Stability Analysis and Improvement in Computational Fluid Dynamics

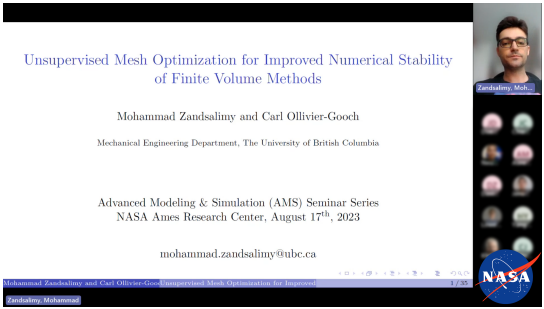
Mohammad Zandsalimy* and Carl Ollivier-Gooch

Mechanical Engineering Department, The University of British Columbia

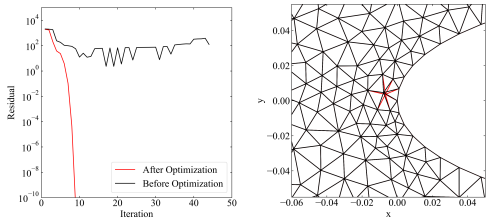
Advanced Modeling & Simulation (AMS) Seminar Series
NASA Ames Research Center, February 27$^{\text{th}}$, 2025

*mohammad.zandsalimy@ubc.ca

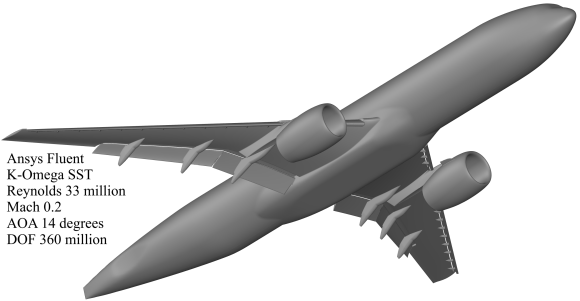
Previously

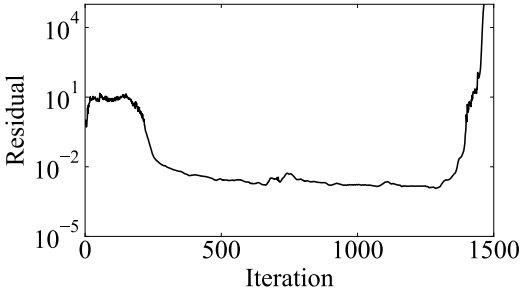


Euler Problem, 7600 Cells, Implicit Euler

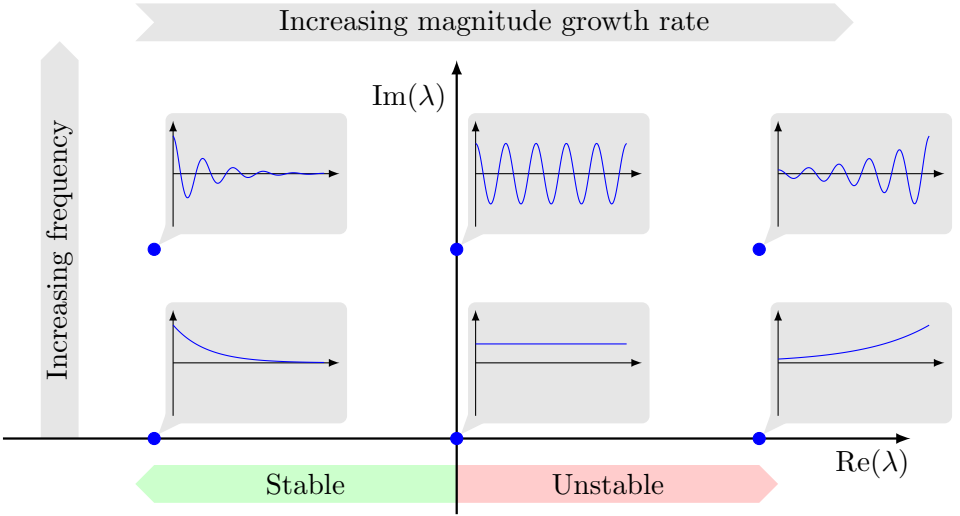


Numerical Stability Issues



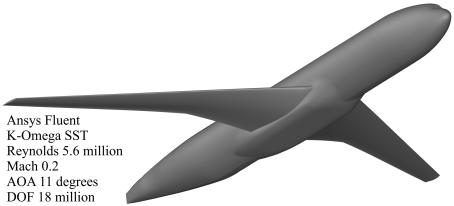


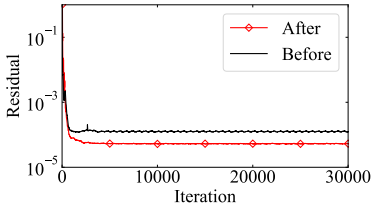
Stability Analysis



Criteria[1]

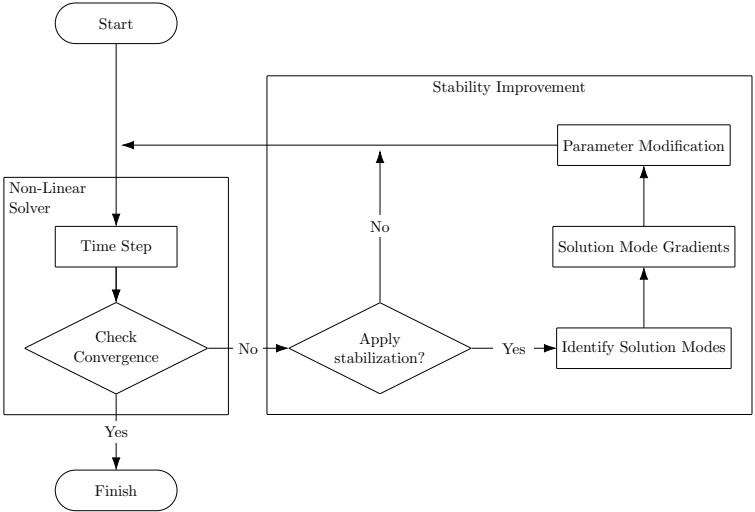
- Computationally feasible for large-scale industrial problems
- Non-invasive to the solver
- Fully automated and user friendly





[1] Mohammad Zandsalimy and Carl Ollivier-Gooch. "CFD Stability Improvement Using Dynamic Mode Decomposition of Solution Update Vectors". In: *Journal of Computational Physics* 513 (2024), p. 113195. ISSN: 0021-9991. DOI: https://doi.org/10.1016/j.jcp.2024.113195.

General Stability Improvement Approach



Solution Mode Identification

- Full Jacobian eigenanalysis of $\boldsymbol{A} \in \mathbb{C}^{n \times n}$

$$\boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{x}, \quad \boldsymbol{x} \in \mathbb{C}^n \tag{1}$$

$$\boldsymbol{y}^H \boldsymbol{A} = \lambda \boldsymbol{y}^H, \quad \boldsymbol{y} \in \mathbb{C}^n \tag{2}$$

- Scales at $\mathcal{O}(n^3)$

- Dynamic Mode Decomposition

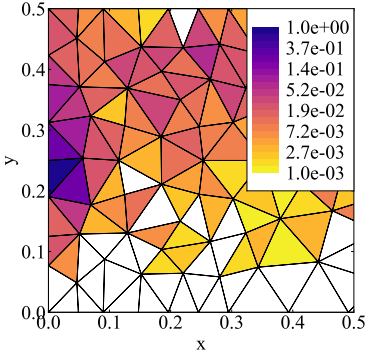
$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k \tag{3}$$

$$\mathcal{K}_1 = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m\} \tag{4}$$

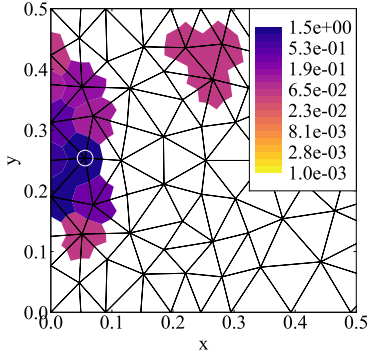
$$\begin{aligned} \mathcal{K}_2 &= \{\boldsymbol{x}_2, \boldsymbol{x}_3, \ldots, \boldsymbol{x}_{m+1}\} \\ &= \boldsymbol{A}\mathcal{K}_1 = \mathcal{K}_1 \boldsymbol{B} + \boldsymbol{r} \end{aligned} \tag{5}$$

- Scales at $\mathcal{O}(m^2 n)$

Vertex Selection



(a) Absolute value of eigenvector



(b) Selection weight measure

Eigenvalue Gradient

- To find the direction and magnitude of vertex movement
- **Direct method** using finite differences
- **Indirect method**,

$$\frac{d\lambda}{d\zeta} = \boldsymbol{y}^H \frac{d\boldsymbol{A}}{d\zeta}\boldsymbol{x} \tag{6}$$

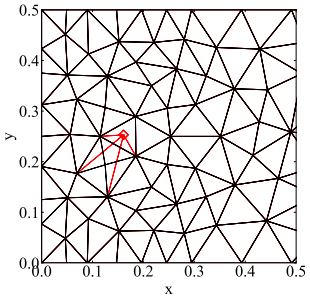
$$\frac{d\boldsymbol{A}}{d\zeta} = \frac{\partial \boldsymbol{A}}{\partial U}\frac{\partial U}{\partial \zeta} + \frac{\partial \boldsymbol{A}}{\partial \zeta} \approx \frac{\partial \boldsymbol{A}}{\partial \zeta} \tag{7}$$

Mesh Modification

- Calculate modification vectors,

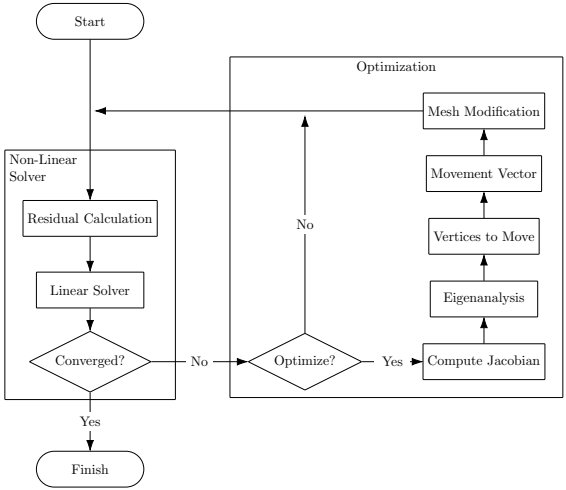
$$\lambda_{\text{new}} = \lambda_{\text{old}} + \frac{d\lambda}{d\zeta}\delta\zeta \tag{8}$$

- Vertex movement limit,



Original Approach[2]

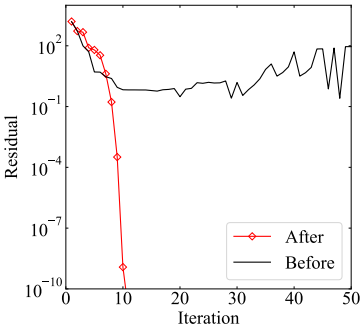
- Improved vertex selection
- Reduced optimization iterations
- Opposing eigenmode resolution

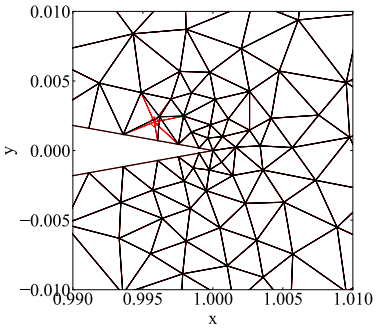


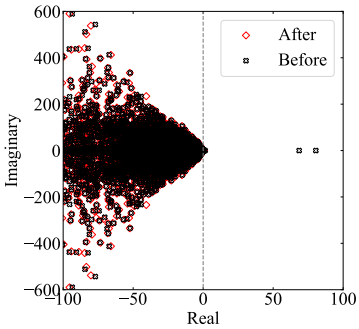
[2] Mohammad Zandsalimy and Carl Ollivier-Gooch. "A novel approach to mesh optimization to stabilize unstructured finite volume simulations". In: *Journal of Computational Physics* 453 (2022), p. 110959. ISSN: 0021-9991. DOI: https://doi.org/10.1016/j.jcp.2022.110959.

Original Approach, 2D Euler

- Solved using ANSLib, 2nd order finite-volume, implicit Euler time-stepping, CFL evolution
- 10 000 DOF

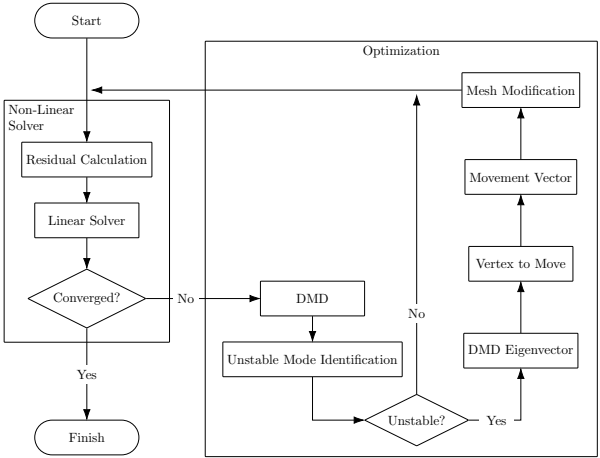






DMD Approach[3]

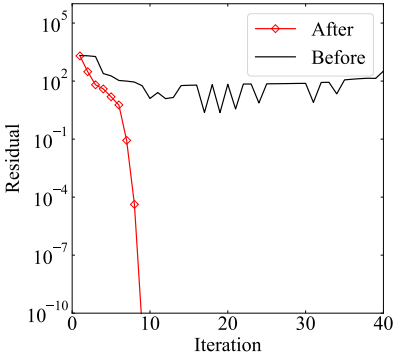
- Fully automatic
- Improves computational complexity
  - Eliminates eigenanalysis
  - Novel vertex movement vectors
- Feasible for large-scale simulations

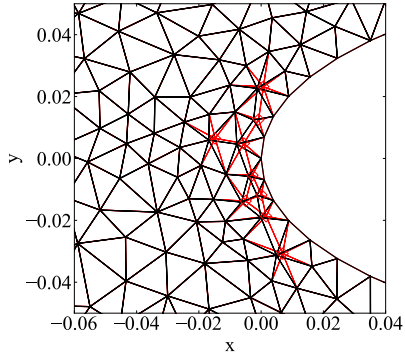


[3] Mohammad Zandsalimy and Carl Ollivier-Gooch. "CFD Stability Improvement Using Dynamic Mode Decomposition of Solution Update Vectors". In: *Journal of Computational Physics* 513 (2024), p. 113195. ISSN: 0021-9991. DOI: https://doi.org/10.1016/j.jcp.2024.113195.

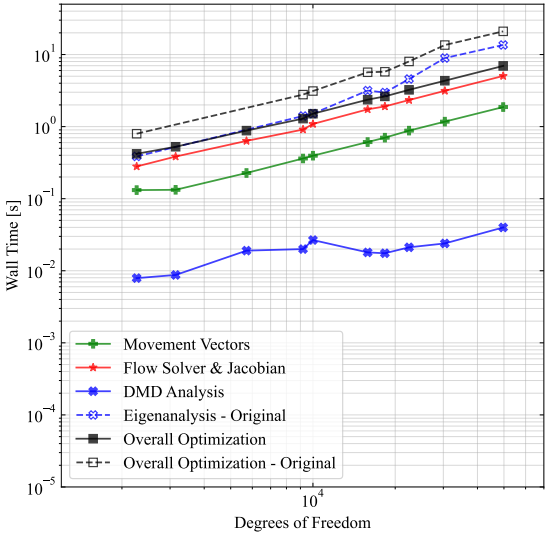
DMD Approach, 2D Euler

- Solved using ANSLib, 2nd order finite-volume, implicit Euler time-stepping, CFL evolution
- 30 000 DOF





Run Time

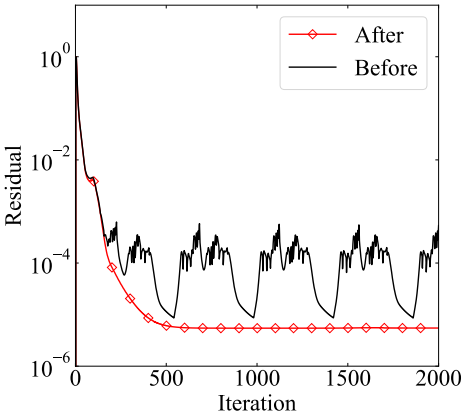

Wall Time [s]
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{4}
Degrees of Freedom
Movement Vectors
Flow Solver & Jacobian
DMD Analysis
Eigenanalysis - Original
Overall Optimization
Overall Optimization - Original

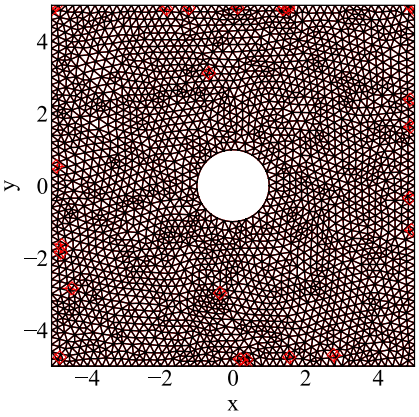
Incentive

- "What if your solver is wrong?"
- Use a commercial third-party flow solver
- Non-invasive to the host software
- Ansys Fluent is a great choice

Original Approach, Modified 2D Cavity

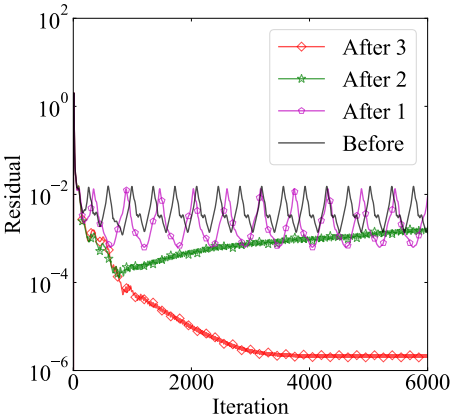
- Reynolds 685 000, $k - \omega$ SST, 26 000 DOF

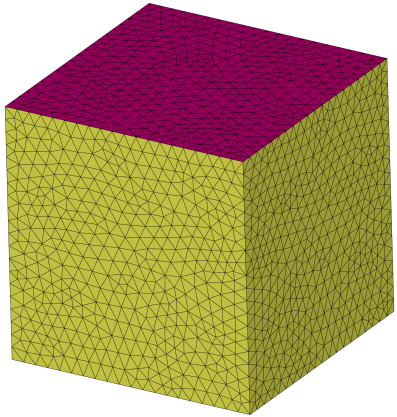




DMD Approach, 3D Cavity

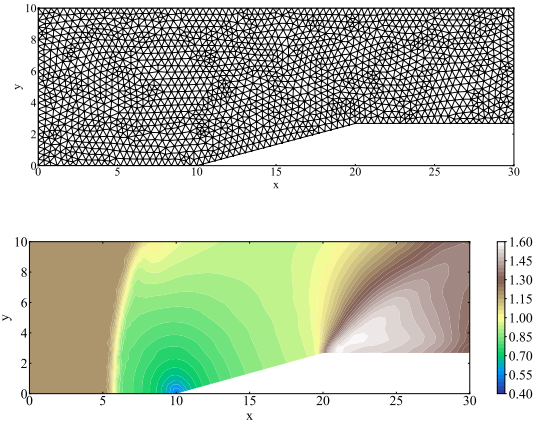
- Reynolds 7000, $k-\omega$ SST, 200 000 DOF





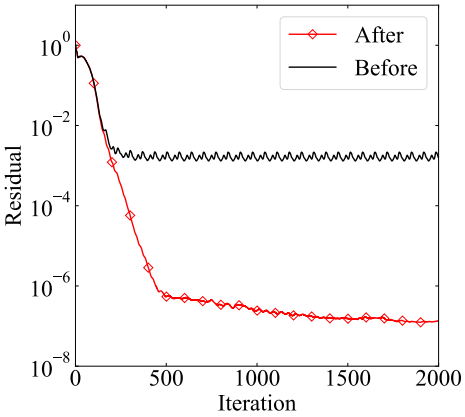
DMD Approach, Compressible Channel

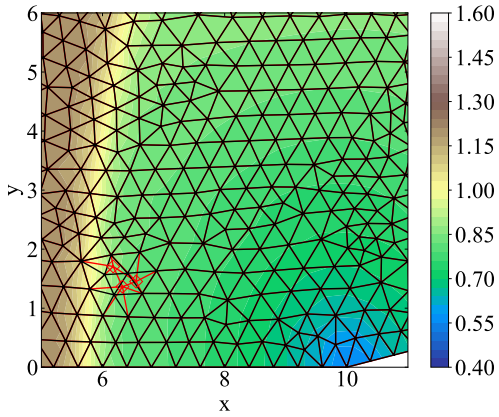
- Mach 1.2, inviscid, 13 000 DOF



DMD Approach, Compressible Channel

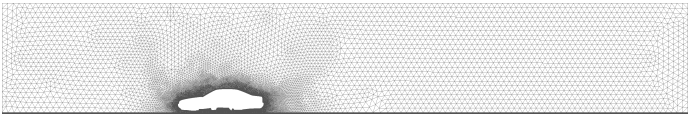
- Mach 1.2, inviscid, 13 000 DOF

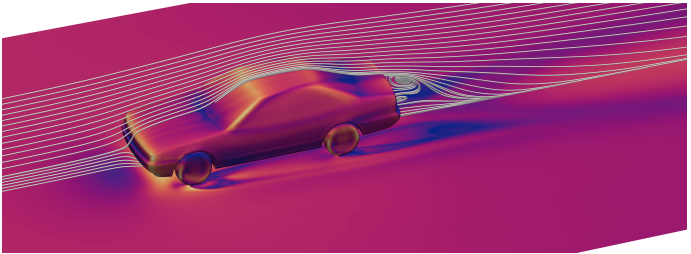




DMD Approach, Automotive

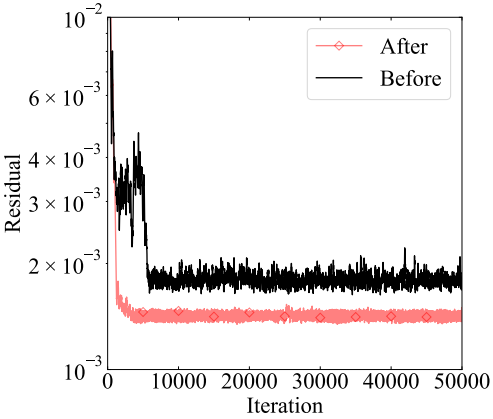
- Reynolds 8.7×10^6, $k - \omega$ SST, 22×10^6 DOF

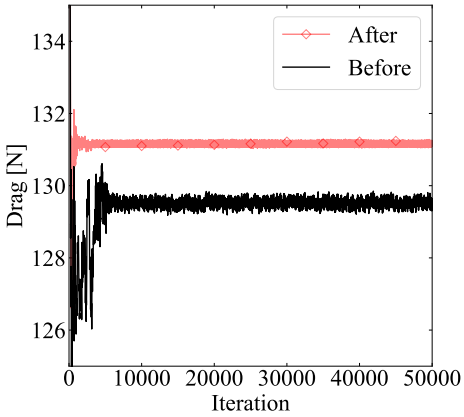




DMD Approach, Automotive

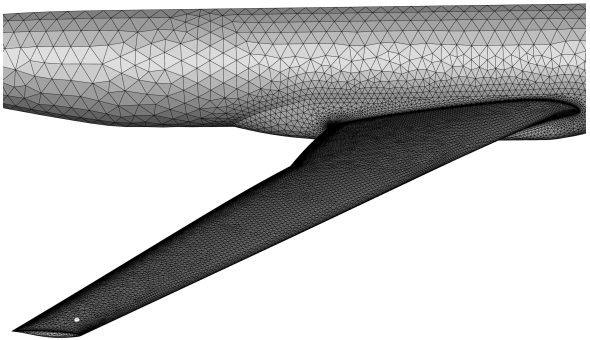
- Reynolds 8.7×10^6, $k - \omega$ SST, 22×10^6 DOF

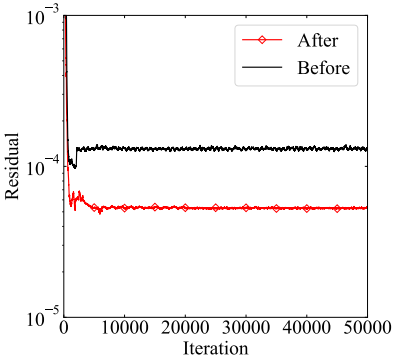




DMD Approach, NASA CRM-HL

- 5th AIAA CFD High Lift Prediction Workshop, Case 1
- Mach 0.2, Reynolds 33×10^6, 11° AOA, $k - \omega$ SST, 18×10^6 DOF

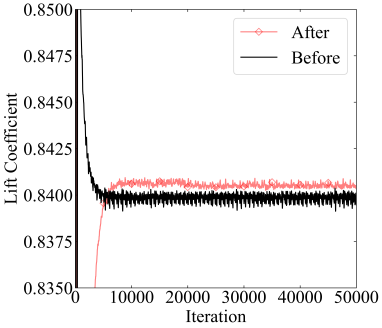


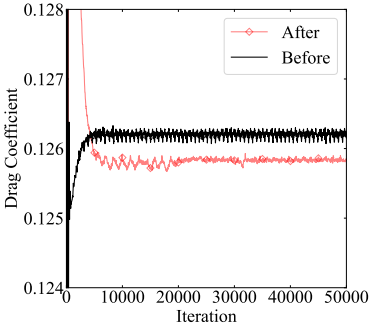


DMD Approach, NASA CRM-HL

- Mach 0.2, Reynolds 33×10^6, 11° AOA, $k - \omega$ SST, 18×10^6 DOF

Team	C_L	C_D
CFD Focus R-001	0.9072	0.0878
CFD Focus R-002	0.9026	0.1109



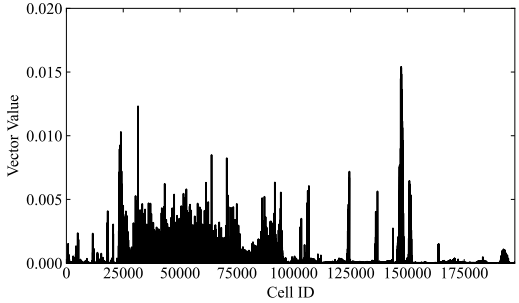


Contributions

- Proven stability improvement at a feasible computational cost
- Automated optimization
- Applicable to third-party closed-source solvers
- Applicable to large-scale industrial problems
- Not limited to CFD

Future Work

- Mesh generation and numerical solution coupling
- Jacobian classification
- Novel solution mode identification methods
- Identify multiple local regions in one vector



Acknowledgment









NSERC
CRSNG



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