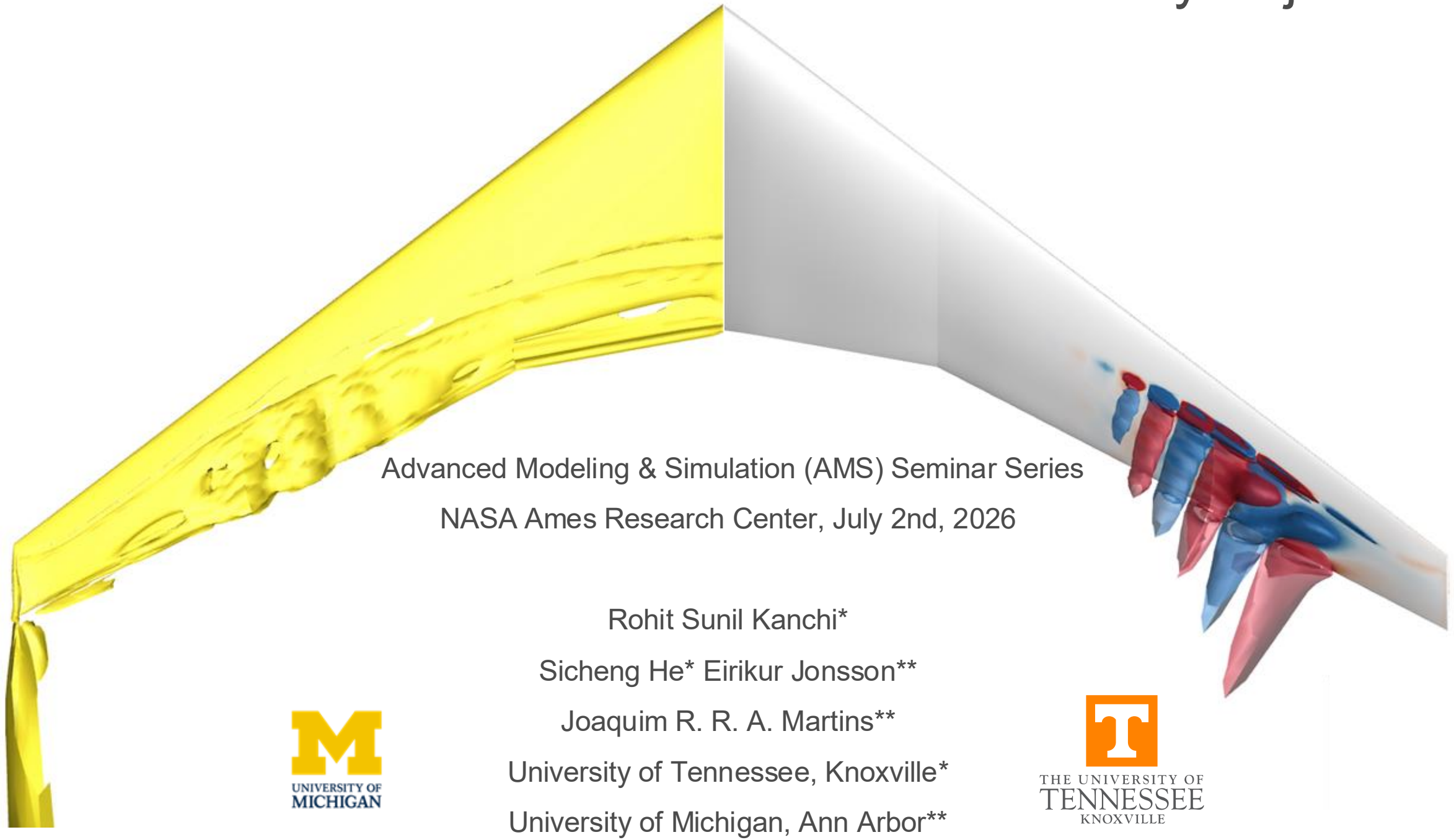
# Transonic Buffet Alleviation via Linear Stability Adjoint



Advanced Modeling & Simulation (AMS) Seminar Series

NASA Ames Research Center, July 2nd, 2026

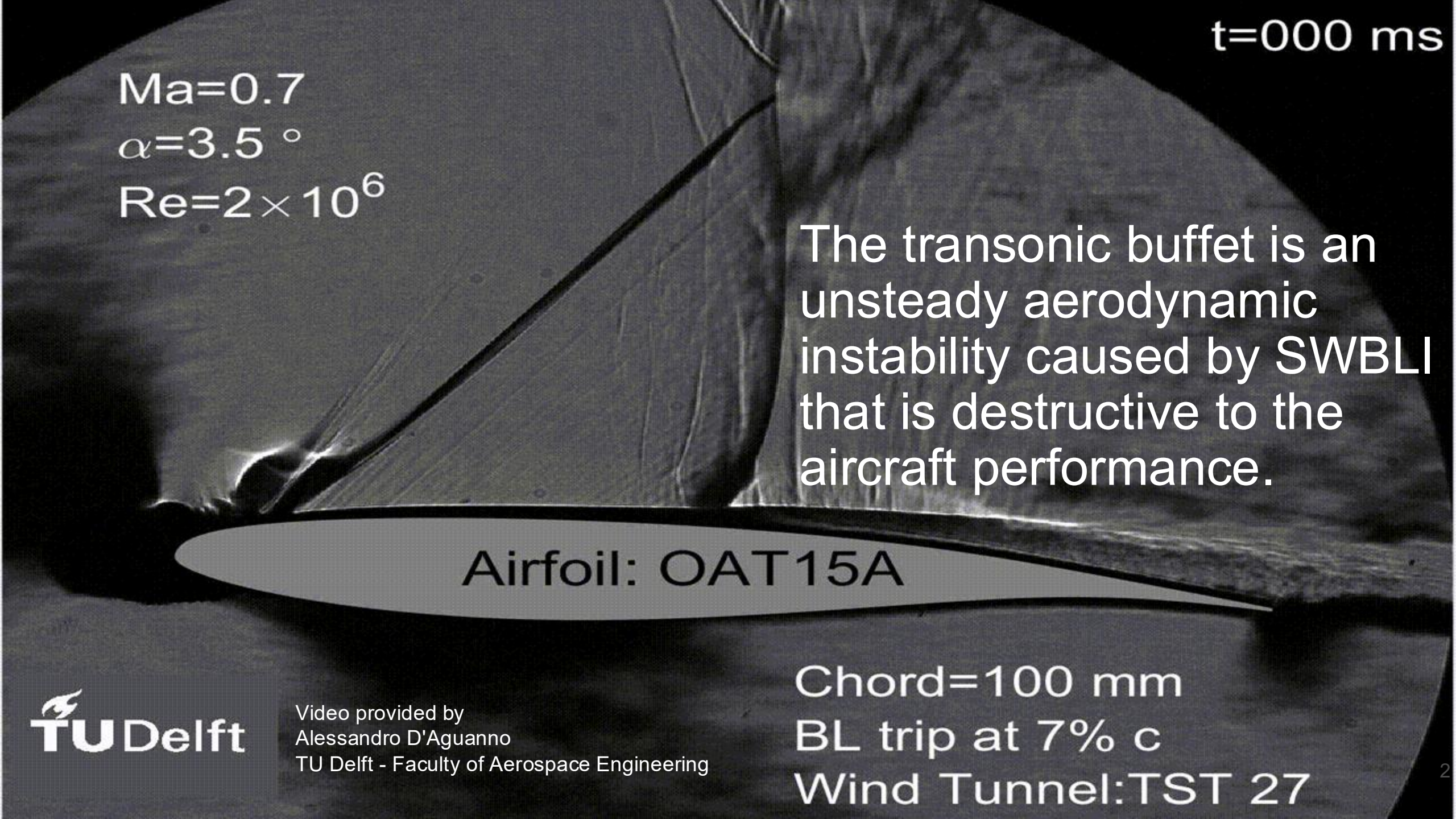

Rohit Sunil Kanchi*

Sicheng He* Eirikur Jonsson**

Joaquim R. R. A. Martins**

University of Tennessee, Knoxville*

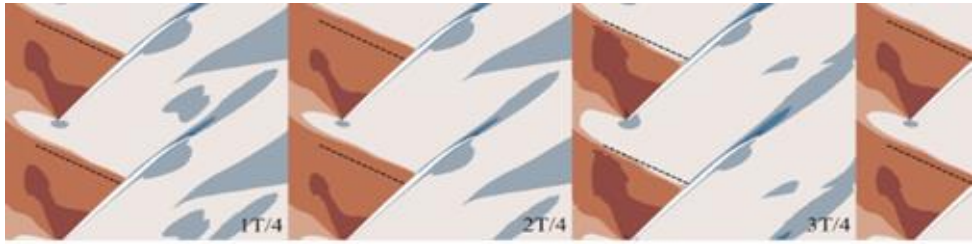
University of Michigan, Ann Arbor**



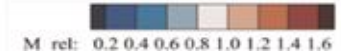

TU Delft

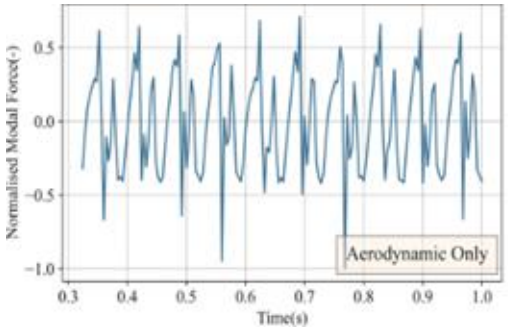
Video provided by
Alessandro D'Aguanno
TU Delft - Faculty of Aerospace Engineering

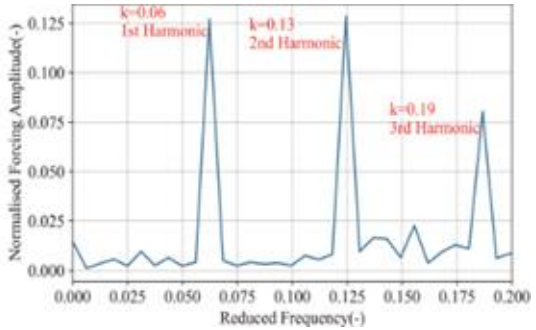
# Buffet affects transonic aircraft and turbomachinery design.

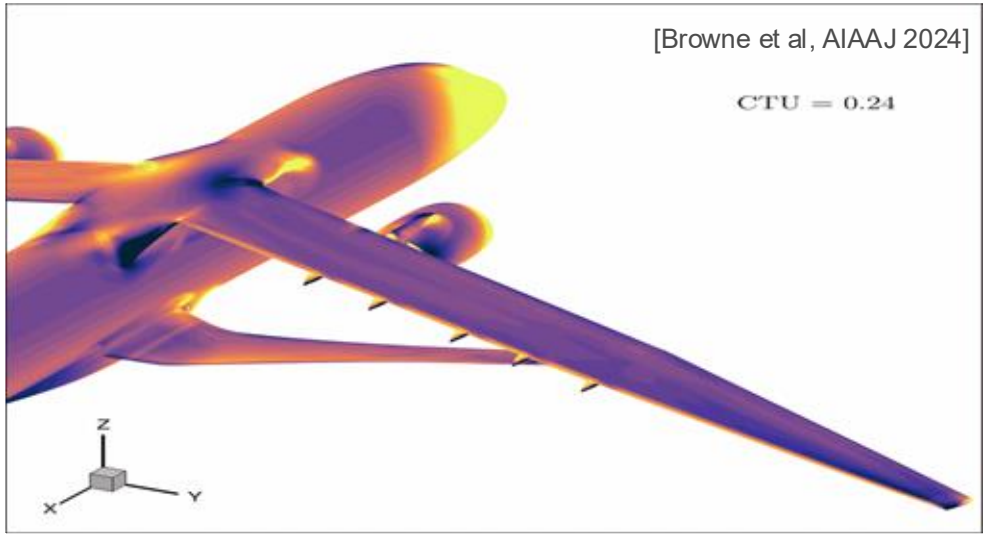


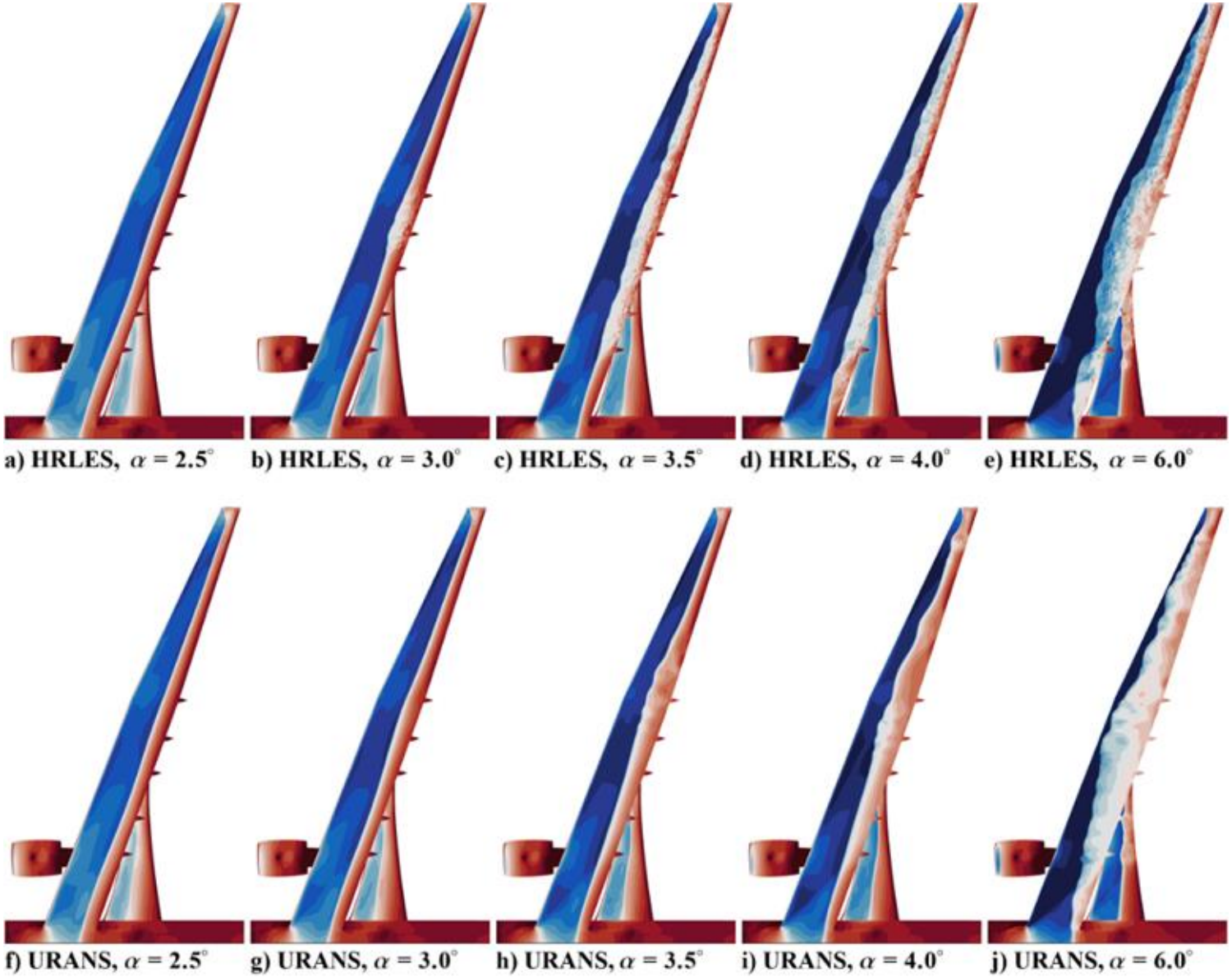

[Gong et al, GPPS 2025]



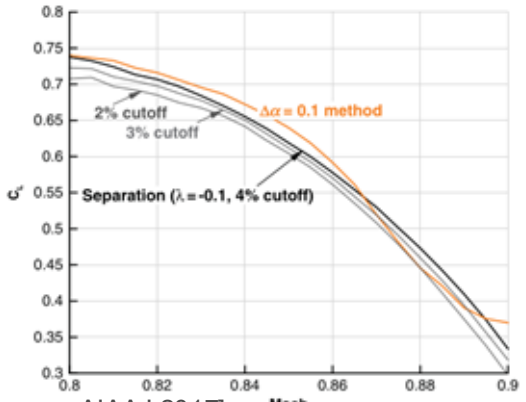




[Browne et al, AIAAJ 2024]

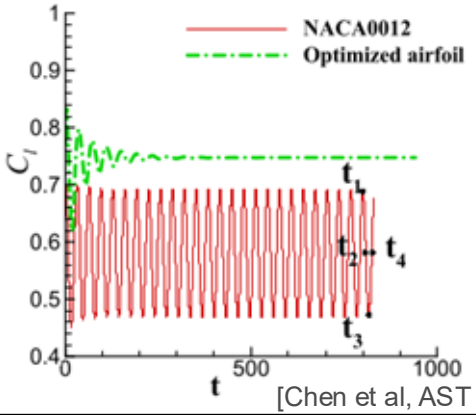
# Large scale buffet-constrained optimization remains limited by costly unsteady methods and indirect stability proxies.

## Flow separation metric
Very cheap ; Heuristic



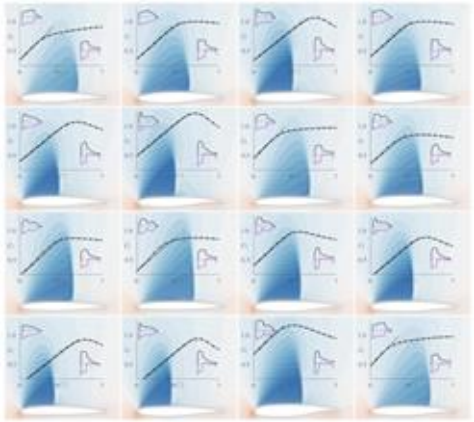

[Kenway, AIAAJ 2017]

## Time accurate simulation
Costly ; First principle



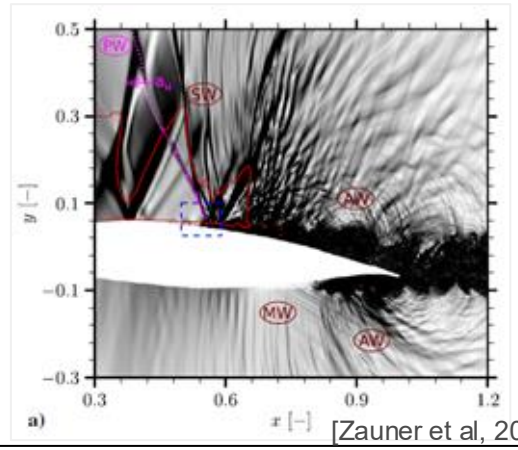

[Chen et al, AST 2022]

## Surrogate buffet constraint
Cheap ; Data-driven

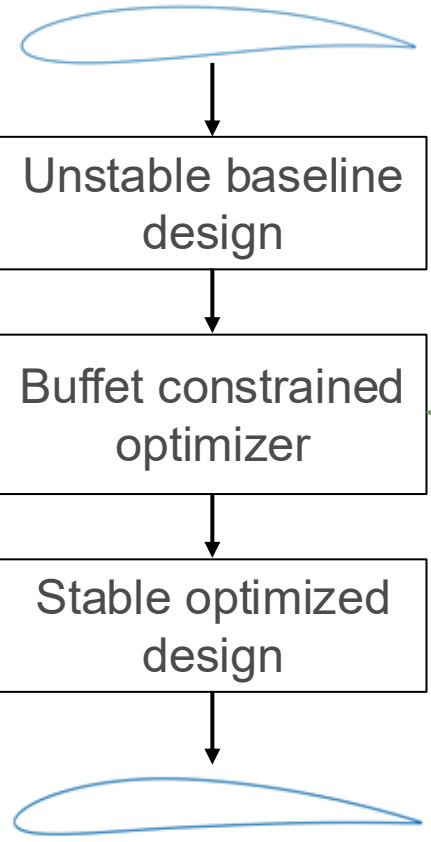


[Li et al, AIAAJ 2022]

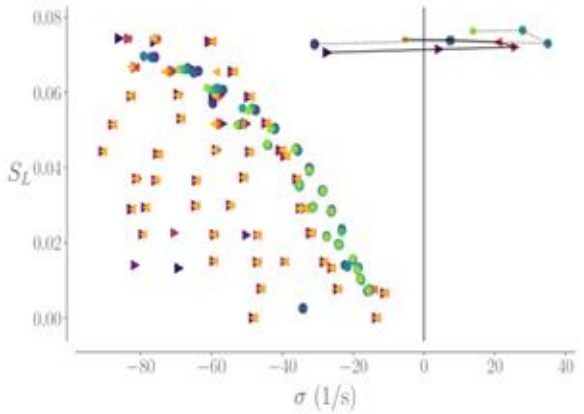
## LES/DNS methods
Very costly ; First principle




[Zauner et al, 2019]




## Linear stability constraint
Moderately cheap ; First principle




$$\frac{\partial \mathbf{w}}{\partial t} + \mathbf{r}(\mathbf{w};\mathbf{x}) = 0$$

$$-\frac{\partial \mathbf{r}}{\partial \mathbf{w}_0}(\mathbf{w}_0;\mathbf{x})\mathbf{q} = \lambda \mathbf{q}$$

# Content

## 1. Buffet simulation and linear stability theory

- Governing equations and linear stability theory
- LST eigenproblem computation methods
- Comparing LST with time accurate analyses

## 2. Adjoint based optimization

- Linear stability adjoint derivatives and verification
- Demonstration of shock control bump design with buffet constraints
- Demonstration of airfoil drag minimization with buffet constraints

## 3. Ongoing work - 3D NASA Common Research Model

- CFD results for the NASA CRM wing only configuration
- Modal sensitivity of the NASA CRM wing to geometric shape

# The linear stability constraint is derived from the primal URANS governing equations.

CFD governing equations

$$\frac{\partial \mathbf{u}(\mathbf{w})}{\partial \mathbf{w}} \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} + \mathbf{r}(\mathbf{w};\mathbf{x}) = \mathbf{0}$$

↓ Infinitesimal perturbation

$$\mathbf{w}(t) = \mathbf{w}_0 + \epsilon \mathbf{q} e^{\lambda t} + O(\epsilon^2)$$

↓ Linearize

$$\hat{\mathbf{r}}(\mathbf{v};\mathbf{w}_0,\mathbf{x}) = \mathbf{0}$$

LST governing equations

$$\mathbf{u}(\mathbf{w}) = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \\ \tilde{\nu} \end{bmatrix} \qquad \mathbf{r}(\mathbf{w};\mathbf{x}) = \begin{bmatrix} \mathbf{r}_{\rho}(\mathbf{w};\mathbf{x}) \\ \mathbf{r}_{\rho u}(\mathbf{w};\mathbf{x}) \\ \mathbf{r}_{\rho v}(\mathbf{w};\mathbf{x}) \\ \mathbf{r}_{\rho E}(\mathbf{w};\mathbf{x}) \\ \mathbf{r}_{\tilde{\nu}}(\mathbf{w};\mathbf{x}) \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{q}_r \\ \mathbf{q}_i \\ \lambda_r \\ \lambda_i \end{bmatrix} \qquad \hat{\mathbf{r}}(\mathbf{v};\mathbf{w}_0,\mathbf{x}) = \begin{bmatrix} \mathbf{J}\mathbf{q}_r + \lambda_r \mathbf{M}\mathbf{q}_r - \lambda_i \mathbf{M}\mathbf{q}_i \\ \mathbf{J}\mathbf{q}_i + \lambda_i \mathbf{M}\mathbf{q}_r + \lambda_r \mathbf{M}\mathbf{q}_i \\ \mathbf{q}_r^\top \mathbf{q}_r + \mathbf{q}_i^\top \mathbf{q}_i - 1 \\ \mathbf{e}_k^\top \mathbf{q}_i \end{bmatrix}$$

$$\lambda = \lambda_r + j\lambda_i$$

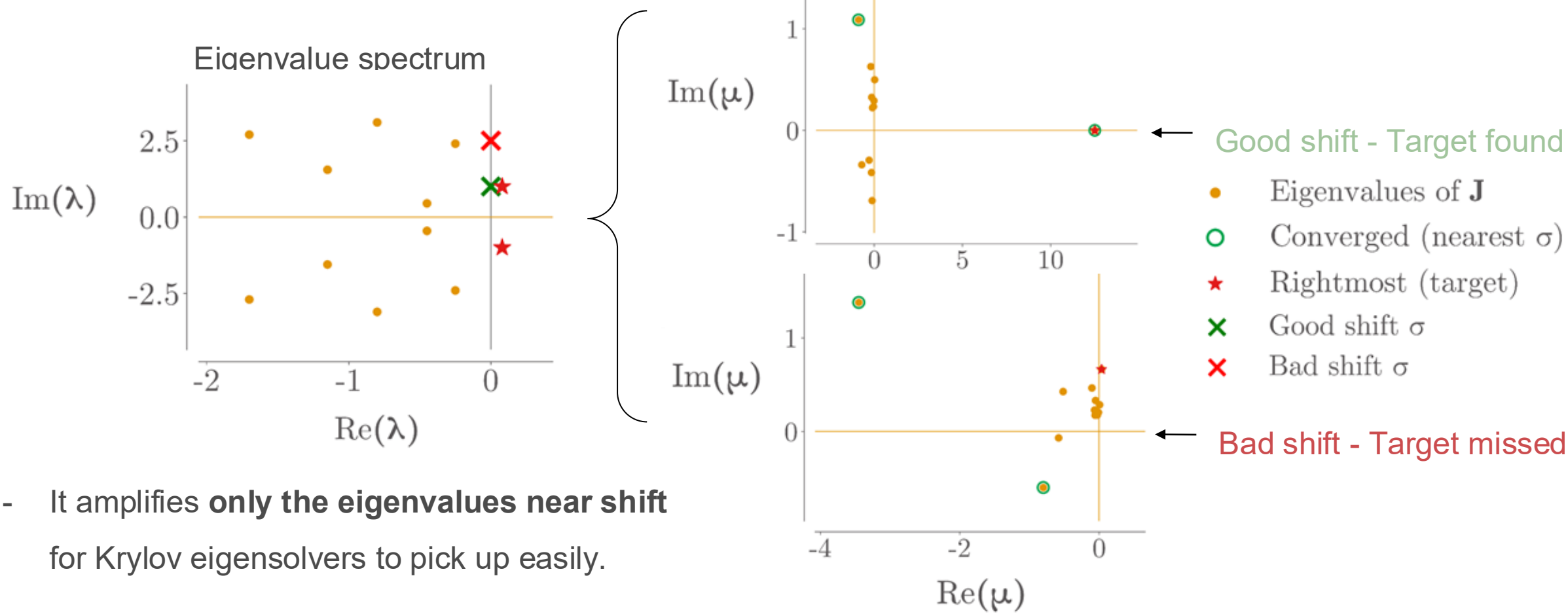
Growth rate: $\lambda_r$ — Frequency: $\lambda_i$

← **Unstable** equilibrium point $\mathfrak{Re}(\lambda) > 0$

→ **Stable** equilibrium point $\mathfrak{Re}(\lambda) < 0$

# Shift invert helps to find eigenvalues in region of shift, but fails to find the correct eigenvalue if not shifted correctly.

Define eigenvalue problem: $\mathbf{J}\mathbf{q} = \lambda\mathbf{q}$ → Shift and invert the operator: $(\mathbf{J} - \boldsymbol{\sigma}\mathbf{I})^{-1}\mathbf{q} = \frac{1}{\lambda - \boldsymbol{\sigma}}\mathbf{q}$ → The transformed eigenvalue is: $\lambda = \boldsymbol{\sigma} + \frac{1}{\mu}$




- It amplifies **only the eigenvalues near shift** for Krylov eigensolvers to pick up easily.

# Time stepper Arnoldi helps to converge to the unstable eigenvalues in one shot requiring no prior estimate of the eigenvalue location.

Define eigenvalue problem:

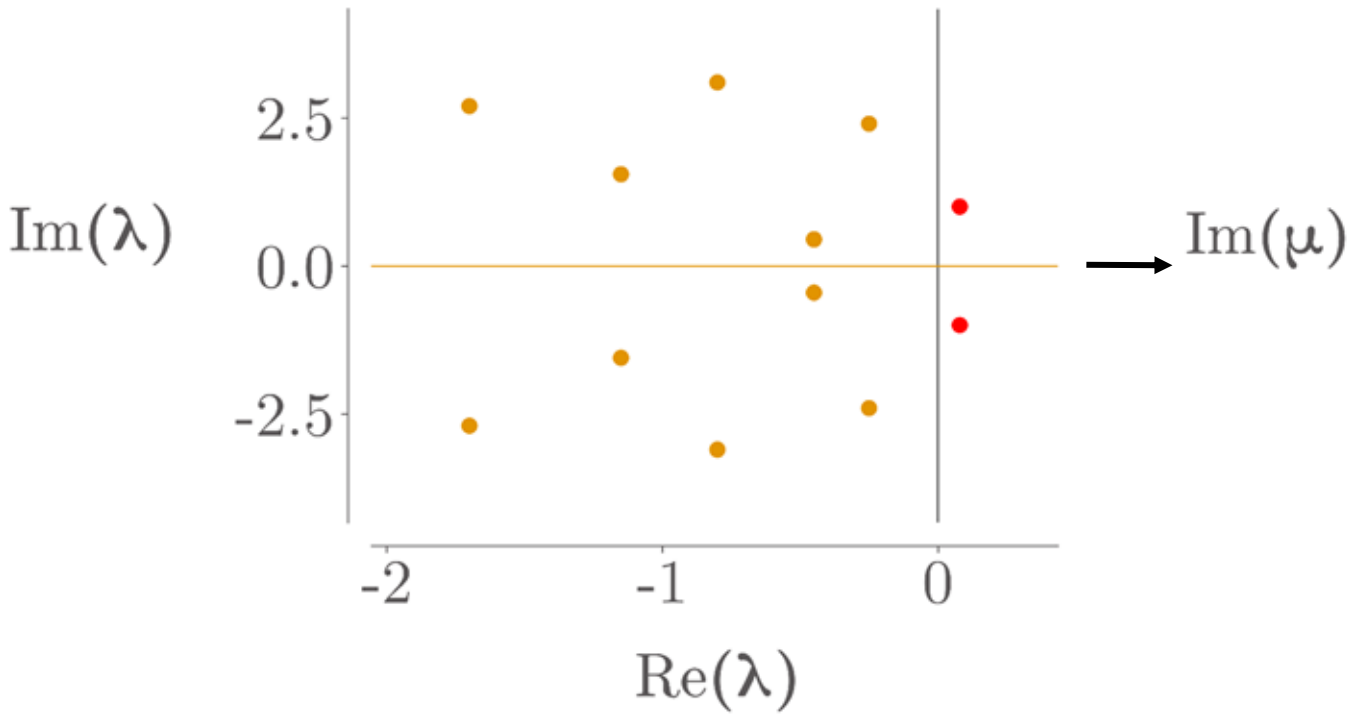
$$\mathbf{J}\mathbf{q} = \lambda\mathbf{q}$$

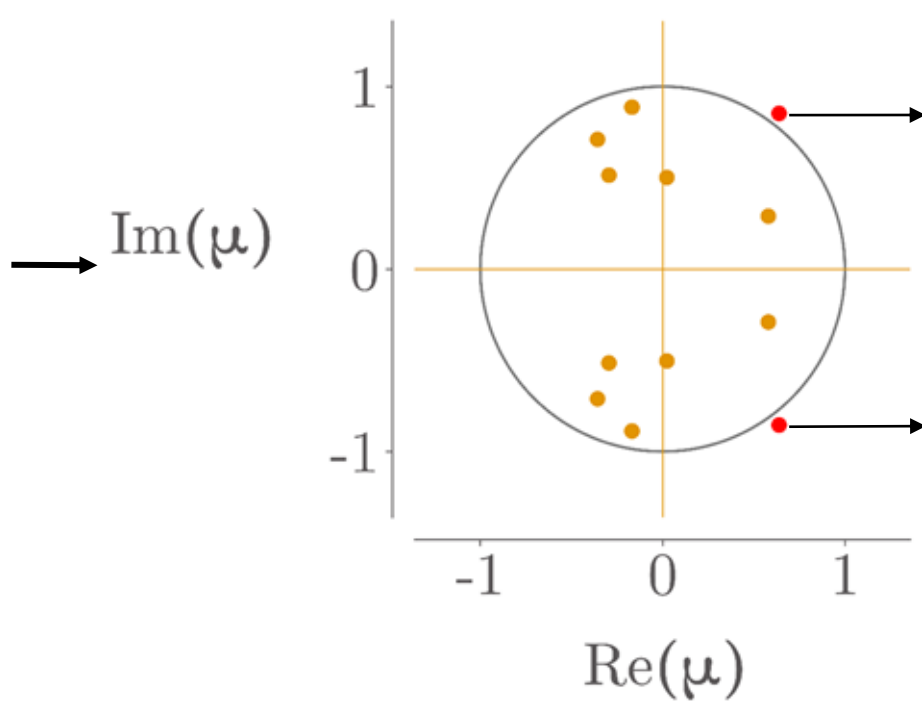
Crank-Nicolson propagator

$$\mathbf{T}(\Delta t) = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{J})(\mathbf{I} - \frac{\Delta t}{2}\mathbf{J})^{-1}$$

Mapped eigenvalue

$$\lambda = \frac{2}{\Delta t}\frac{\mu - 1}{\mu + 1}$$




Large $\mu$ magnitude

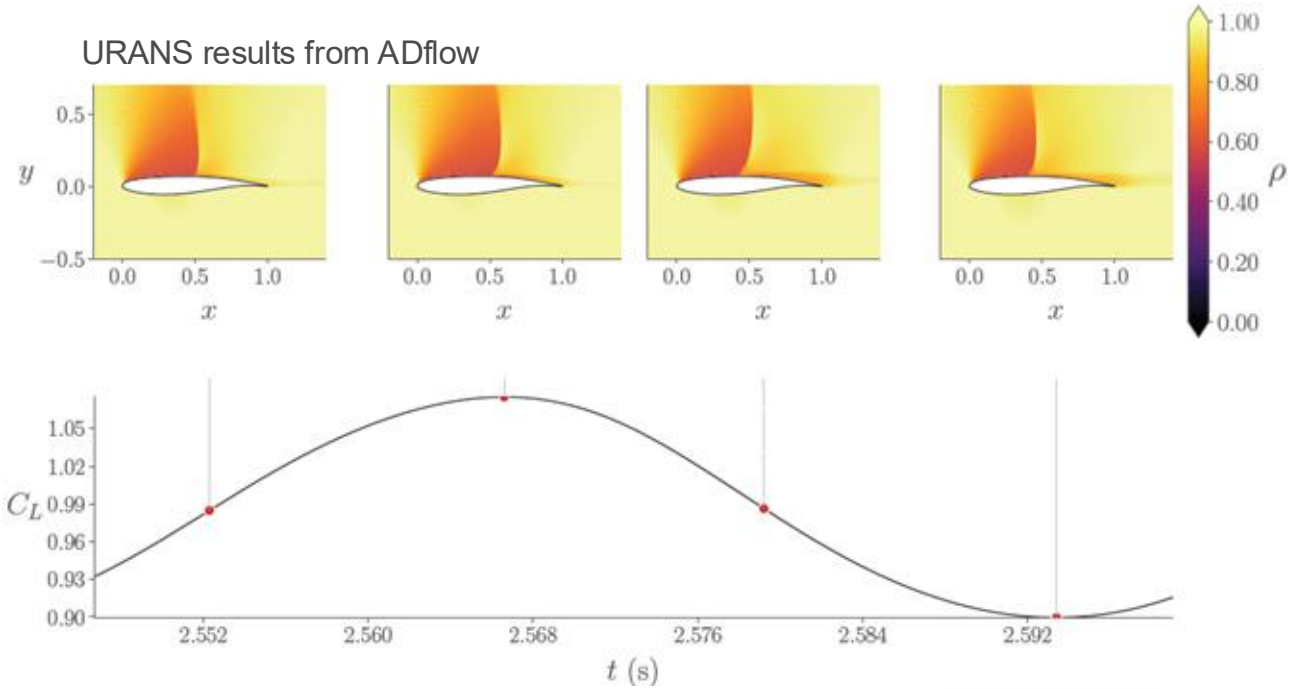
Large growth

Dominates Krylov subspace

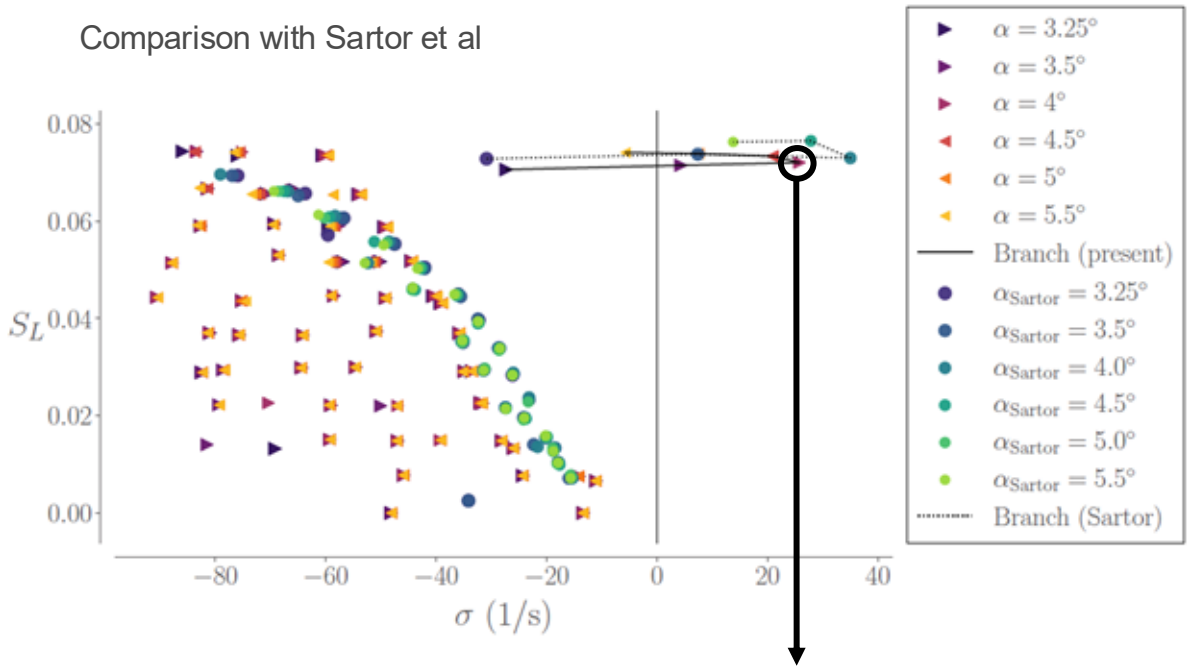
Instantly picked up by eigensolver

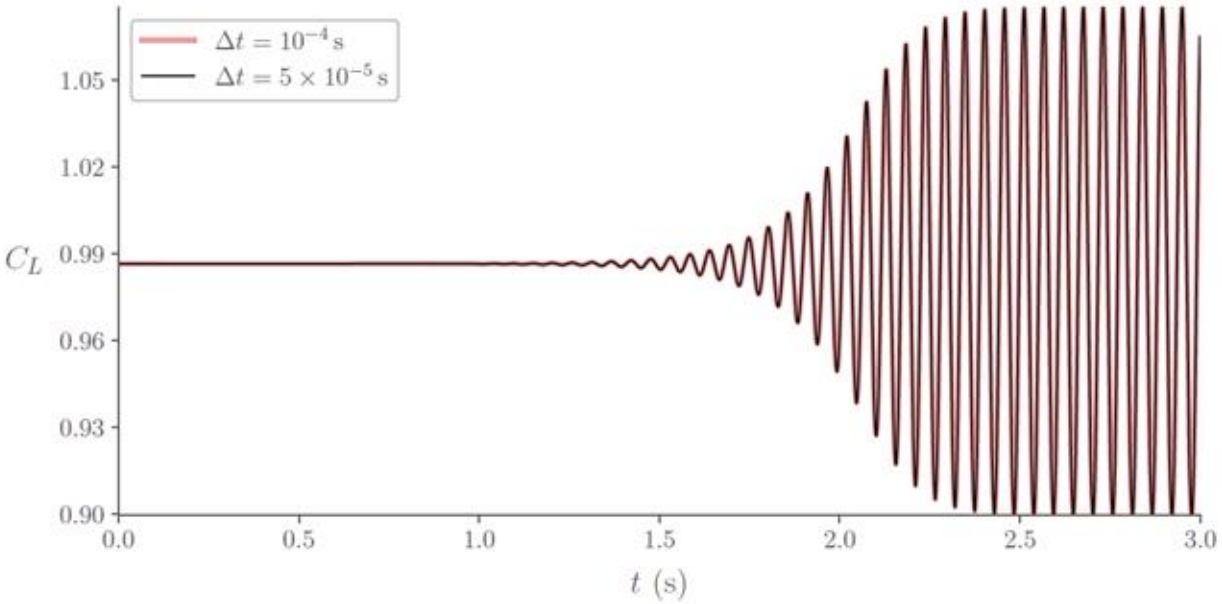
# URANS results agree well with the LST perturbation theory.

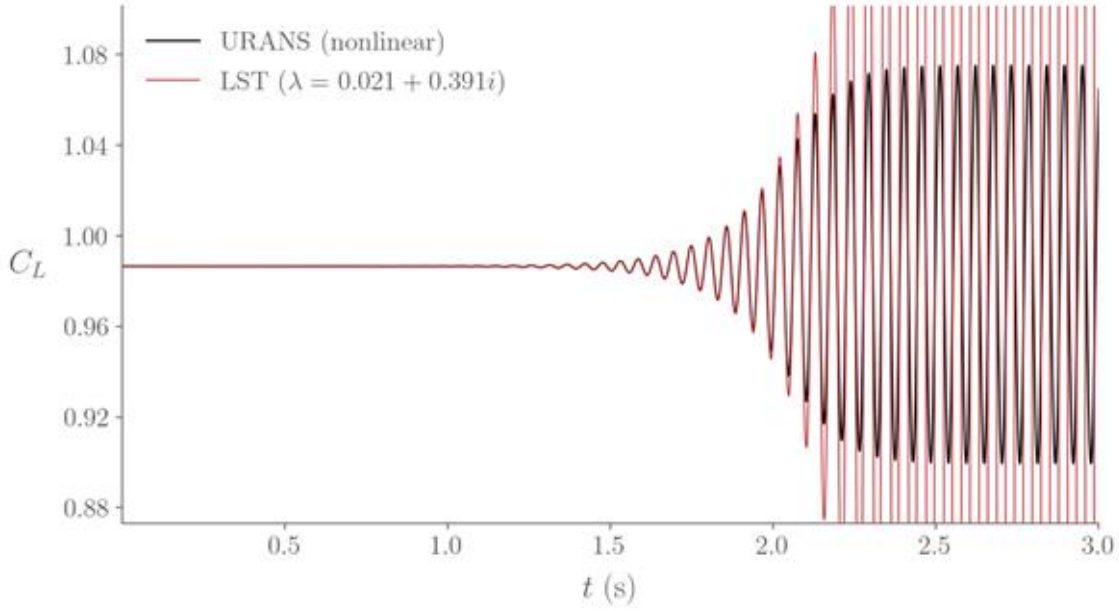
URANS results from ADflow




Comparison with Sartor et al



# Content

## 1. Buffet simulation and linear stability theory

- Governing equations and linear stability theory
- LST eigenproblem computation methods
- Comparing LST with time accurate analyses

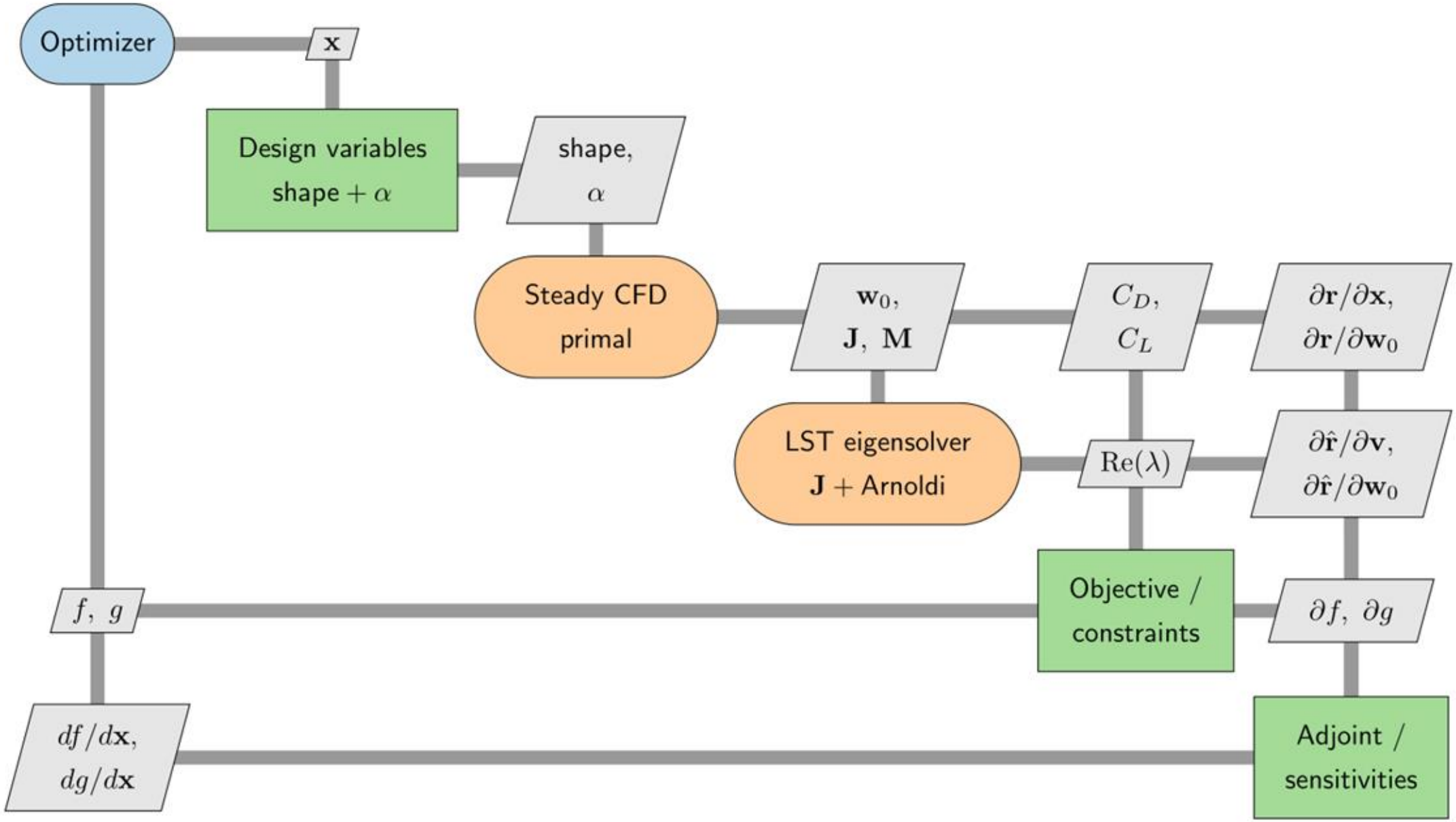
## 2. Adjoint based optimization

- Linear stability adjoint derivatives and verification
- Demonstration of shock control bump design with buffet constraints
- Demonstration of airfoil drag minimization with buffet constraints

## 3. Ongoing work - 3D NASA Common Research Model

- CFD results for the NASA CRM wing only configuration
- Modal sensitivity of the NASA CRM wing to geometric shape

# Design optimization framework



# Adjoint based method helps us to compute derivatives in a computationally tractable manner for the LST derivatives.

Design variables (Eg: shape, flight condition)

$$\mathbf{x}$$

CFD Solver

$$\mathbf{r}(\mathbf{w}_0, \mathbf{x}) = 0 \ , \ \mathbf{J} = \frac{\partial \mathbf{r}}{\partial \mathbf{w}_0}$$

Eigenvalue problem

$$\mathbf{J}\mathbf{q} = \lambda \mathbf{q}$$

$$\hat{\mathbf{r}}(\mathbf{v}; \mathbf{w_0}, \mathbf{x}) = 0$$

$$\mathbf{v} = [\mathbf{q}; \lambda]$$

Objective function

$$f = f(\mathbf{v}, \mathbf{w}_0; \mathbf{x})$$

Solve top adjoint

$$\left(\frac{\partial \hat{\mathbf{r}}}{\partial \mathbf{v}}\right)^{\mathsf{T}} \boldsymbol{\psi}_{\hat{\mathbf{r}}} = \left(\frac{\partial f}{\partial \mathbf{v}}\right)^{\mathsf{T}}$$

Solve bottom adjoint

$$\left(\frac{\partial \mathbf{r}}{\partial \mathbf{w}_0}\right)^{\mathsf{T}} \boldsymbol{\psi}_{\mathbf{r}} = \left(\frac{\partial \hat{\mathbf{r}}}{\partial \mathbf{w}_0}\right)^{\mathsf{T}} \boldsymbol{\psi}_{\hat{\mathbf{r}}}$$

Compute total derivative:

$$\frac{\mathrm{d}f}{\mathrm{d}\mathbf{x}} = -\boldsymbol{\psi}_r^{\mathsf{T}} \frac{\partial \mathbf{r}}{\partial \mathbf{x}} - \boldsymbol{\psi}_{\hat{r}}^{\mathsf{T}} \frac{\partial \hat{\mathbf{r}}}{\partial \mathbf{x}}$$

We have an analytic form for the top adjoint if: $f = \Re(\lambda)$.

The top adjoint is then:

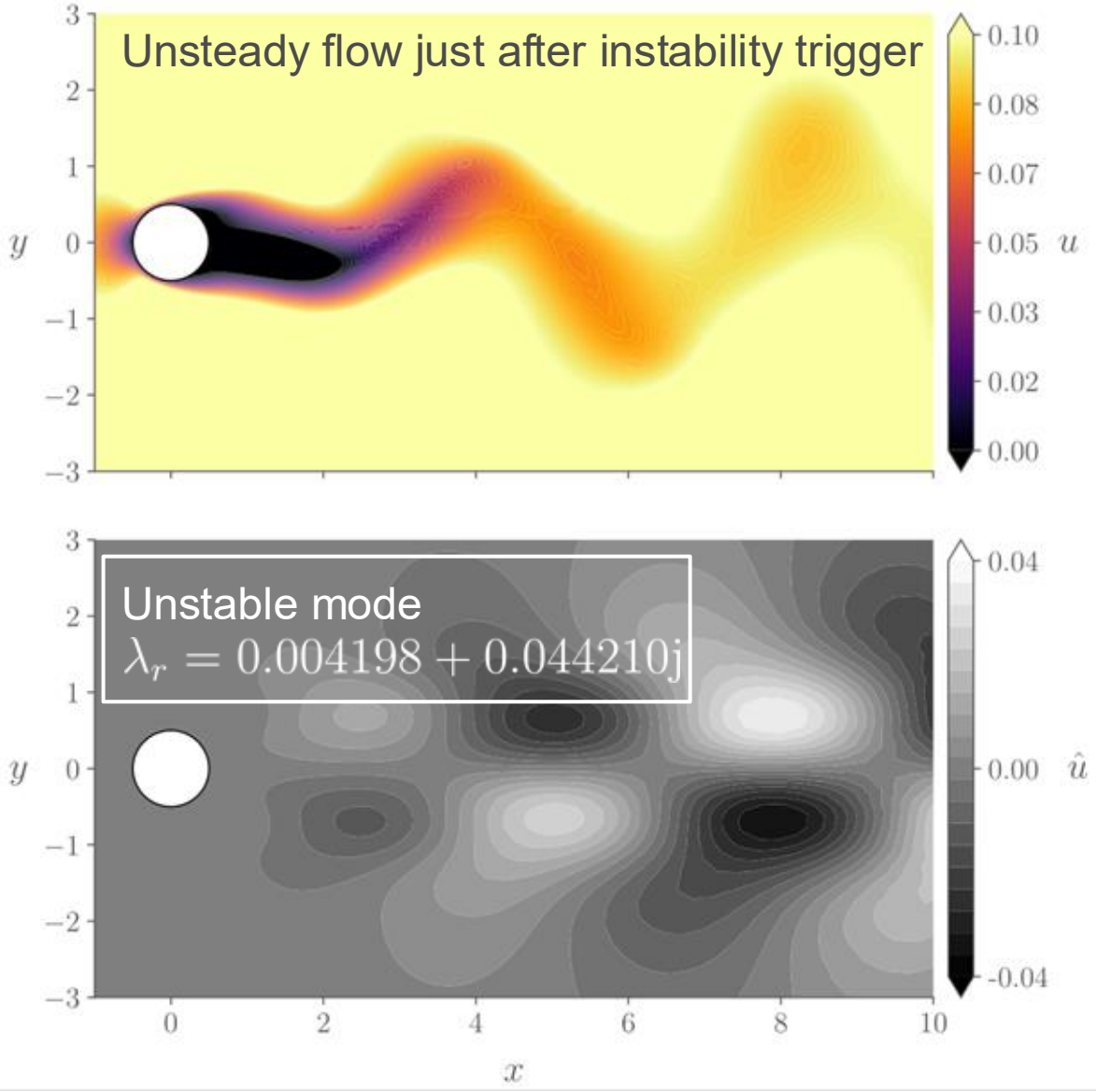
$$\boldsymbol{\psi}_{\hat{\mathbf{r}}} = \begin{bmatrix} \mathbf{u}_r^{\mathsf{T}} & \mathbf{u}_i^{\mathsf{T}} & 0 & 0 \end{bmatrix}$$

$$\mathbf{J}^{\mathsf{T}}\mathbf{u} = \lambda^* \mathbf{u}$$

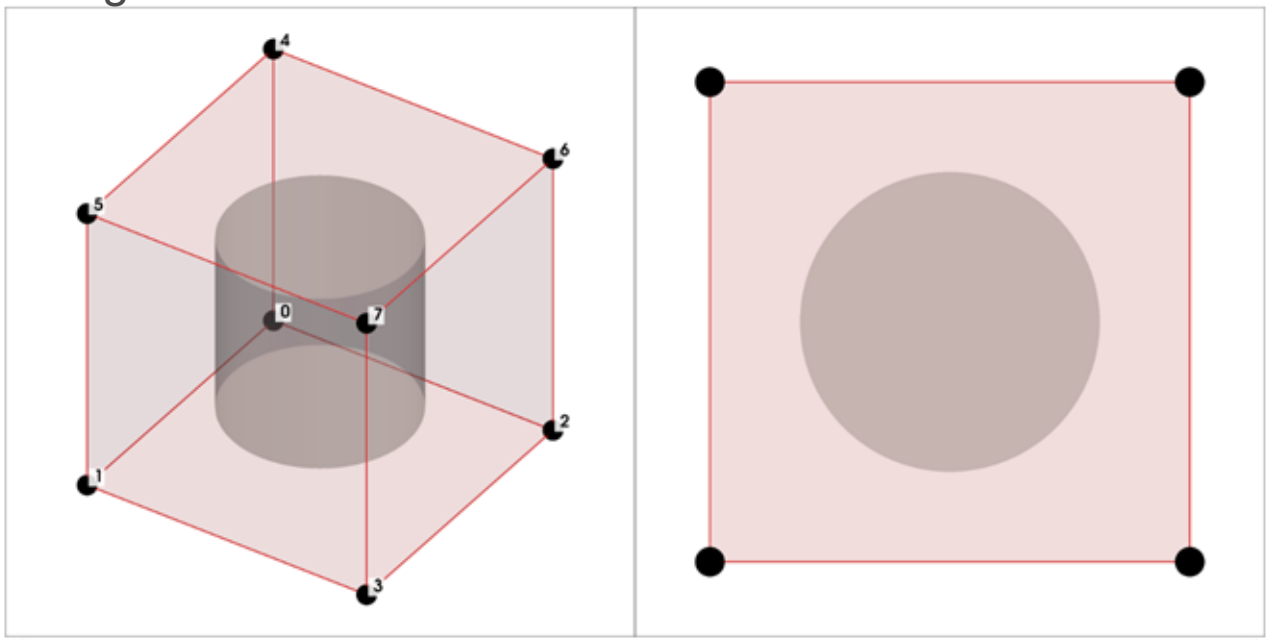
$$\mathbf{u}^*\mathbf{q} = -1$$

[He et al, MSSP 2023]

# The linear stability adjoint derivatives agree with the finite difference derivatives.




Design variables are the vertices of the FFD box




| Component $i$ | Adjoint $\partial\lambda_r/\partial x_i$ | FD $\partial\lambda_r/\partial x_i$ | Relative difference |
|---|---|---|---|
| 0 | $-1.18147 \times 10^{-3}$ | $-1.18143 \times 10^{-3}$ | $3.40 \times 10^{-5}$ |
| 1 | $-8.02607 \times 10^{-4}$ | $-8.02640 \times 10^{-4}$ | $4.19 \times 10^{-5}$ |
| 2 | $+1.18159 \times 10^{-3}$ | $+1.18163 \times 10^{-3}$ | $3.38 \times 10^{-5}$ |
| 3 | $+8.02458 \times 10^{-4}$ | $+8.02424 \times 10^{-4}$ | $4.29 \times 10^{-5}$ |
| 4 | $-1.13997 \times 10^{-3}$ | $-1.13994 \times 10^{-3}$ | $2.88 \times 10^{-5}$ |
| 5 | $-8.39036 \times 10^{-4}$ | $-8.39075 \times 10^{-4}$ | $4.64 \times 10^{-5}$ |
| 6 | $+1.13984 \times 10^{-3}$ | $+1.13987 \times 10^{-3}$ | $2.94 \times 10^{-5}$ |
| 7 | $+8.38973 \times 10^{-4}$ | $+8.38931 \times 10^{-4}$ | $5.00 \times 10^{-5}$ |

# Content

## 1. Buffet simulation and linear stability theory

- Governing equations and linear stability theory
- LST eigenproblem computation methods
- Comparing LST with time accurate analyses

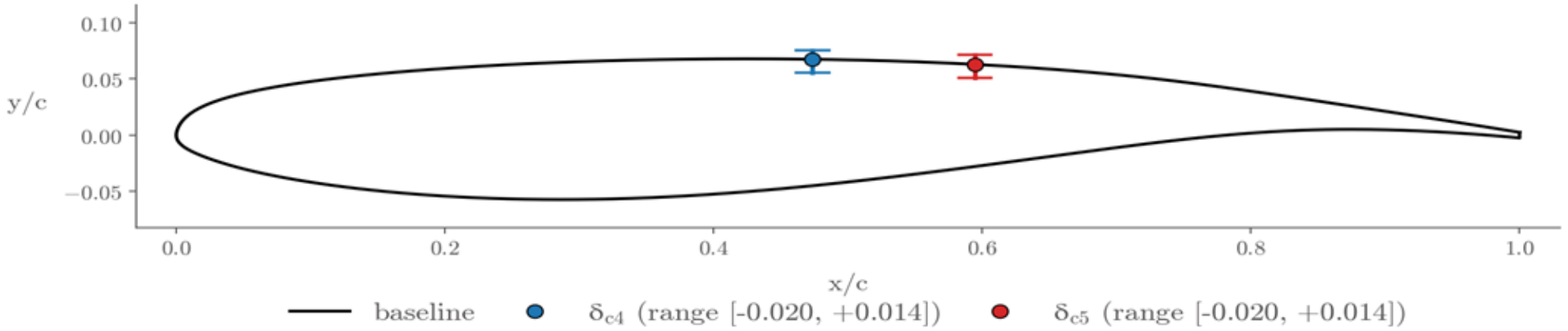
## 2. Adjoint based optimization

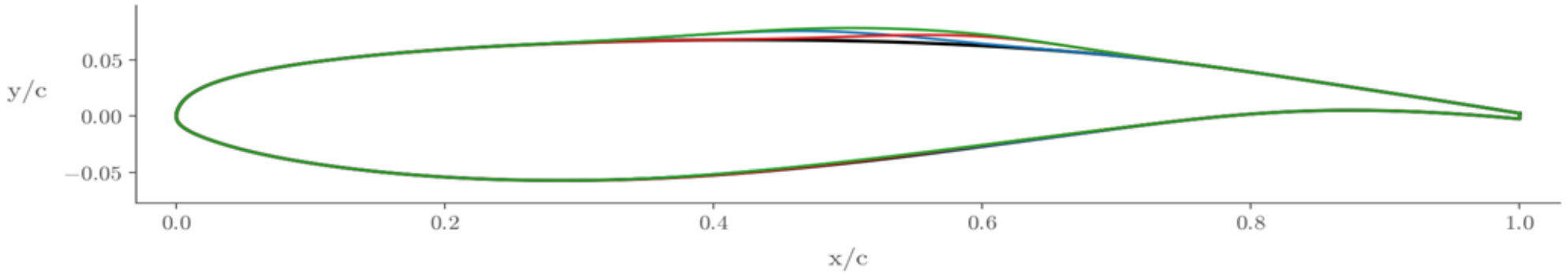
- Linear stability adjoint derivatives and verification
- Demonstration of shock control bump design with buffet constraints
- Demonstration of airfoil drag minimization with buffet constraints

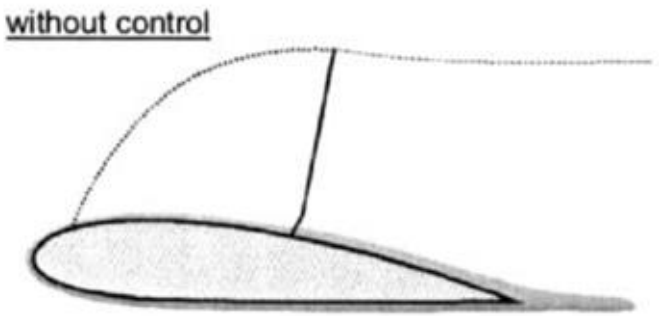
## 3. Ongoing work - 3D NASA Common Research Model

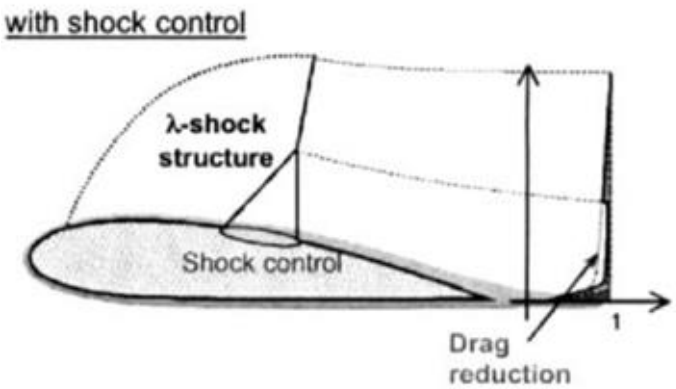
- CFD results for the NASA CRM wing only configuration
- Modal sensitivity of the NASA CRM wing to geometric shape

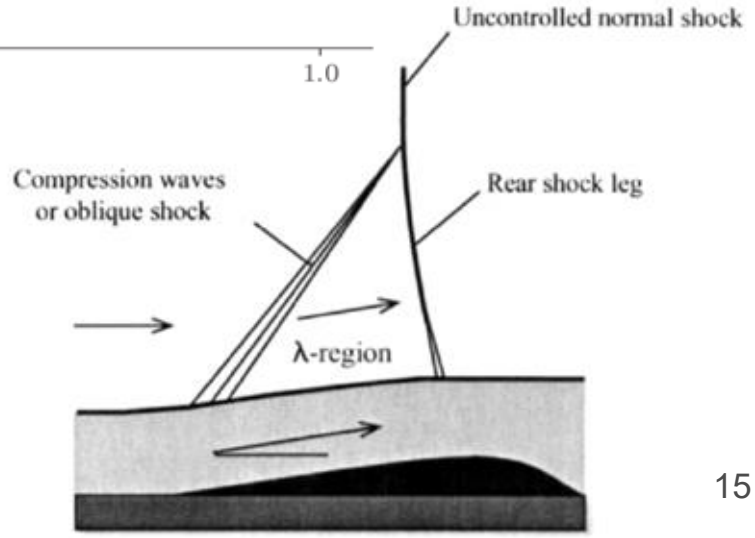
# Shock control bump design could be explored through buffet constrained optimization methods.



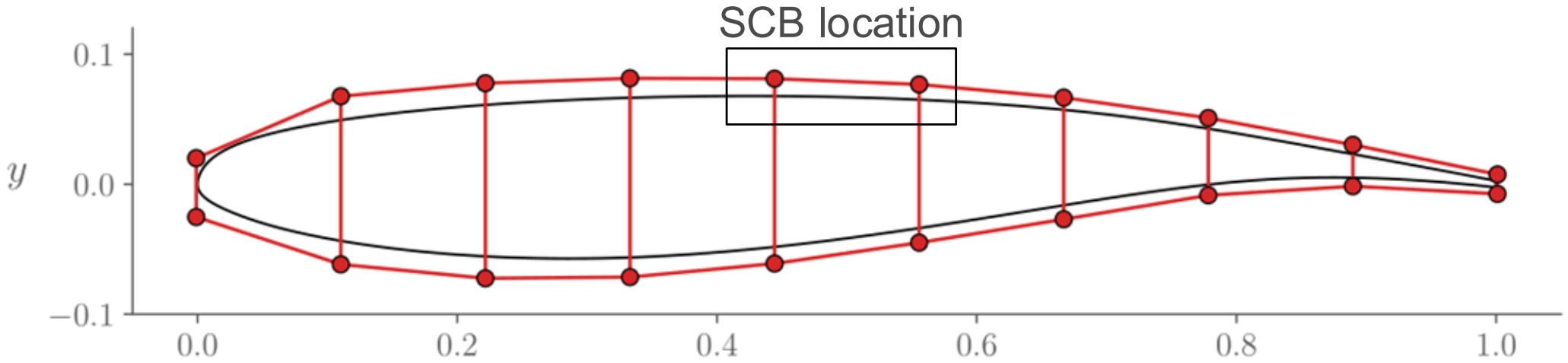



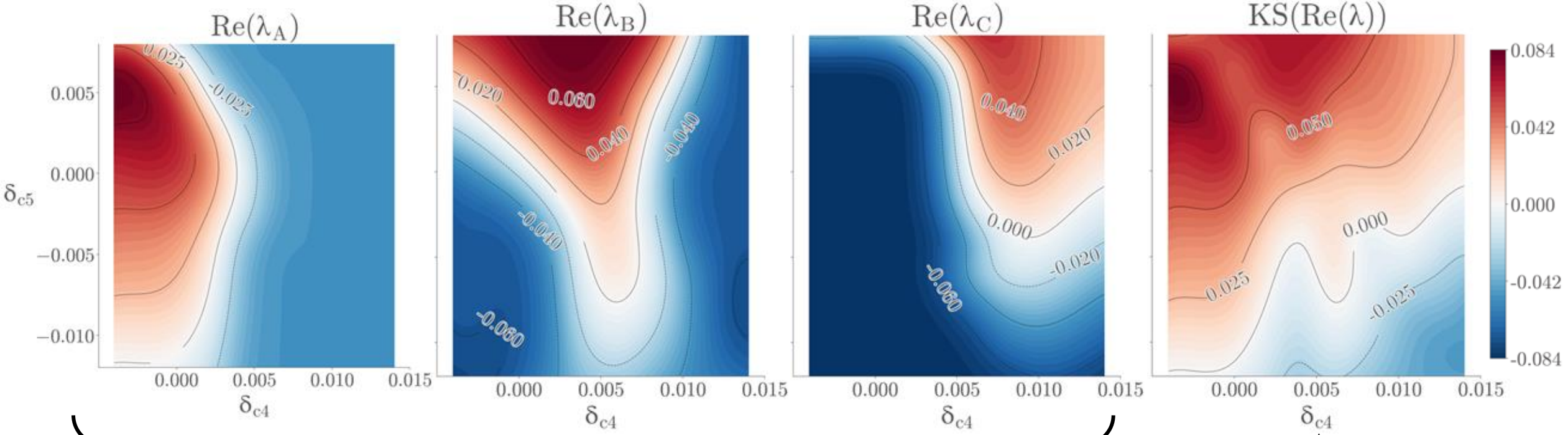

Babinsky and Harvey, Cambridge University Press, 2012

# The buffet constrained optimization. We setup two optimizations here - with and without the buffet constraint.

| Quantity | Specification |
|---|---|
| Objective | minimize $C_D$ |
| Design variables | 2 FFD shape DVs ($\delta_{c4}, \delta_{c5}$ at upper–surface chord stations $i = 4, 5$), $y$–displacements, bounds $[-0.02, +0.014]$, DV scale 20; angle of attack fixed at $\alpha = 4°$ |
| Lift constraint | $C_L = C_L^* >= 0.915$ |
| Buffet constraint | (a) none; (b) KS(Re($\lambda$)) $\leq 0$; |
| Optimizer | SLSQP via pyOptSparse, acc $= 10^{-6}$, maxiter $= 500$ |



# Mode swapping can cause non-smoothness. KS aggregation mitigates this issue by smoothening the constraint.




$$\mathrm{KS}_\rho(\lambda_{r,1},\ldots,\lambda_{r,N}) \coloneqq g_{\max} + \frac{1}{\rho}\ln\sum_{i=1}^{N} e^{\rho(\lambda_{r,i}-g_{\max})}$$

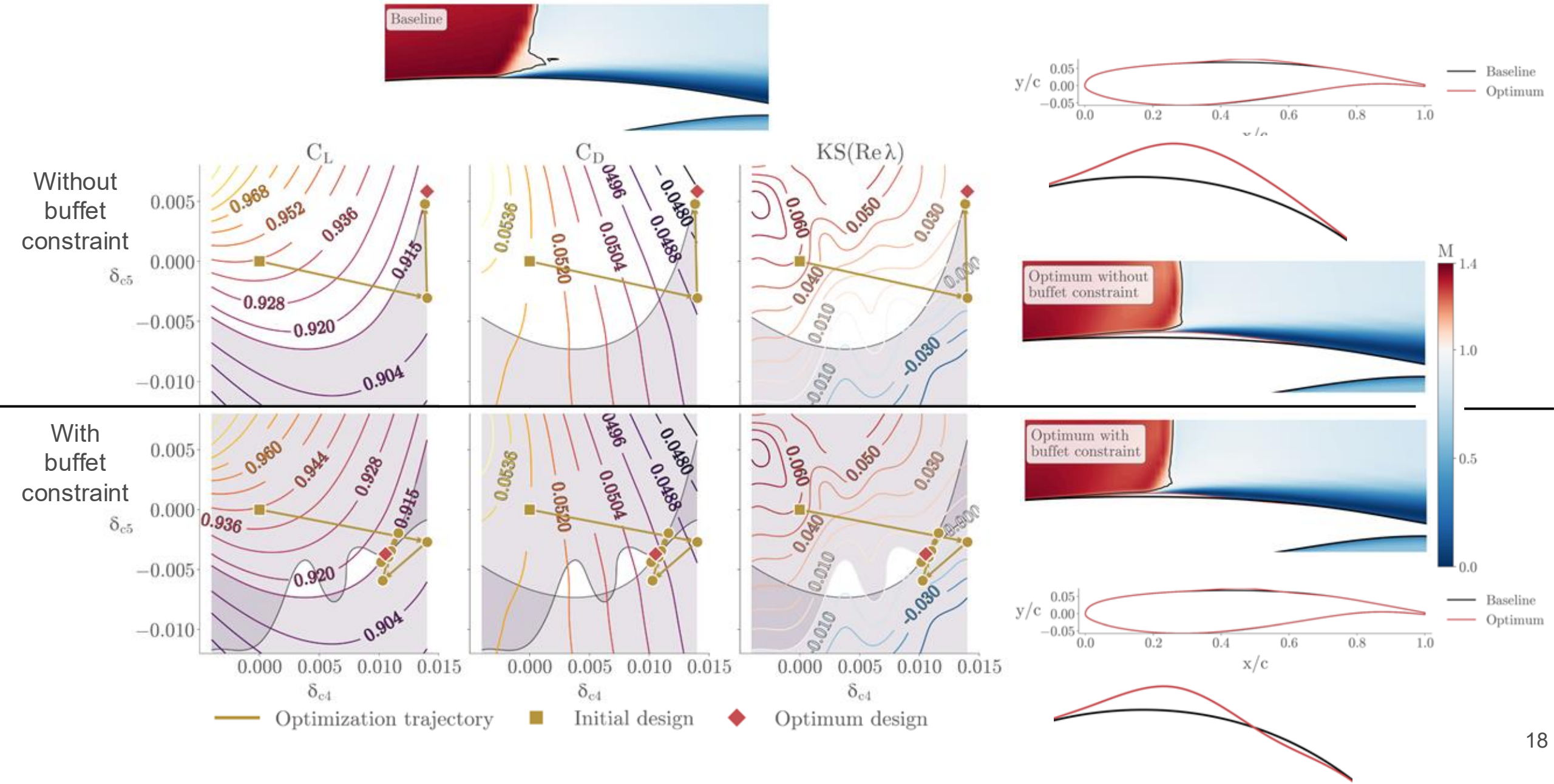
$$g_{\max} \coloneqq \max_i \lambda_{r,i}$$

$$\frac{\mathrm{d}\,\mathrm{KS}_\rho}{\mathrm{d}\mathbf{x}} = \sum_{i=1}^{N} w_i \frac{\mathrm{d}\lambda_{r,i}}{\mathrm{d}\mathbf{x}}, \qquad w_i \coloneqq \frac{e^{\rho(\lambda_{r,i}-g_{\max})}}{\sum_{j=1}^{N} e^{\rho(\lambda_{r,j}-g_{\max})}}$$

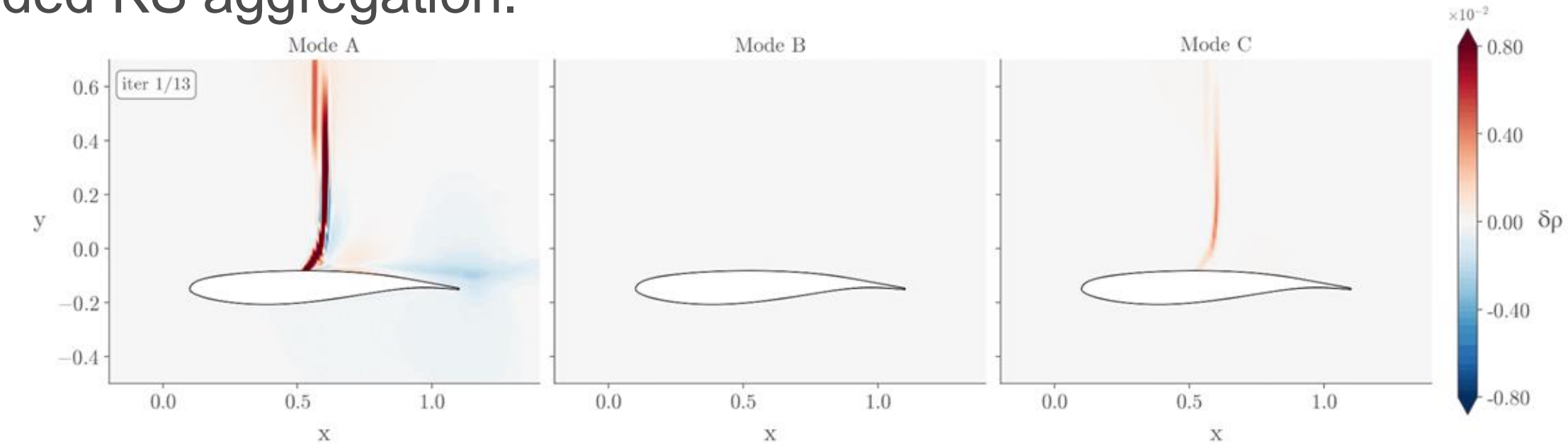
We used Modal assurance criterion (MAC) for A,B and C mode tracking:

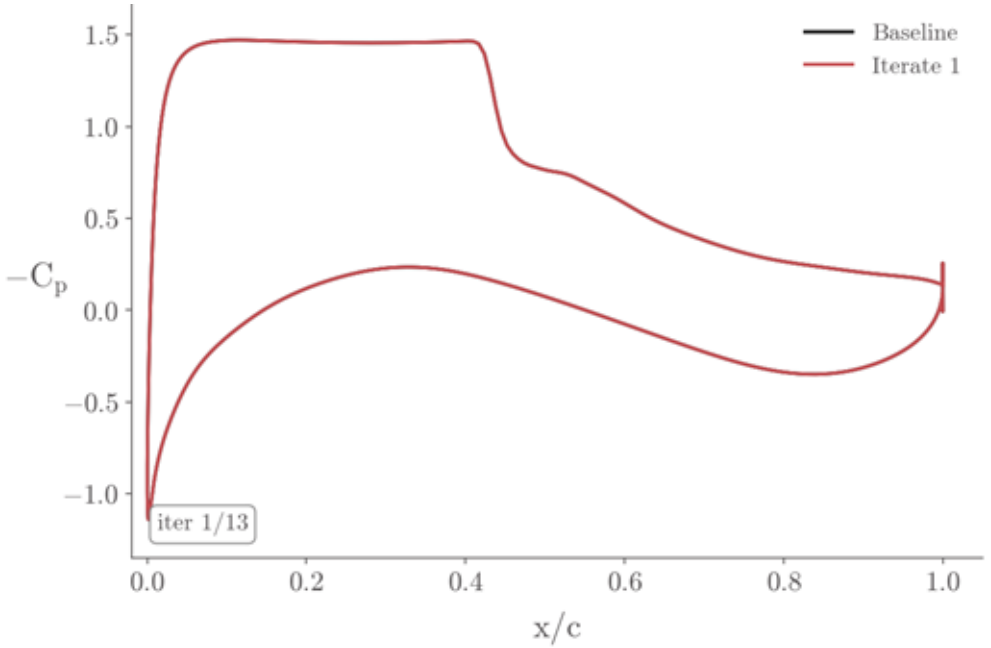
$$\frac{|\mathbf{q}_1^{\mathrm{H}}\mathbf{q}_2|^2}{(\mathbf{q}_1^{\mathrm{H}}\mathbf{q}_1)(\mathbf{q}_2^{\mathrm{H}}\mathbf{q}_2)}$$

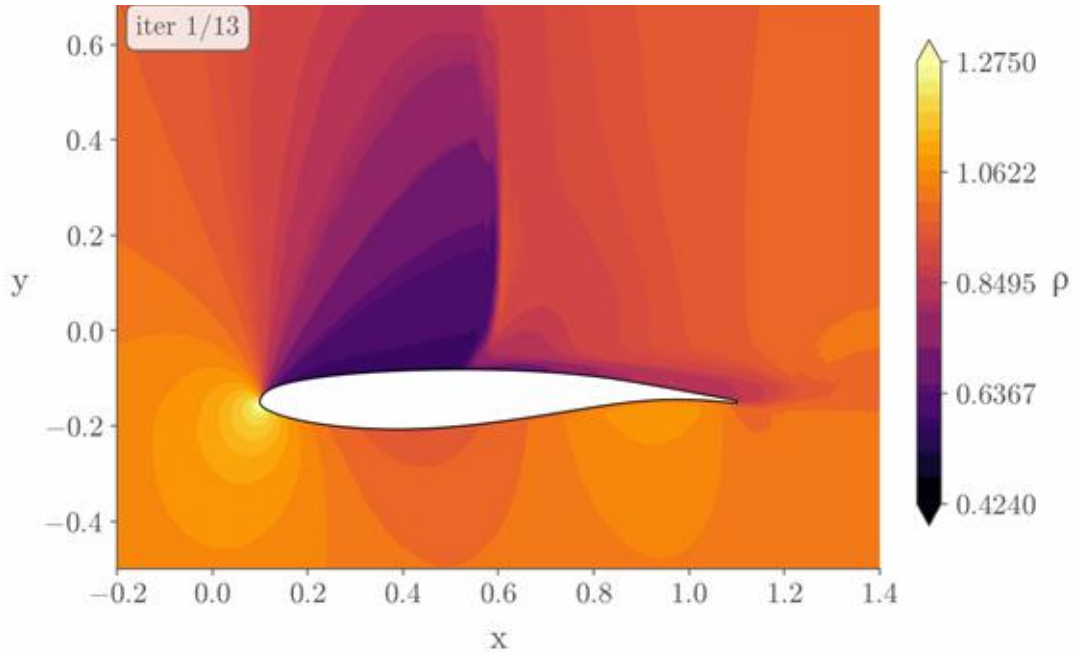
# The stability constrained optimization discovers the SCB shape.



During optimization, different eigen branches got activated. Thus, we needed KS aggregation.



# Content

## 1. Buffet simulation and linear stability theory

- Governing equations and linear stability theory
- LST eigenproblem computation methods
- Comparing LST with time accurate analyses

## 2. Adjoint based optimization

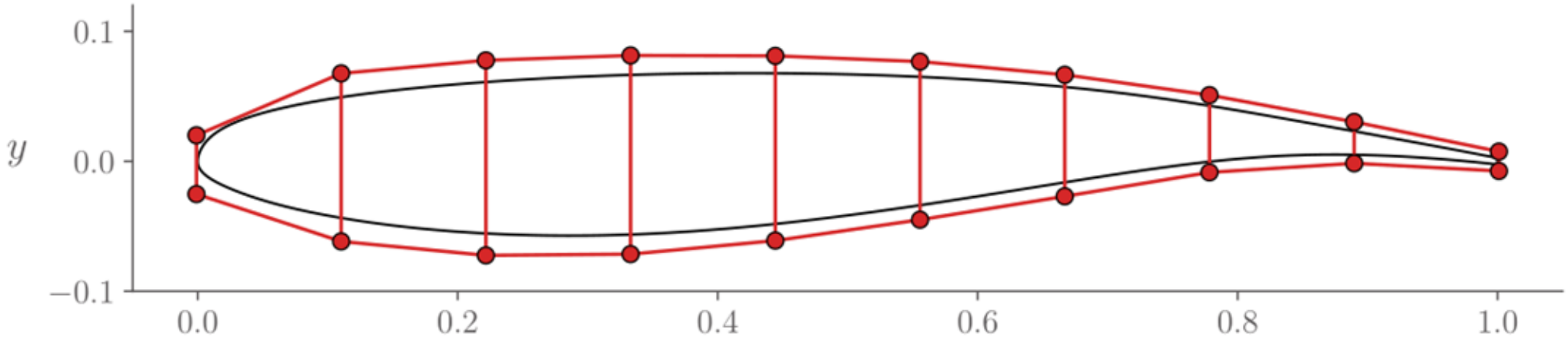
- Linear stability adjoint derivatives and verification
- Demonstration of shock control bump design with buffet constraints
- Demonstration of airfoil drag minimization with buffet constraints

## 3. Ongoing work - 3D NASA Common Research Model

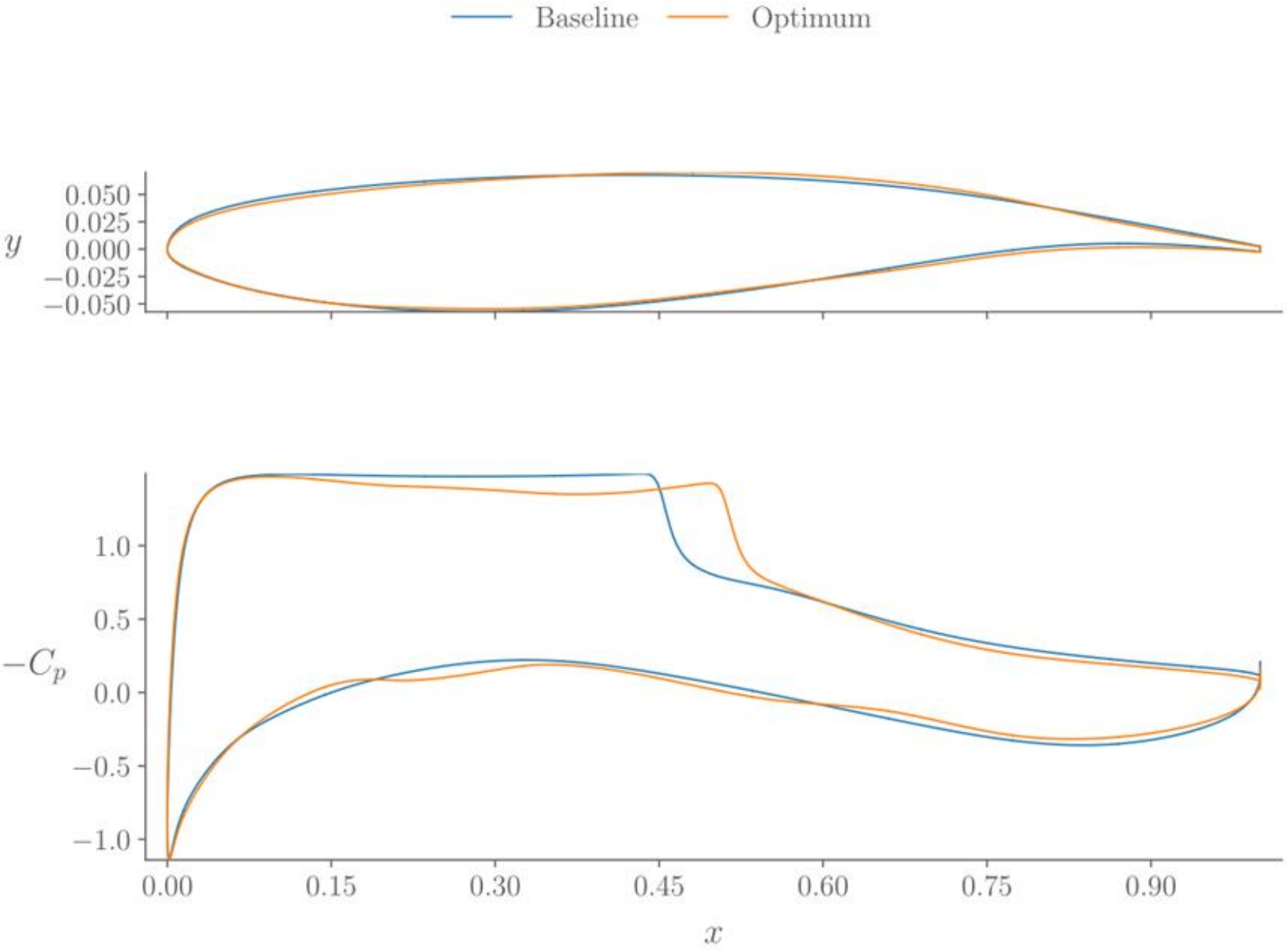
- CFD results for the NASA CRM wing only configuration
- Modal sensitivity of the NASA CRM wing to geometric shape

# We set up buffet constrained drag minimization problem here and used the same FFD as SCB case but add all the DVs.

| Quantity | Specification |
|---|---|
| Objective | minimize $C_D$ |
| Design variables | 40 FFD shape DVs ($y$–displacements, bounds $\pm 0.05$) + $\alpha \in [3°, 5°]$ |
| Lift constraint | $C_L = C_L^* = 0.98$ (equality) |
| Buffet constraint | $\mathrm{Re}(\lambda) \leq 0$ (LST adjoint, single $\Delta t = 5$ s in eigsolve) |
| Volume constraint | $V/V_0 \in [1.0, 1.08]$ |
| Thickness constraints | 200 stations, each $t \geq 0.1\, t_{\text{baseline}}$ |
| Symmetry constraints | LE/TE pairing and $z$–spanwise symmetry (linear) |
| Optimizer | SLSQP via pyOptSparse, acc $= 10^{-6}$, maxiter $= 500$ |



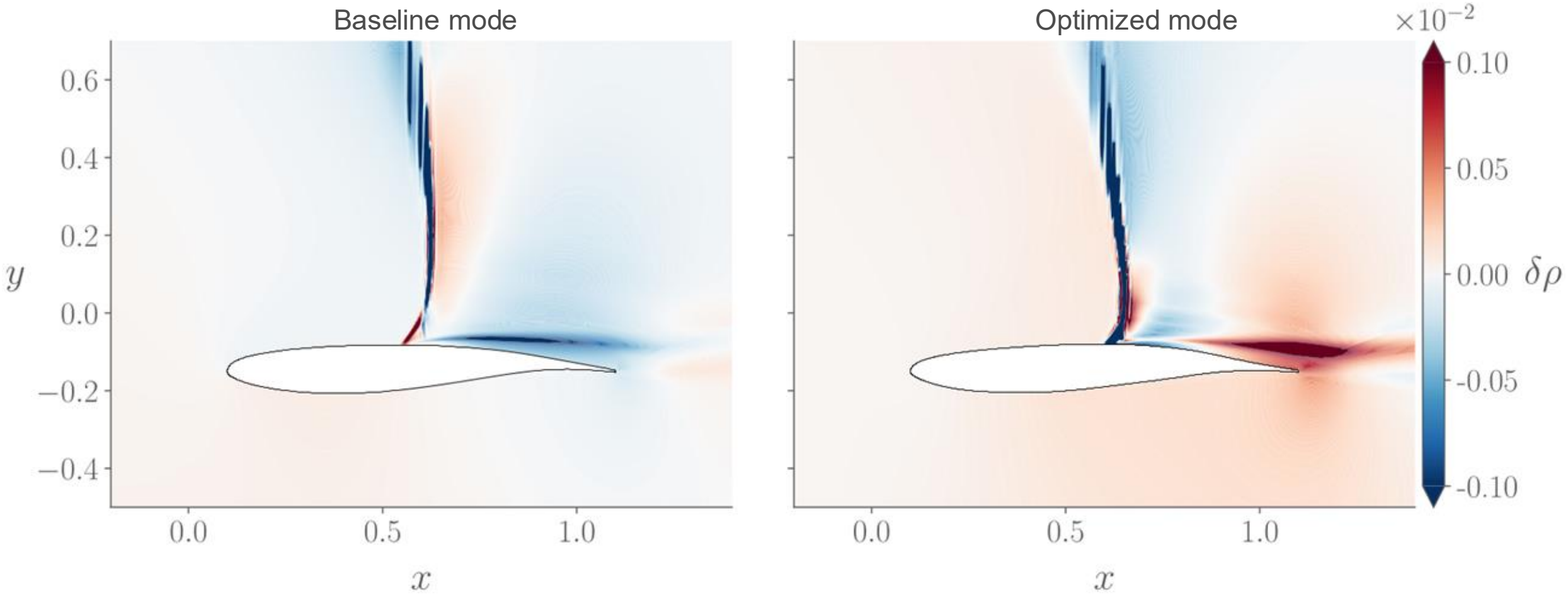
# Buffet constrained drag minimization results.



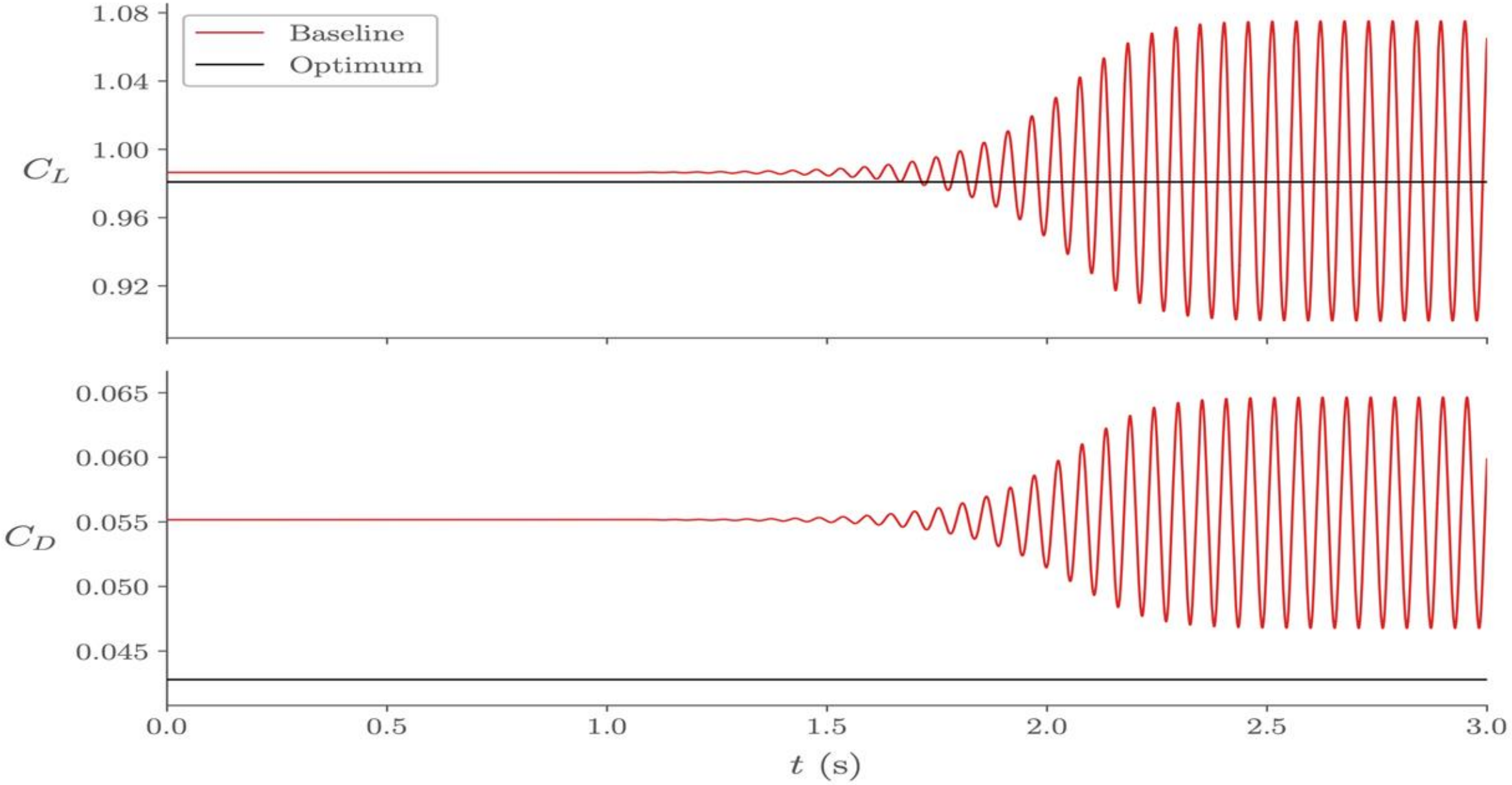

| Quantity | Baseline | Optimum | Δ |
|---|---|---|---|
| $C_D$ | 0.05516 | 0.04279 | −22.4% (−124 counts) |
| $C_L$ | 0.9865 | 0.9809 | −0.6% |
| $\alpha$ | 4.000° | 3.906° | −0.094° |
| $\mathrm{Re}(\lambda)$ | +0.02121 | $-3.74 \times 10^{-4}$ | sign flip (stable) |
| $\mathrm{Im}(\lambda)$ | +0.39129 | +0.48230 | +23% |

- The optimizer found a stable optimum point after 72 iterations.
- The minute changes in profile indicate strong sensitivity to shape.

# The baseline and unstable modes presented. The lambda structure at shock foot is nearly absent in the optimized design.



# Unsteady simulations were run for the baseline and optimized designs and prove that the stable point found is indeed stable.



# Content

## 1. Buffet simulation and linear stability theory

- Governing equations and linear stability theory
- LST eigenproblem computation methods
- Comparing LST with time accurate analyses

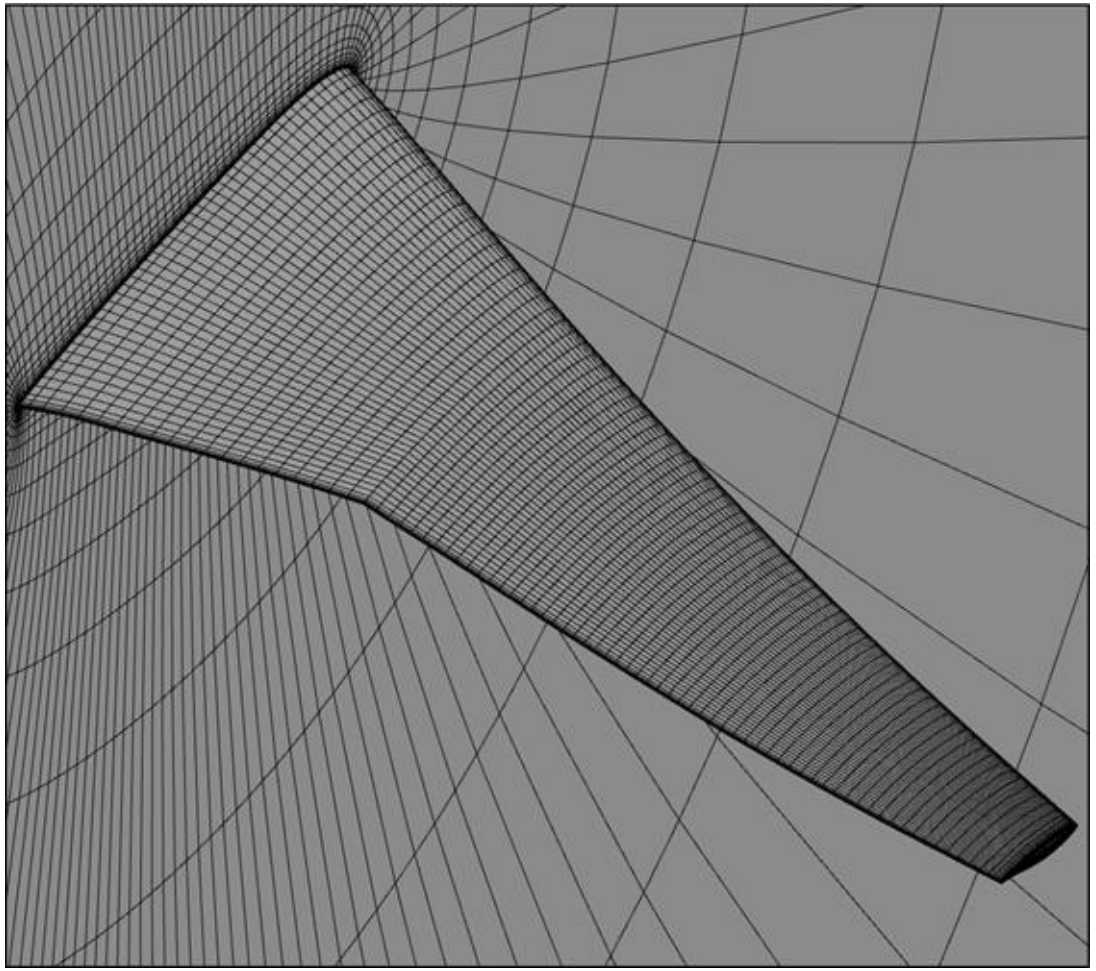
## 2. Adjoint based optimization

- Linear stability adjoint derivatives and verification
- Demonstration of shock control bump design with buffet constraints
- Demonstration of airfoil drag minimization with buffet constraints

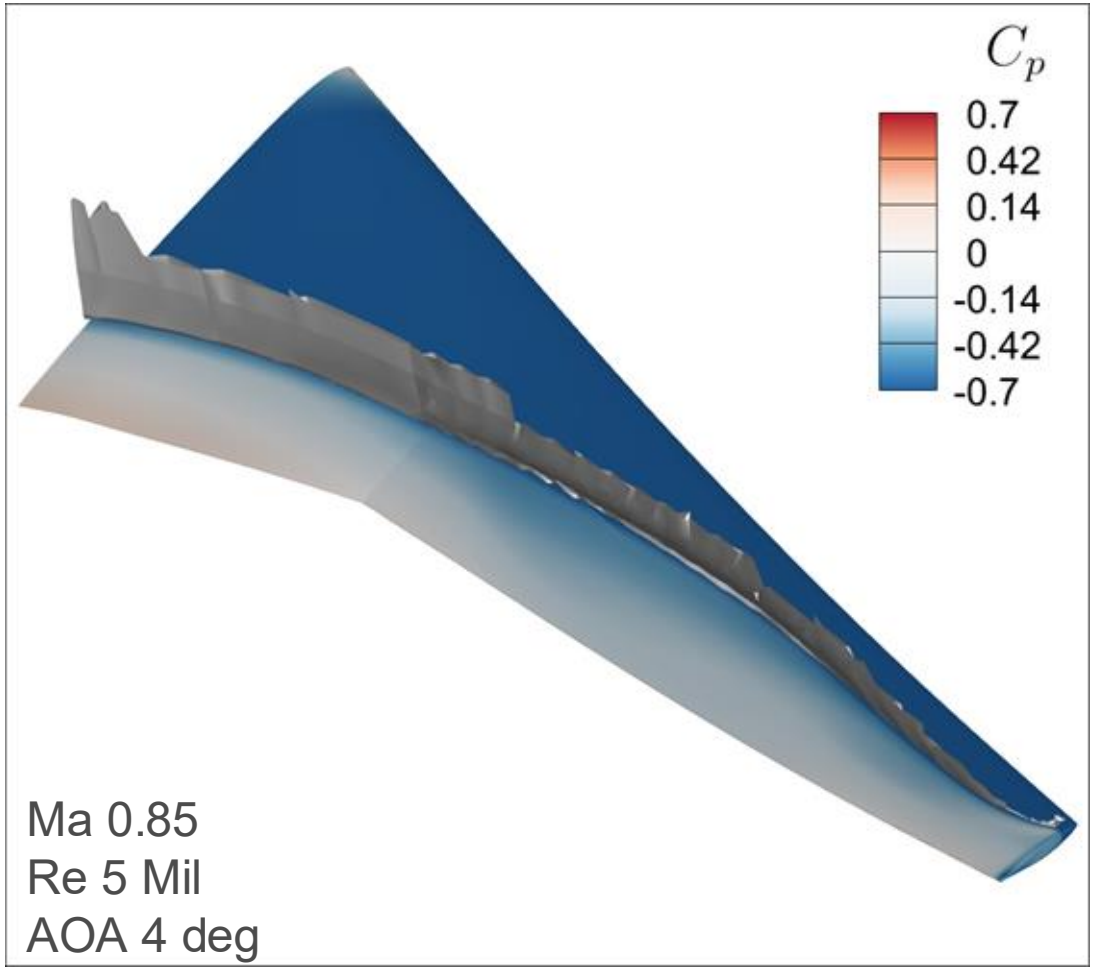
## 3. Ongoing work - 3D NASA Common Research Model

- CFD results for the NASA CRM wing only configuration
- Modal sensitivity of the NASA CRM to geometric wing shape

# NASA-CRM steady state results.

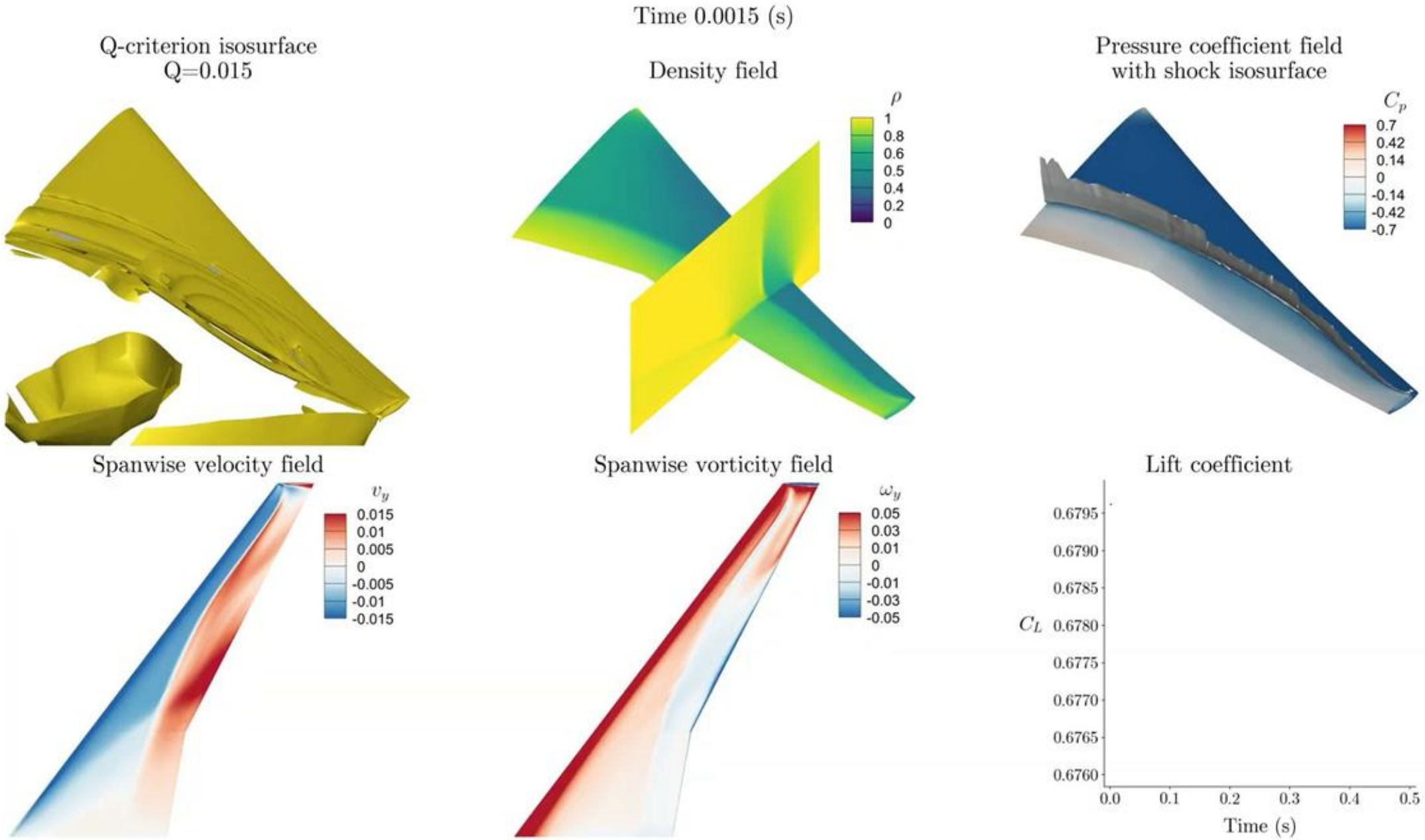


The NASA Common research model wing [Lacy et al, 2016].

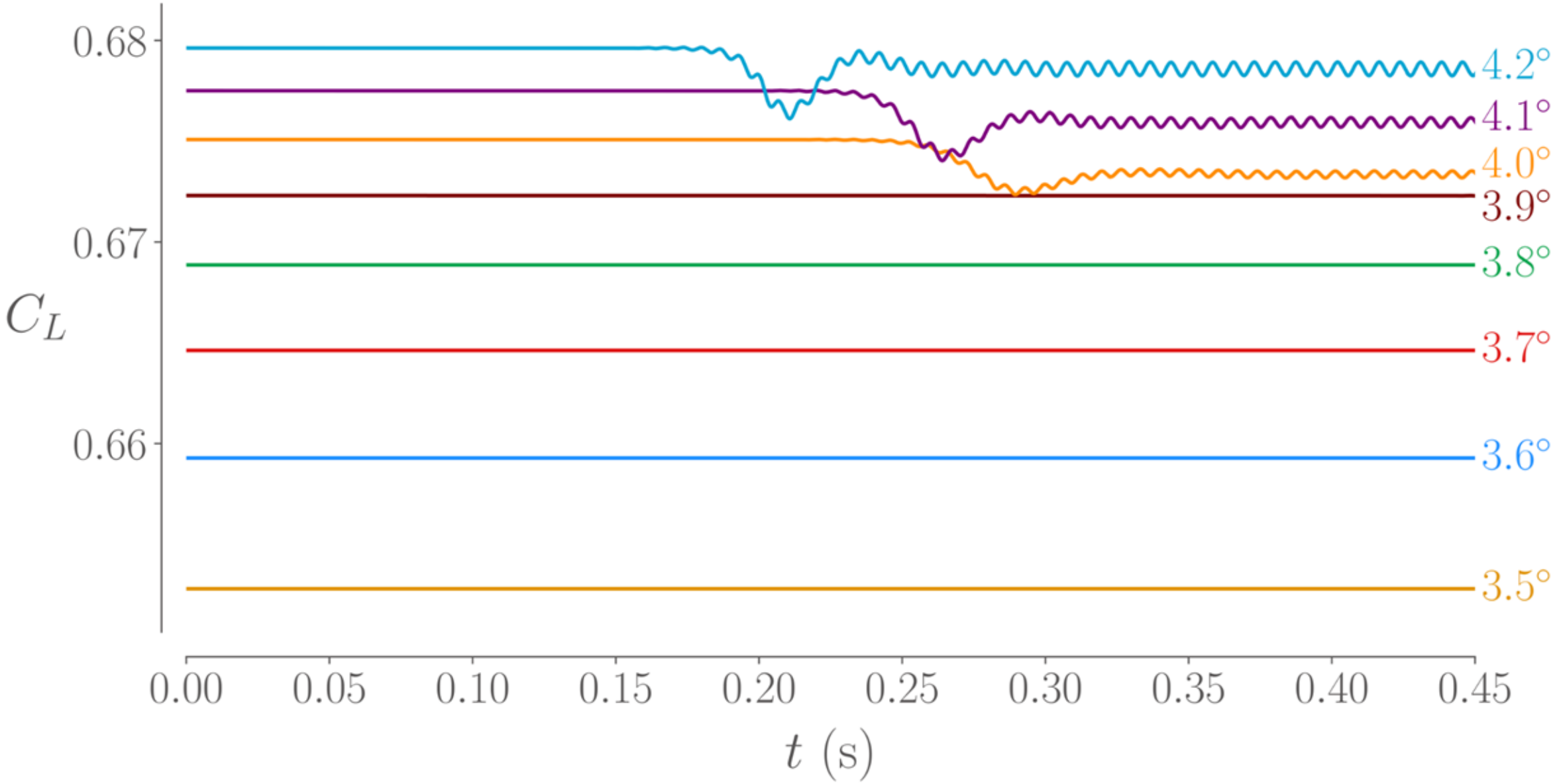



Steady state pressure contours and shock isosurface

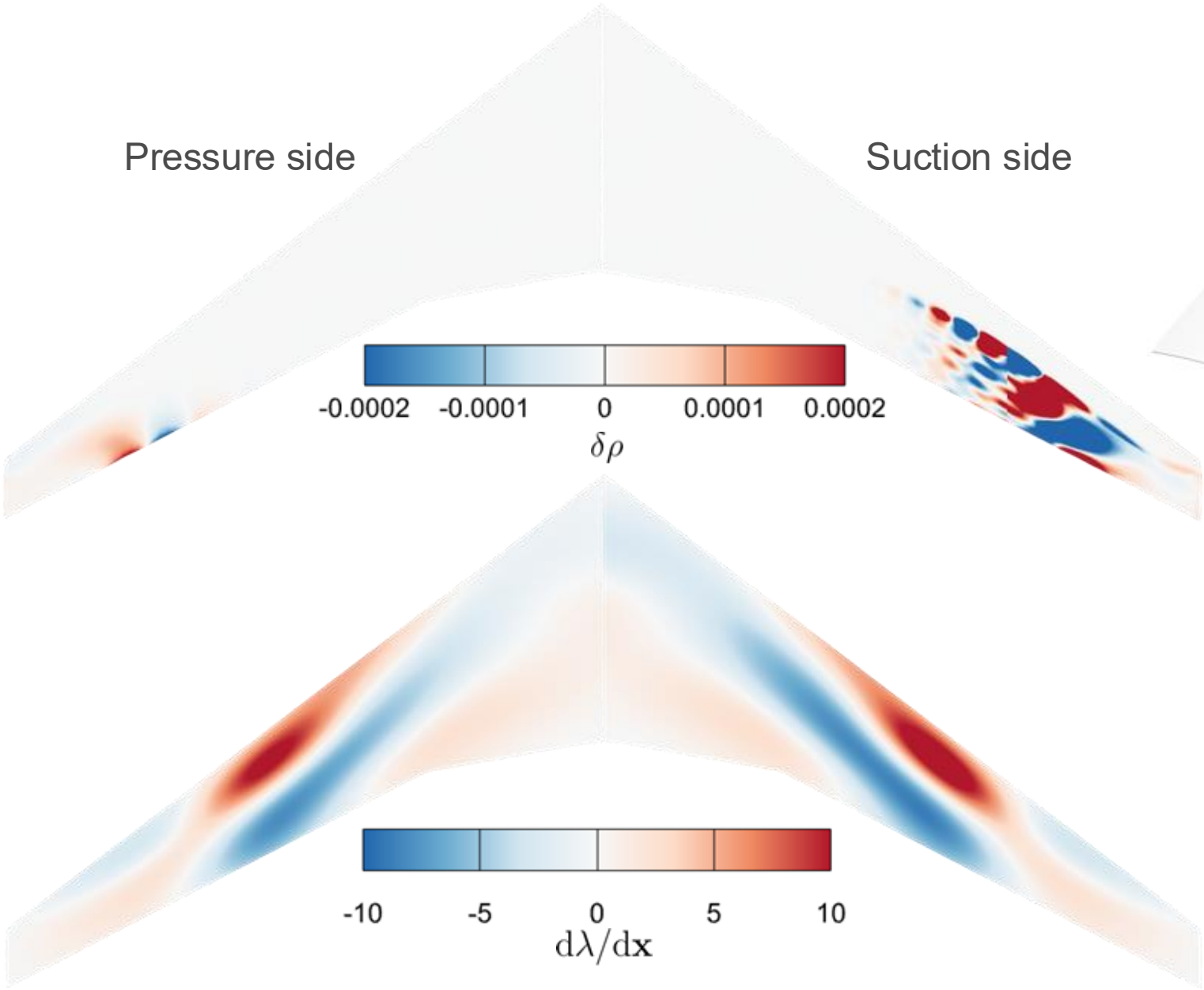
# NASA-CRM time accurate results for transonic buffet.

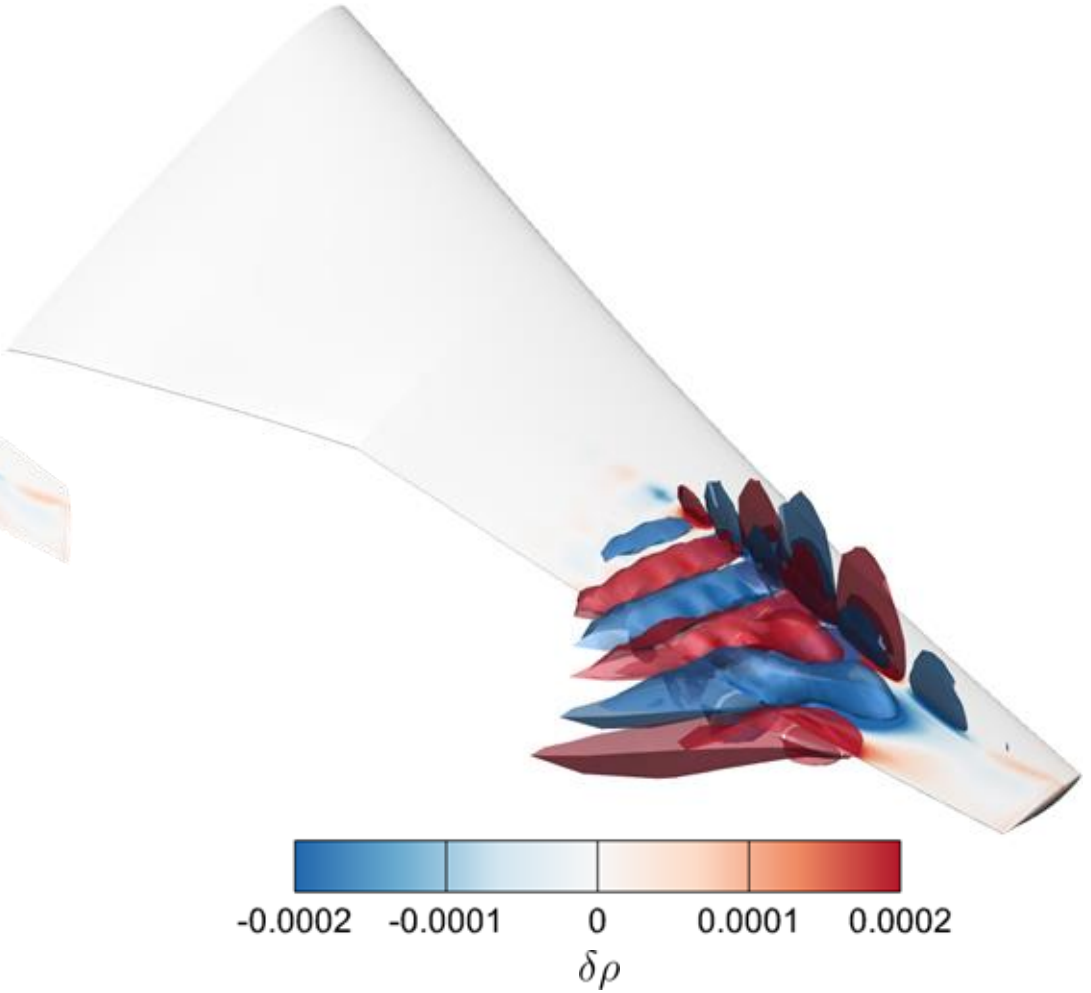


# Buffet onset time accurate lift coefficient response curves were obtained at different angles of attack



# NASA-CRM buffet eigenmodes and spatial sensitivity.

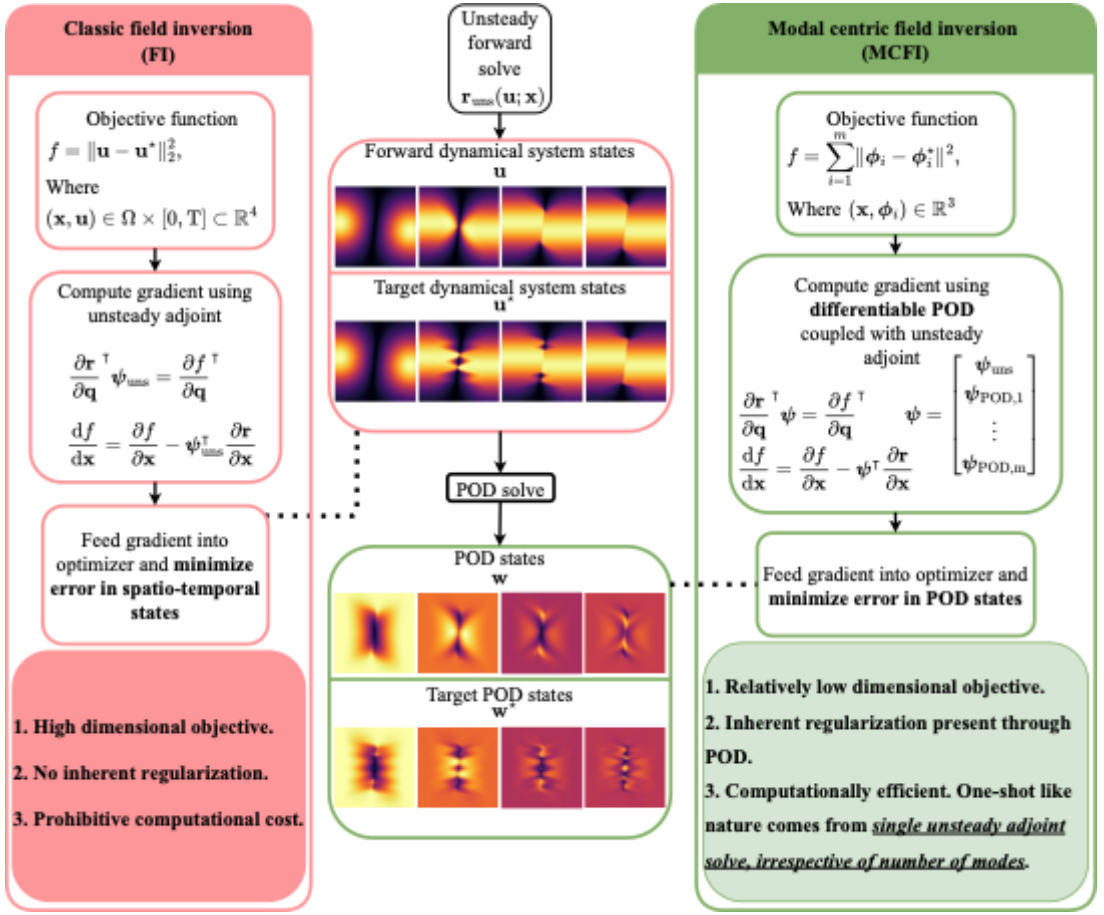


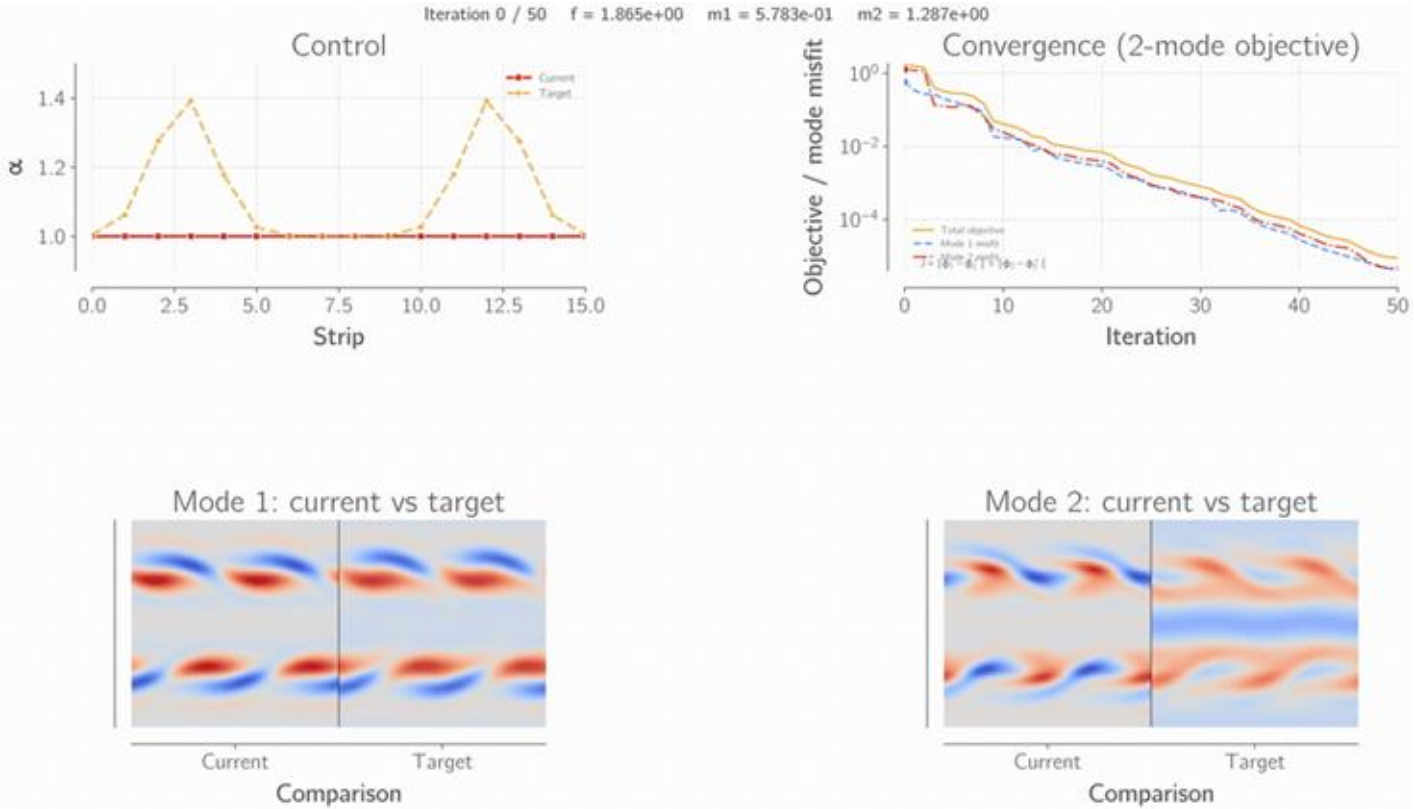



# Summary

- Introduced a new scalable method to compute the buffet constraint derivative using an adjoint method based.
- Verified the correctness of adjoint based method derivatives by comparing against finite difference derivatives.
- Demonstrated that buffet constrained optimizations can rediscover the shock control bump naturally.
- Demonstrated a 20 design variable buffet constrained drag minimization. We reduced the drag by 22.4% and removed the buffet instability.
- Presented buffet and for first time, the unstable eigenmode's spatial sensitivity for NASA CRM case using the adjoint method.

# More ventures into differentiable modal analysis from the XD² research group - Proper Orthogonal Decomposition.




XD² Lab
Extreme-Condition Dynamics & Design Lab

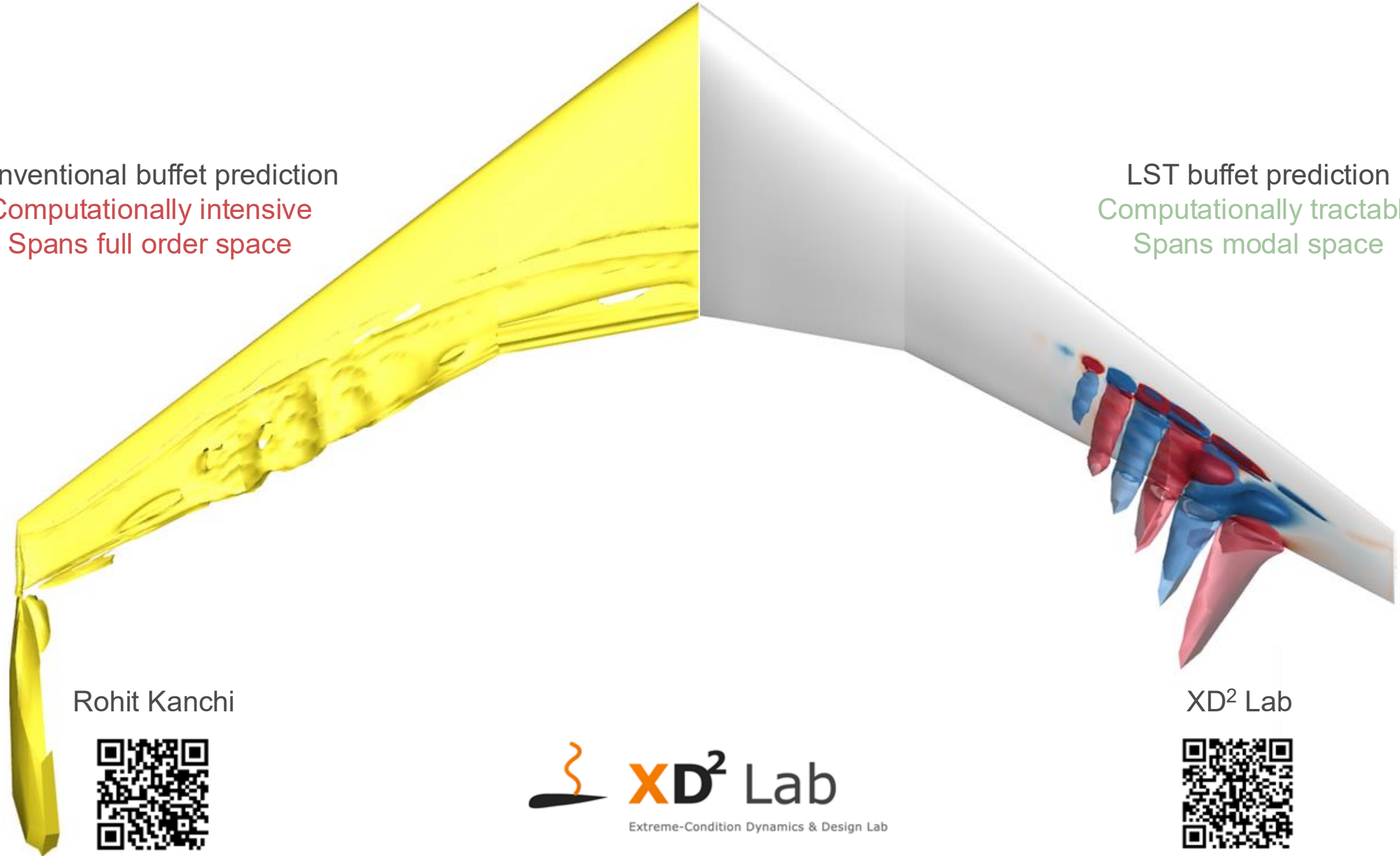
# More ventures into differentiable modal analysis from the XD$^2$ research group - Resolvent Analysis.



XD$^2$ Lab
Extreme-Condition Dynamics & Design Lab

Conventional buffet prediction
Computationally intensive
Spans full order space

LST buffet prediction
Computationally tractable
Spans modal space



Rohit Kanchi

XD² Lab
Extreme-Condition Dynamics & Design Lab

XD² Lab