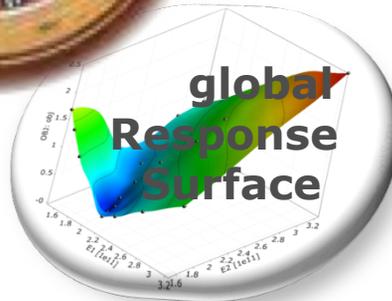
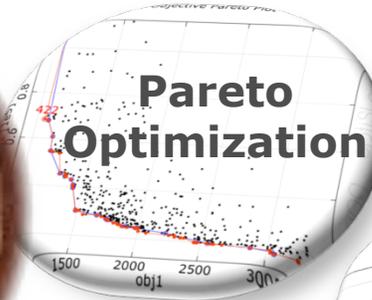
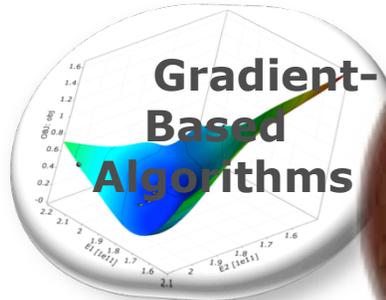
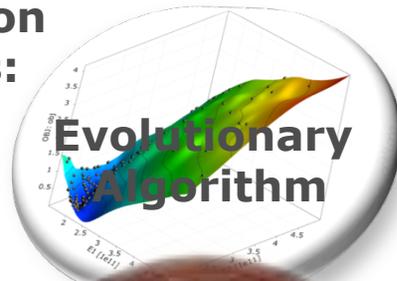


## Pareto Optimization

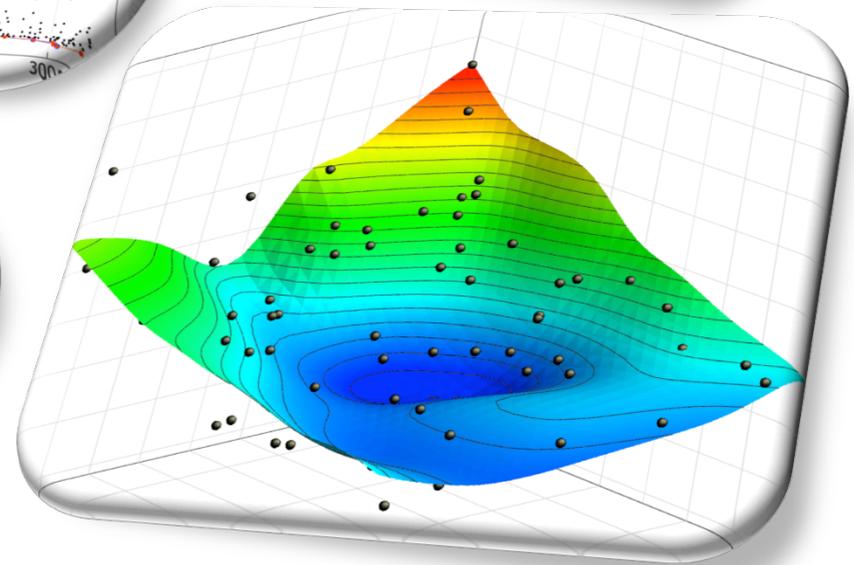
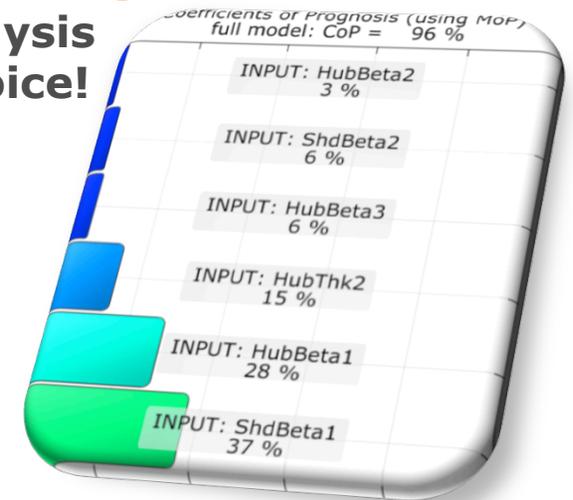


# Which Optimization Algorithm is the right one?

**Optimization Algorithms:**



**Sensitivity Analysis allows best choice!**



## Optimizer Selection Wizzard

- An optimizer is automatically suggested depending on the parameter properties, the defined criteria as well as user specified settings

**Optimization Wizard**

**Additional information**  
Additional information about the task. Used to recommend an algorithm.

Number of parameter: 2

Number of objectives: 1

**Parameter type**

Pure continuous  
 Has discrete

**Discrete type**

Ordered  
 Nominal

Analysis status: Preoptimized

Constraints violations: Not set

Failed designs: None

Solver noise: None

< Back Next > Cancel Help

**Optimization Wizard**

**Optimization method**  
Specify the optimization method

Optimization method

Response surface method

Adaptive Response Surface Method (ARSM)

Natural inspired optimization algorithms

Evolutionary Algorithm (EA) - local  
 Evolutionary Algorithm (EA) - global  
 Particle Swarm Optimization (PSO) - local  
 Particle Swarm Optimization (PSO) - global  
 Stochastic Design Improvement (SDI)

Gradient based optimization

Non-Linear Programming by Quadratic Lagrangian (NLPQL)

Additional options

Use Previous Data As Starting Point(s)

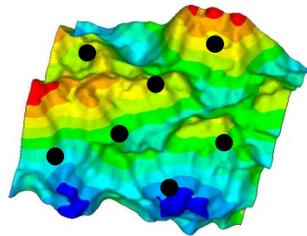
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# Sensitivity, Optimizaion and Identification

1) Start with a sensitivity study

2) Identify the important parameters and responses using meta model technique

- understand the problem
- reduce the problem

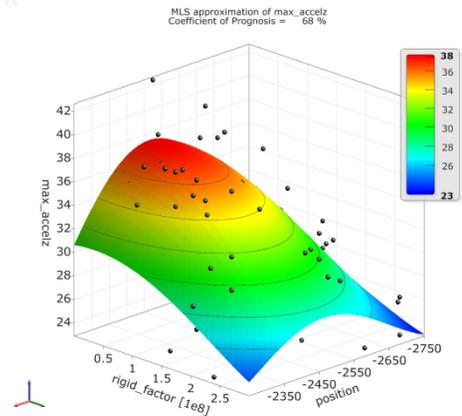
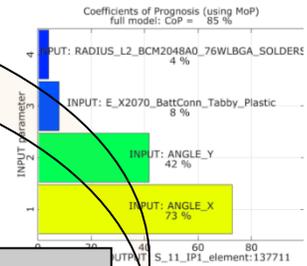


Scan the whole Design Space

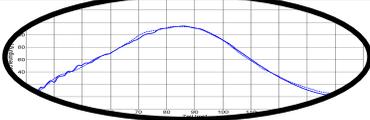


optiSLang

Understand the Problem using CoP/MoP



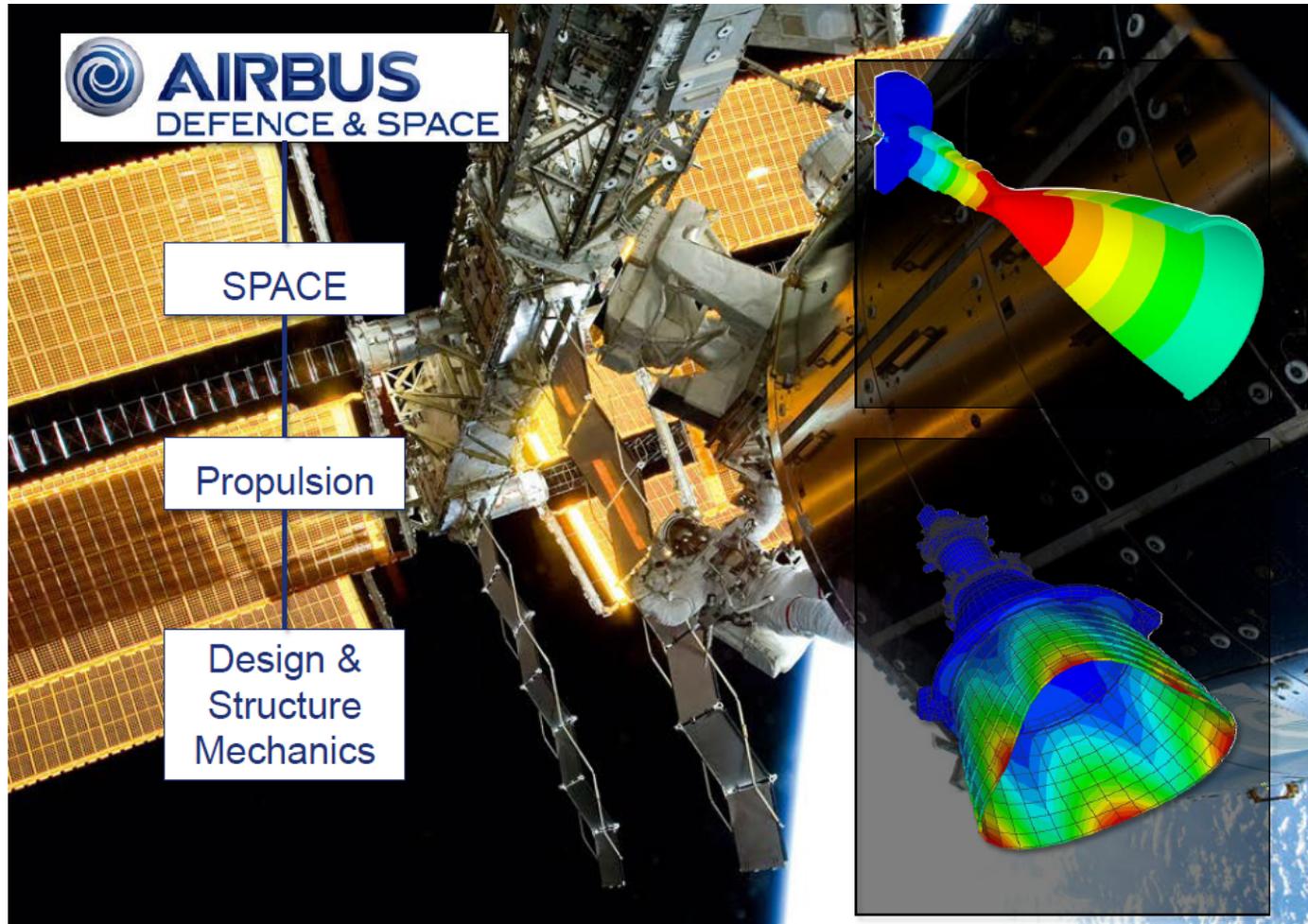
Search for Optima



3) Run gradient based or biological based optimization algorithms at optimal meta model (MOP)

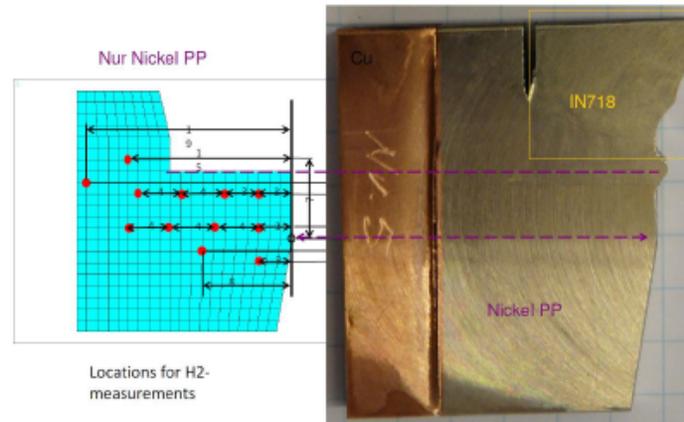
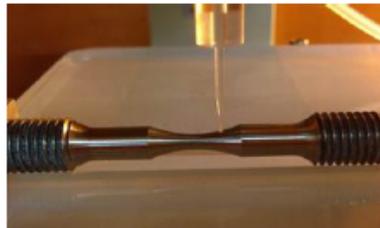
4) Run an RSM based, gradient based or biological based optimization algorithms using additional CAE solver runs

Airbus DS has developed an internal procedure based on finite elements method to simulate the hydrogen diffusion inside galvanic nickel parts of combustion chambers for rocket engines.



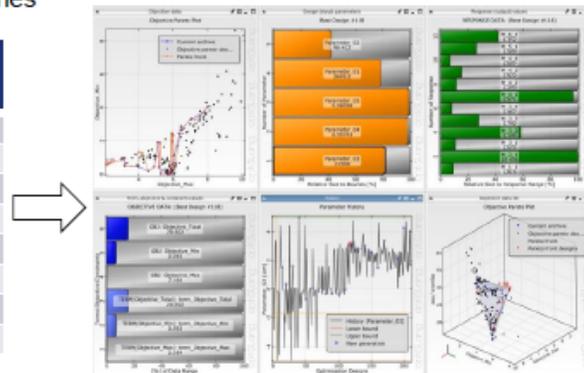
## Parameter Identification of input parameter based on test

- Parameters A1, A2 and A3 determined through permeation tests on material samples.
- The remaining parameters are back-calculated using the evolutionary optimization algorithm of OPTISLANG™ in order to get the same H-content as measured on tensile samples (left) and on a combustion chamber slice (right) after a certain storage time.



Initial parameter set and variation boundaries

Parameter	Value	Lower bound	Upper bound
$x_1$	3500	0	50000
$x_2$	100	0	10000
$x_3$	-10800	-50000	0
$x_4$	0.5	-1	1
$A_1$	3500	-	-
$A_2$	6880	-	-
$A_3$	13500	-	-
$A_4$	0.5	-1	1



Evolutionary optimization algorithm with OPTISLANG™

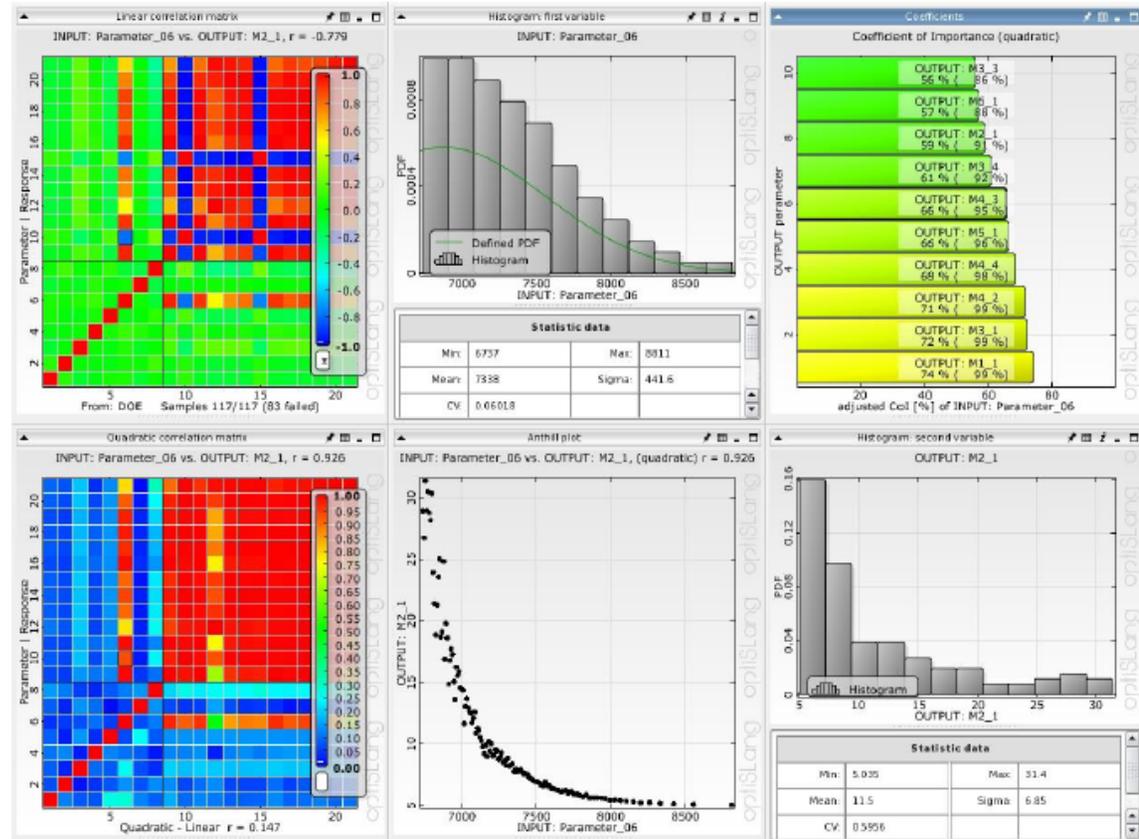
Optimized set

Parameter for S	Value	Parameter for D	Value
$x_1$	22218 ppt	$A_1$	11766 mm <sup>2</sup> /s
$x_2$	1800 K	$A_2$	6880 K
$x_3$	-10155 K	$A_3$	10155 K
$x_4$	0.0K/MPa	$A_4$	0.0 K/MPa

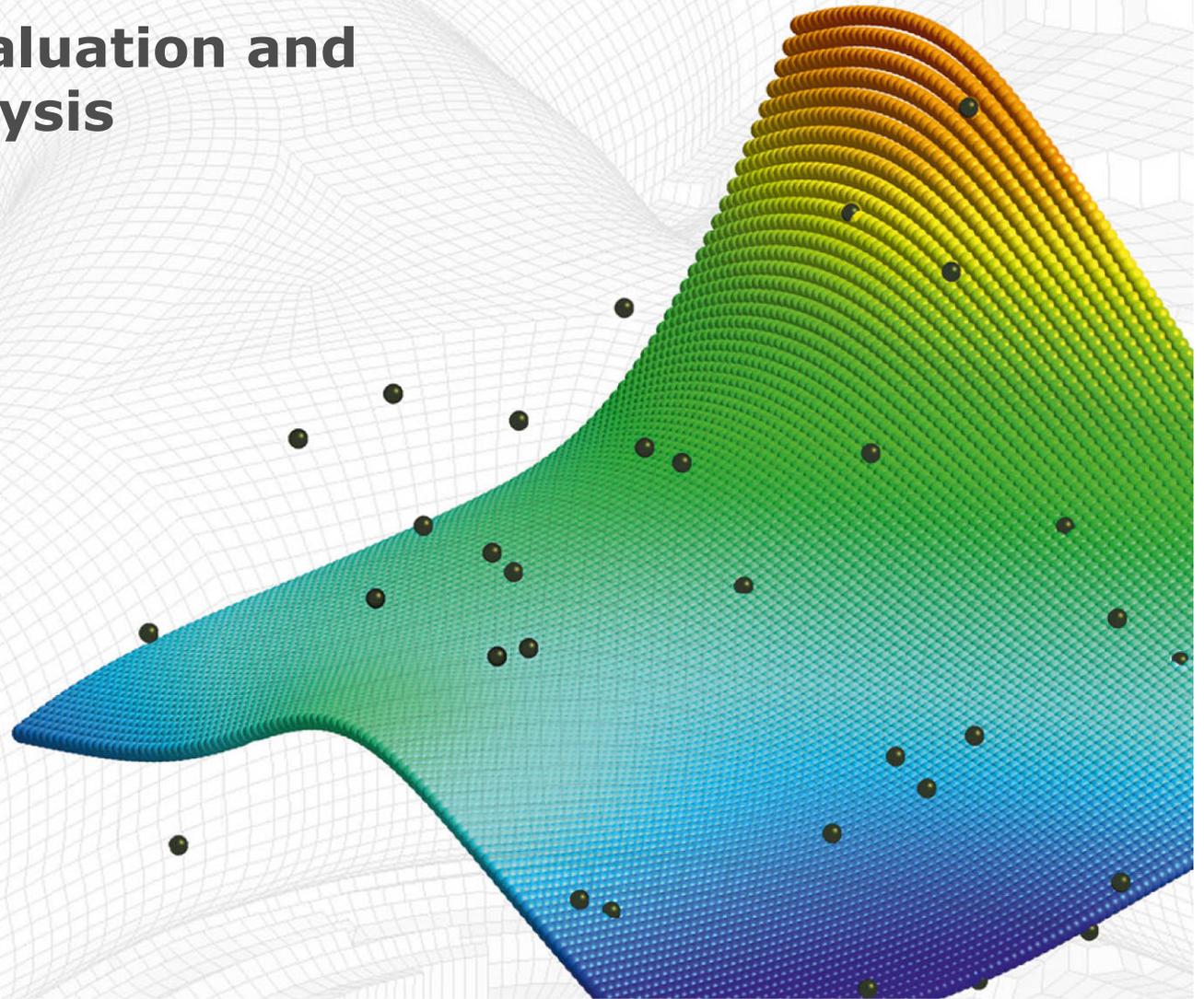


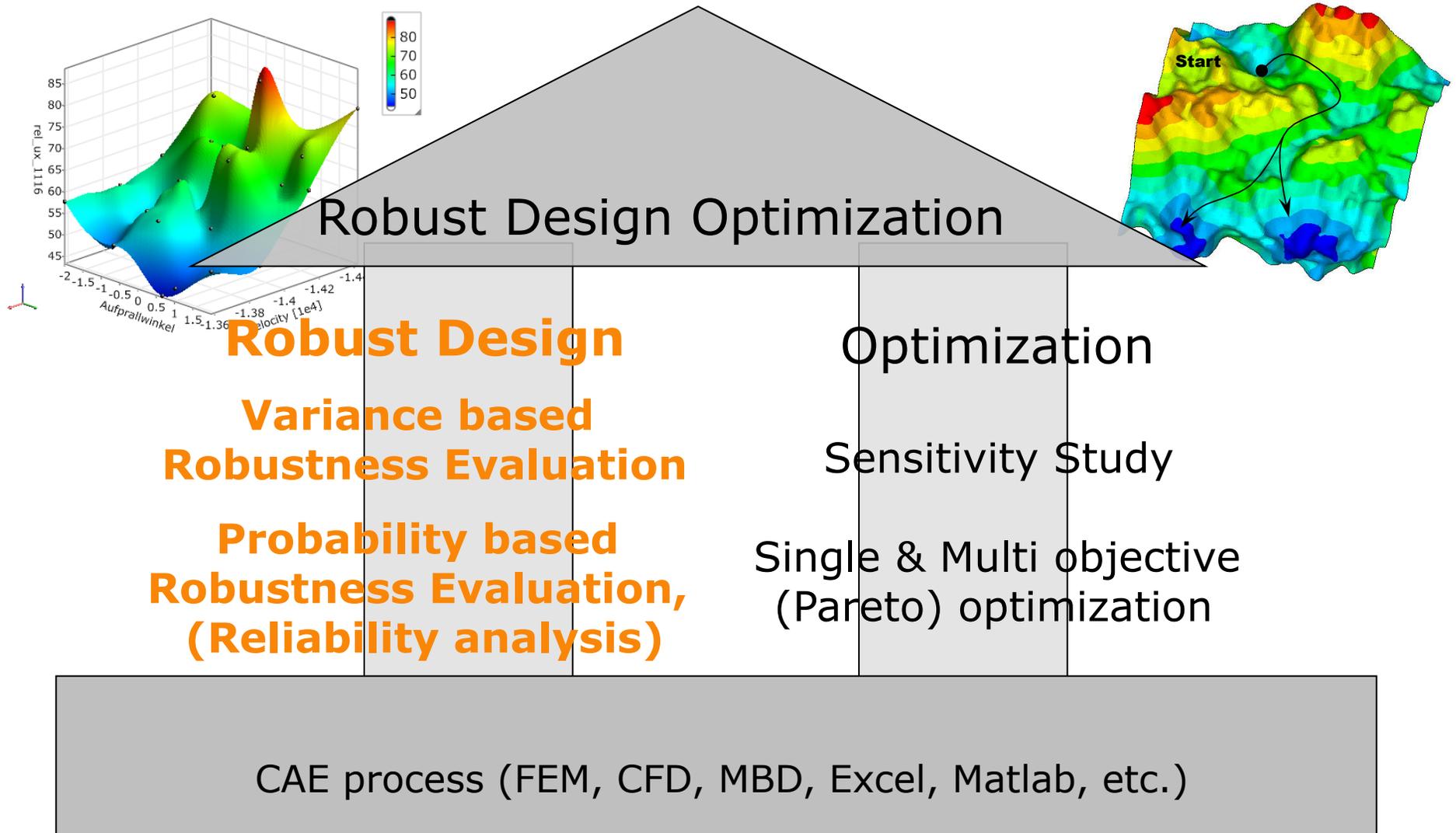
# Robustness Evaluation is used to investigate the sensitivity of input parameter uncertainty

- Uncertainty of parameter A2 dominates the scatter of important response values
- a CV of 6% of inputs leads to CV of 60% at important output
- therefore measures of parameter A2 needs to be most reliable!



# Robustness Evaluation and Reliability Analysis





## Which Robustness measure we should use?

### Variance based RDO

- Safety margins of all critical responses are larger than a specified sigma level (e.g. **Design for Six Sigma**)

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

### Reliability based RDO

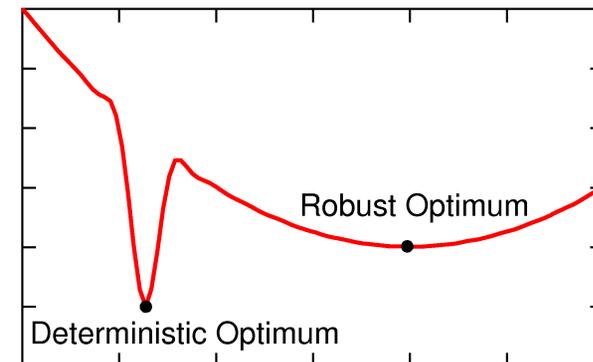
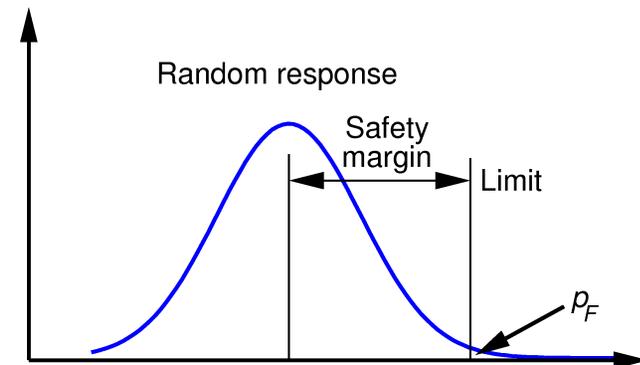
- Failure probability with respect to given limit states is smaller as required value

$$p_F \leq p_F^{target}$$

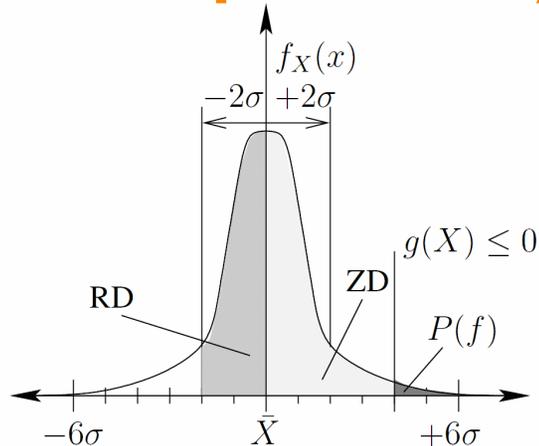
### Taguchi based RDO

- Taguchi loss functions
- Modified objective function

$$f(y) = \frac{k}{N} \sum y_i^2 = k(\bar{y}^2 + \sigma_y^2)$$



## Failure probability for Six Sigma design



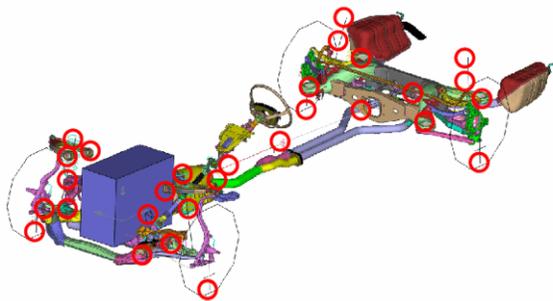
The statement six sigma results in 3.4 defects out of a million introduces a “safety distance” of 1.5 sigma shift for long term effects!

Therefore the target of virtual prototyping is a **6-1.5=4.5 Sigma design proof**.

Sigma level	Variation	Probability of failure	Defects per million (short term)	Defects per million (long term - $\pm 1.5\sigma$ shift)
$\pm 1\sigma$	68.26	3.1 E-1	317,400	697,700
$\pm 2\sigma$	95.46	4.5 E-2	45,400	308,733
$\pm 3\sigma$	99.73	2.7 E-3	2,700	66,803
$\pm 4\sigma$	99.9937	6.3 E-5	63	6,200
$\pm 5\sigma$	99.999943	5.7 E-7	0.57	233
<b><math>\pm 6\sigma</math></b>	<b>99.99999998</b>	<b>2.0 E-9</b>	<b>0.002</b>	<b>3.4</b>

## Essential input are Uncertainties and Tolerances

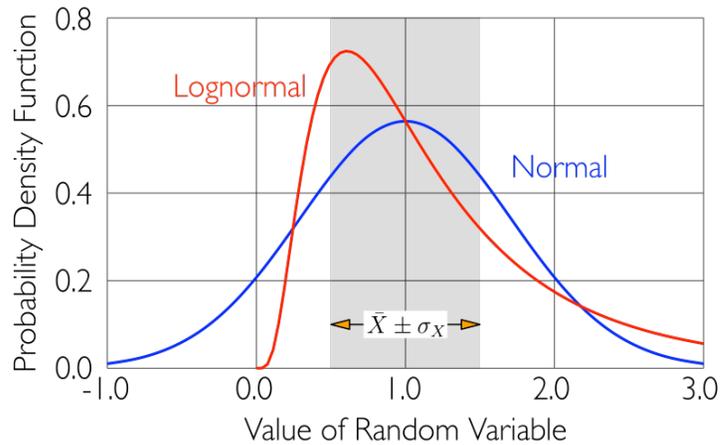
- Design variables
- Material, geometry, loads, constrains,...
- Manufacturing
- Operating processes (misuse)
- Resulting from Deterioration
- ...



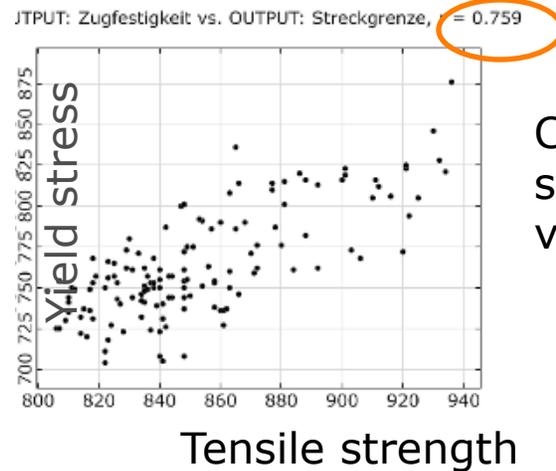
Property	SD/Mean %
<b>Metallic materiales, yield</b>	<b>15</b>
Carbon fiber rupture	17
Metallic shells, buckling strength	14
Bond insert, axial load	12
Honeycomb, tension	16
Honeycomb, shear, compression	10
Honeycomb, face wrinkling	8
Launch vehicle , thrust	5
<b>Transient loads</b>	<b>50</b>
Thermal loads	7.5
Deployment shock	10
<b>Acoustic loads</b>	<b>40</b>
Vibration loads	20

# Definition of Uncertainties

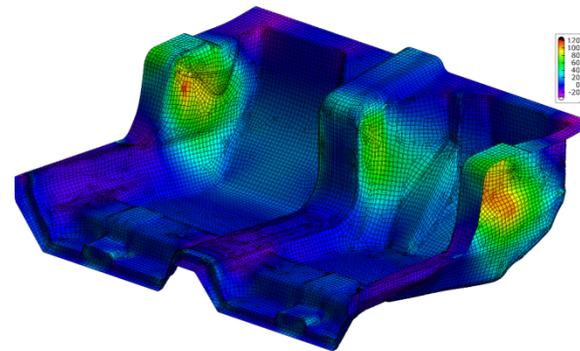
Distribution functions define variable scatter



Correlation is an important characteristic of stochastic variables.



Correlation of single uncertain values

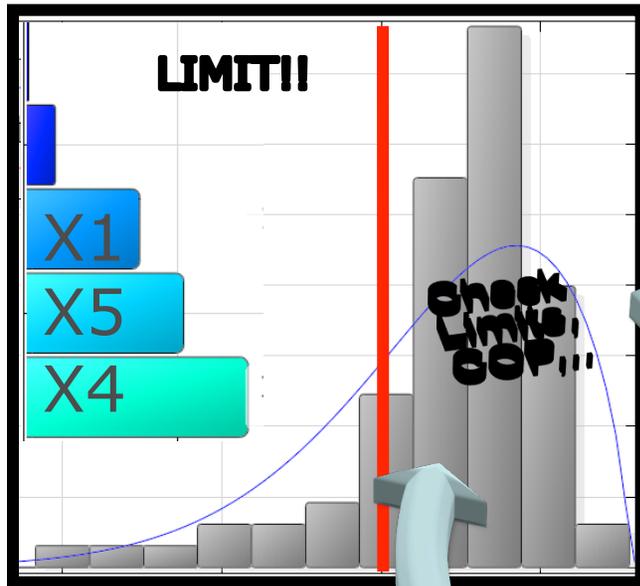


Spatially correlated field values

**Translate know how about uncertainties into proper scatter definition**

# Robustness Evaluation

Robust Design??

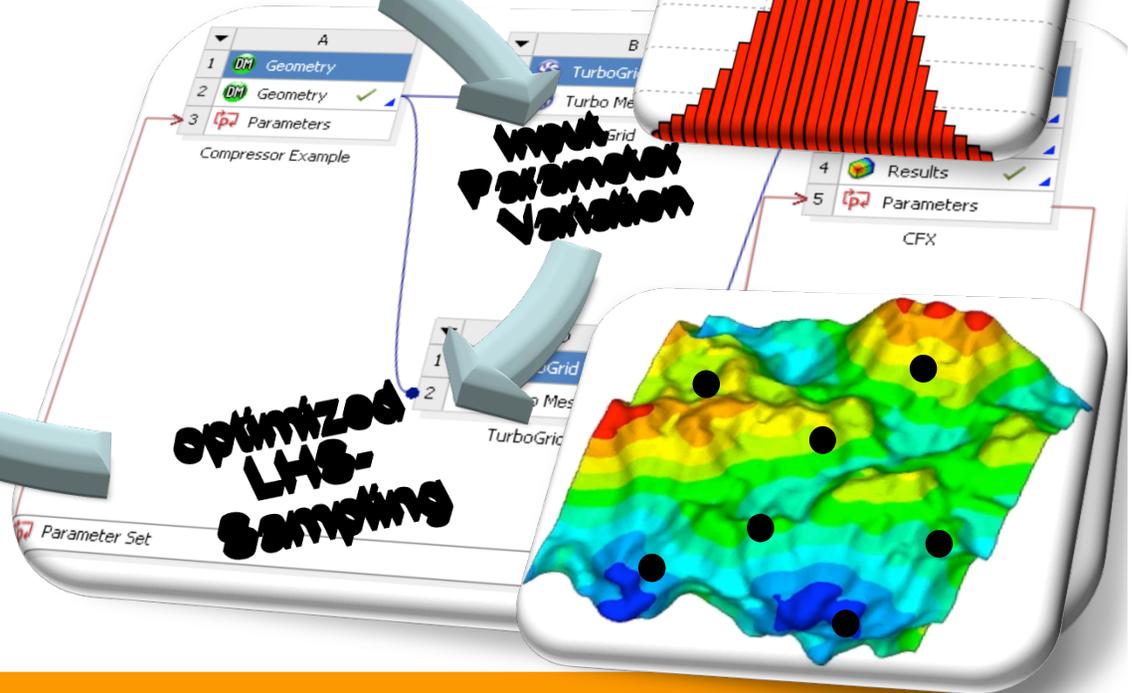


User Interaction

Response Parameter Variation

Optimized LHS-Sampling

Input Parameter Variation



## Reliability Analysis

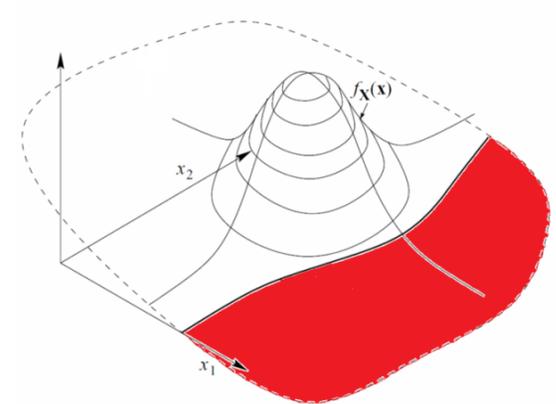
- Robustness evaluation reliable estimate relatively high probabilities ( $\pm 2\sigma$ , like 1% of failure)
- Reliability analysis verify rare event probabilities ( $\geq 3\sigma$ , smaller than 1 out of 1000)

**Monte Carlo Sampling is the safest and most robust way to calculate small event probabilities, but at the same prohibitive expensive**

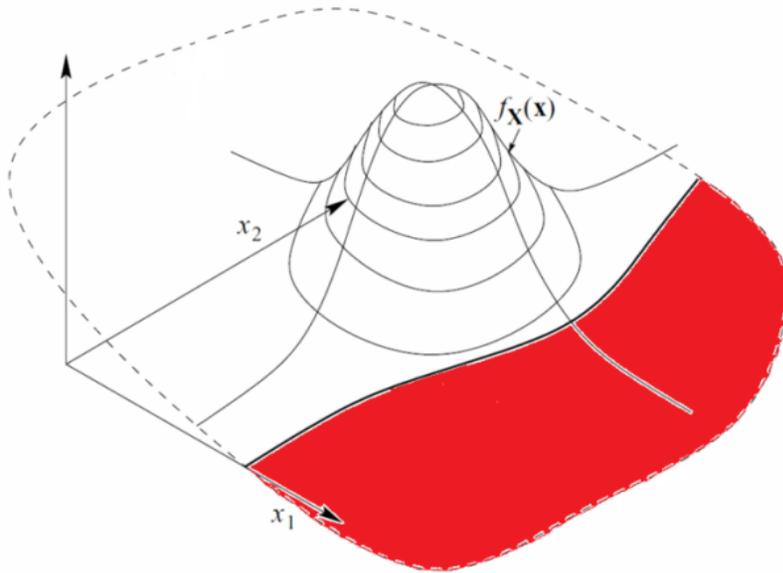
**There is no one magic algorithm to estimate probabilities with “minimal” sample size.**

**All “effective” algorithms will try to learn about the failure domain and have the risk to learn unreliable information**

**Therefore it is recommended to use two different algorithms to verify rare event probabilities**



## Reliability analysis

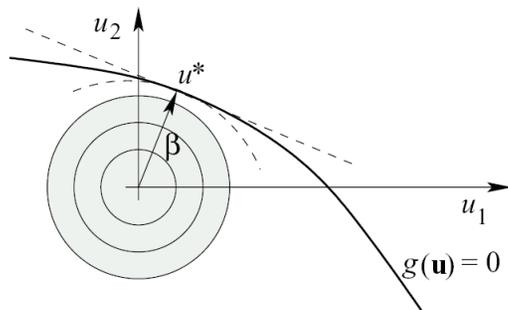


$$\begin{aligned}
 P_F &= P[\mathbf{X} : g(\mathbf{X}) \leq 0] \\
 &= \int_{g(\mathbf{x}) \leq 0} \cdots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
 &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I(g(\mathbf{x})) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}
 \end{aligned}$$

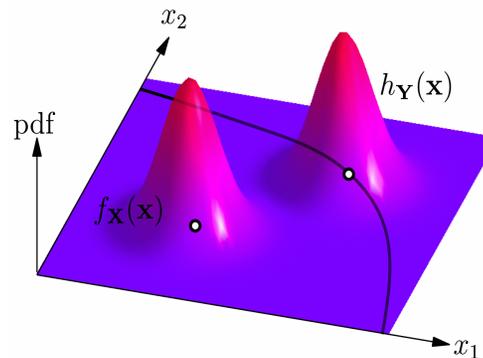
- Limit state function  $g(\mathbf{x})$  divides random variable space  $\mathbf{X}$  in safe domain  $g(\mathbf{x}) > 0$  and failure domain  $g(\mathbf{x}) \leq 0$
- Multiple failure criteria (limit state functions) are possible
- Failure probability is the probability that at least one failure criteria is violated (at least one limit state function is negative)
- Integration of joint probability density function over failure domain

# Reliability Analysis Algorithms

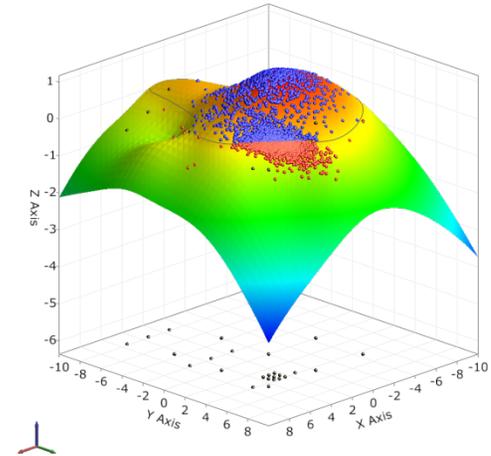
**Gradient-based algorithms = First Order Reliability algorithm (FORM)**



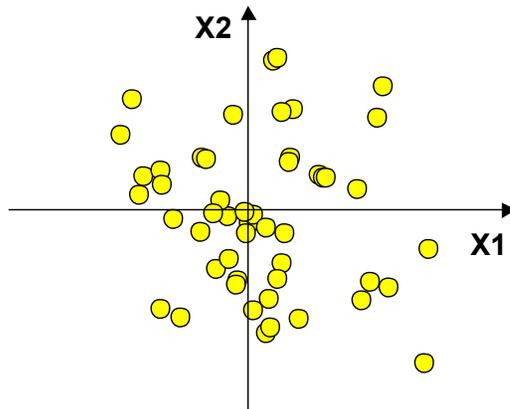
**ISPUD Importance Sampling using Design Point**



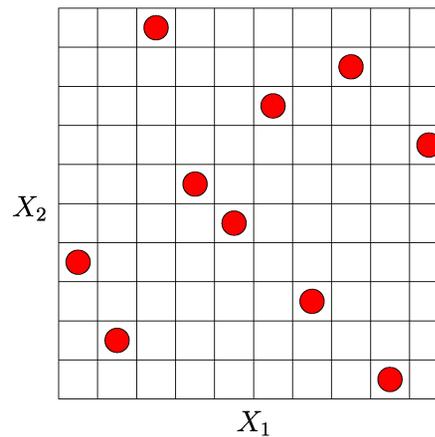
**Adaptive Response Surface Method**



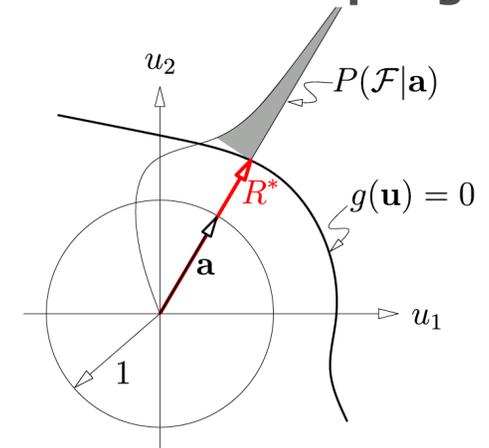
**Monte Carlo Sampling**



**Latin Hypercube Sampling**

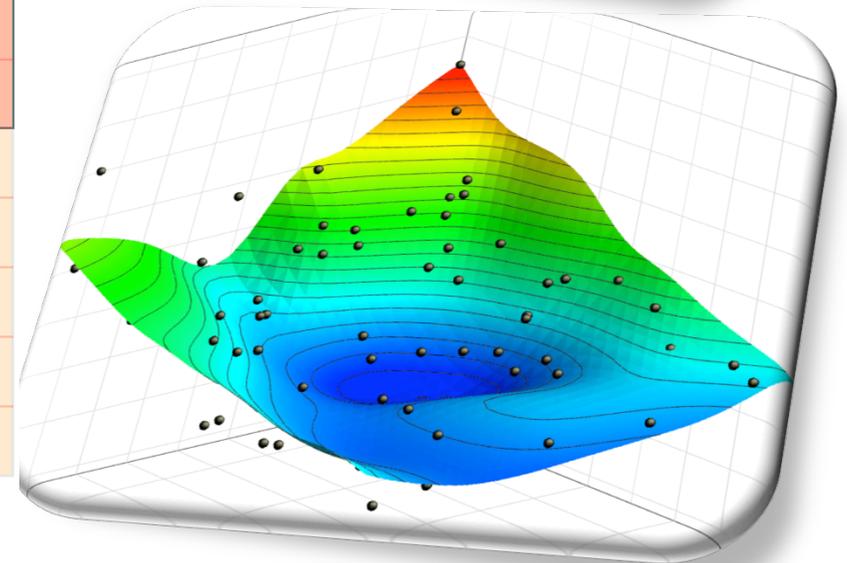
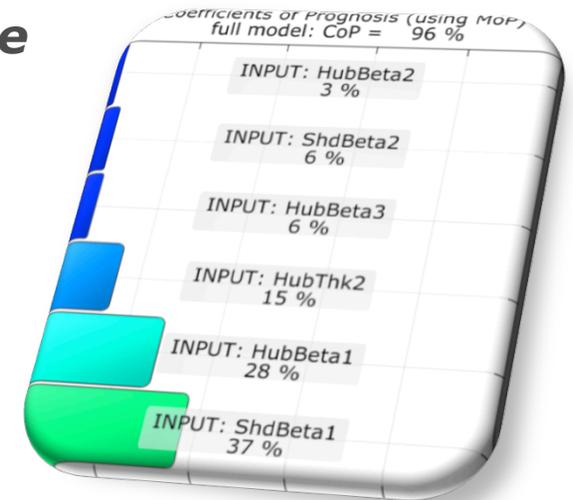
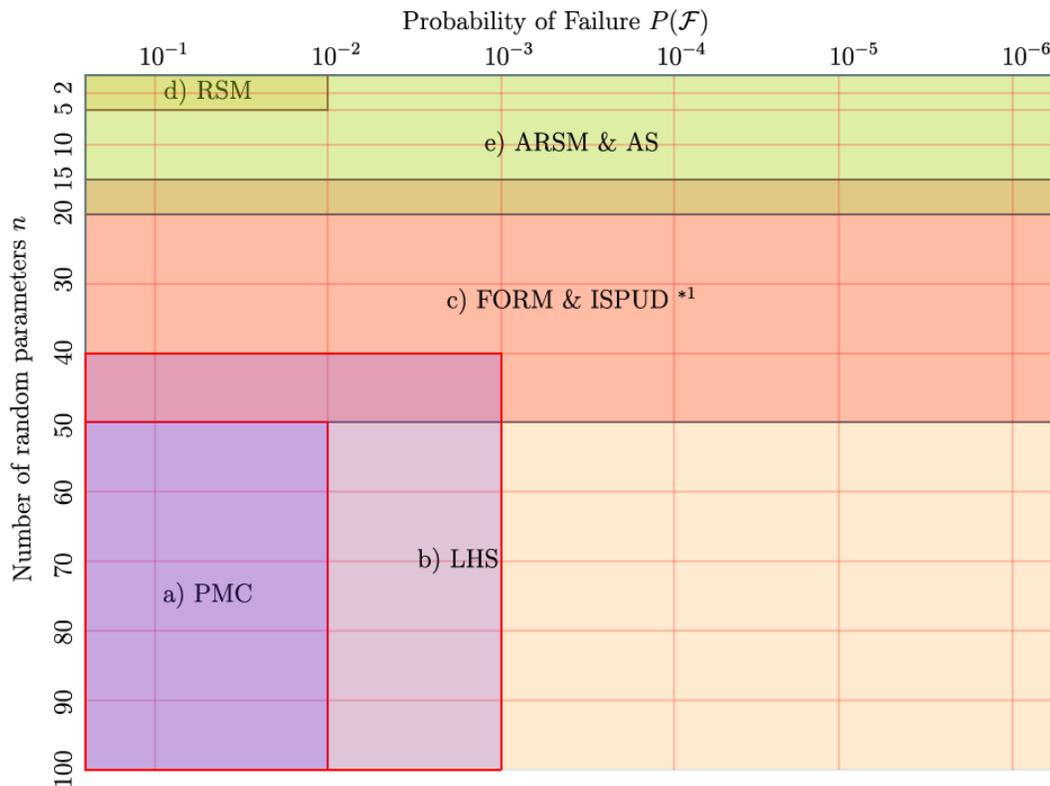


**Directional Sampling**



# How choosing the right algorithm?

**Robustness Analysis provide the knowledge to choose the appropriate algorithm**



## Robustness & Reliability Algorithms



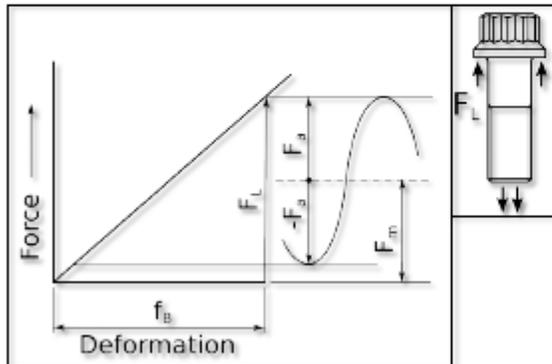
# Reliability of bolts of a rocket combustion chamber

Test condition needs to be translated to flight conditions

## Problem illustration

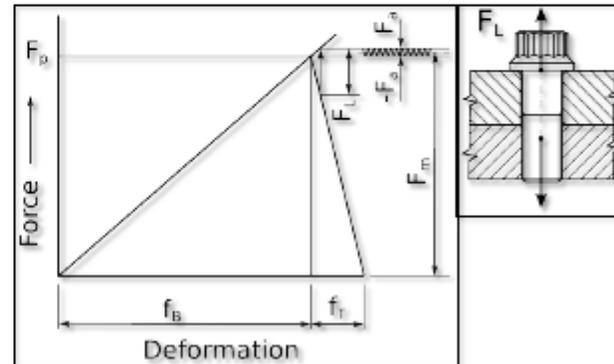
### Life in test:

- No pretension
- High amplitude
- Low number of cycles until failure  $N_{f,test}$



### Life during flight:

- High pretension
- Low amplitude
- High number of cycles until failure  $N_{f,flight}$



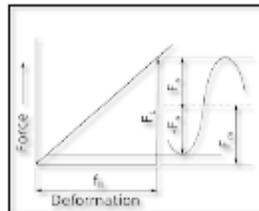
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# Reliability of bolts of a rocket combustion chamber

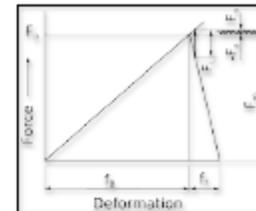
Sensitivity analysis identifies the most important parameter related to life estimation of test and flight conditions

## Sensitivity of parameters

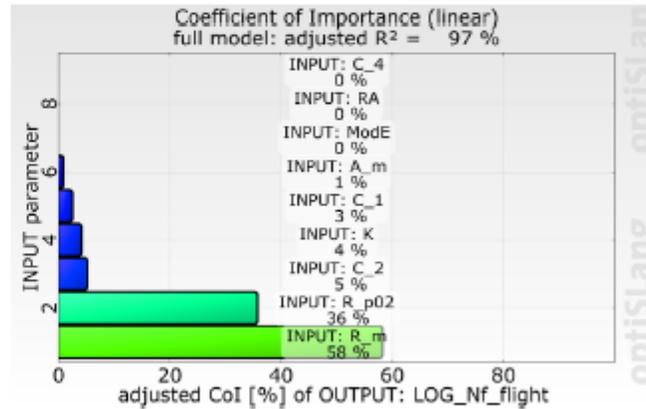
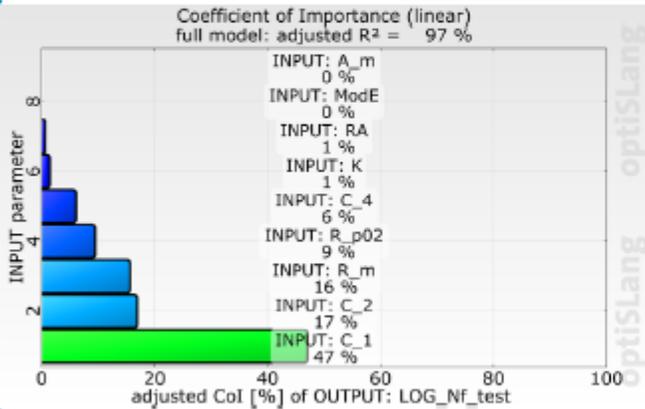
### On test conditions



### On flight conditions



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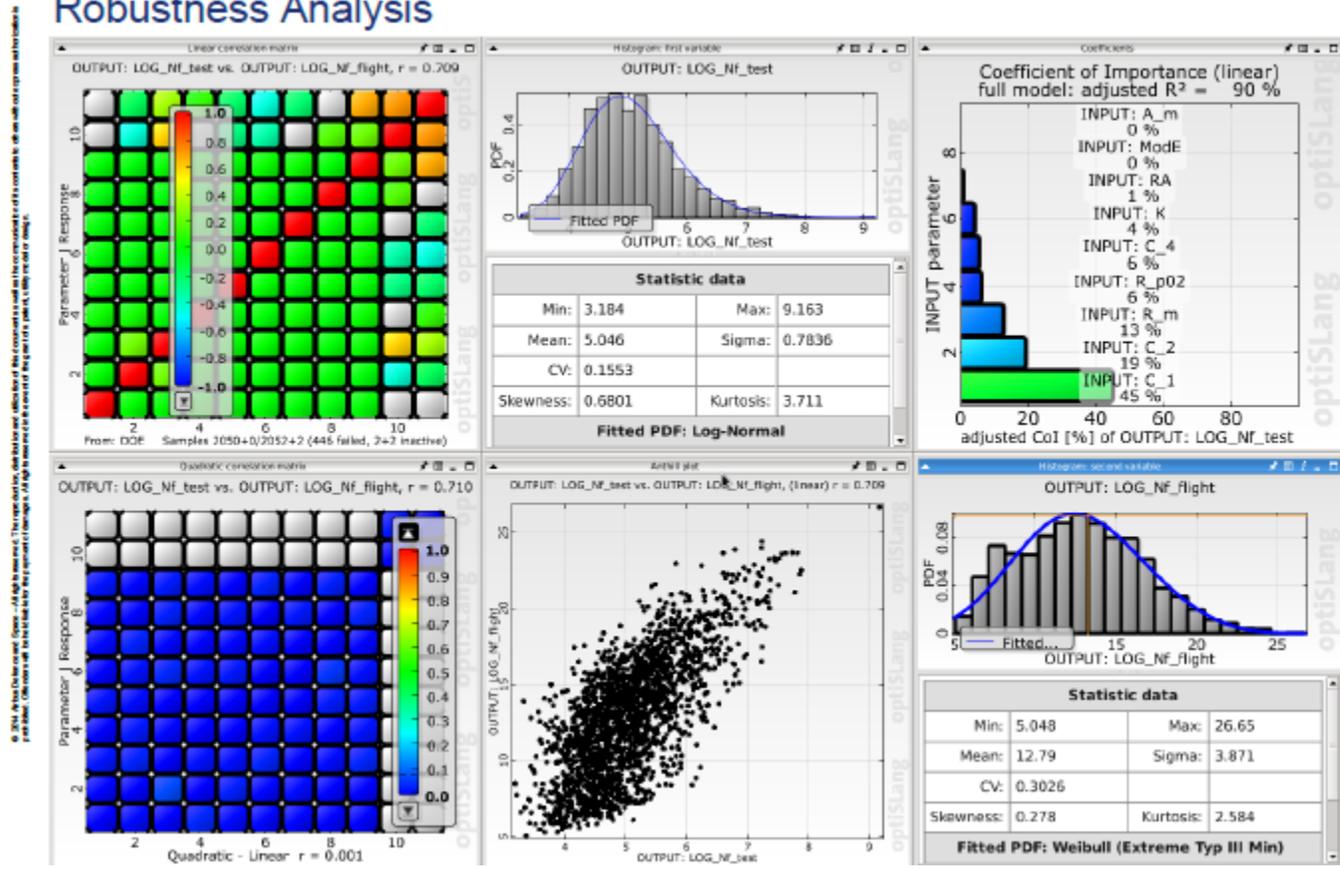
Coffin Manson:

$$\Delta \epsilon = C_1 \cdot \frac{R_m - \sigma_m}{E} \cdot N_f^{-C_2} + D_u \cdot C_3 \cdot N_f^{-C_4}$$

# Reliability of bolts of a rocket combustion chamber

After definition of uncertainties of all important parameters Robustness Evaluation estimate the uncertainty of the relation between flight and test life

## Robustness Analysis

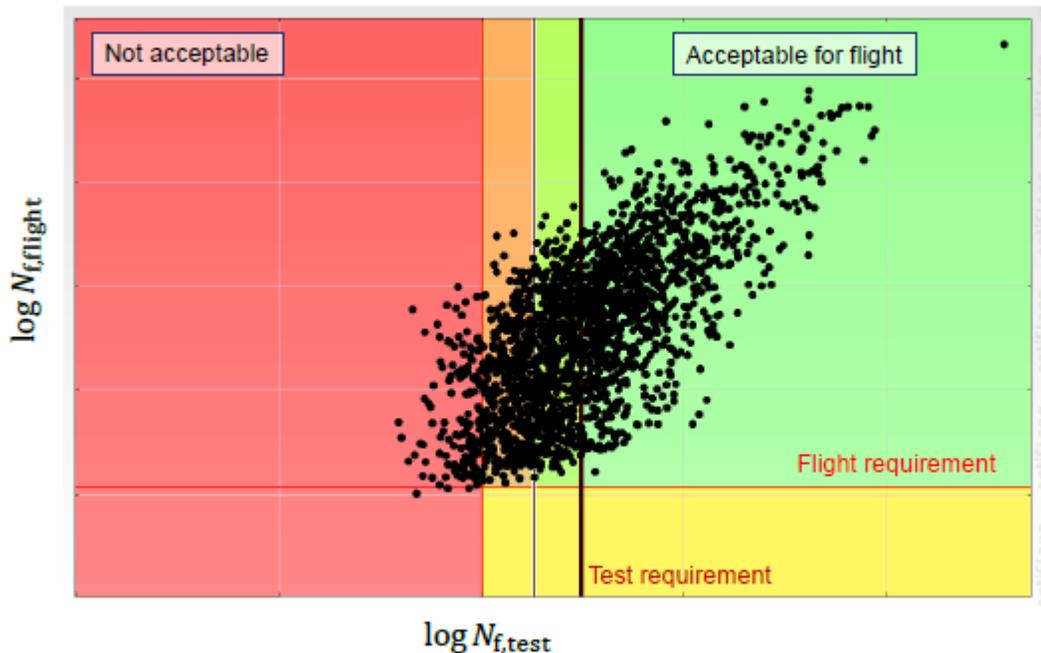


# Reliability of bolts of a rocket combustion chamber

Initial test requirement was proven to have sufficient safety margin

For bolts which fail the initial test requirements, reliability analysis can be used to certify sufficient life under flight conditions!

## Robustness Analysis - Setting test criteria

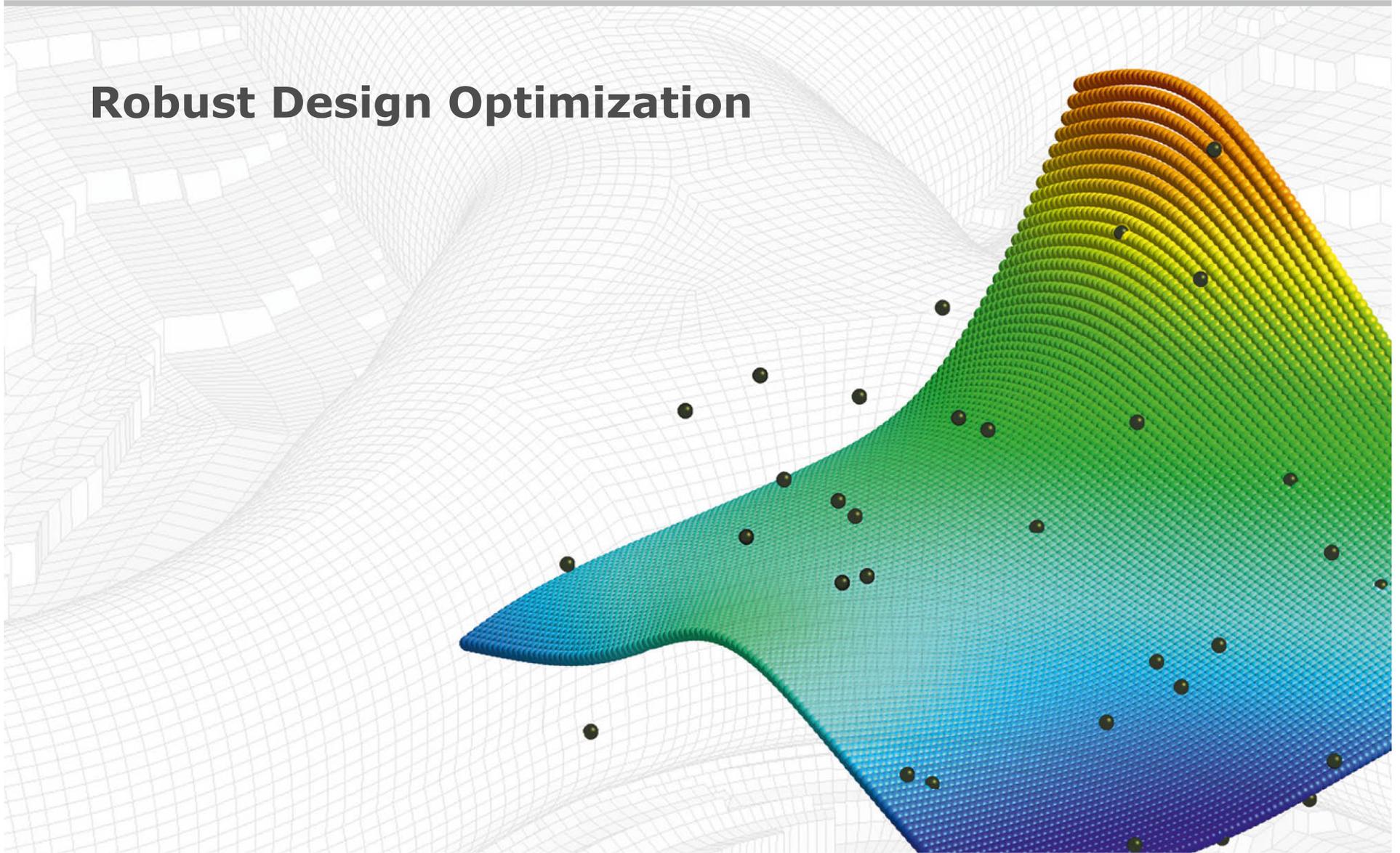


The project was used to certify the life of available bolts.

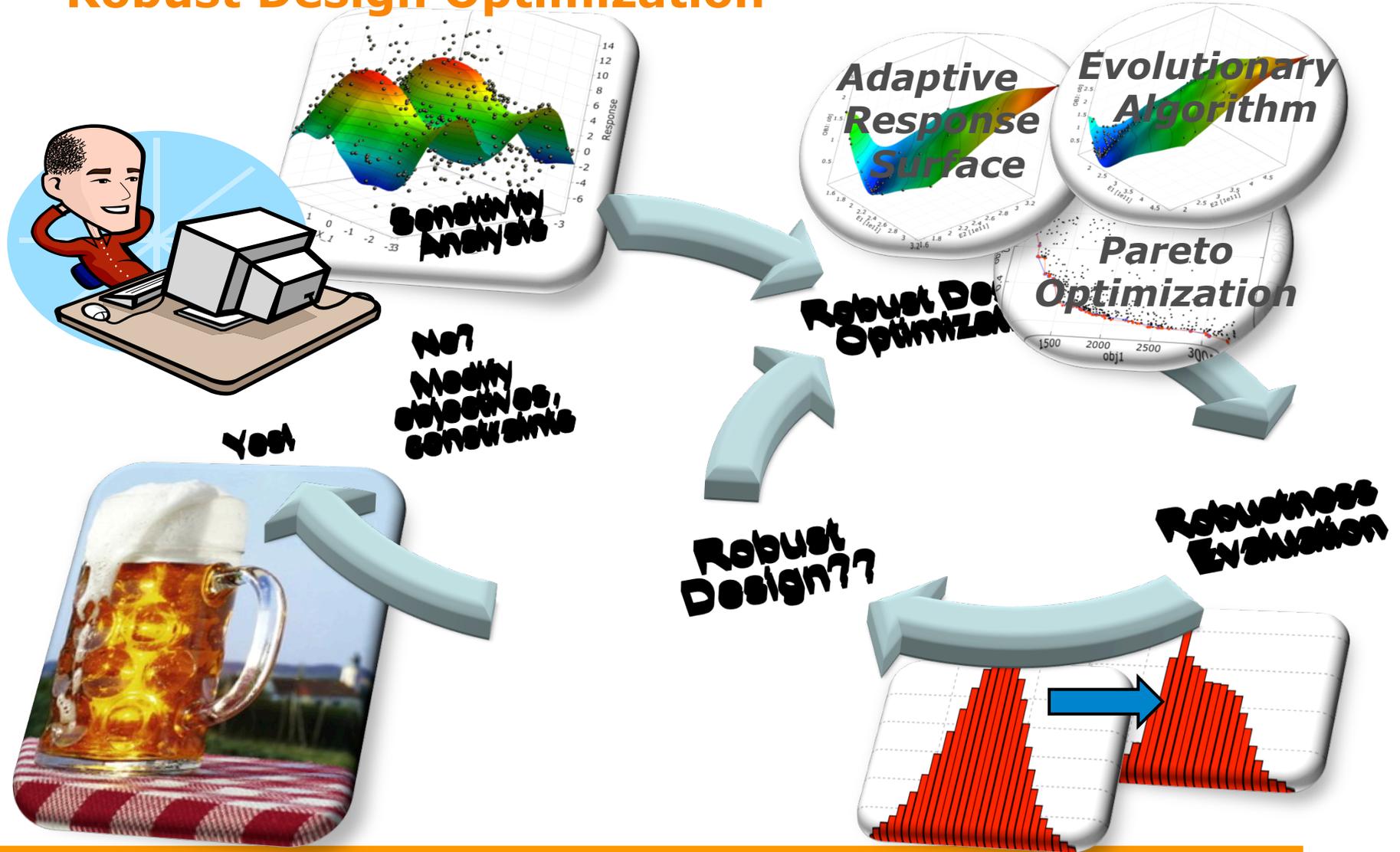
Cost's to order new bolts were avoided!

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# Robust Design Optimization



# Robust Design Optimization



## Challenges of RDO in Virtual Prototyping

- With improvements in parametric modeling, CAE (software) and CPU (hardware) there seems to be no problem to establish RDO (DfSS) product development strategies by using stochastic analysis
- There are many research paper or marketing talks about RDO/DfSS.
- But why industrial papers about successful applications are so rare?

Where is the problem with RDO?



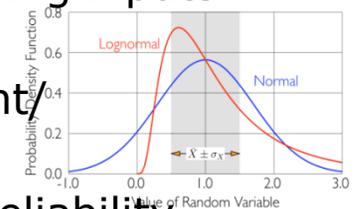
## Successful RDO needs a balance between:

- **Reliable definition of uncertainties**

- ⇒ many scattering variables (in the beginning) of an RDO task
- ⇒ best translation of input scatter to suitable parametric including distribution functions and correlations between scattering inputs

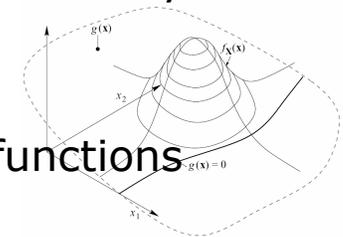
- **Reliable stochastic analysis methodology**

- ⇒ efficient and reliable methodology to sort out important/unimportant variables
- ⇒ because all RDO algorithms will estimate robustness/reliability measurements with minimized number of solver runs the proof of the reliability of the final RDO design is absolutely mandatory!



- **Reliable Post Processing**

- ⇒ Filter of insignificant/unreliable results
- ⇒ Reliable estimation of variation using fit of distribution functions



- **User Friendliness**

- ⇒ establish automatic flows of best practice which minimize the user input „ease of use“ and maximize the „safe of use“
- ⇒ **Finally non experts of stochastic analysis need be able to perform RDO**

## When and How to apply stochastic analysis?

- When material, geometry, process or environmental scatter is significantly affecting the performance of important response values
- When significant scatter of performance is seen in reality

**and** there is doubt that safety distances may be too small or safety distances should be minimized for economical reasons.

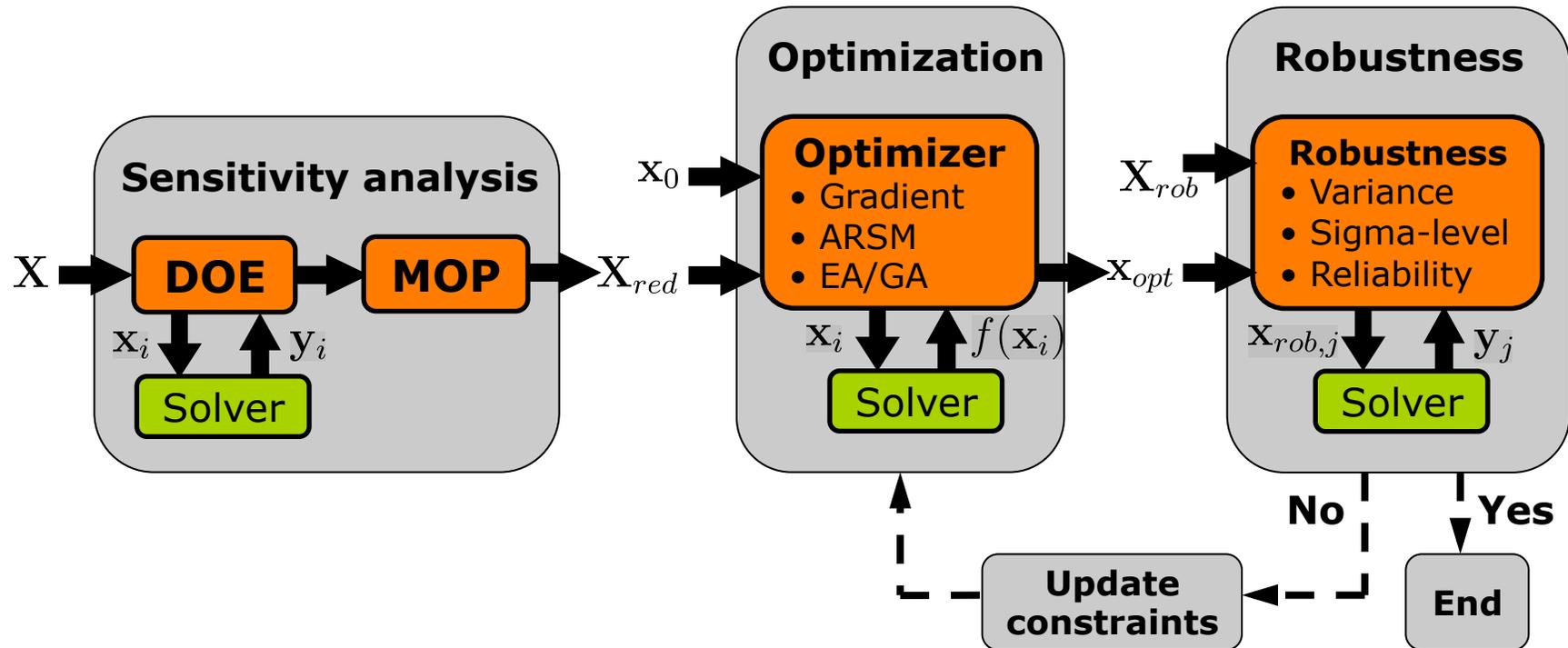
- **Iterative RDO** strategies using optimization steps with safety margins in the design space and checks of robustness in the space of scattering variables

**or**

- **Automatic (Loop in Loop) RDO** strategies estimating variance based or probability based measurements of variation for every candidate in the optimization space

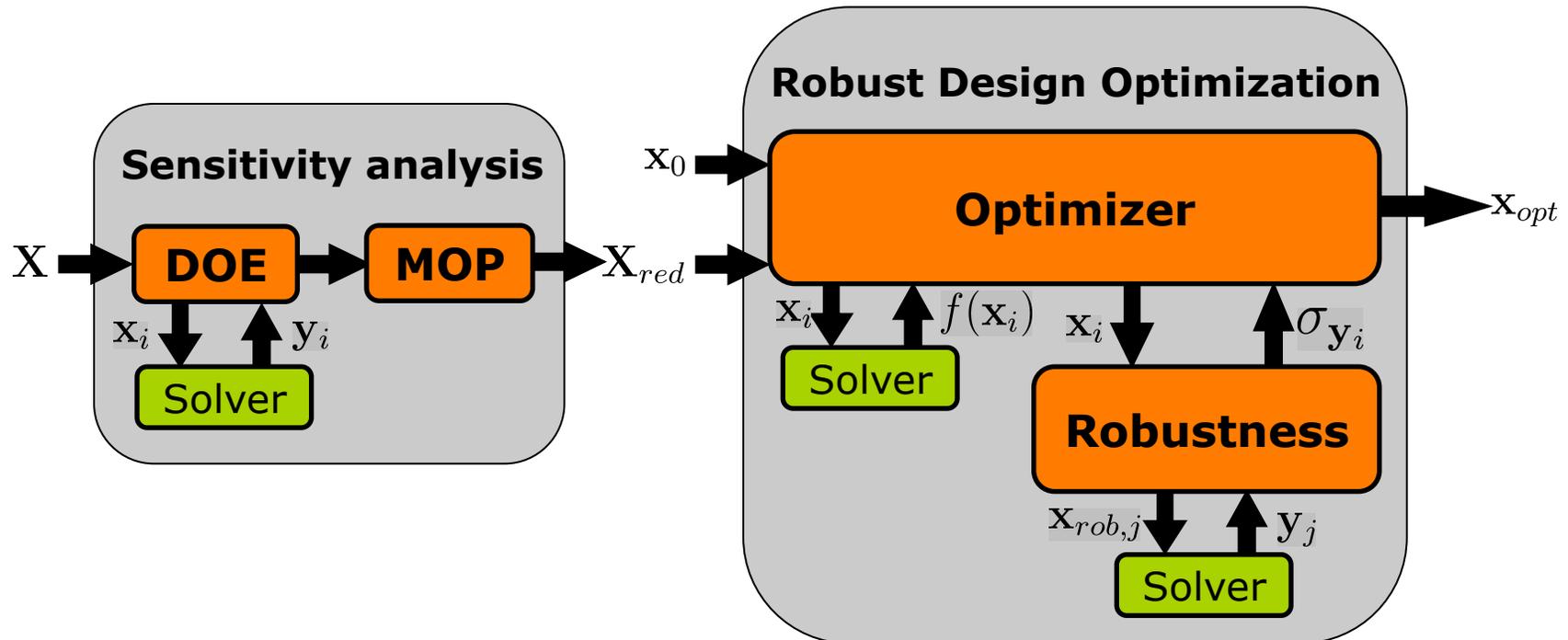
are possible RDO strategies.

## Iterative Robust Design Optimization



- Sensitivity analysis gives reduced optimization variable space  $X_{red}$
- Optimizer determines optimal design  $x_{opt}$  by direct solver calls
- Robustness evaluation
  - Robust optimum – end of iteration
  - Non-robust optimum - update constraints and repeat optimization + robustness evaluation

## Simultaneous Robust Design Optimization



- Sensitivity analysis gives reduced optimization variable space  $\mathbf{X}_{red}$
- Optimizer determines optimal design  $\mathbf{x}_{opt}$  by direct solver calls with simultaneous robustness evaluation for every design
- Each robustness evaluation determines robustness values by direct solver calls

## Benefits of Robustness Evaluation

- 1) Estimation of result variation:** By comparison of the variation with performance limits, we can answer the question: Is the design robust against expected material, environmental and test uncertainties? By comparison of the variation with test results, we can verify the variation prediction quality of the model.
- 2) Identify the most important input scatter which are responsible for the response scatter and quantify their influence.
- 3) Due to robustness evaluation, possible problems are identified early in the development process and design improvements are much cheaper than late in the development process.
- 4) Side effect: Validation of the modeling quality (quantification of numerical noise and identification of modeling errors)

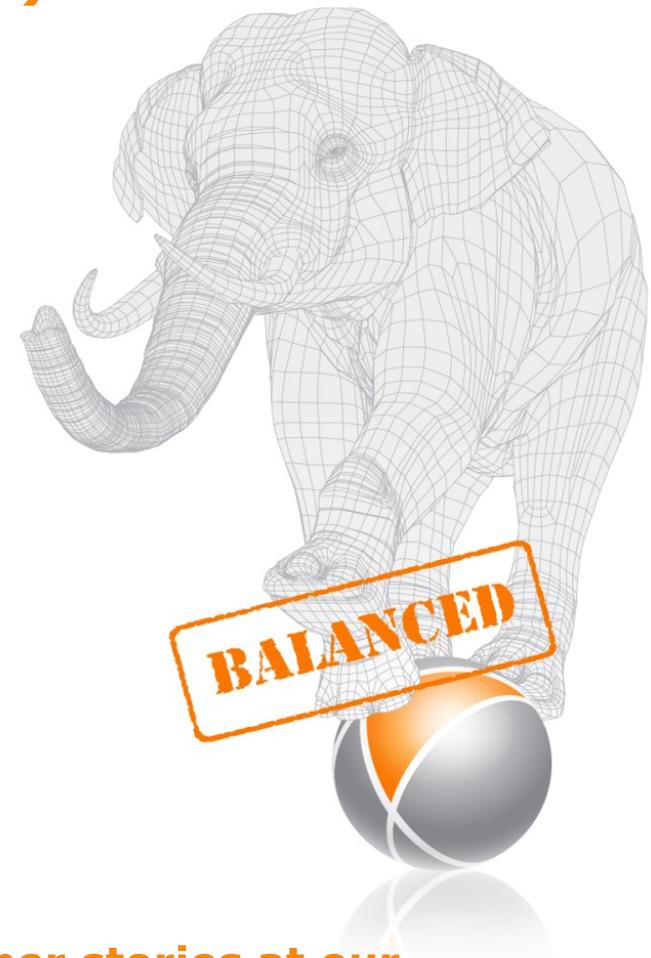
## Summary

- Highly optimized structures tend to loose robustness
- Variance-based robustness analysis can estimate sigma level
- Reliability analysis is necessary to proof small failure probabilities
- Use specific robustness/reliability measurements
- Stochastic analysis needs a balance between input definitions, stochastic analysis method and post processing
- Because all RDO strategies will try to minimize solver runs for robustness measures, a final proof of robustness/reliability is mandatory
- Carefully translation and introduction of material scatter is crucial
- Start with robustness evaluation, continue with iterative RDO approach using safety distances
- Iterative optimization/variance-based Robustness Evaluation with final reliability proof is often our method of choice

## Robust Design Optimization (RDO) in virtual product development

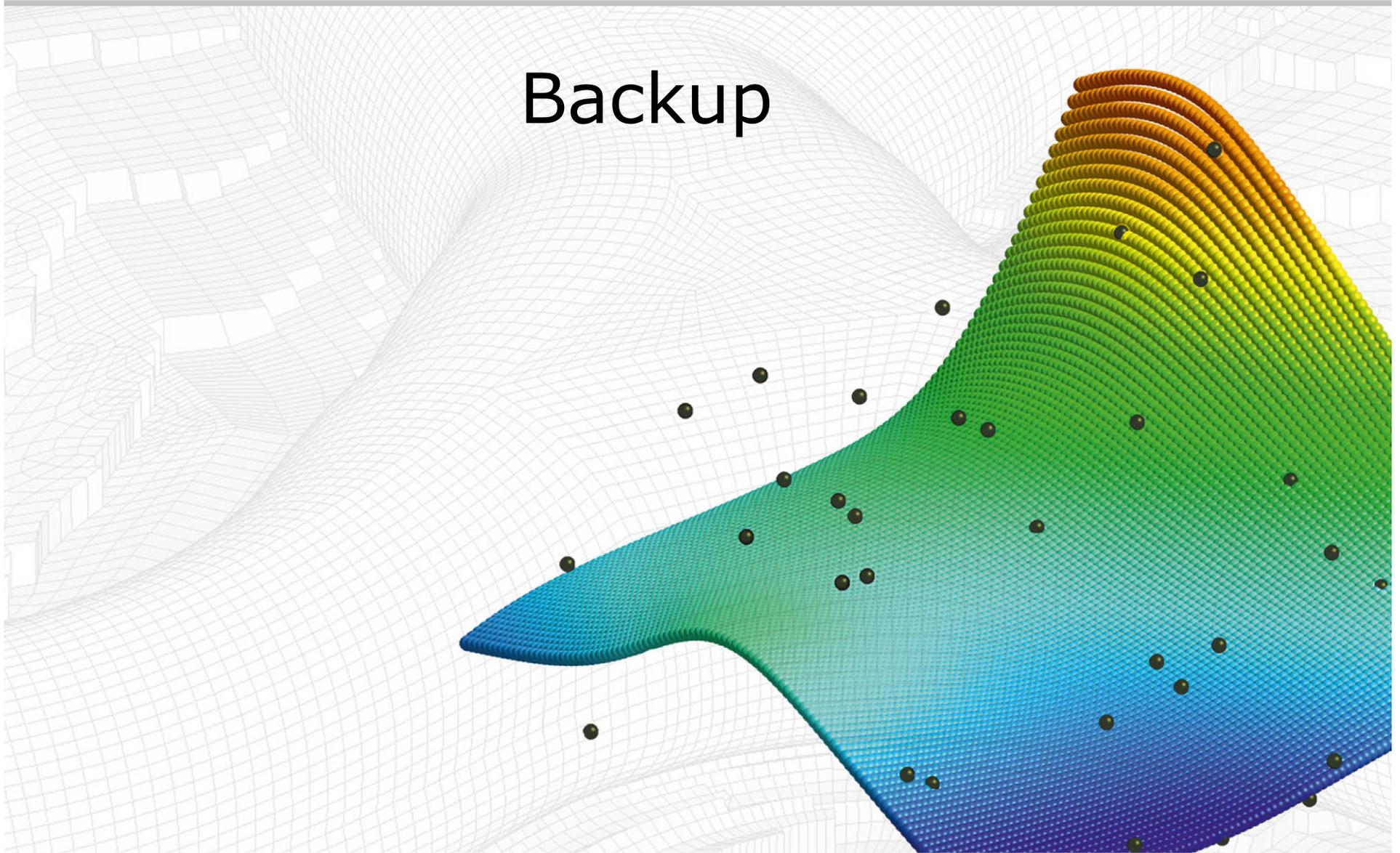
**optiSLang enables you to:**

- **Quantify risks**
- **Identify optimization potentials**
- **Adjust safety margins without limitation of input parameters**
- **Secure resource efficiency**
- **Improve product performance**
- **Save time to market**



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# Backup



## What's the Difference to Others?

### Methodology

- Sensitivity analysis and optimization for large (number of variables) non-linear problems
- COP/MOP to minimize the number of solver calls
- Optimization with robust defaults (ARSM, EA, GA, PARETO)
- Complete methodology suite to run robustness evaluation, reliability analysis and robust design optimization

### Key applications

- Optimization with large number of parameter (>10) having non linear effects, noise, design failure
- Model update and parameter identification using sensitivity study and optimization
- oS & SOS have completed the functionality for robustness evaluation, reliability analysis and robust design optimization to be used in production

**We do not just offer a tool, we deliver a process.**

**We are the ones who implemented RDO at different industries.**

## Our Customers agree

**BOSCH**

**DAIMLER**

**SIEMENS**



## Trash in , Trash out

- When asking the question about the influence of uncertainties we need to collect the best available knowledge about expected uncertainties with the best possible translation into the statistical definition of the CAE-model. Illustrating that with a translation to a stress calculation: Nobody would question that a reliable stress calculation can only be achieved by using a reliable value of the Young's modulus, otherwise the calculated stress value is not confident. The same question arises for the stochastic analysis itself. If we have no trustable information on the essential input uncertainties and no suitable approach to translate this information into adequate definitions of a set of scattering parameters, we should not perform a stochastic analysis. In such a case this analysis would lead to useless estimates of the variations, sensitivities etc.
- There is no reason that normal distribution is the "best" estimate. If we know only lower and upper bound uniform distribution is the best translation
- In sharp contrast to optimization task, the verification of product safety with a simplified robustness evaluation is only possible, if the unimportance of the neglected uncertain inputs is proven or their effect is covered sufficiently by safety factors.

## We need to define confident robustness measures

- By defining an RDO task measures of variation will be including into the optimization objectives and/or constraints. These statistical measures like mean value, standard deviation, higher order moments, safety margins or probabilities of exceeding a critical event are outcomes of the stochastic analysis.
- Note that all of these measures are estimates and their confidence has to be proven. This is similar to the verification of the mesh quality of a finite element analysis: the verification of that the variance estimates is necessary in order to trust in the predicted robustness of an investigated design. Everybody agrees that evaluating only 10 sample points will not lead to a confident assessment of a six sigma design. A six sigma design requires the proof that the probability of its failure is not larger than three out of a million realizations. 10 sample points are sufficient only to estimate roughly a mean value and a standard deviation, but the projection to a small event probability related to a six sigma design has an almost unpredictable large error.

## We need to define confident robustness measures

- At the same time it is a real challenge for any real world RDO problem to balance between the number of design runs spend for the estimation of the variation and the necessary accuracy of the robustness measures to drive the design in the right direction. Therefore, all RDO strategies need to estimate variation values with a minimal number of solver calls. To reach this goal, some methods make assumptions about the linearity of the problem or use response surface approximations spanned in the space of the scattering parameters whereby the final proof of robustness is of urgent need to proof the targeted robustness and reliability requirements.
- If the knowledge is vague about the importance of the uncertainties and their best available representation in a CAE model, a verification of the robustness at current product lines is strongly recommended before extrapolating robustness measures to future designs.

## RDO is not just a small extension of an optimization task

- Often, in marketing or scientific publications the RDO task is simplified by assuming that the robustness space as a subspace of the space spanned by the optimization parameters. The suggested RDO strategies based on this simplification allow to recycle solver runs from the optimization algorithms for the robustness evaluation and reduce the additional effort of RDO compared to deterministic optimization to a minimum. Unfortunately, for real world engineering applications outside the scatter of the optimization parameters also other important uncertain parameters like loading conditions or material properties have to be taken into account in order to obtain an engineering meaningful robustness assessment. As a consequence we often need to deal with different domains of the optimization and the robustness parameters. Thus usually design runs in the optimization domain cannot be recycled directly to estimate the robustness criteria and vice versa.

## RDO is not just a small extension of an optimization task

- Therefore, we should expect that substantial engineering robustness evaluations or RDO tasks always need to consider a significant amount of additional information for the input uncertainty, which will start with a large number of uncertain parameters and will need significant additional CPU requirements. Therefore, double checking of availability of the knowledge about the uncertainties and their best representation in an uncertainty model, the careful planning of a suitable algorithmic RDO workflow and careful checking of suitable measures for design robustness is recommended.
- Consequently, it is recommended to start with an **iterative RDO** approach using decoupled optimization and robustness steps including an initial sensitivity analysis in the domain of the optimization parameters as well as a subsequent sensitivity evaluation in the domain of uncertain parameters. This iterative approach helps to better understand the variable importance and the complexity of the RDO task in order to adjust the necessary safety margins. Only with this knowledge and if the iterative approach did not converge successfully a **simultaneous RDO** task should and can be defined.