Partially-Averaged Navier-Stokes Approach for Unsteady Separated Flows

A. Petrocchi and G. N. Barakos

CFD Laboratory, University of Glasgow, James Watt South Building, G12 8QQ
Glasgow, Scotland, UK
www.gla.ac.uk/cfd

Towards Effective Flow Control and Mitigation of Shock Effects In Aeronautical Applications.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No EC grant 860909.

Advanced Modelling and Simulation (AMS) Seminar Series

NASA Ames Research Center, July 11th, 2024

Agenda

- Intro
- Equations and CFD Solver
- PANS: circular cylinder
- PANS: transonic buffet
- Some ongoing work
- Conclusions

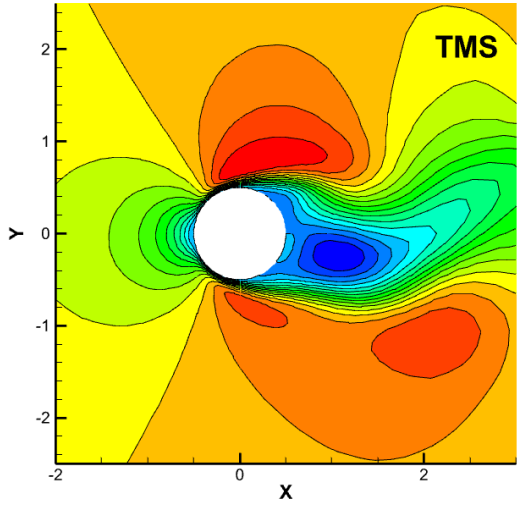
Motivation

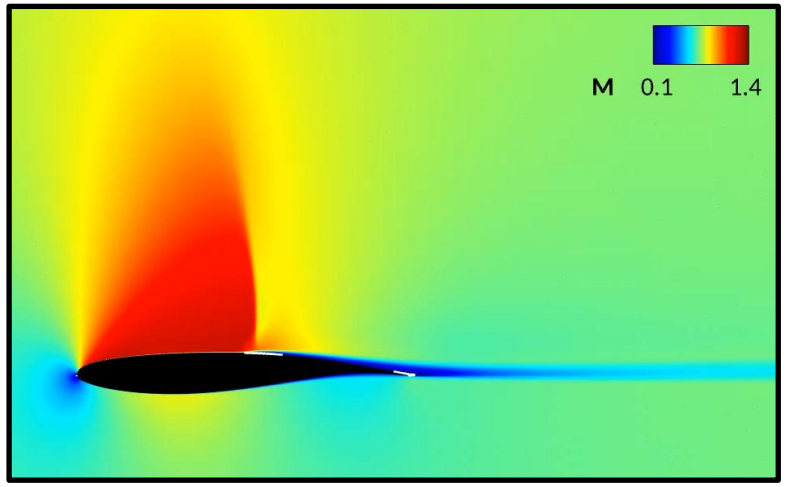
Motivation

- RANS cannot predict separated flows
- RANS are strongly **dependent** on the adopted **turbulence model**
- DES-family approaches work well for **fixed point separation** but not for **shallow separation**
- LES and DNS are computationally **expensive** for complex cases at **high Re**

Motivation

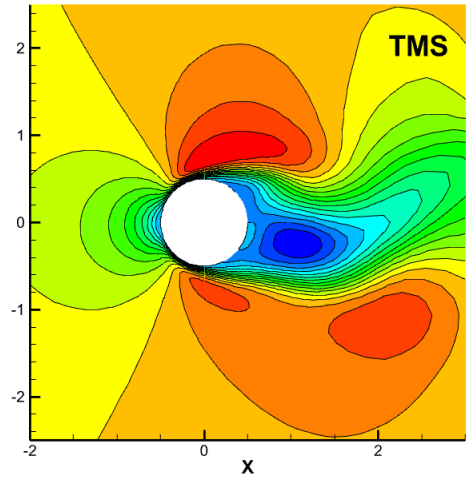
- RANS do not predict well separated flows

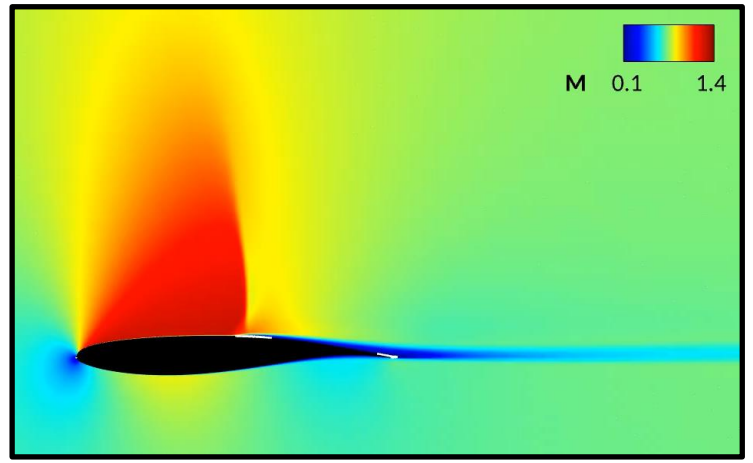




Motivation

- RANS do not predict well separated flows
- RANS are **strongly dependent** on the adopted **turbulence model**



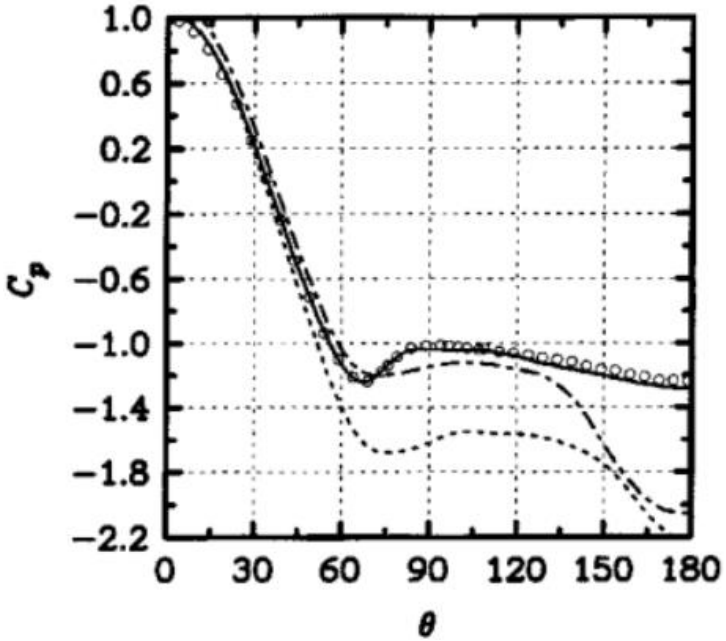


Motivation

- RANS do not predict well separated flows
- RANS are **strongly dependent** on the adopted **turbulence model**
- DES methods:
  - work well for **fixed point separation**
  - less for **shallow separation**



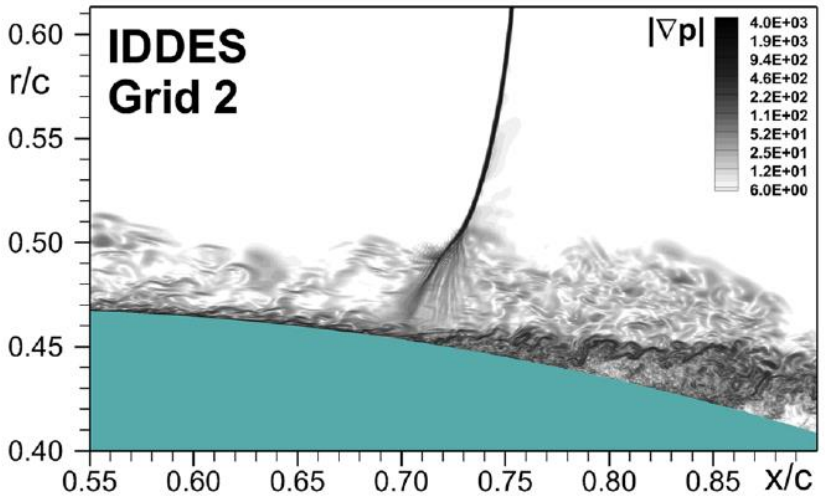
Figures from *

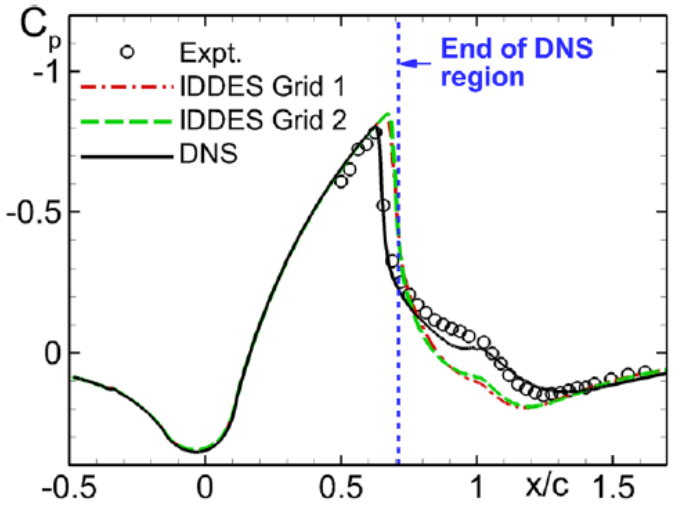


*A. Travin, M. Shur, M. Strelets, and P. Spalart. Detached-Eddy Simulations Past a Circular Cylinder. Flow, Turbulence and Combustion, 63:293–313, 2000.

Motivation

- RANS do not predict well separated flows
- RANS are **strongly dependent** on the adopted **turbulence model**
- DES methods:
  - work well for **fixed point separation**
  - less well for **shallow separation**





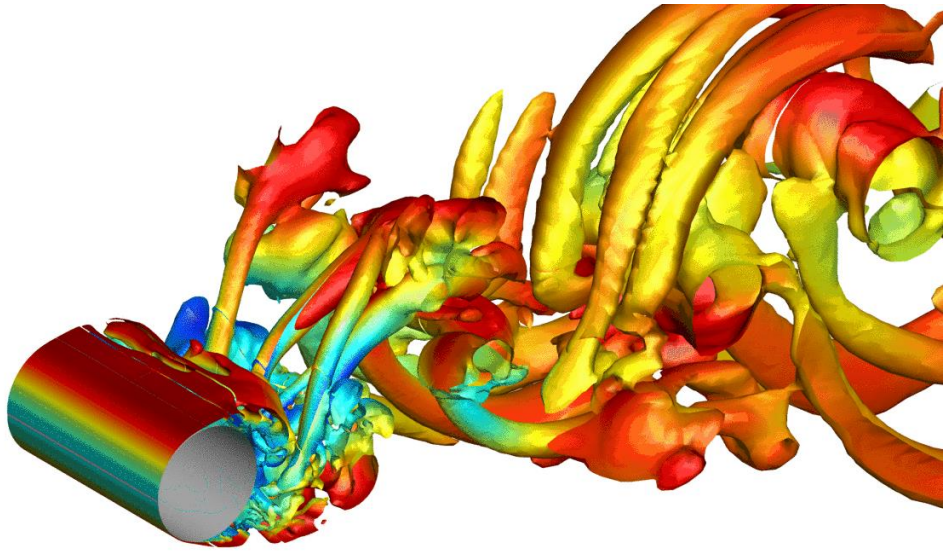
*P. Spalart et al. Large-Eddy and Direct Numerical Simulations of the Bachalo-Johnson Flow with Shock-Induced Separation. Flow, Turbulence and Combustion, 2017.

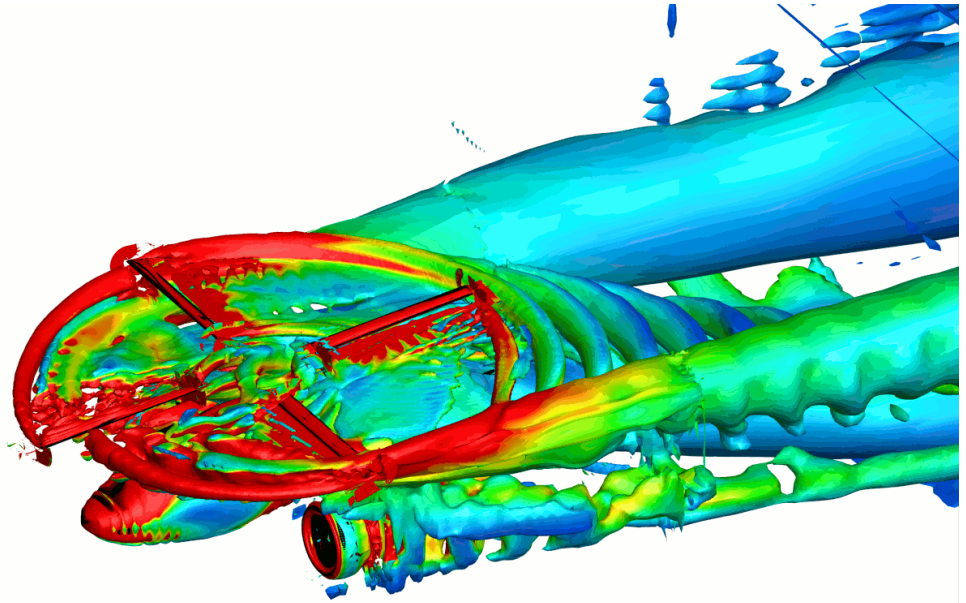
Motivation

- RANS do not predict well separated flows
- RANS are **strongly dependent** on the adopted **turbulence model**
- DES methods:
  - work well for **fixed point separation**
  - less well for **shallow separation**
- LES and DNS are computationally **expensive** for complex cases at **high Re**
- We want something **accurate and flexible → PANS**

Equations and CFD Solver

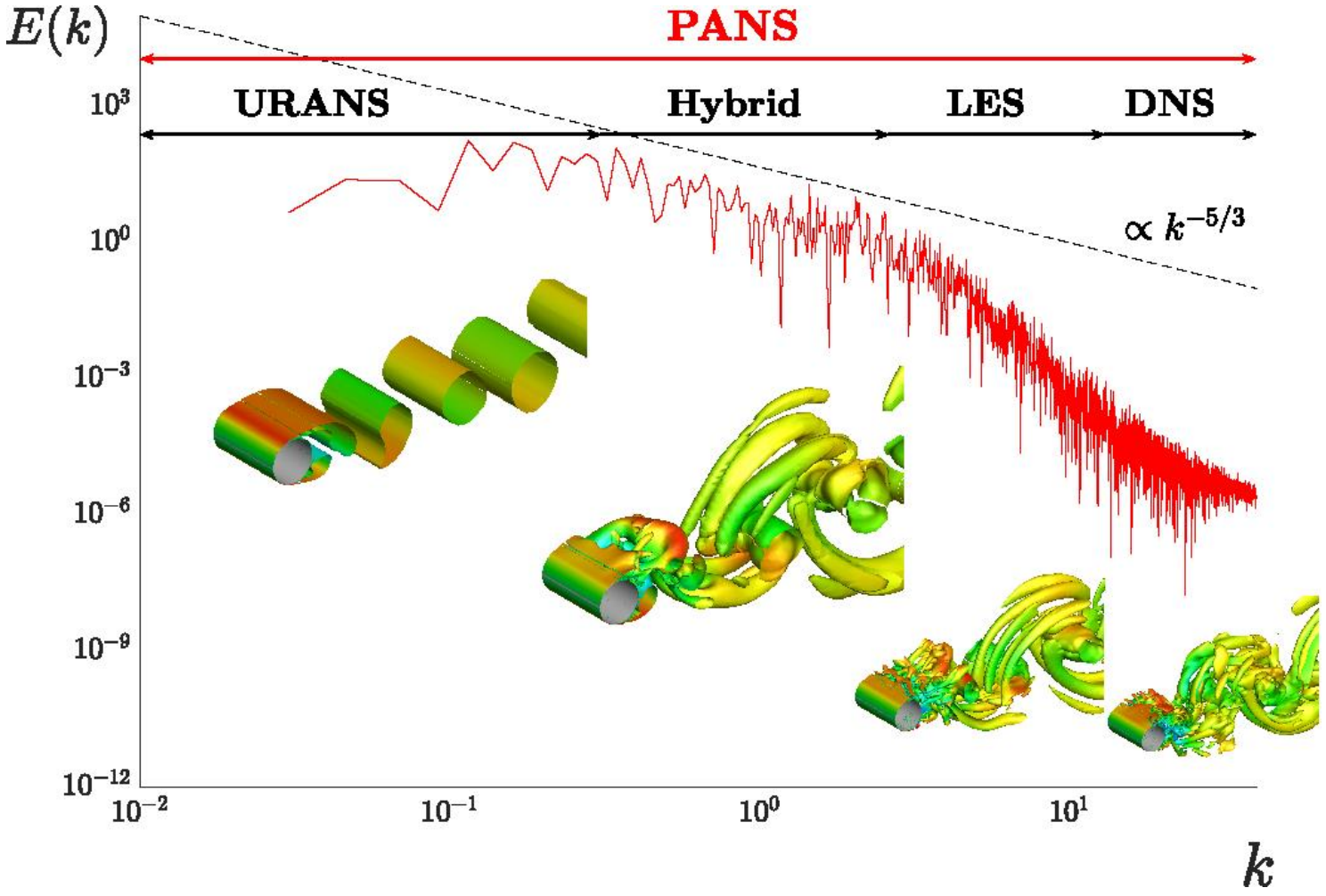
- R. Steijl, G. Barakos, K. Badcock, A framework for CFD analysis of helicopter rotors in hover and forward flight, Int. J. Numer. Methods Fluids 51 (2006) 819–847.
- Biava, M., Woodgate, M., and Barakos, G. N., “Fully implicit discrete-adjoint methods for rotorcraft applications,” AIAA Journal, Vol. 54, 2016, pp. 735–749.





Partially-Averaged Navier-Stokes*

PANS is a bridging model between RANS and DNS



* S.S. Girimaji and K.S. Abdol-Hamid, Partially-averaged Navier Stokes model for turbulence: implementation and validation. AIAA Aerospace Sciences Meeting and Exhibit (2005). Reno, NE, USA.

Partially-Averaged Navier-Stokes*

Eddy viscosity 2-eq model Boussinesq's approximation $(\text{k-}\epsilon,\ \text{k-}\omega)$

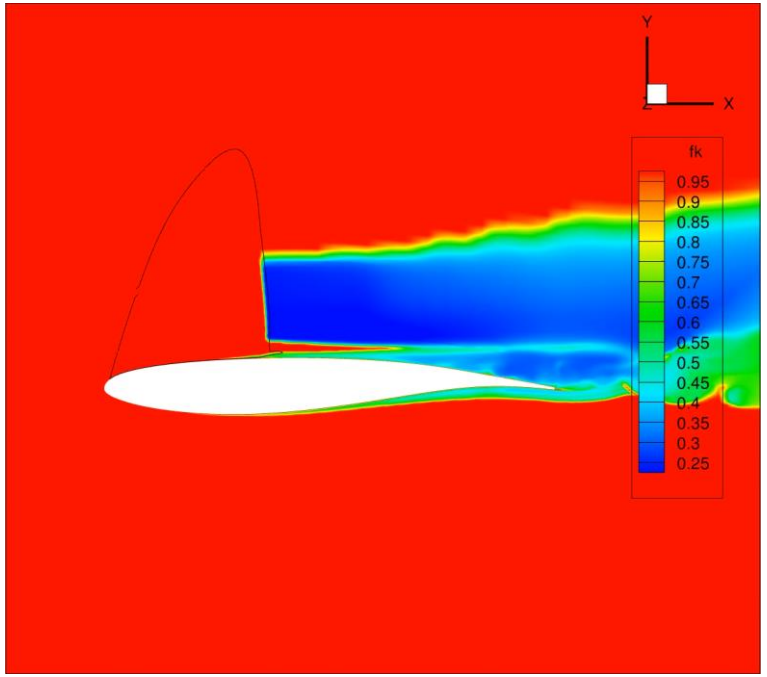
- Unresolved-to-total ratios

$$f_k = \frac{k_u}{k} \quad f_\epsilon = \frac{\epsilon_u}{\epsilon}$$

$$f_\omega = \frac{\omega_u}{\omega} = \frac{f_\epsilon}{f_k}$$

$$\nu_{PANS} = \frac{f_k^2}{f_\epsilon}\nu_{RANS} = \frac{f_k}{f_\omega}\nu_{RANS}$$

$\frac{f_k^2}{f_\epsilon} \rightarrow 1$



- Application to $k-\omega$ SST model**

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho U_j k}{\partial x_j} = \overbrace{P_k}^{\text{production}} \overbrace{-\beta^* \rho k \omega}^{\text{destruction}} + \overbrace{\frac{\partial}{\partial x_j}\left[\left(\mu + \mu_t \sigma_k \frac{f_\omega}{f_k}\right)\frac{\partial k}{\partial x_j}\right]}^{\text{diffusion}}$$

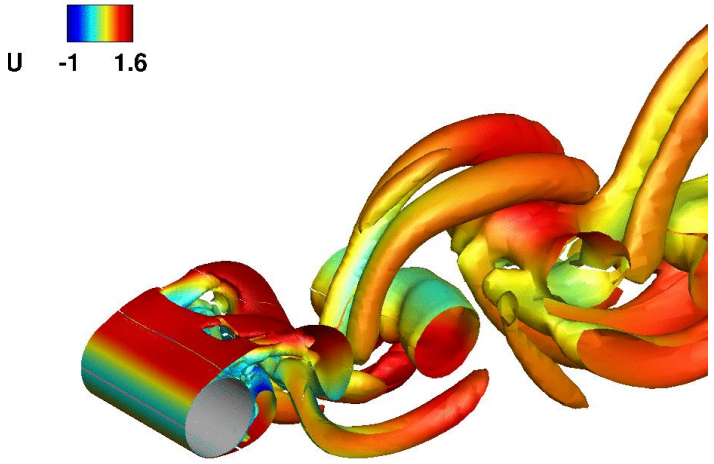
$$\frac{\partial \rho \omega}{\partial t} + \frac{\partial \rho U_j \omega}{\partial x_j} = \underbrace{\frac{\gamma}{\nu_t} P_k}_{\text{production}} \underbrace{-\left(\gamma\beta^* - \frac{\gamma\beta^*}{f_\omega} + \frac{\beta}{f_\omega}\right)\rho\omega^2}_{\text{destruction}} + \underbrace{\frac{\partial}{\partial x_j}\left[\left(\mu + \mu_t \sigma_\omega \frac{f_\omega}{f_k}\right)\frac{\partial \omega}{\partial x_j}\right]}_{\text{diffusion}} + \underbrace{2\frac{f_\omega}{f_k}(1 - F_1)\frac{\rho\sigma_{\omega 2}}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}}_{\text{cross-diffusion}}$$

* S.S. Girimaji and K.S. Abdol-Hamid, Partially-averaged Navier Stokes model for turbulence: implementation and validation. AIAA Aerospace Sciences Meeting and Exhibit (2005). Reno, NE, USA.

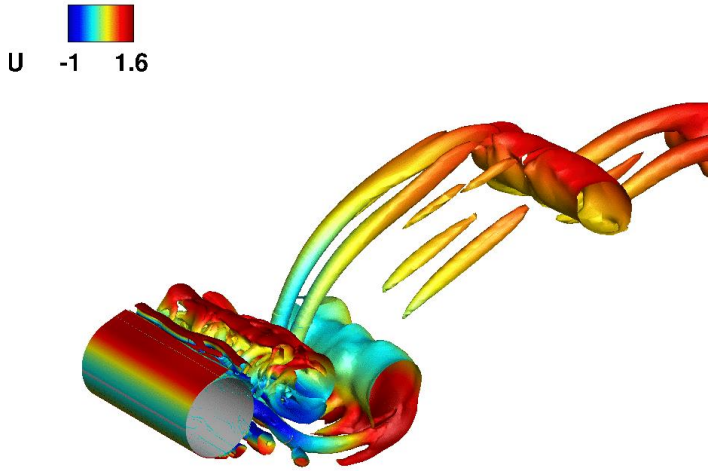
Partially-Averaged Navier-Stokes

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho U_j k}{\partial x_j} = \overbrace{P_k}^{\text{production}} \overbrace{-\beta^* \rho k \omega}^{\text{destruction}} + \overbrace{\frac{\partial}{\partial x_j}\left[\left(\mu + \mu_t \sigma_k \frac{f_\omega}{f_k}\right)\frac{\partial k}{\partial x_j}\right]}^{\text{diffusion}}$$

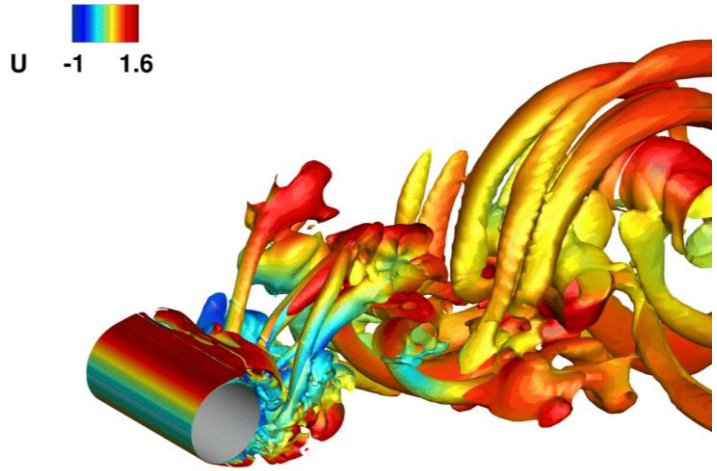
$$\frac{\partial \rho \omega}{\partial t} + \frac{\partial \rho U_j \omega}{\partial x_j} = \underbrace{\frac{\gamma}{\nu_t} P_k}_{\text{production}} \underbrace{-\left(\gamma\beta^* - \frac{\gamma\beta^*}{f_\omega} + \frac{\beta}{f_\omega}\right)\rho\omega^2}_{\text{destruction}} + \underbrace{\frac{\partial}{\partial x_j}\left[\left(\mu + \mu_t \sigma_\omega \frac{f_\omega}{f_k}\right)\frac{\partial \omega}{\partial x_j}\right]}_{\text{diffusion}} + \underbrace{2\frac{f_\omega}{f_k}(1 - F_1)\frac{\rho\sigma_{\omega 2}}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}}_{\text{cross-diffusion}}$$



$f_k = 0.6$



$f_k = 0.4$

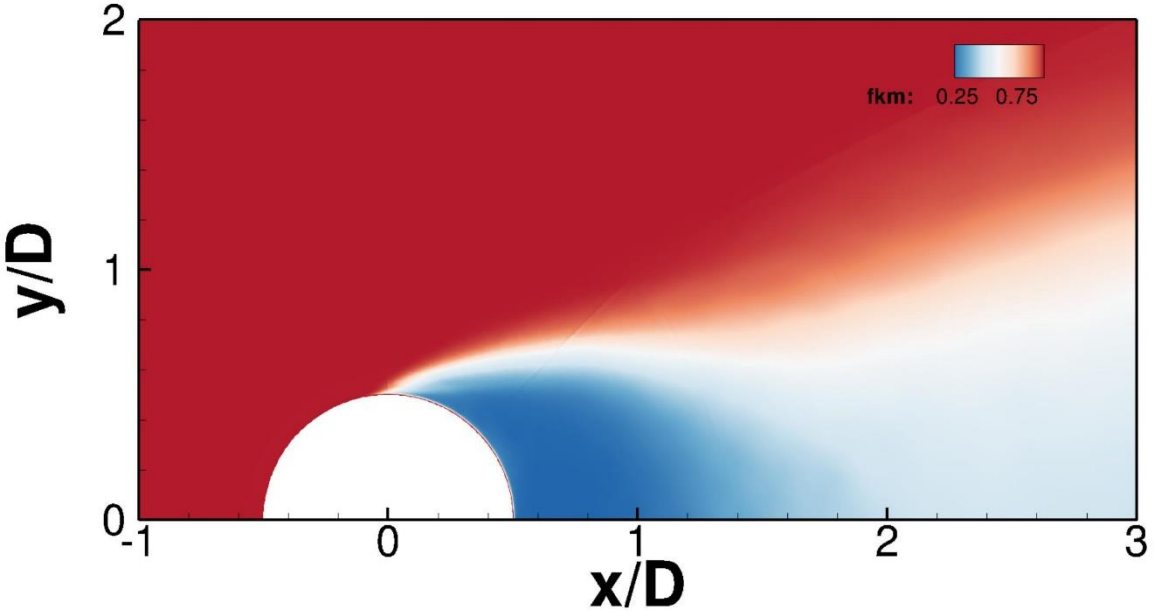


$f_k = 0.2$

Estimates of f_k

Three approaches in literature:

- **Constant** $f_k = const$
- **Static** $f_k = f(x)$
  - Preliminary RANS solution
- **Dynamic** $f_k = f(x, t)$
  - Runtime evaluation



Approach	Constant	Static	Dynamic
Pros	• Easy implementation • No commutation error	• Optimal use of grid • One estimate of f_k required • Ok for fixed separation	• Optimal use of grid • **Optimal for statistically unsteady flows**
Cons	• **Near wall problems** • f_k is case-grid sensitive • **Δx, Δt required too fine**	• No consensus on estimates • Not optimal for statistically unsteady flows	• Harder implementation • Possibly higher costs

Estimates of f_k

1) Compute the unresolved Kolmogorov length-scale

$$\eta_u = \left(\frac{\nu_u^3}{\epsilon_u}\right)^{1/4}$$

$$= C_\mu^{3/4} \frac{f_k^{3/2}}{f_\epsilon} \boxed{\frac{k^{3/2}}{\epsilon}} \quad l_t$$

turbulence length scale

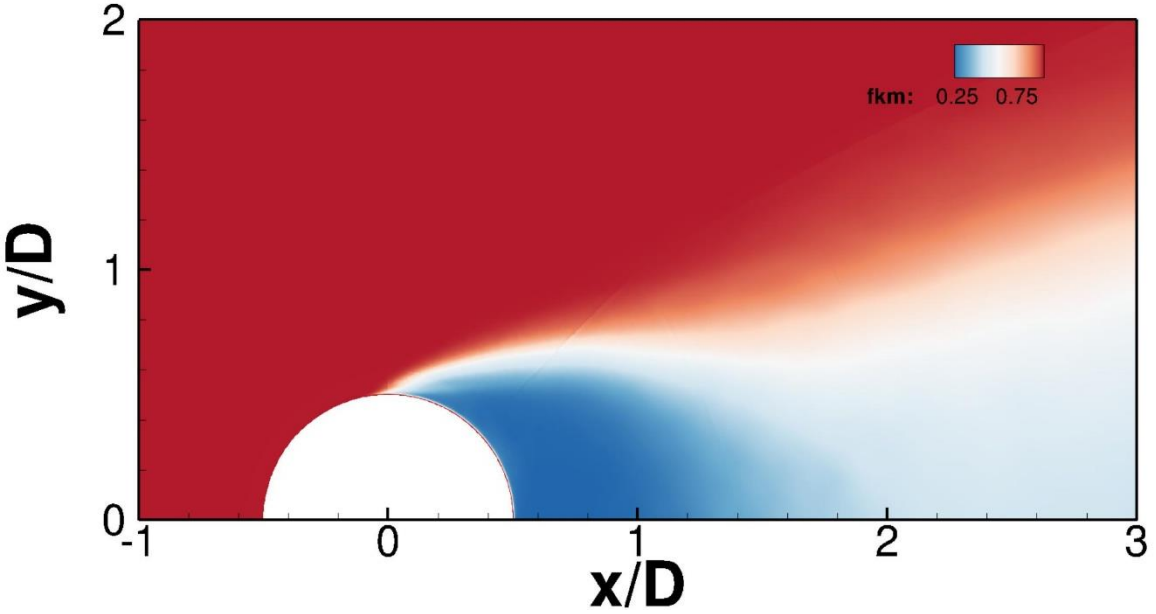
2) Compare it to the grid size

$$\eta_u = \Delta$$

3) Find the optimum

$$\boxed{f_k = C_\mu^{-1/2} (\Delta / l_t)^{2/3}}$$

Grid-dependent Flow-dependent

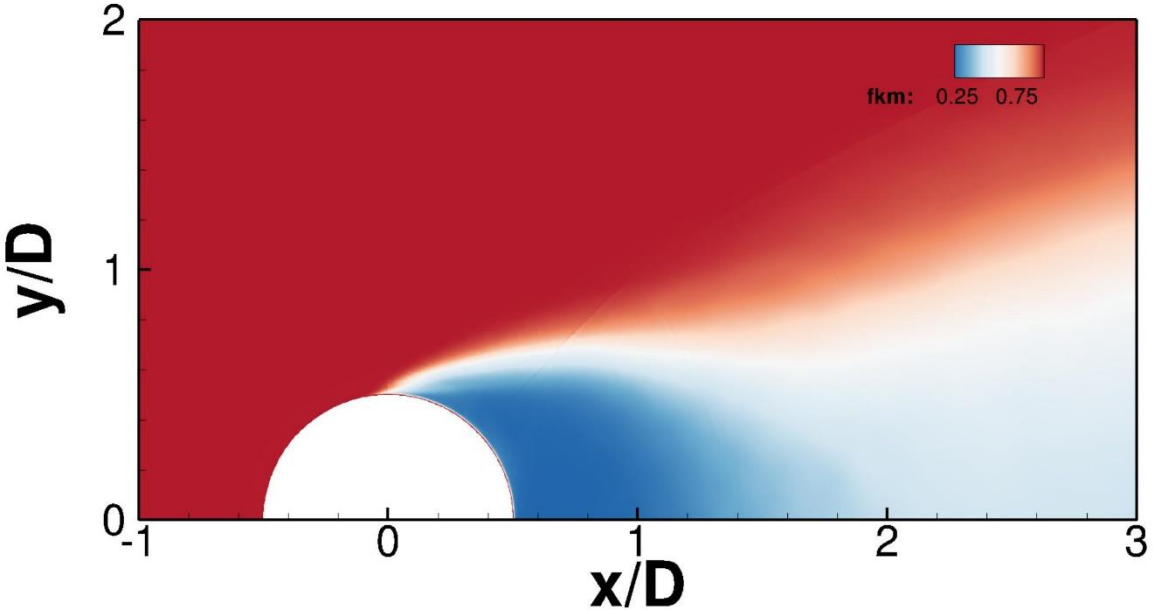


S.S. Girimaji. Partially-averaged Navier-Stokes model for turbulence: Fixed Point Analysis and Comparison With Unsteady Partially Averaged Navier Stokes. Journal of Applied Mechanics, Transactions ASME, 73:422–429, 2006.

Estimates of f_k

Dynamic estimates*, based on:

- Local grid size: $\Delta = (\Delta x\ \Delta y\ \Delta x)^{1/3}$
- Turbulent lengthscale: $\Lambda_u = k/(C_\mu \omega)$



$$f_k = C_{\mathrm{PANS}} \left(\frac{\Delta}{\Lambda_u} \right)^{2/3}$$

$$f_k = \frac{1 + \tanh\left(2\pi\left(\Lambda - 0.5\right)\right)}{2}, \qquad \Lambda = \frac{1}{1 + \left(\frac{\Lambda_u}{\Delta} \right)^{4/3}}$$

*M. Klapwijk, T. Lloyd, and G. Vaz. On the accuracy of partially averaged Navier–Stokes resolution estimates. *International Journal of Heat and Fluid Flow*, 80:1–10, 2019.

HMB3 Solver*,**

Finite Volume Method

URANS, LES, DES, SAS & PANS available

Variety of turbulence & transition models

Multi-block structured/unstructured grids

- Chimera overset grid method
- Moving/Deforming grids, Sliding Planes

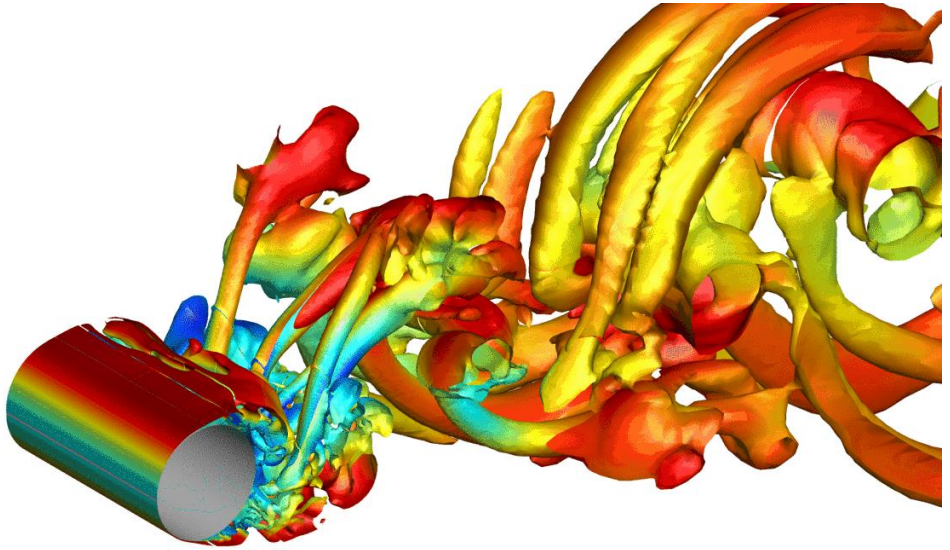
Fully-Implicit time marching/Frequency domain

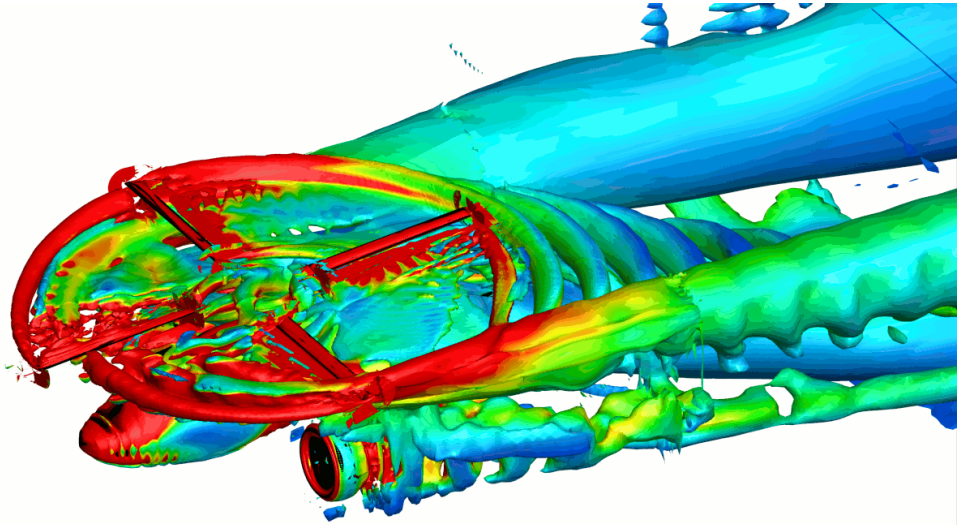
Osher, Roe, LM-Roe, AUSM+/UP Schemes

3rd, 4th –order MUSCL schemes

Central differences for viscous fluxes

High-performance parallel computing

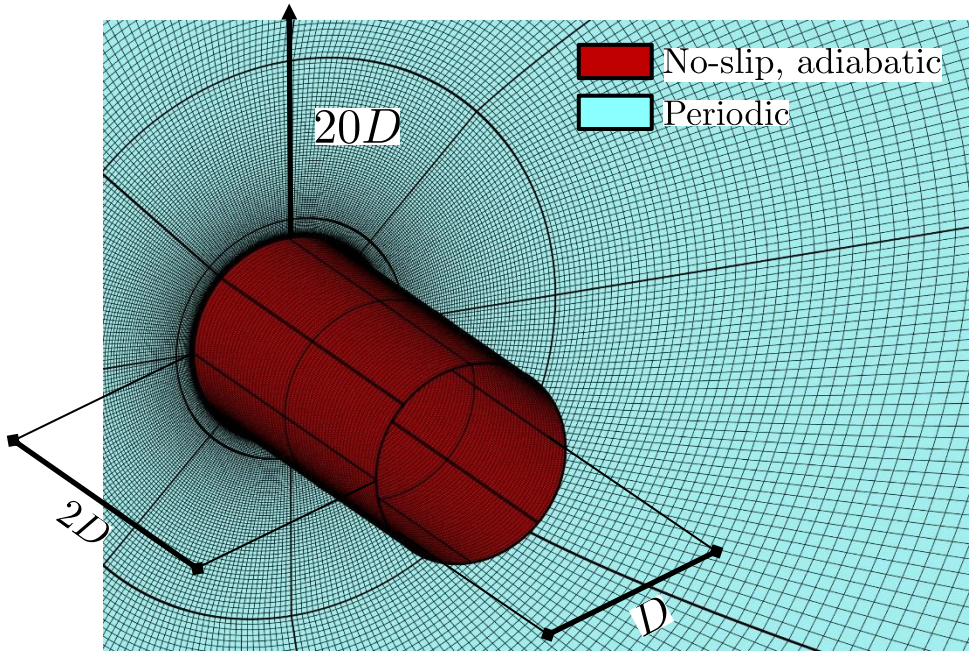


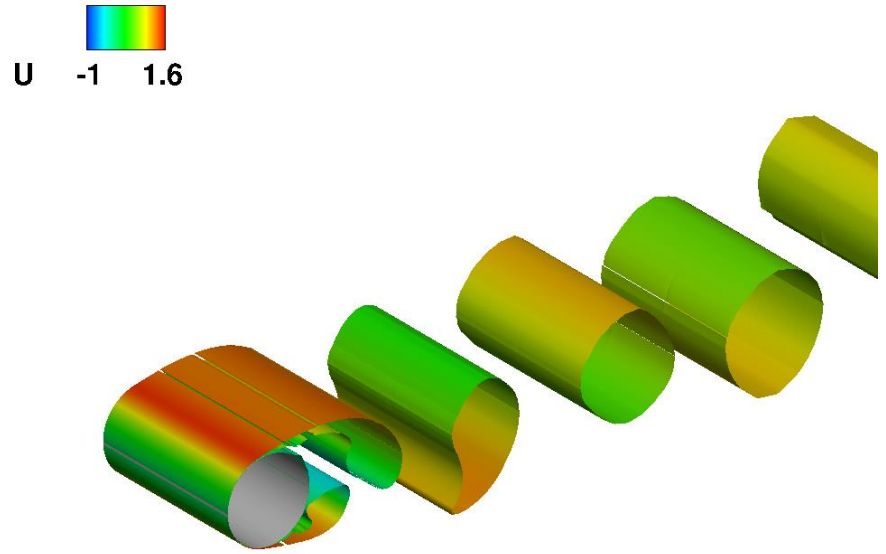


* R. Steijl, G. Barakos, K. Badcock, A framework for CFD analysis of helicopter rotors in hover and forward flight, Int. J. Numer. Methods Fluids 51 (2006) 819–847.
** Biava, M., Woodgate, M., and Barakos, G. N., "Fully implicit discrete-adjoint methods for rotorcraft applications," *AIAA Journal*, Vol. 54, 2016, pp. 735–749.

Flow Around a Circular Cylinder

- A. Petrocchi, G. N. Barakos, "Dynamic fk Estimates for Partially-Averaged Navier-Stokes Around a Circular Cylinder". In: AIAA SCITECH 2024 Forum, 8-12 January 2024, Orlando, FL, US.

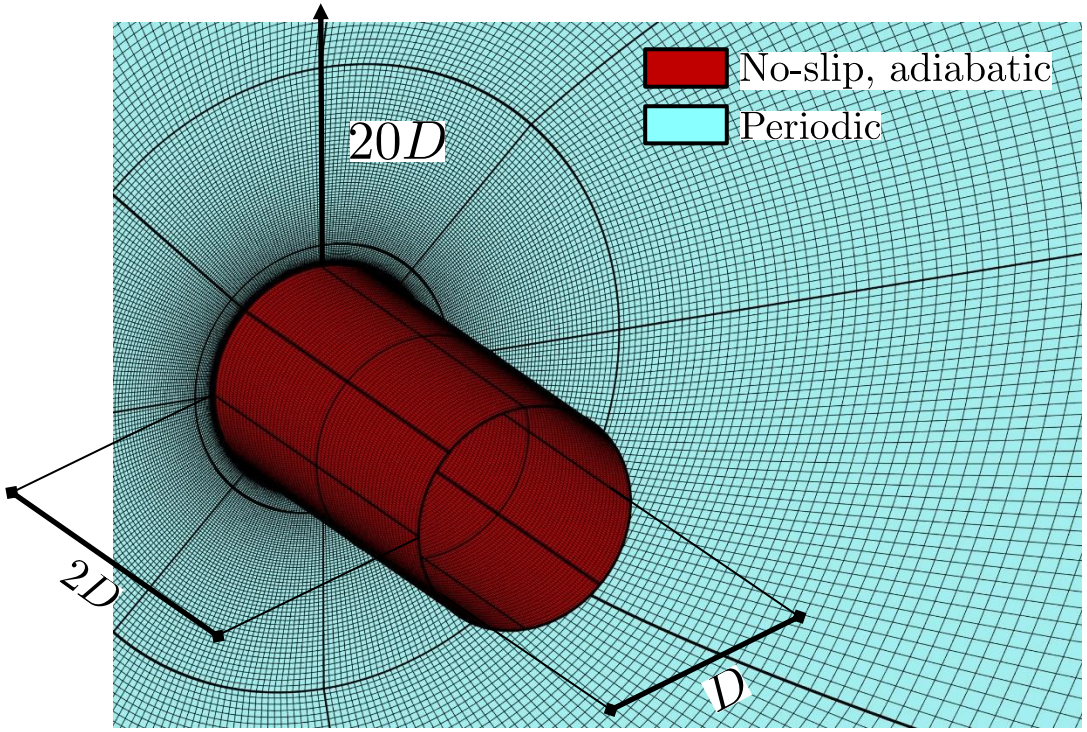




CFD Test Case

Circular cylinder*

- $\mathrm{Re}_D = \rho \mathrm{U}_\infty D/\mu = 5 \times 10^4$
- Laminar BL and turbulent wake
- Vortex shedding (VS) with $\mathrm{St} = 0.18 \div 0.22$
- $\mathrm{M} = 0.3$
- $\frac{L_z}{D} = 2$ (spanwise)
- $\frac{L_y}{D} = 20$ (wall-normal)
- $\Delta t \frac{U_\infty}{D} = 0.01$ (approx. 500 steps/vortex shedding period), if not differently specified.



Fine grid

x1.5

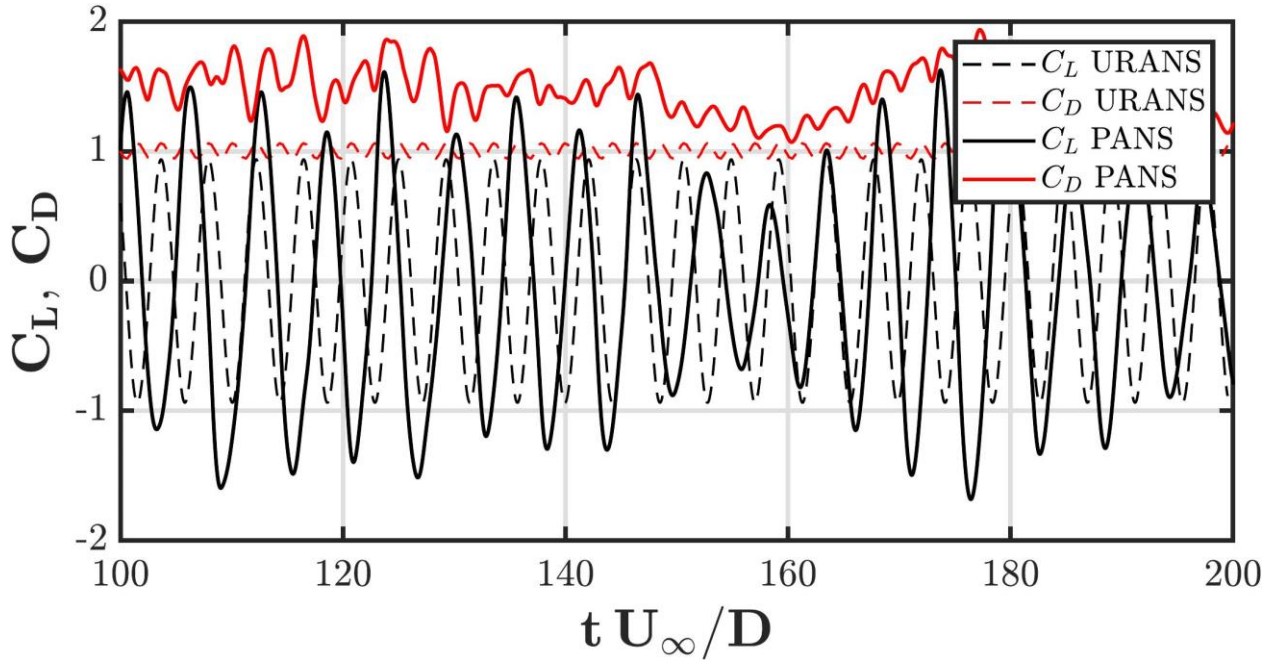
Grid #	N_θ	N_y	N_z	N_{tot}
Coarse	136	99	32	3.8×10^5
Medium	208	135	40	1.1×10^6
Fine	304	201	60	3.7×10^6

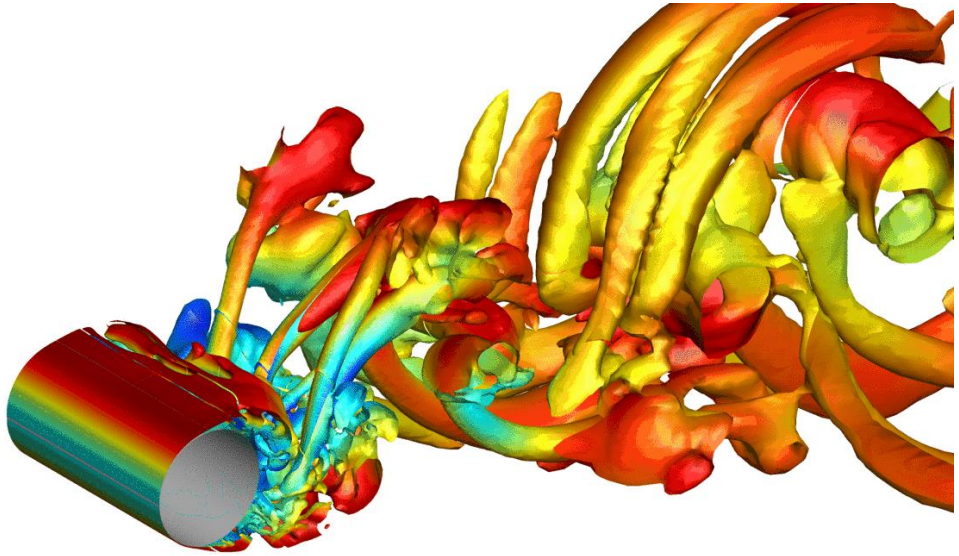
*C. Norberg. An experimental investigation of the flow around a circular cylinder: Influence of aspect ratio. *Journal of Fluid Mechanics*, 258:287–316, 1994.

CFD Test Case

Circular cylinder*

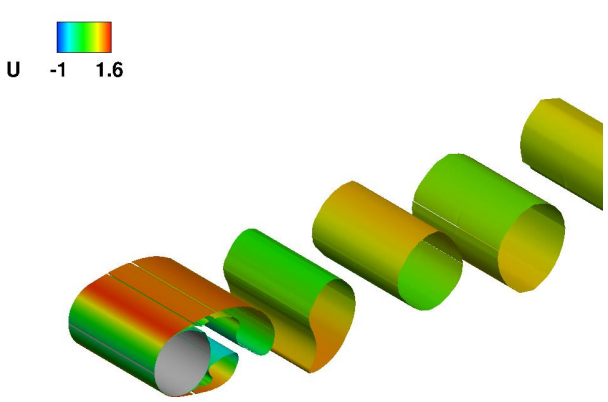
- $\mathrm{Re}_D = \rho \mathrm{U}_\infty D/\mu = 5 \times 10^4$
- Laminar BL and turbulent wake
- Vortex shedding (VS) with $\mathrm{St} = 0.18 \div 0.22$
- $\mathrm{M} = 0.3$
- $\frac{L_z}{D} = 2$ (spanwise)
- $\frac{L_y}{D} = 20$ (wall-normal)
- $\Delta t \frac{U_\infty}{D} = 0.01$ (approx. 500 steps/vortex shedding period), if not differently specified.
- Simulation time: **at least** 20 VS periods



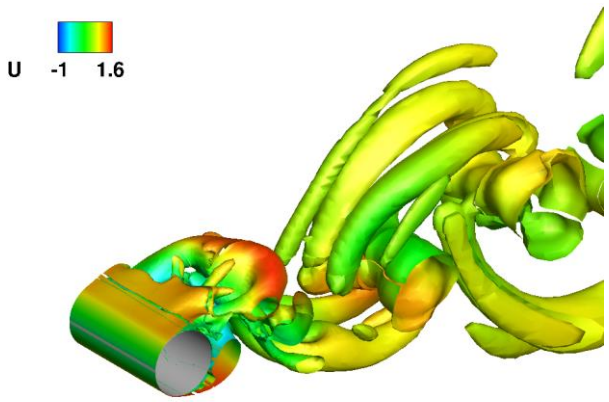


*C. Norberg. An experimental investigation of the flow around a circular cylinder: Influence of aspect ratio. *Journal of Fluid Mechanics*, 258:287–316, 1994.

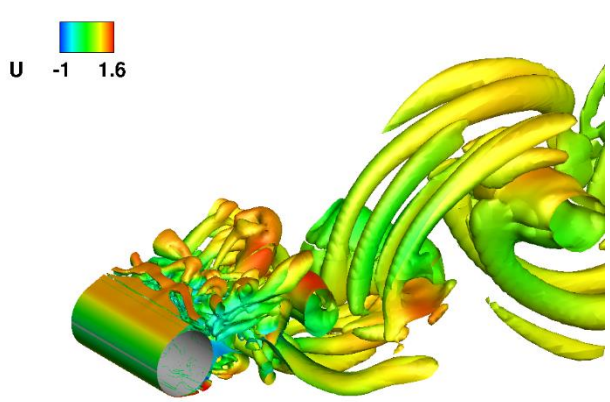
Constant f_k



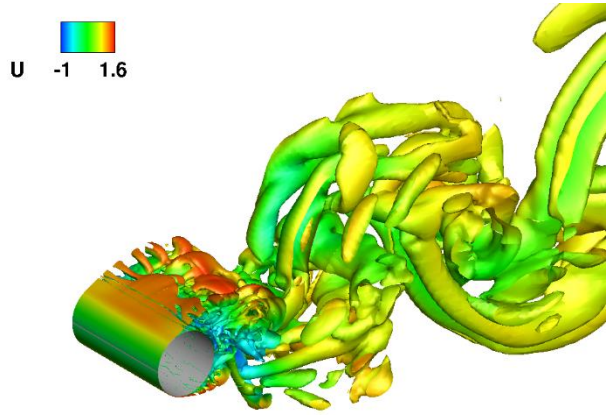
$f_k = 1$



$f_k = 0.6$



$f_k = 0.4$

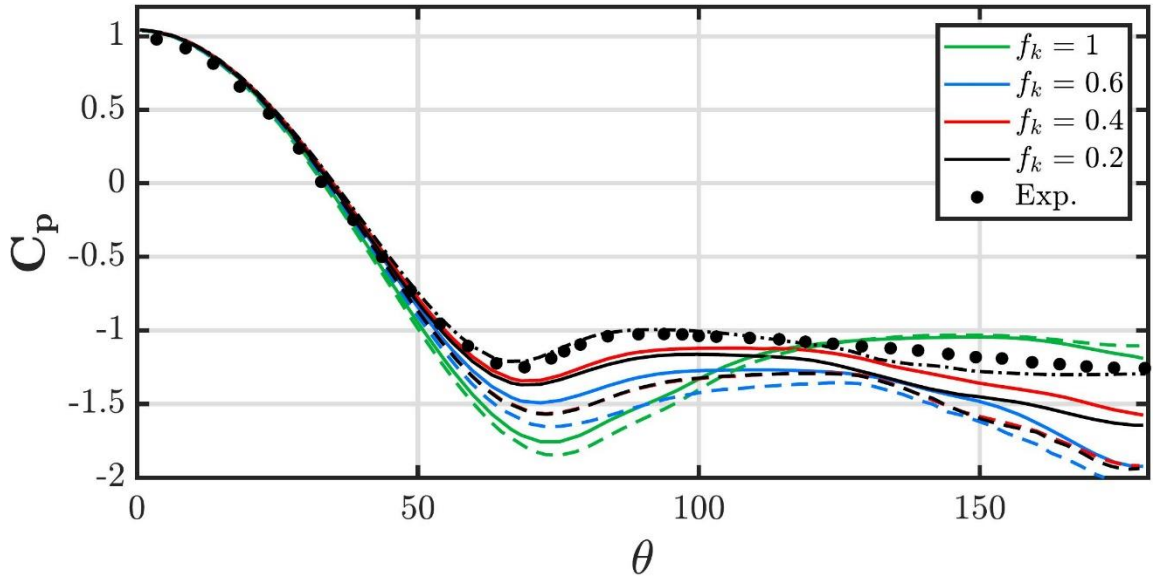


$f_k = 0.2$

Smaller f_k:

- Higher resolution in the wake

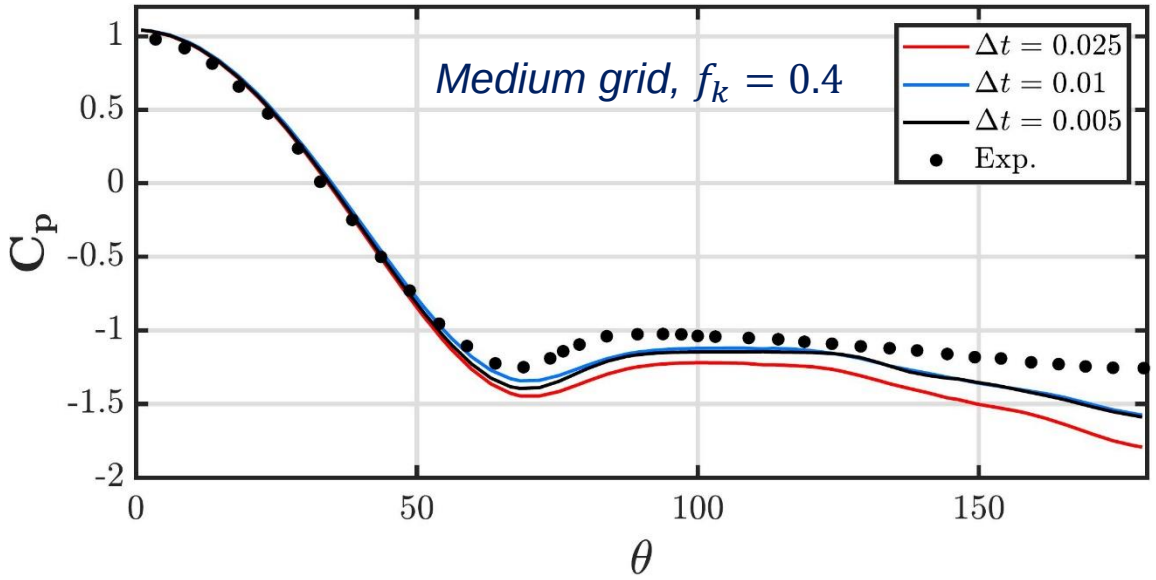
Constant f_k

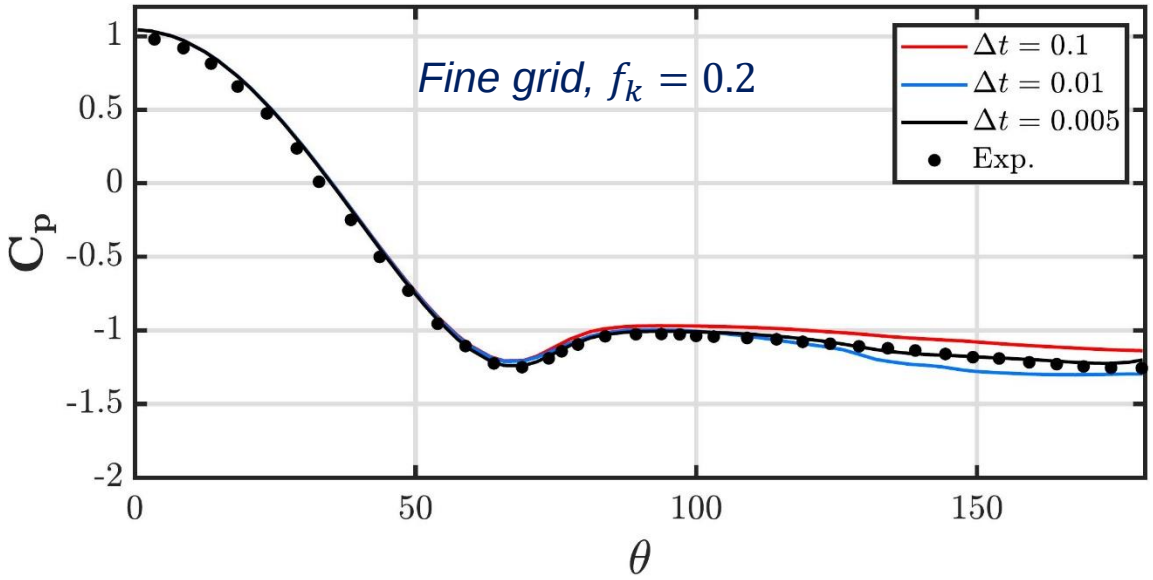


Smaller f_k:

- Higher resolution in the wake
- Reduced μ_T → early BL separation

Constant f_k

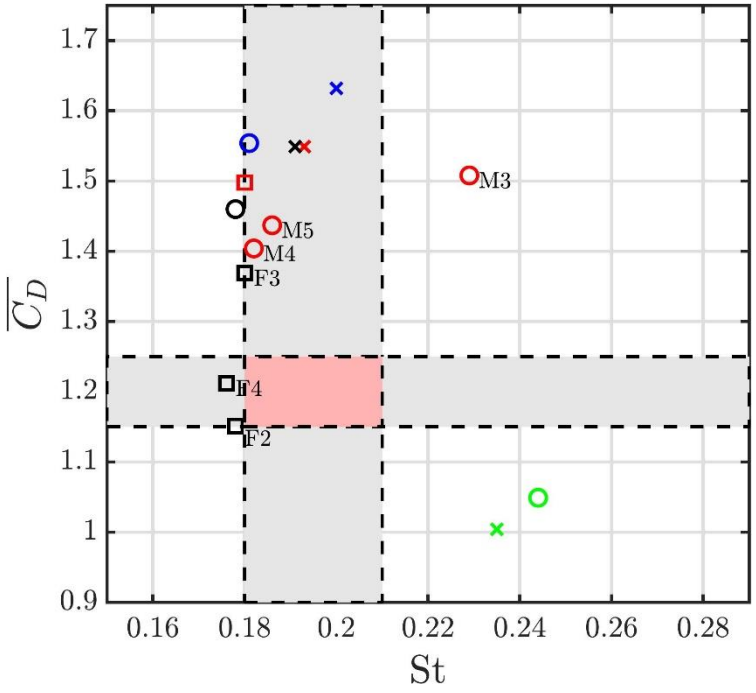


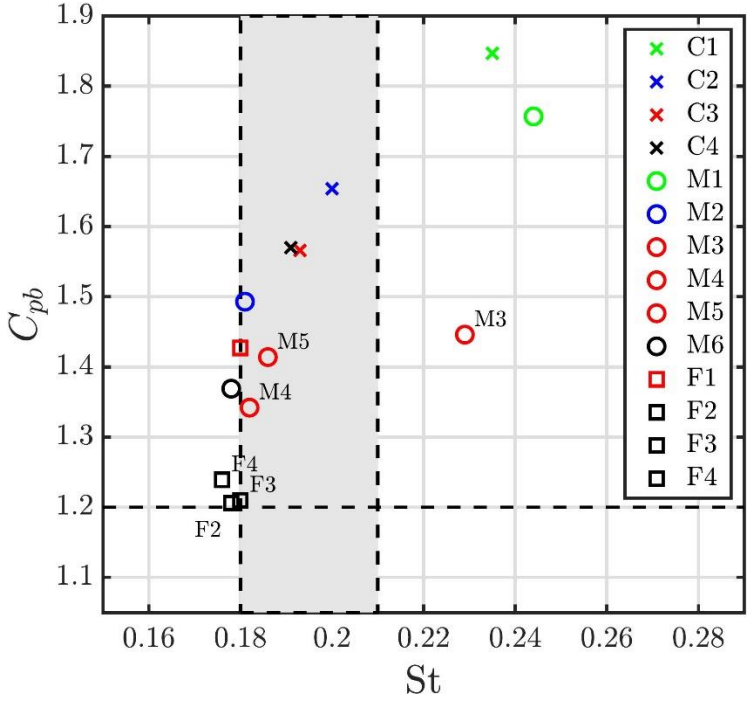


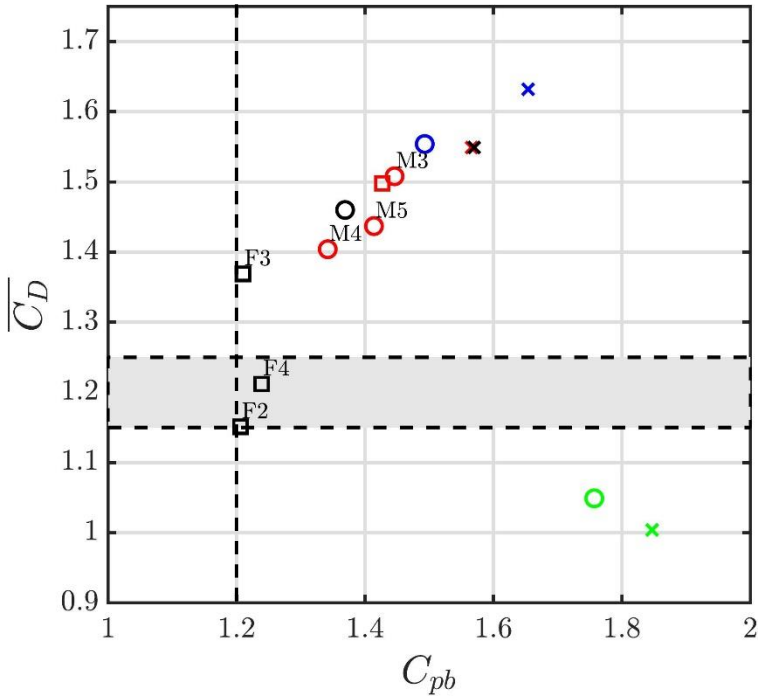
Smaller f_k:

- Higher resolution in the wake
- Reduced μ_T → early BL separation
- Smaller Δt required

Constant f_k

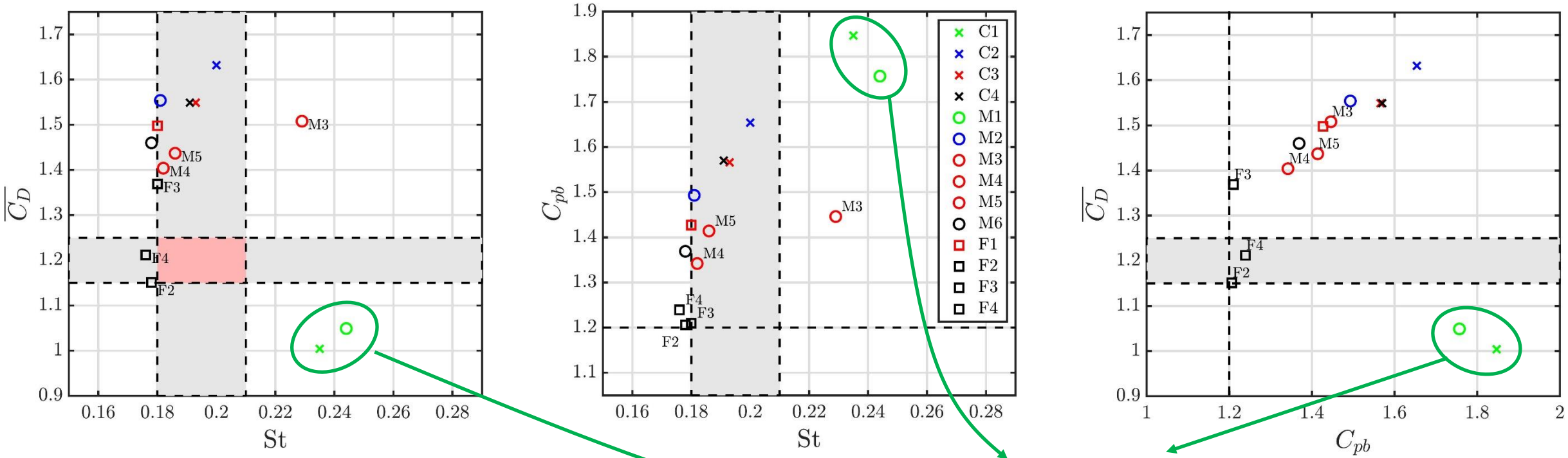






- Big scatter in the results

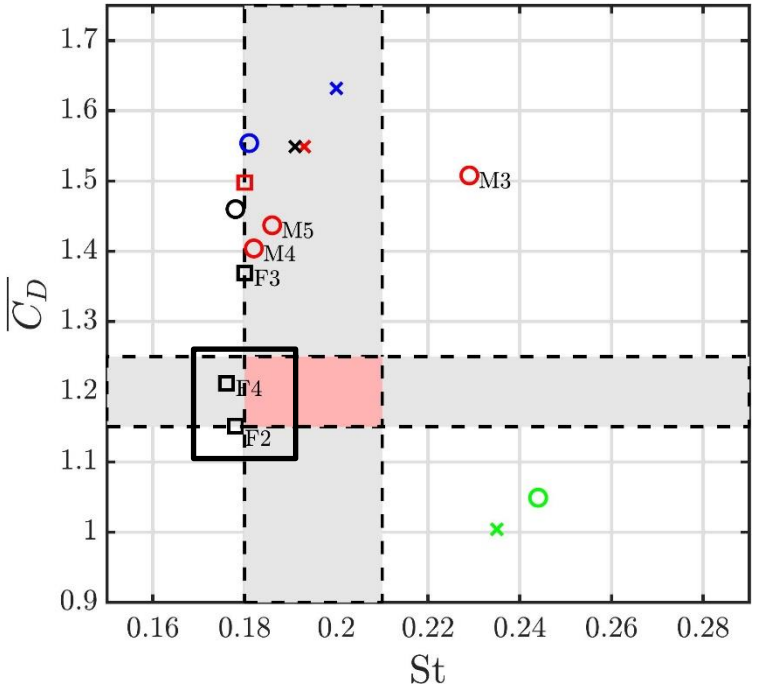
Constant f_k

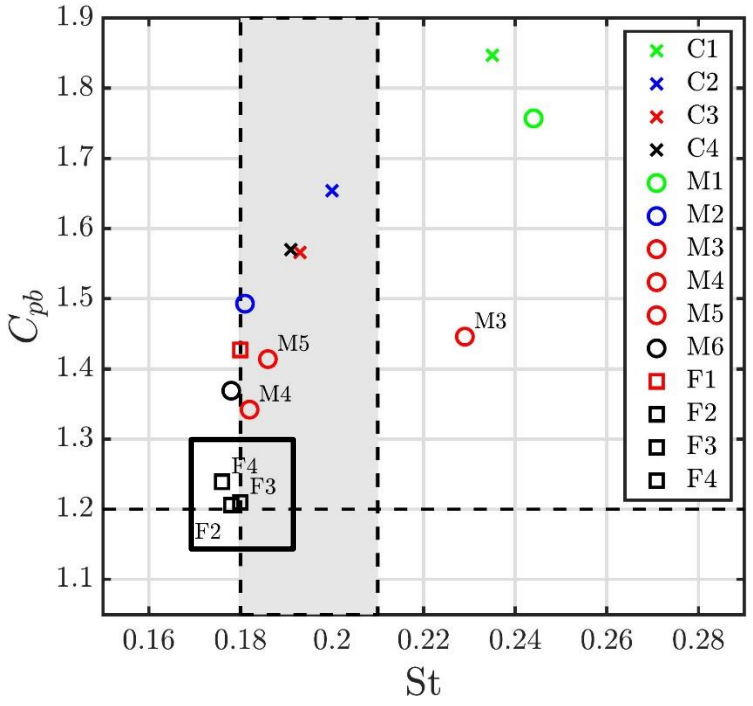


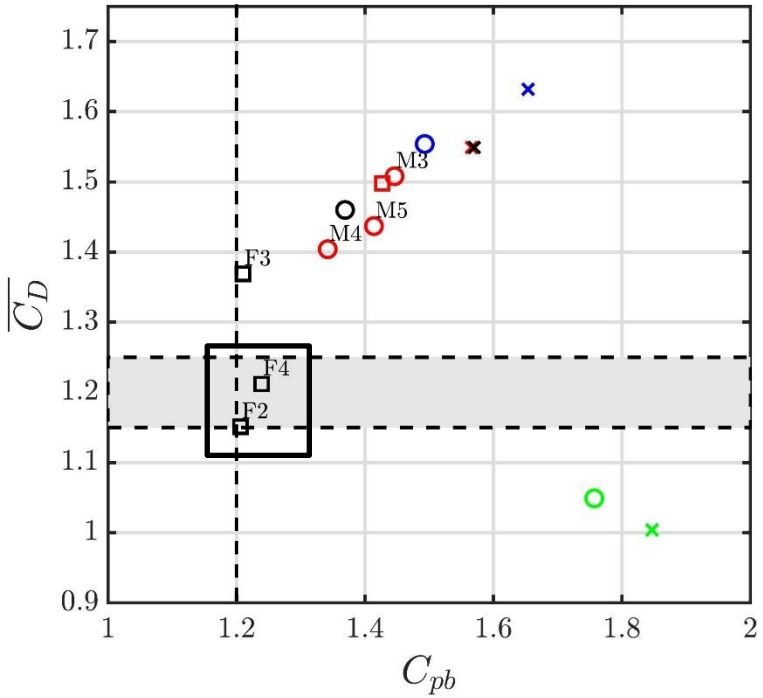
URANS

- Big scatter in the results
- PANS results improved wrt URANS

Constant f_k

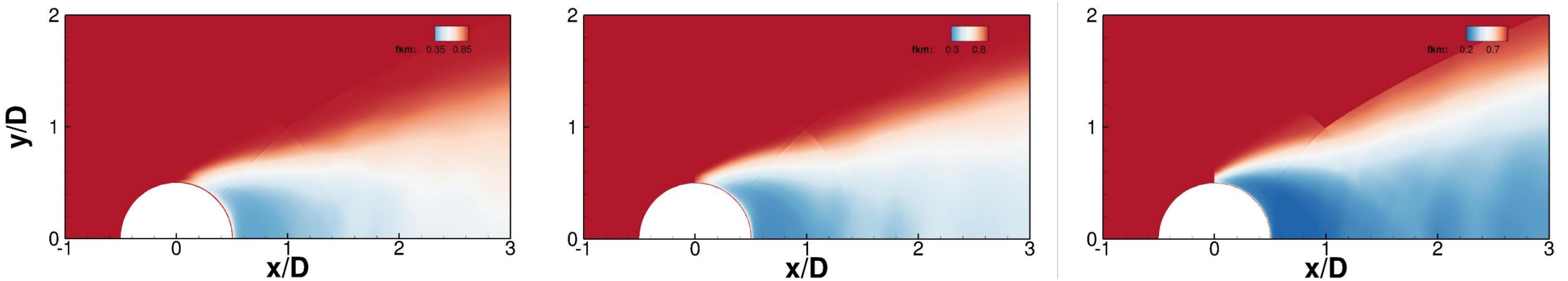






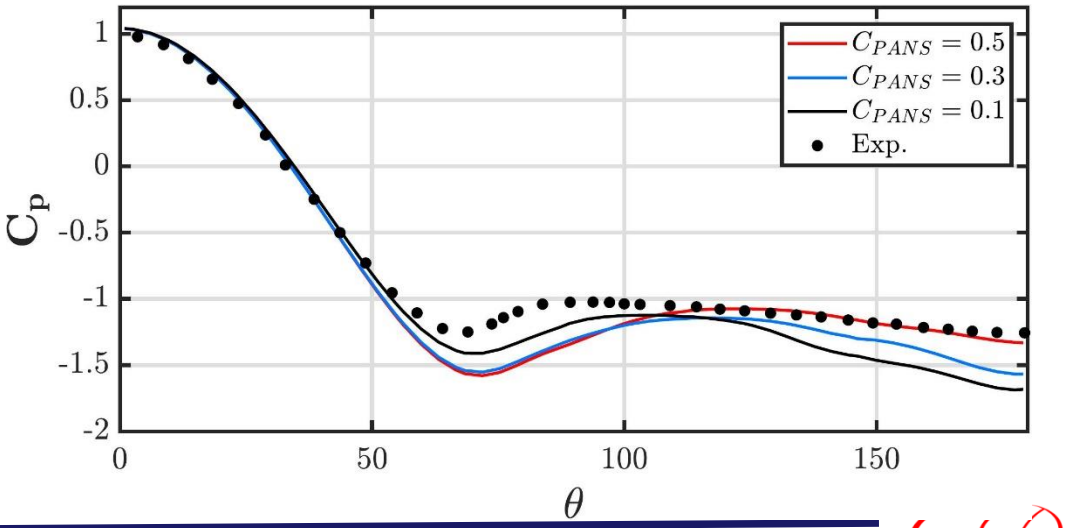
- Big scatter in the results
- PANS results improved wrt URANS
- Fine mesh and $\Delta t \rightarrow$ Good agreement

Dynamic estimate: Case 1

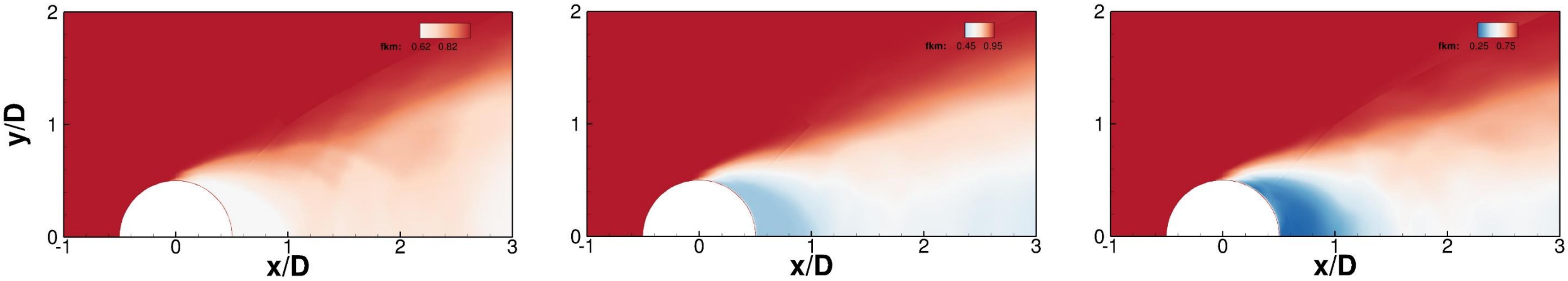


- f_k is lowered in region of high turbulence and near the wall
- In the boundary layer, $f_k \simeq 1$
- f_k is proportional to C_{PANS}
- C_{PANS} must be tuned → Main drawback
- The prediction benefits from lower C_{PANS}

$$f_k = C_{\mathrm{PANS}} \left(\frac{\Delta}{\Lambda_u} \right)^{2/3}$$

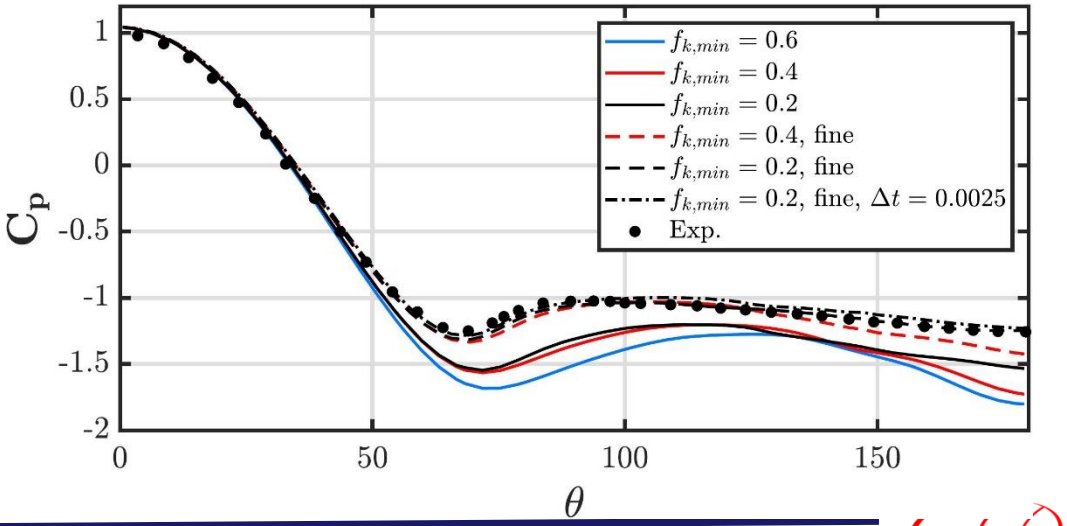


Dynamic estimate: Case 2

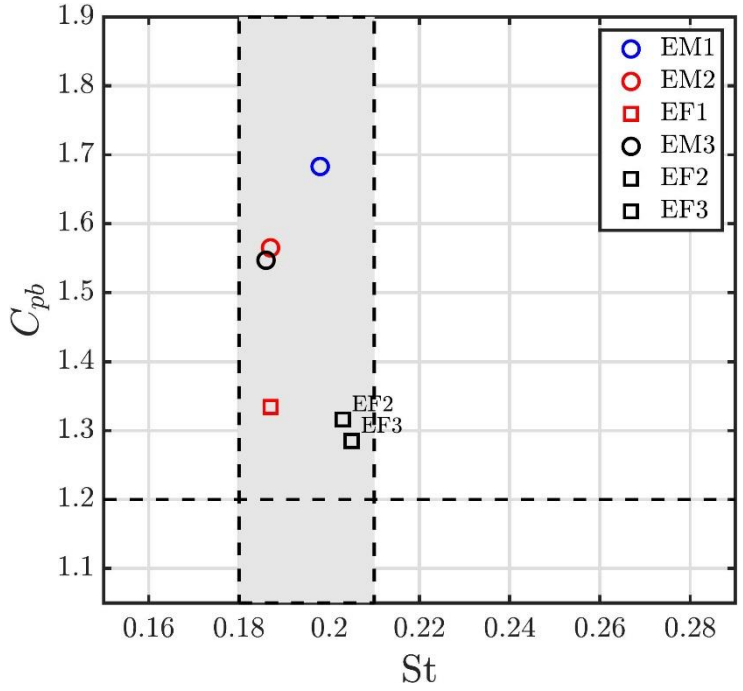


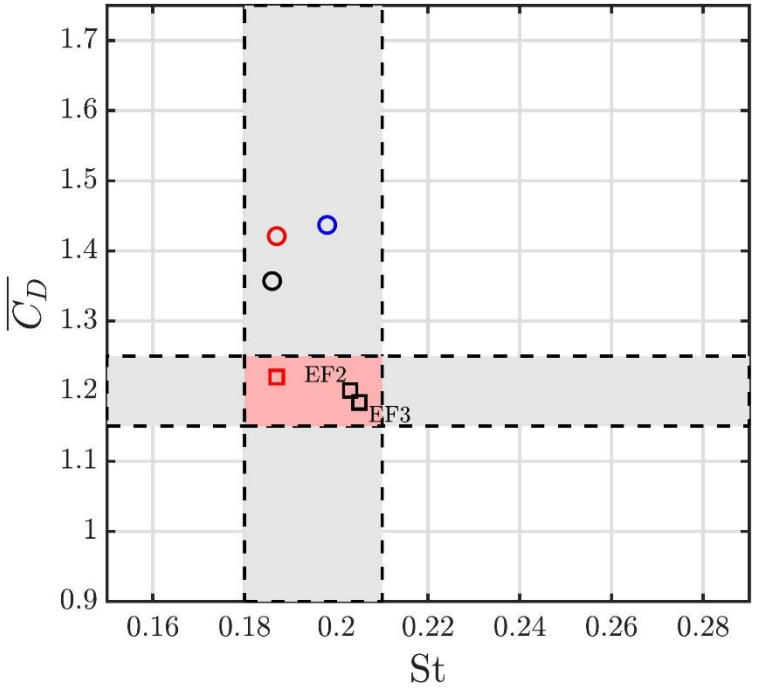
$$f_k = \frac{1 + \tanh\left(2\pi\left(\Lambda - 0.5\right)\right)}{2}, \quad \Lambda = \frac{1}{1 + \left(\frac{\Lambda_u}{\Delta}\right)^{4/3}}$$

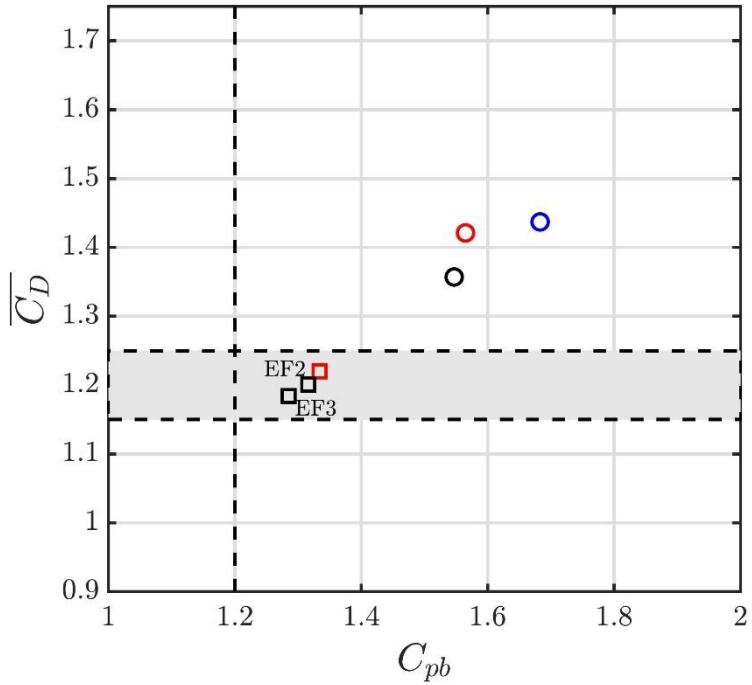
- Similar behaviour
- $f_{k,min}$ clips the value of f_k
- Small $f_{k,min}$ improves the prediction



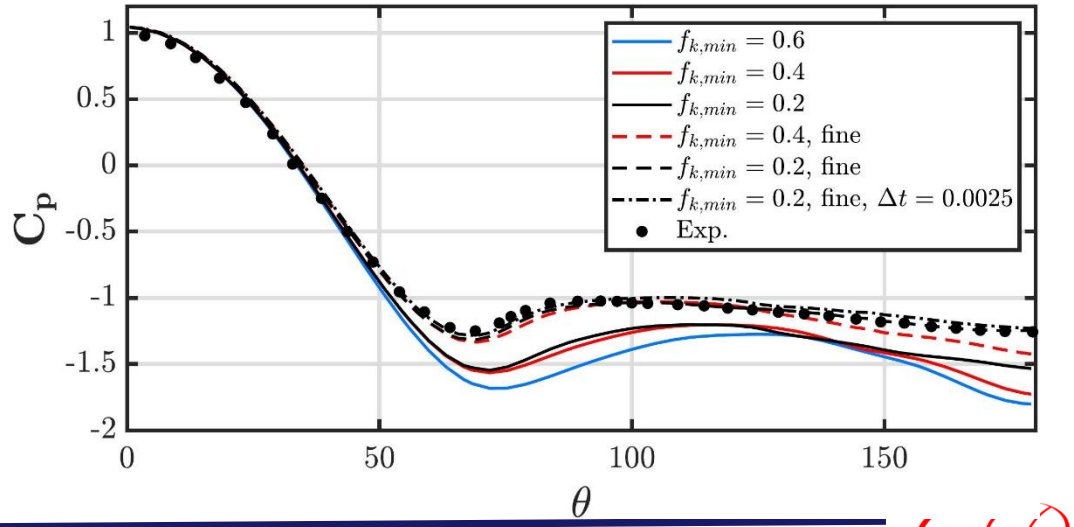
Dynamic estimate: Case 2





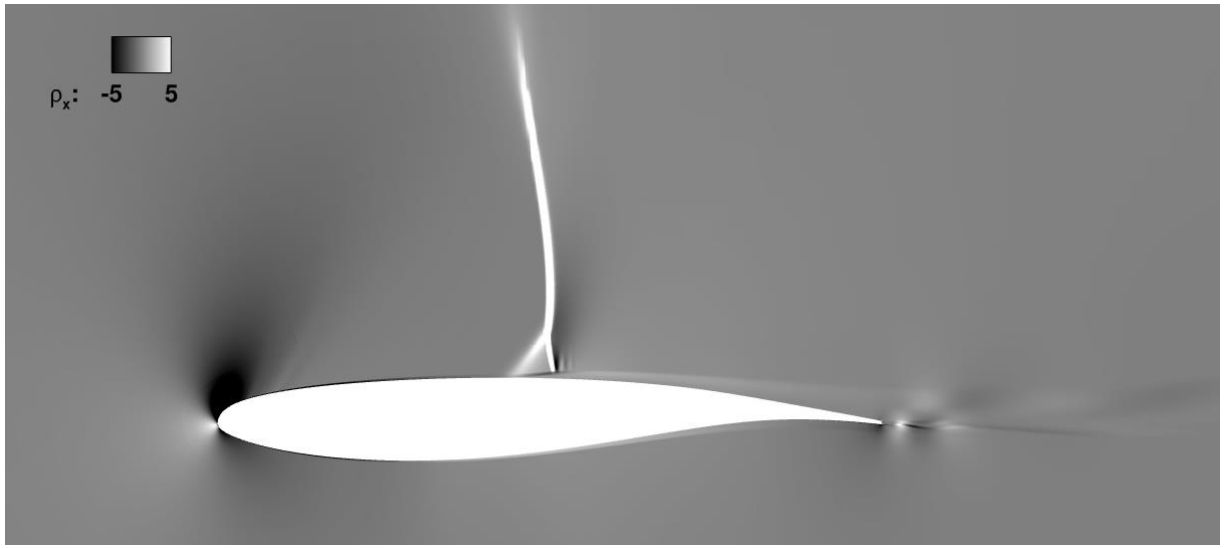


- Similar behaviour
- $f_{k,min}$ clips the value of f_k
- Small $f_{k,min}$ improves the prediction
- Fine grid leads to better prediction



Transonic Buffet Flow

- A.Petrocchi,G.N.Barakos,"Transonic Buffet Simulation Using A Partially-Averaged Navier-Stokes Approach". In: 8th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2022), Oslo, Norway, 5-9 June 2022.
- A.Petrocchi, G.N.Barakos, "Transonic Buffet Simulation using a Partially-Averaged Navier-Stokes Approach", Aerospace Science and Technology.



Transonic Buffet

Buffet: "*self-sustained shock oscillations around aerofoils and wings in transonic conditions*"

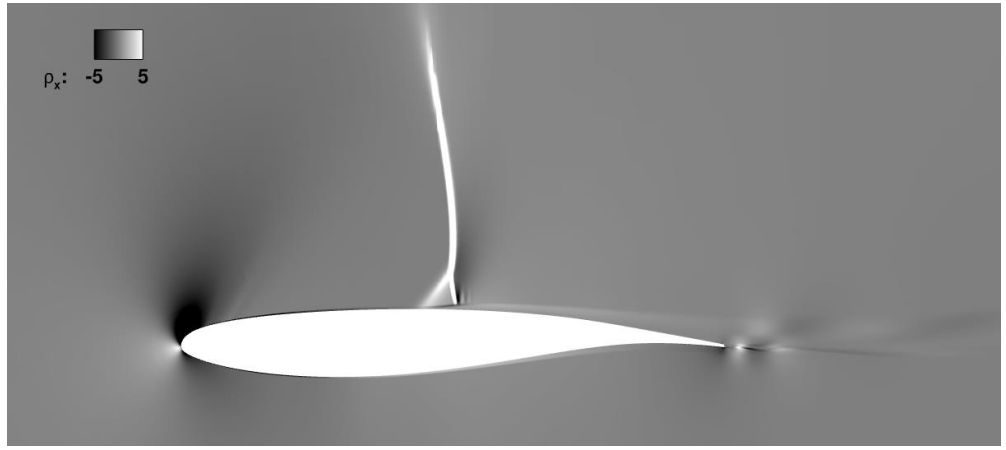
Physics and causes of buffet are not fully understood

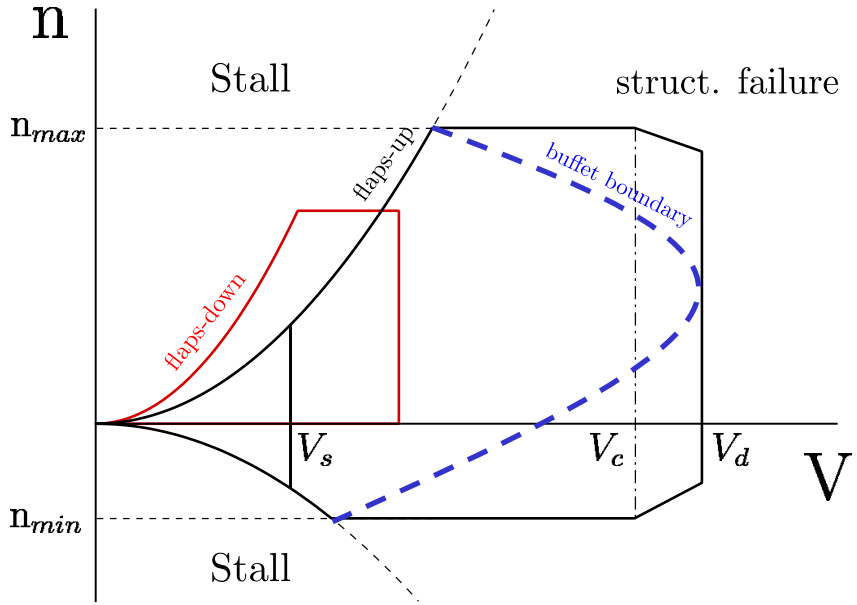
CFD of buffet is challenging

The buffet boundary limits the flight envelope

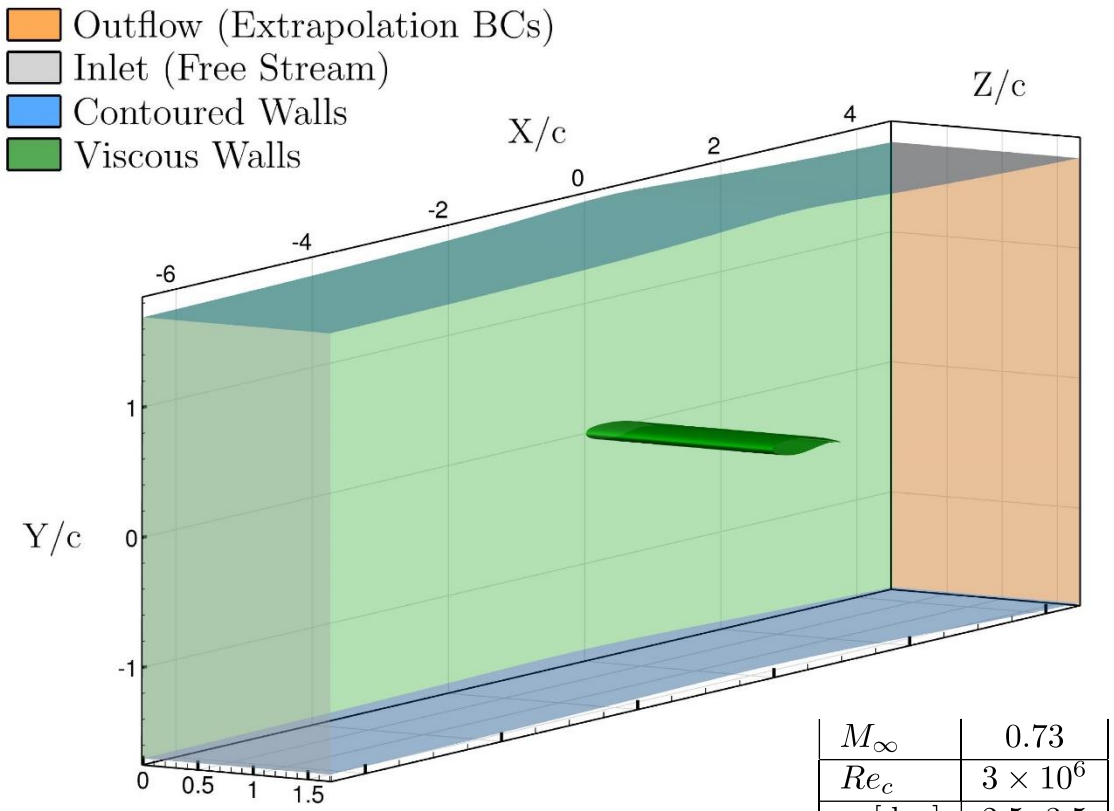
Emergency manoeuvres and **gusts** → the aircraft can cross the buffet boundary

- An angle of attack increase
- A Mach number increase



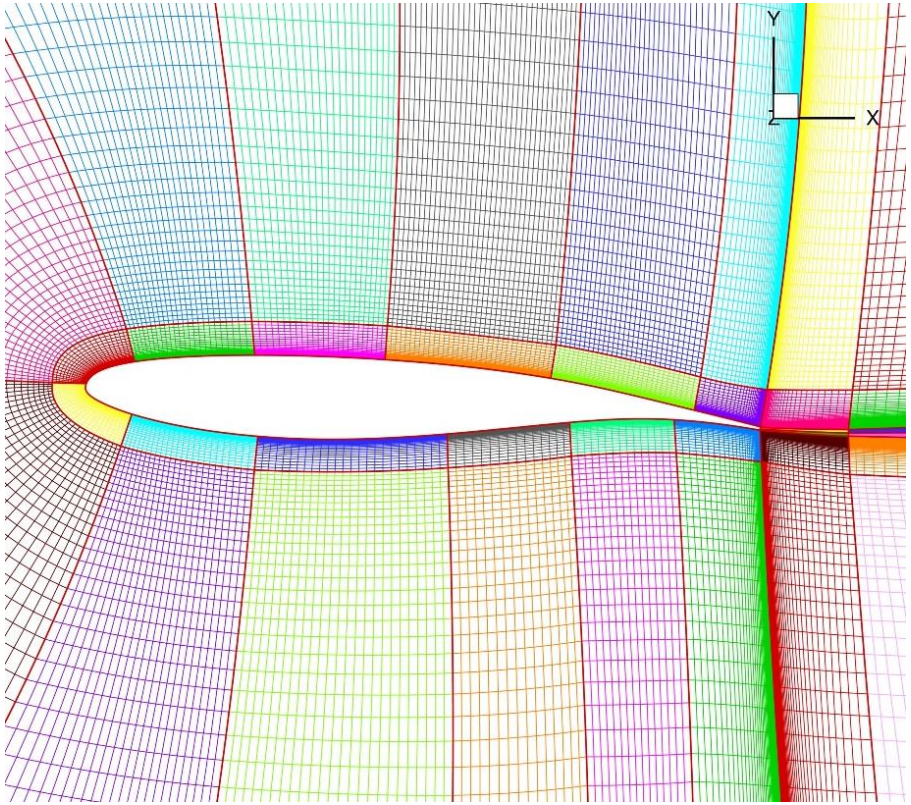


OAT15A test case



M_∞	0.73
Re_c	3×10^6
α [deg]	2.5, 3.5

Upper and lower walls (slip walls) shape extracted from a preliminary 2D simulation



Implicit time-marching scheme ($\Delta t = 0.01\ t^*$)

Osher's scheme with MUSCL extrapolation

Central difference for viscous fluxes

Convergence crit.: 3 ord. reduction of the residual

*L. Jacquin, P. Molton, S. Deck, B. Maury, and D. Soulevant, "Experimental study of shock oscillation over a transonic supercritical profile," AIAA Journal, vol. 47, pp. 1985–1994, 2009.

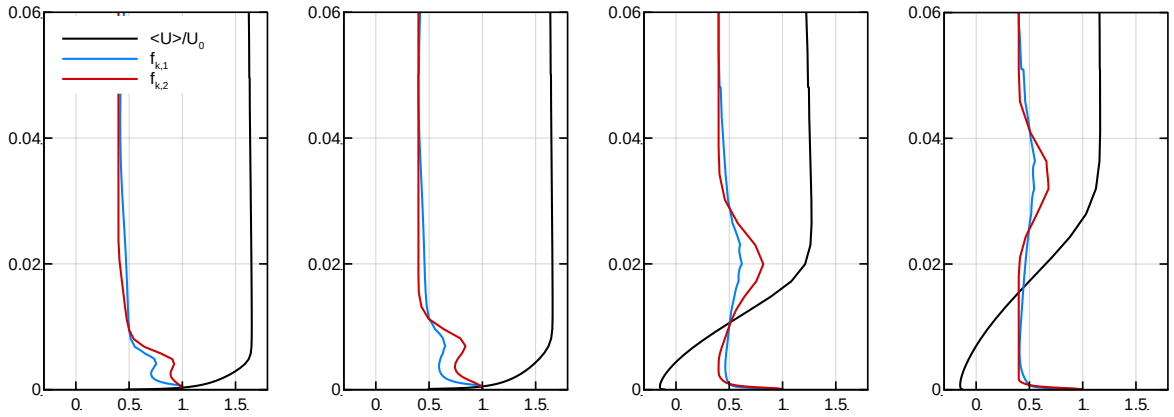
Estimates of f_k

Both based on local level of turbulence and grid size

Very similar profiles

The first one has an additional constant C_{PANS}

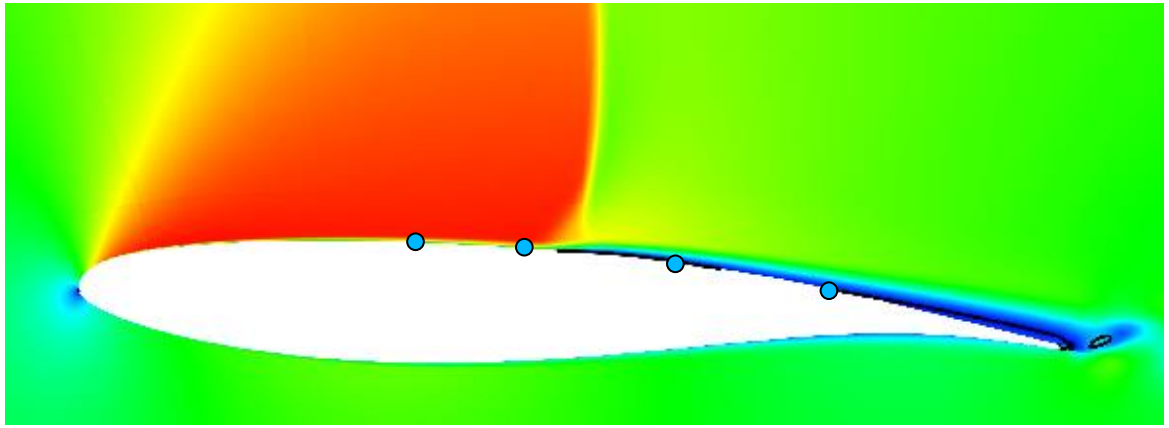
The second allows for slightly later transition in the BL



$x/c = 0.28$ $x/c = 0.45$ $x/c = 0.60$ $x/c = 0.75$

$$f_{k,1} = C_{\text{PANS}} \left(\frac{\Delta}{L_t} \right)^{2/3} \quad *$$

$$f_{k,2} = \frac{1 + \tanh(2\pi(\Lambda - 0.5))}{2}, \; \Lambda = \frac{1}{1 + \left(\frac{L_t}{\Delta} \right)^{4/3}} \quad **$$

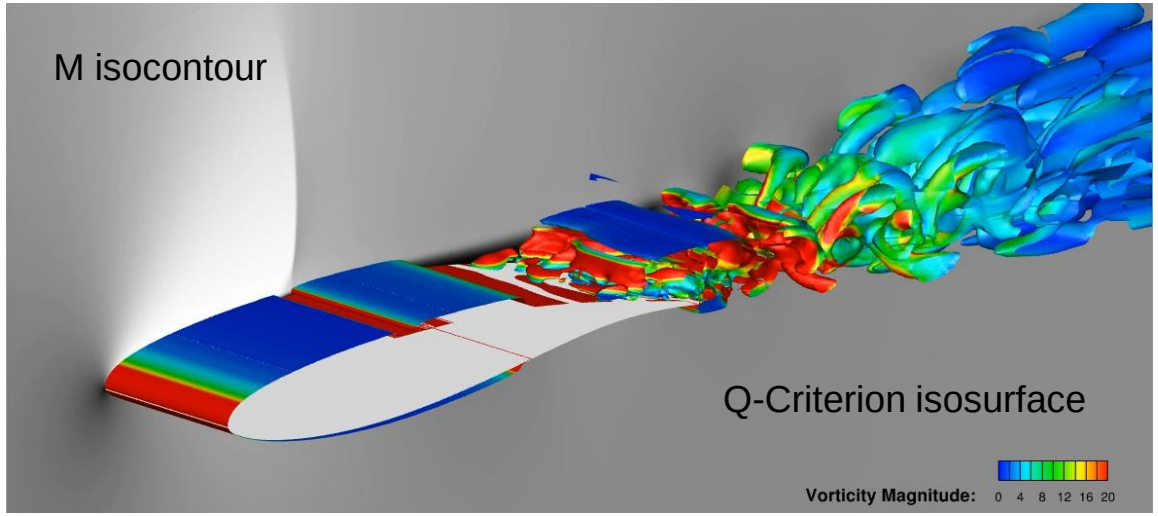


*D. Luo, C. Yan, and X. Wang. Computational study of supersonic turbulent-separated flows using partially averaged Navier-stokes method. *Acta Astronautica*, 107:234–246, 2015.

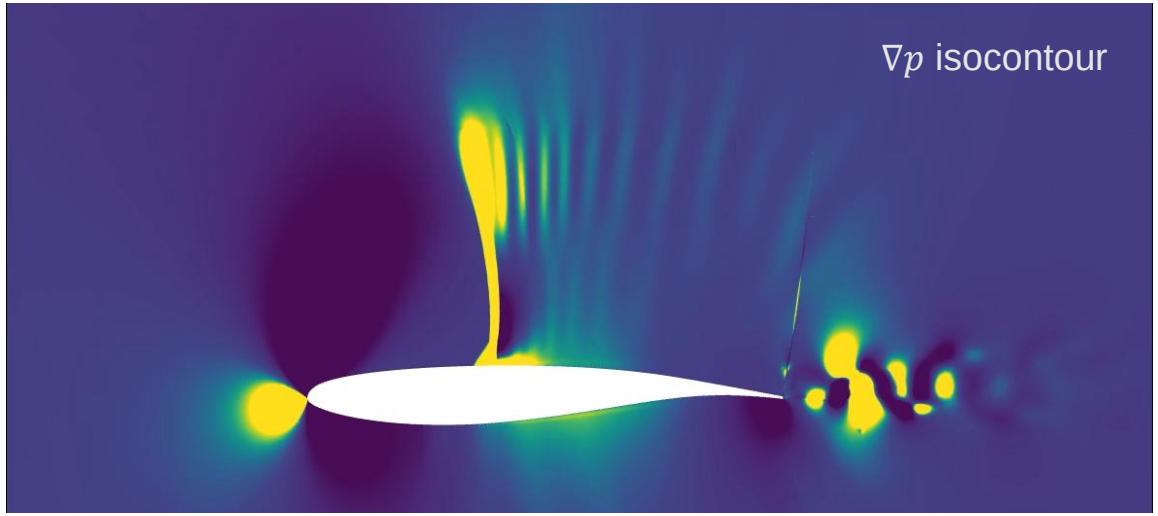
**A. Elmiligui, K.S. Abdol-Hamid, S.J. Massey, and S. Paul Pao. Numerical Study of Flow Past a Circular Cylinder Using RANS, Hybrid RANS/LES and PANS Formulations. In *22nd Applied Aerodynamics Conference and Exhibit*, Providence, Rhode Island, 2004.

Buffet with periodic BCs

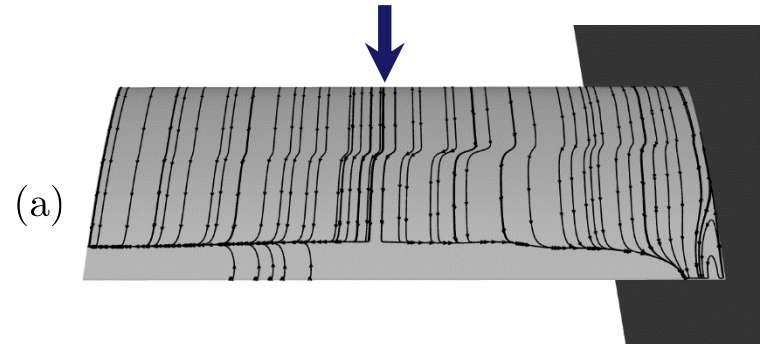
- Predicted alternated separation/reattachment
- Downstream shock position: attached flow and 2D structures
- Upstream motion: separated flow and crossflow detected

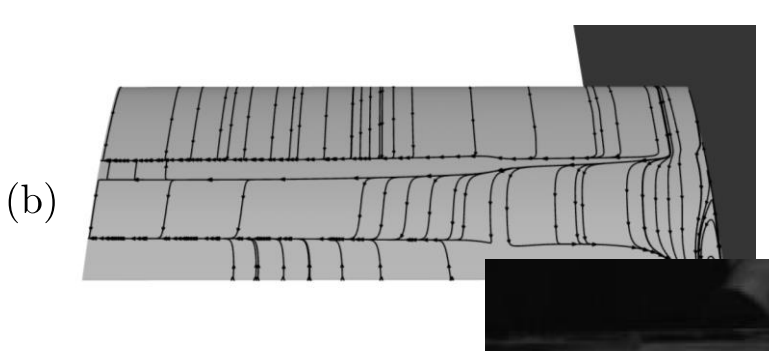


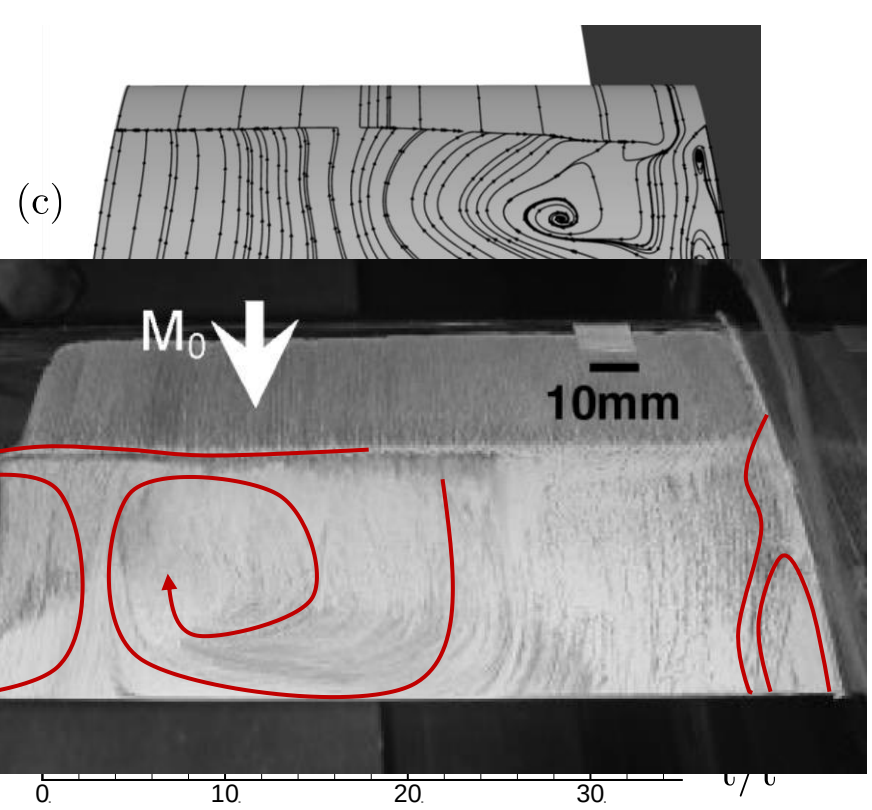
- Predicted pressure waves from the TE
- On both pressure and suction side



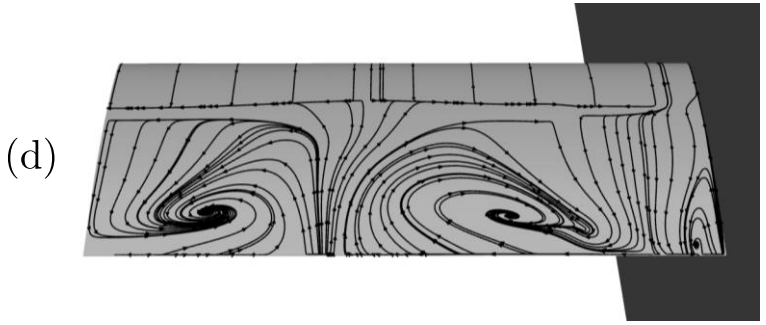
3D Buffet Dynamics

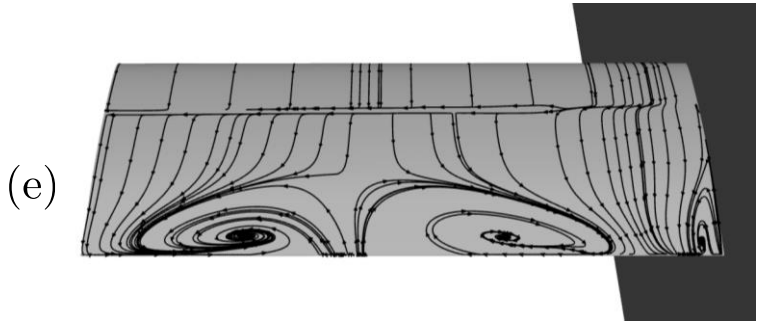


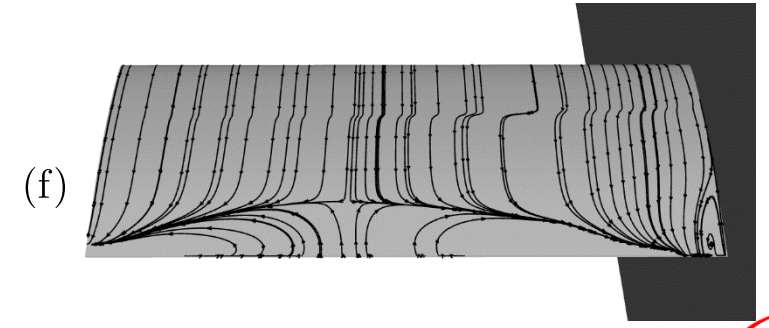




a. Most downstream position
b. Two distinct separated flow regions
c. Vortical structures propagate in the separated BL
d. Fully developed buffet cells
e. Flow reattachment
f. Downstream motion of the shock







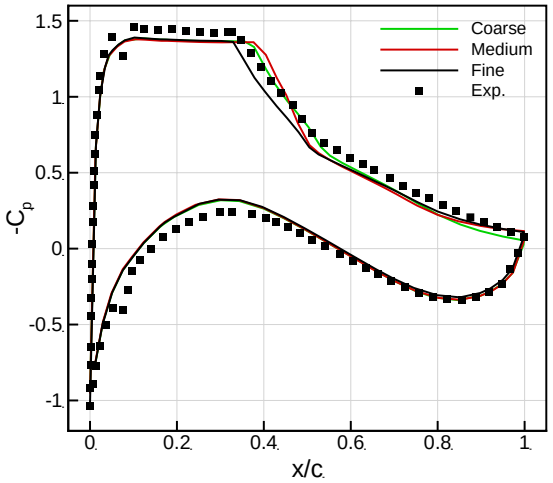
Grid Resolution Study

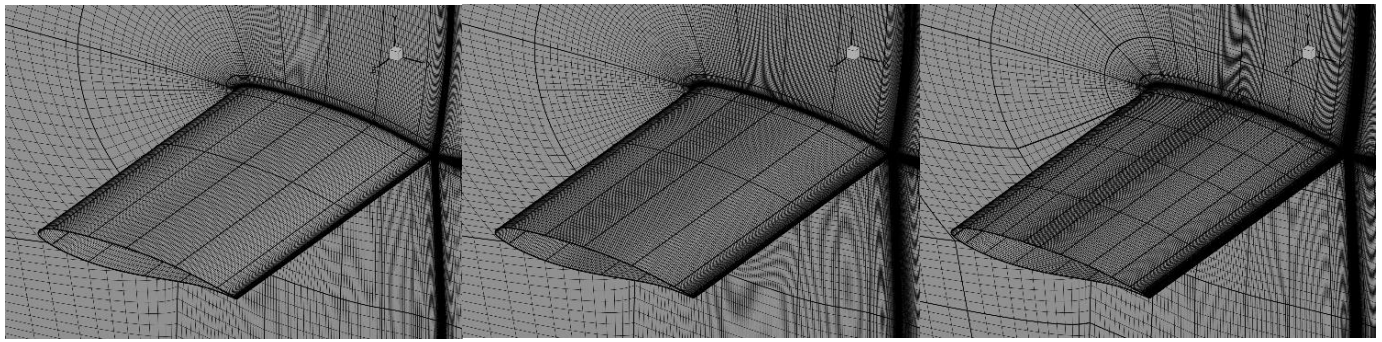
High resolution needed to capture the buffet cells

Shock oscillations present in every case

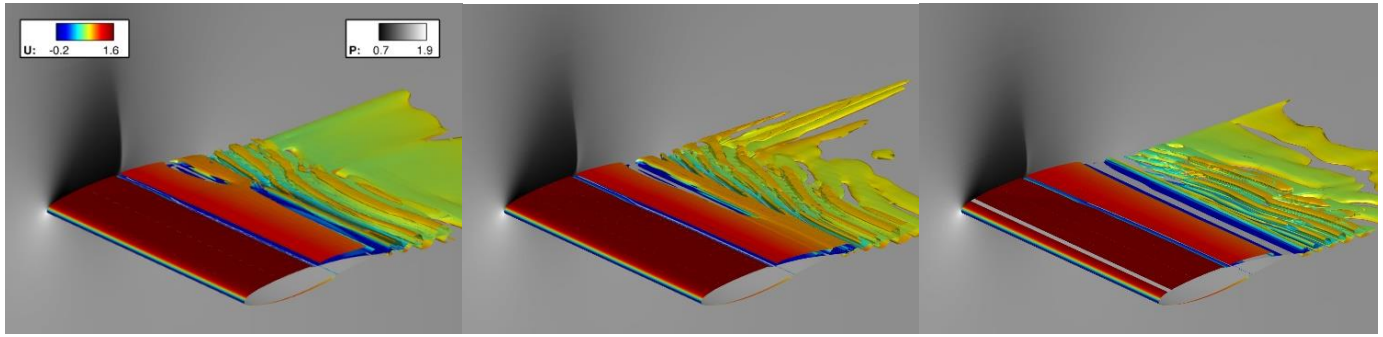
The influence in the mean quantities is limited

The trailing edge C_p improves with finer grids

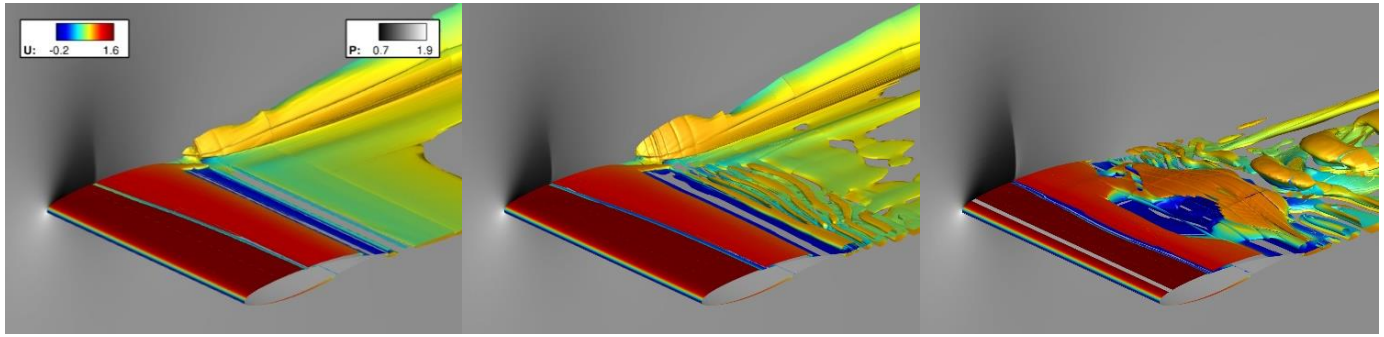




Most downstream shock position



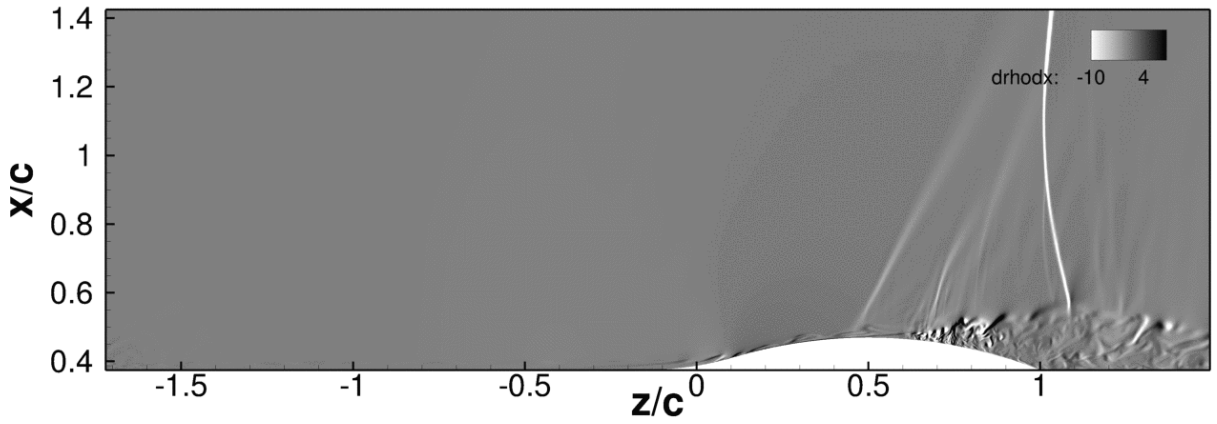
Most upstream shock position

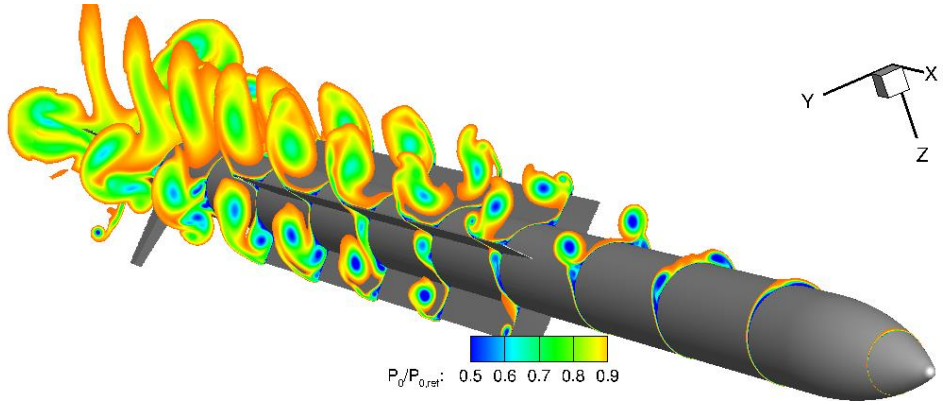


Coarse Medium Fine

Summary

- PANS improves the capability to predict transonic buffet wrt URANS
- PANS works at both pre- and post- buffet conditions
- Strong 3D component
- PANS is able to capture the correct flow physics when a finer discretisation is used
- The method can be applied at different resolutions
- Further study to provide guidelines on the usage of PANS will be provided





Future Work and Conclusions

New f_k estimate: runtime statistics

- Λ_u is based on modelled quantities
- C_{PANS} replaces $1/\sqrt{C_\mu}$ in the original formulation*
- To avoid the dependency on C_{PANS}, we evaluate Λ (based on total quantities)
- k is computed at each iteration:
- $\overline{u_i'^2}$ are computed through:

$$f_k = C_{\mathrm{PANS}} \left(\frac{\Delta}{\Lambda_u} \right)^{2/3}$$

$$f_k = C_\mu^{-1/2} \left(\frac{\Delta}{\Lambda} \right)^{2/3}$$

$$k = \frac{1}{2} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)$$

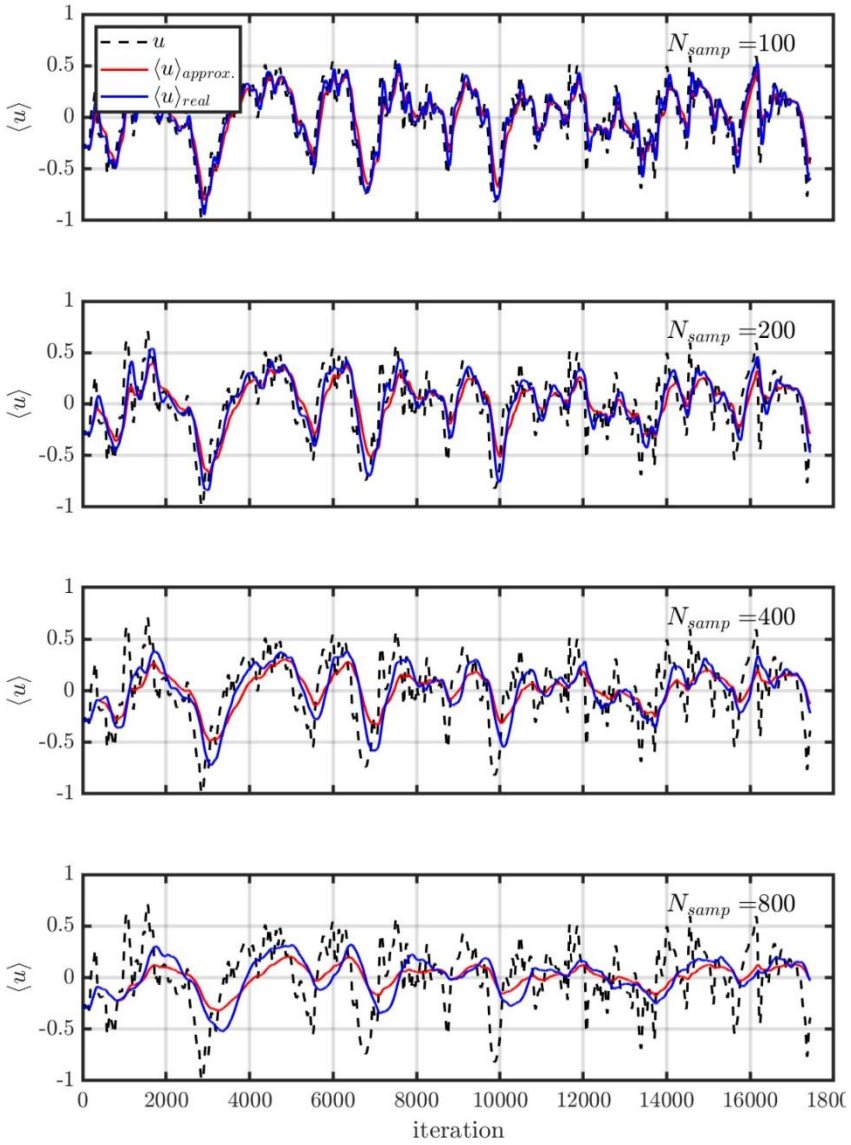
$$\mu(x, N_s)_{i+1} \simeq \frac{\mu(x, N_s)_i (N_s - 1) + x_i}{N_s}$$

$$\sigma^2(x, N_s)_{i+1} \simeq \frac{\sigma^2(x, N_s)_i (N_s - 1) + (x_i - \mu(x, N_s)_{i+1})^2}{N_s}$$

*D. Luo, C. Yan, and X. Wang. Computational study of supersonic turbulent-separated flows using partially averaged Navier-stokes method. *Acta Astronautica*, 107:234–246, 2015.

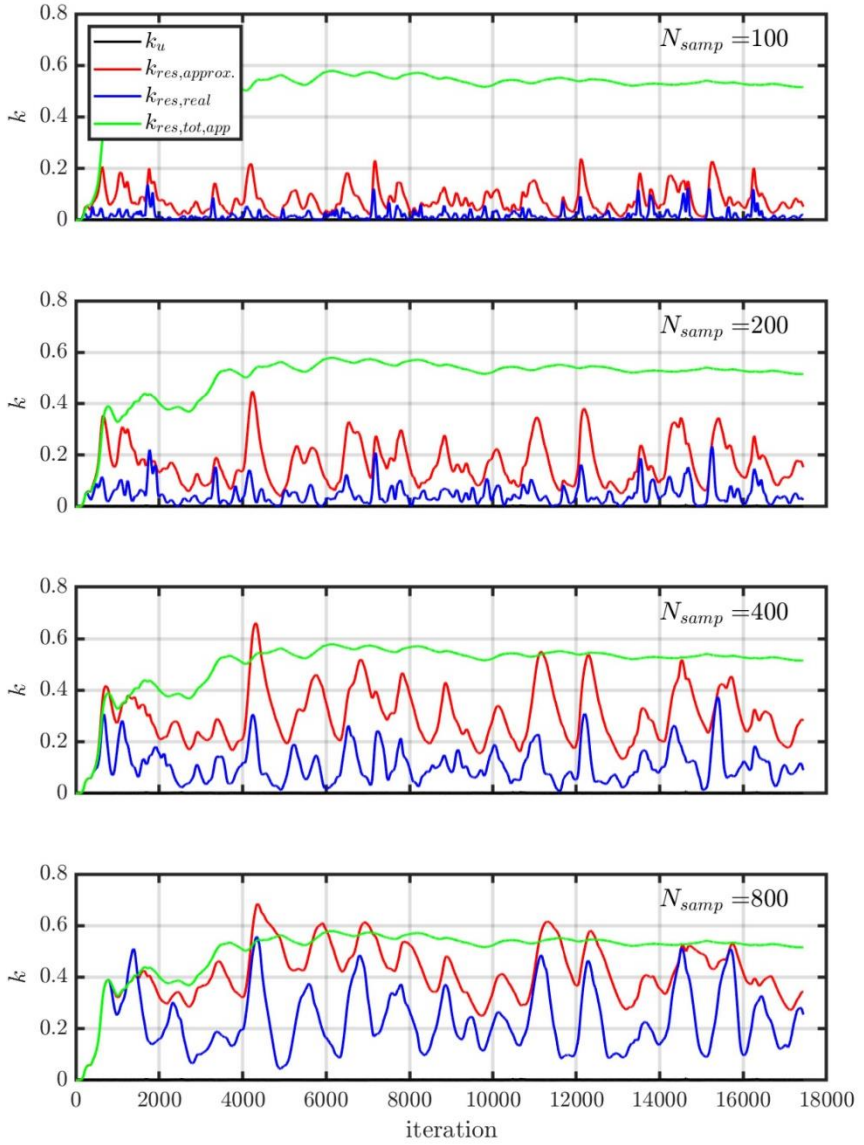
Runtime statistics

- Statistics are computed over N_S timesteps to neglect the contribution from large oscillations (VS)
- Small $N_S \rightarrow$ the mean velocity follows the instantaneous field
- High $N_S \rightarrow$ small oscillations are filtered out

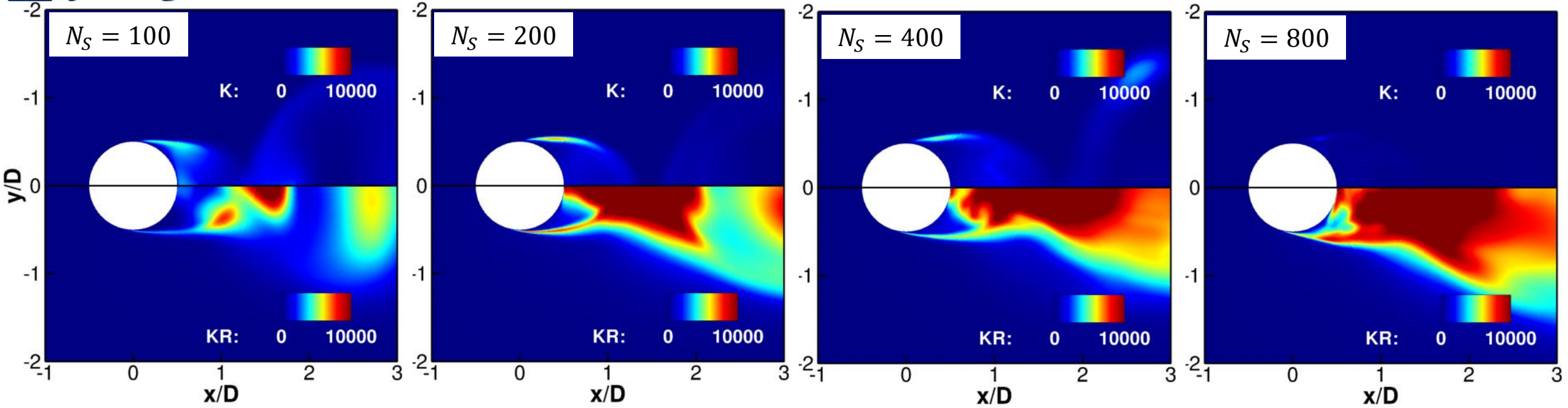


Runtime statistics

- Statistics are computed over N_S timesteps to neglect the contribution from large oscillations (VS)
- Small N_S → the mean velocity follows the instantaneous field
- High N_S → small oscillations are filtered out
- k increases with N_S
- k is overpredicted by the adopted formula

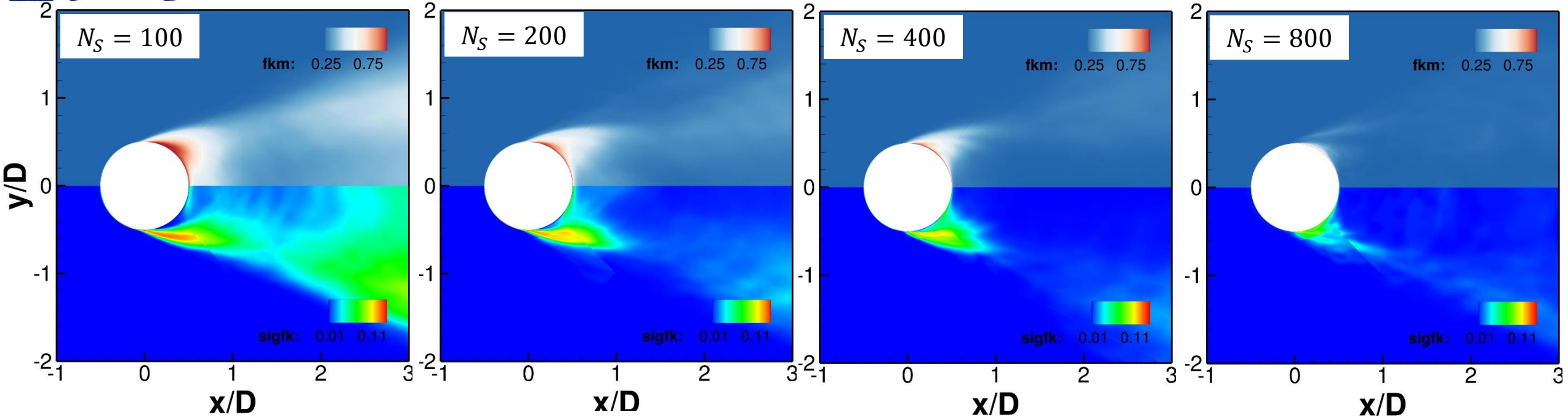


Results

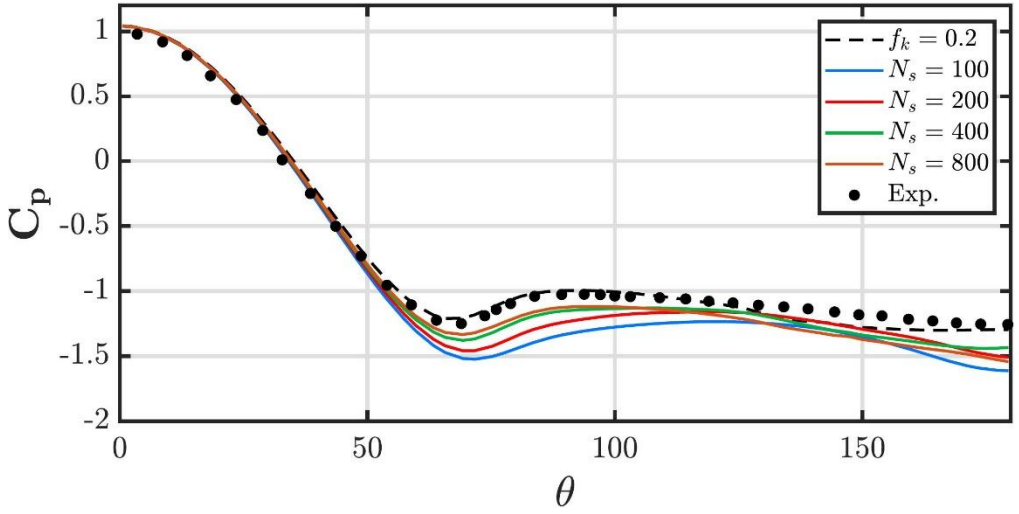


- High $N_S \rightarrow k_{res} \uparrow$ and $k_{mod} \downarrow$

Results



- High $N_S \rightarrow k_{res} \uparrow$ and $k_{mod} \downarrow$
- High $N_S \rightarrow$ fast transition to SRS in the wake
- $f_k \rightarrow 0$ upstream
- High $N_S \rightarrow f_k$ no longer adapts to VS $\left(\overline{f_k'^2} \simeq 0\right)$
- $N_S = 800$ does not preserve a RANS in the BL



New f_k estimate: timestep constraint

- The available formulations are based on Δ but not on Δt

$$f_{k,x} = C_\mu^{-1/2} \left(\frac{\Delta}{\Lambda} \right)^{2/3}$$

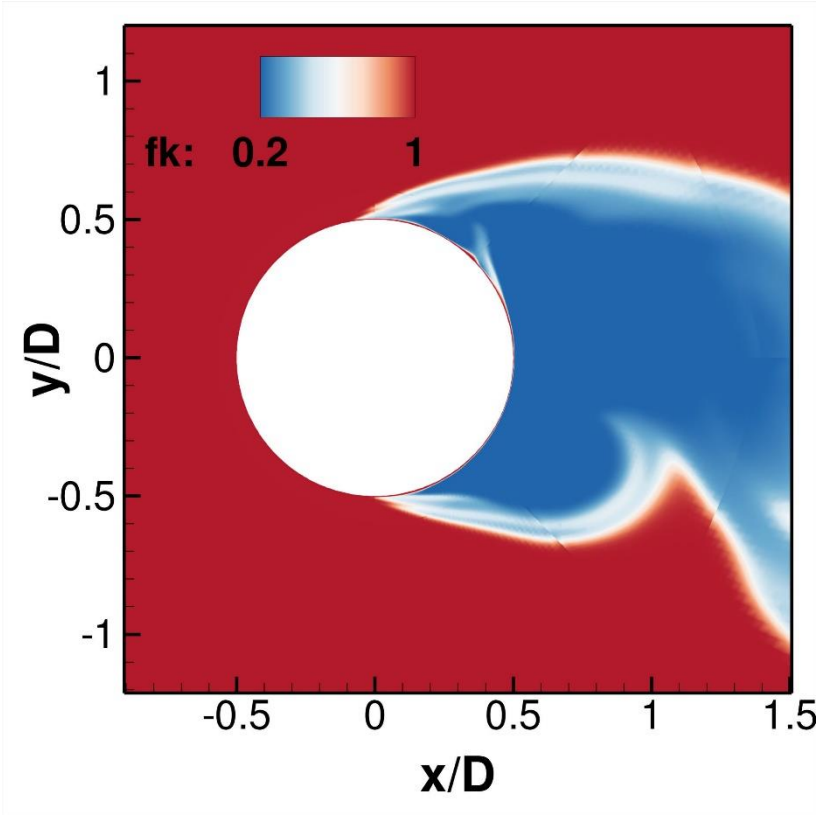
- Δt can be accounted for by formulating a similar estimate

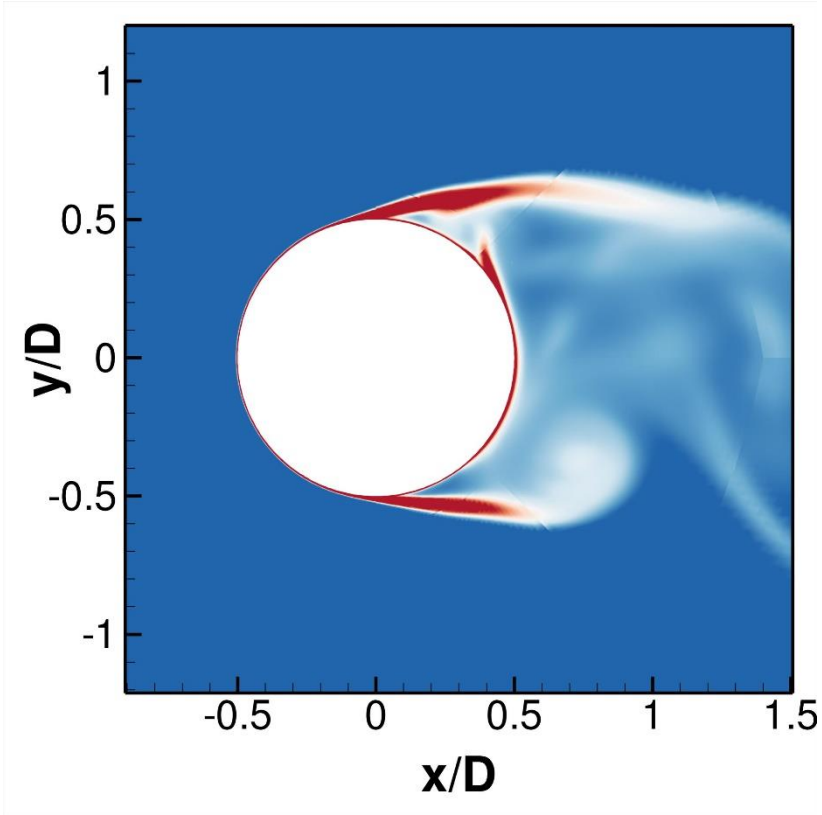
$$f_{k,t} = C_\mu^{-1/2} \frac{\Delta t}{T_\Lambda}$$

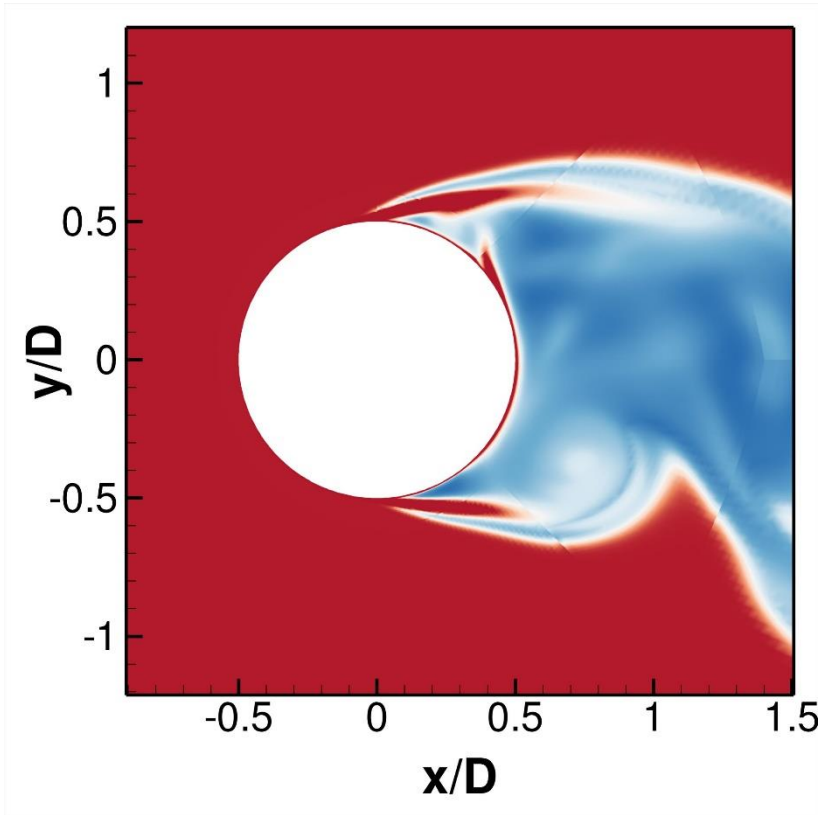
- These are then combined by

$$f_k(\Delta, \Delta t) = \max \left[f_{k,x}(\Delta), f_{k,t}(\Delta t) \right]$$

Results with time constraint







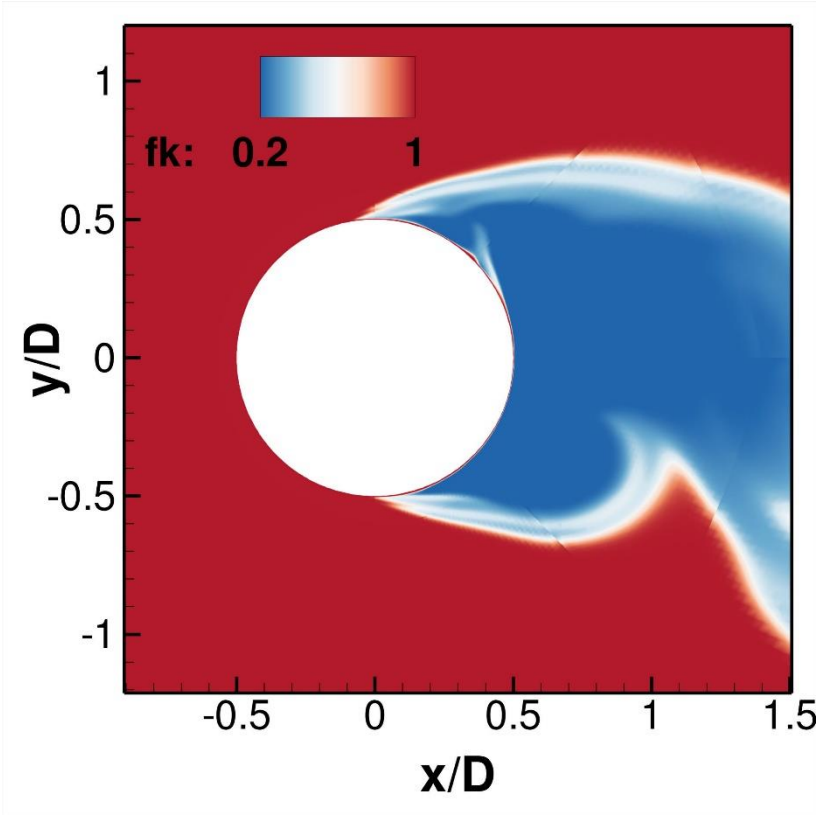
$$f_{k,x} = C_\mu^{-1/2} \left(\frac{\Delta}{\Lambda} \right)^{2/3}$$

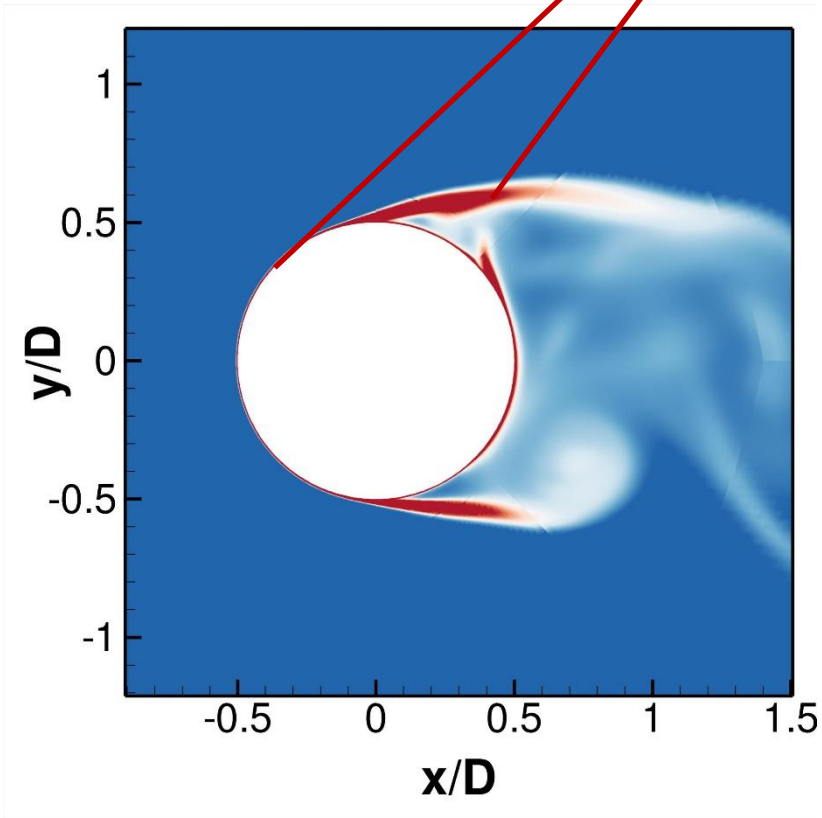
$$f_{k,t} = C_\mu^{-1/2} \frac{\Delta t}{T_\Lambda}$$

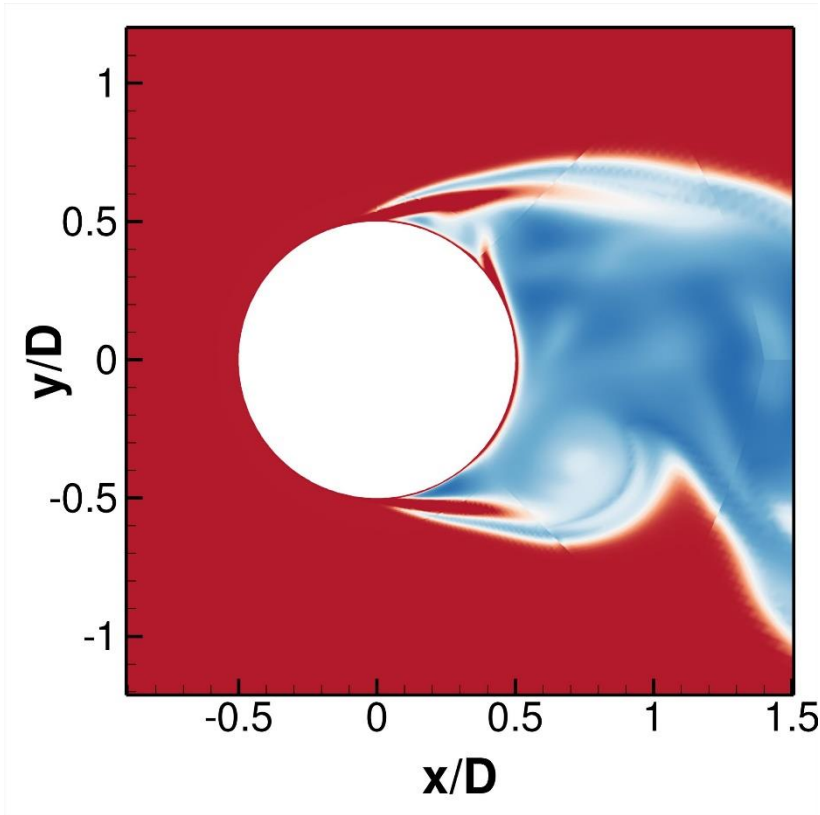
$$f_k(\Delta, \Delta t) = \max \left[f_{k,x}(\Delta), f_{k,t}(\Delta t) \right]$$

Results with time constraint

Small timescales associated with VS and BL





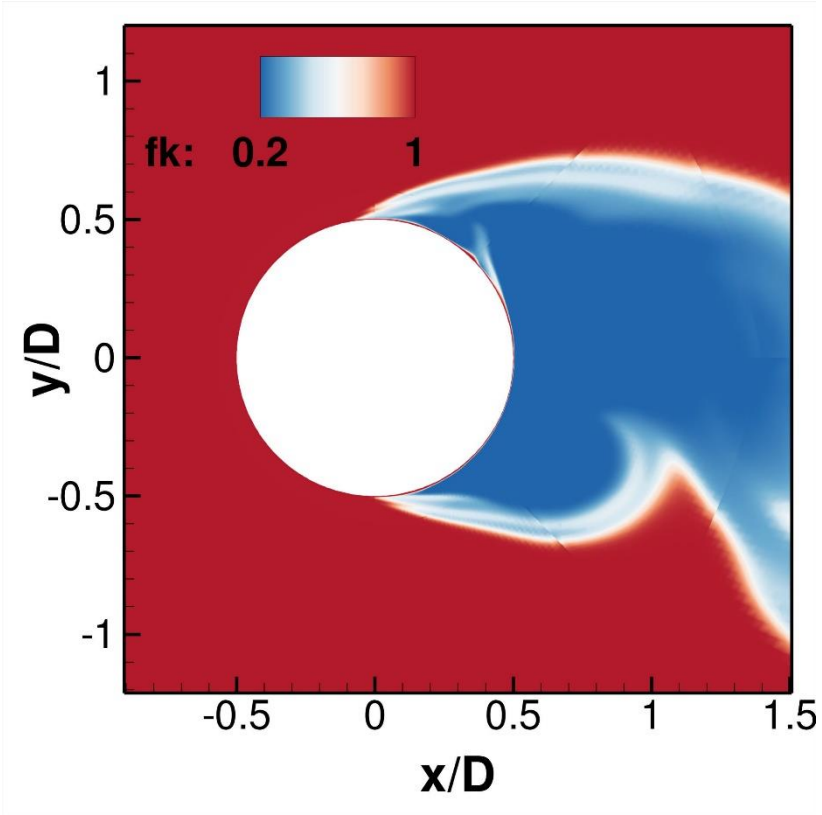


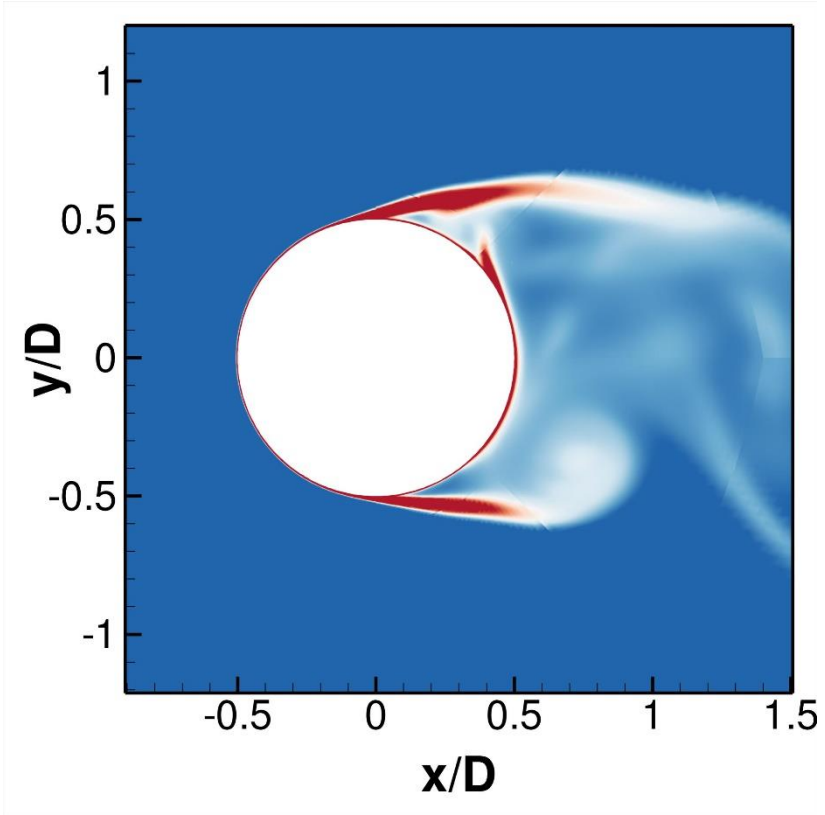
$$f_{k,x} = C_\mu^{-1/2} \left(\frac{\Delta}{\Lambda} \right)^{2/3}$$

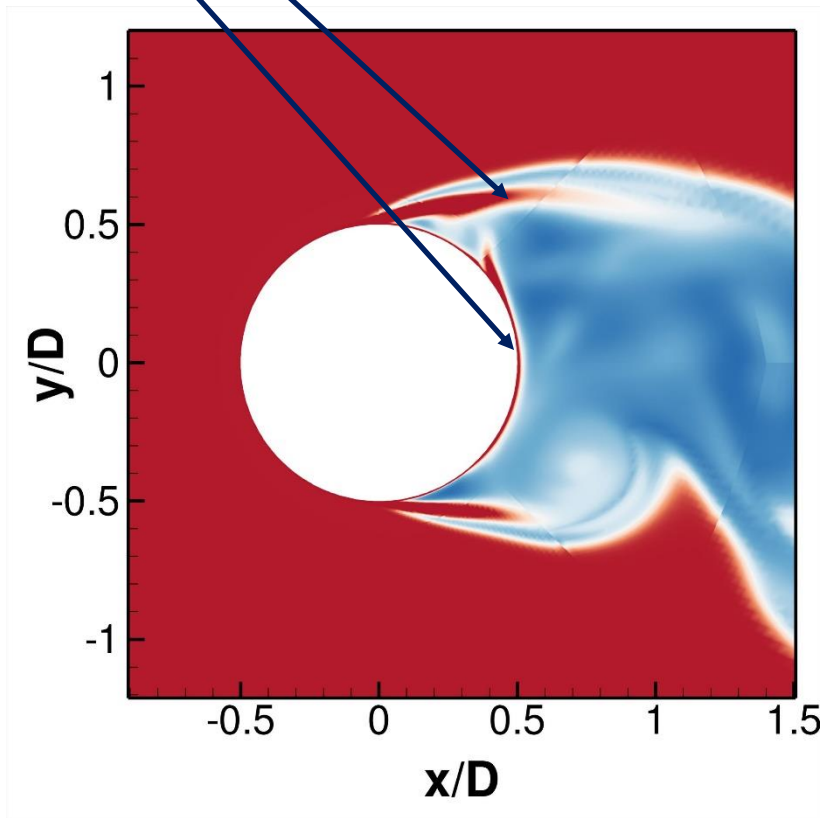
$$f_{k,t} = C_\mu^{-1/2} \frac{\Delta t}{T_\Lambda}$$

$$f_k(\Delta, \Delta t) = \max \left[f_{k,x}(\Delta), f_{k,t}(\Delta t) \right]$$

Results with time constraint





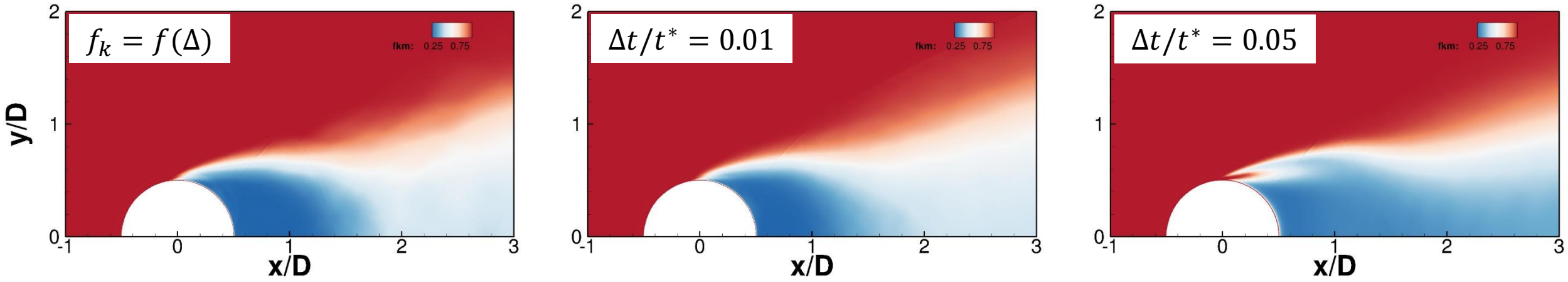


$$f_{k,x} = C_\mu^{-1/2} \left(\frac{\Delta}{\Lambda} \right)^{2/3}$$

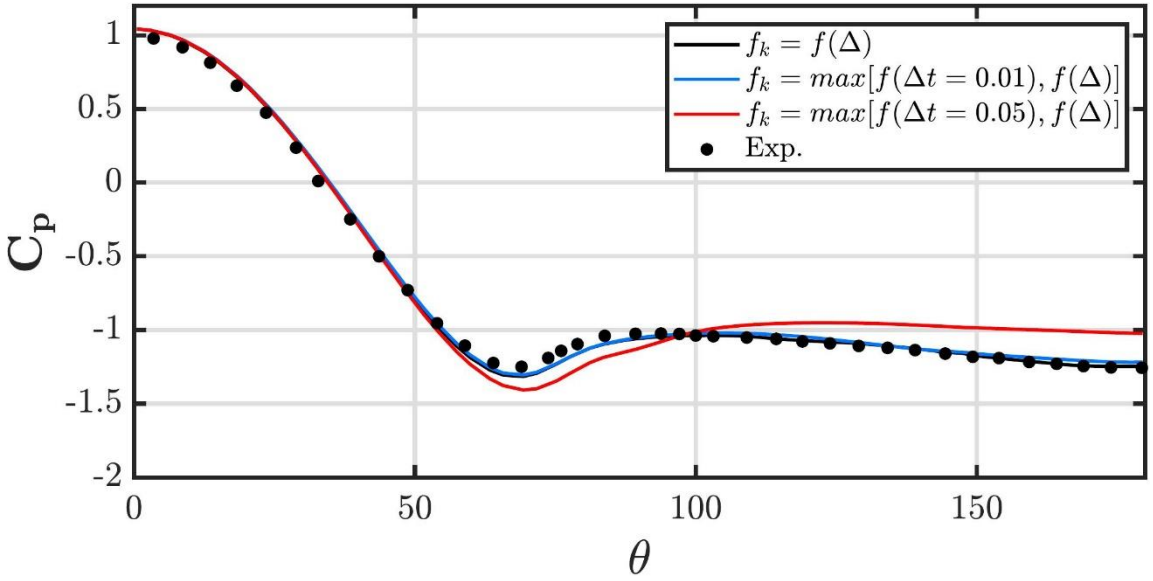
$$f_{k,t} = C_\mu^{-1/2} \frac{\Delta t}{T_\Lambda}$$

$$f_k(\Delta, \Delta t) = \max \left[f_{k,x}(\Delta), f_{k,t}(\Delta t) \right]$$

Results with time constraint



- Only the case at $\Delta t/t^* = 0.05$ is affected by the new estimate
- It reflects in a later BL separation
- If $\Delta t/t^*$ is small enough, the constraint is not helpful
- The estimate can be used to determine the optimal timestep



Conclusions and Future Work

- PANS improved results wrt URANS
- For constant f_k approach, big dependency on
  - Grid
  - Timestep
  - f_k
- Dynamic f_k reduced the effort for calibration but still need fine grids
- The new proposed estimates are promising as
  - Further delay transition to SRS
  - Can neglect the contribution from large self-induced flow oscillations
  - Account for the CFD timestep
- Application of new estimates to cases with shallow separation

Thank you!

A. Petrocchi and G. N. Barakos

James Watt School of Engineering, University of Glasgow

University of Glasgow

TEAMAero



Towards Effective Flow Control and Mitigation of Shock Effects In Aeronautical Applications.



This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No EC grant 860909.