Advanced Modeling & Simulation (AMS) Seminar Series
NASA Ames Research Center
May 16th, 2024

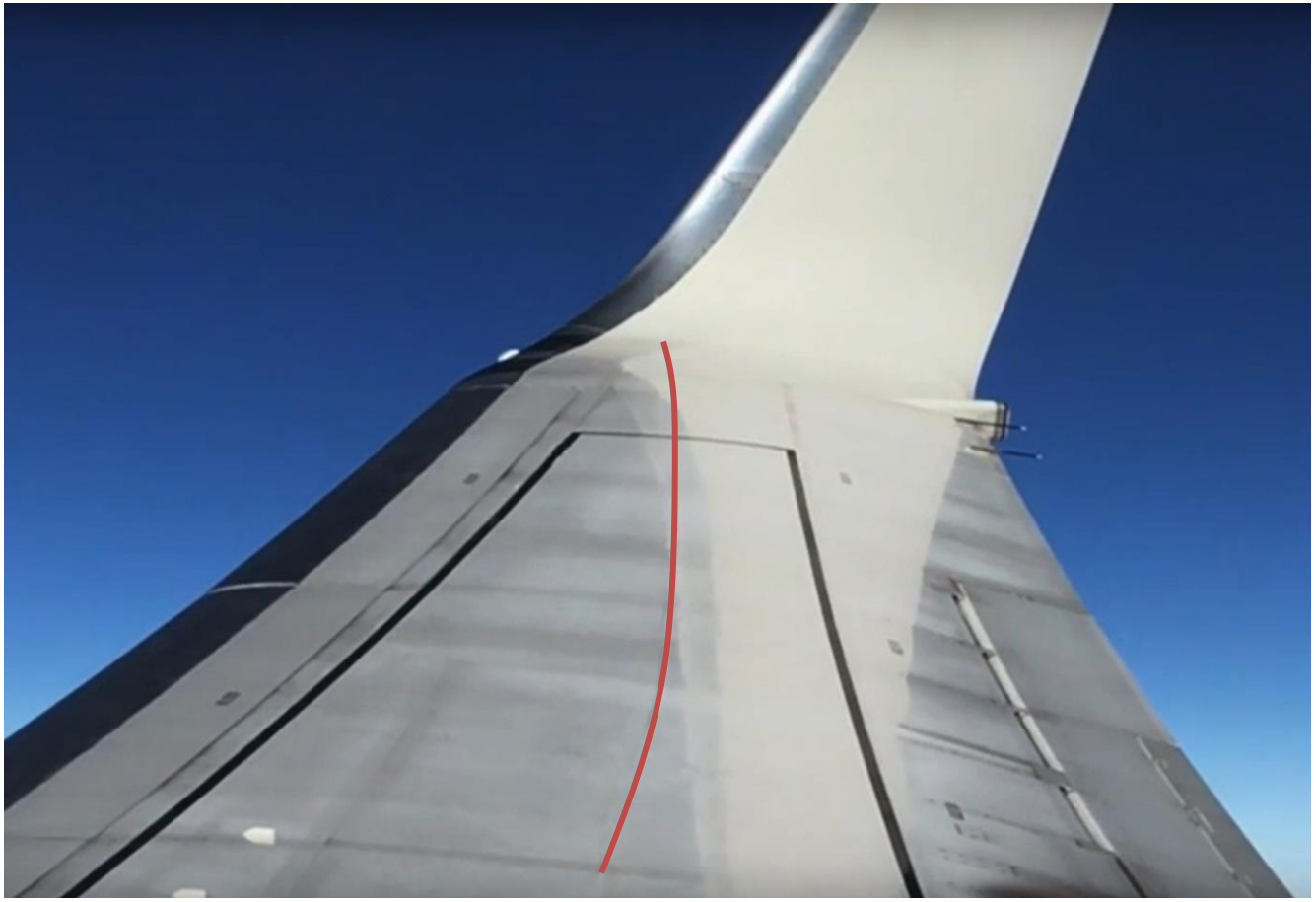
On Modal Methods for the Study of Transonic Buffet on Wings

Sebastian Timme

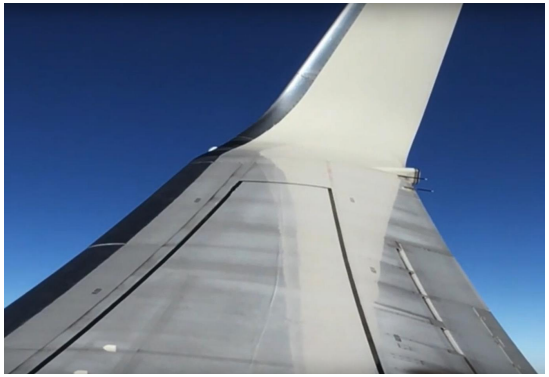


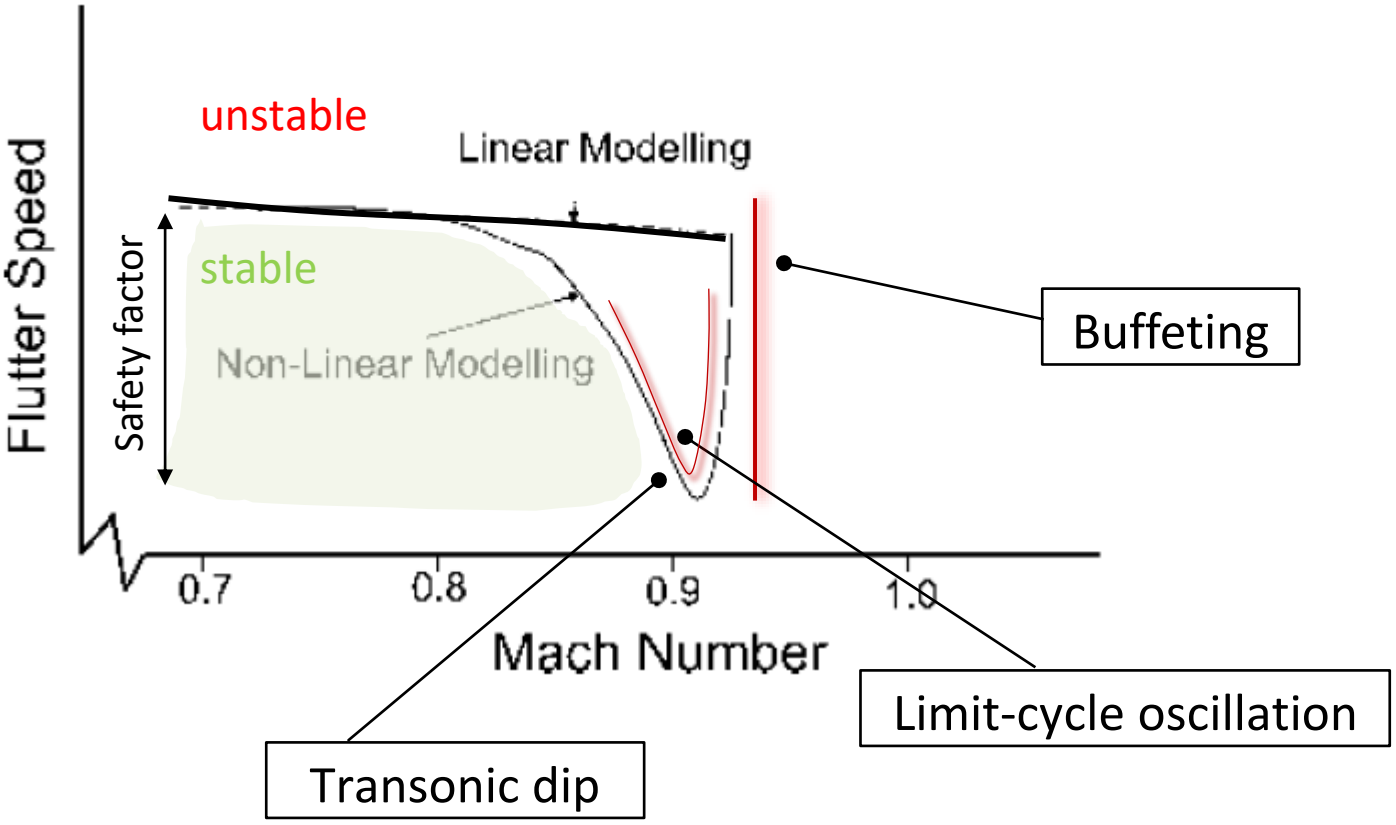
UNIVERSITY OF
LIVERPOOL

Common Interest



Loads & Aeroelastics





(adapted from Wright and Cooper, 2007, Wiley)

Outline

Idea of Modal Methods and Overview of Transonic Buffet

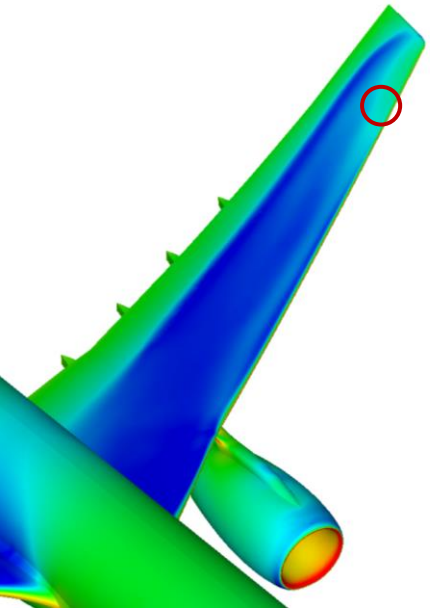
Global Stability (incl. solver technology)

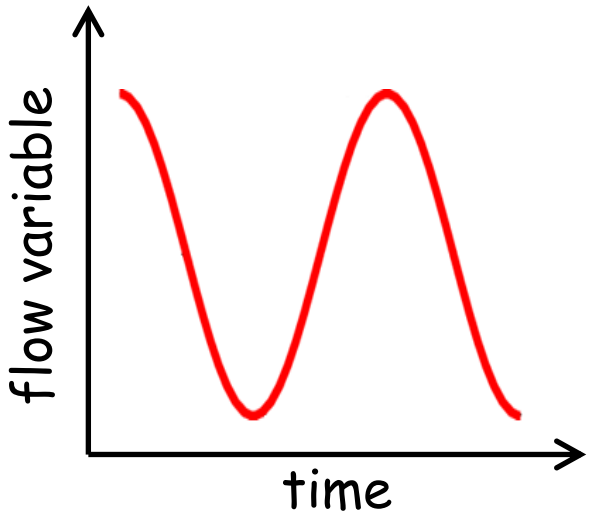
Resolvent Analysis

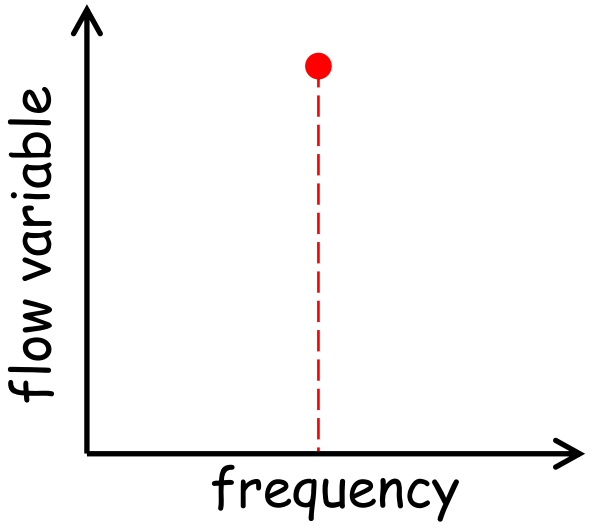
FSI Global Stability

Summary

Basics

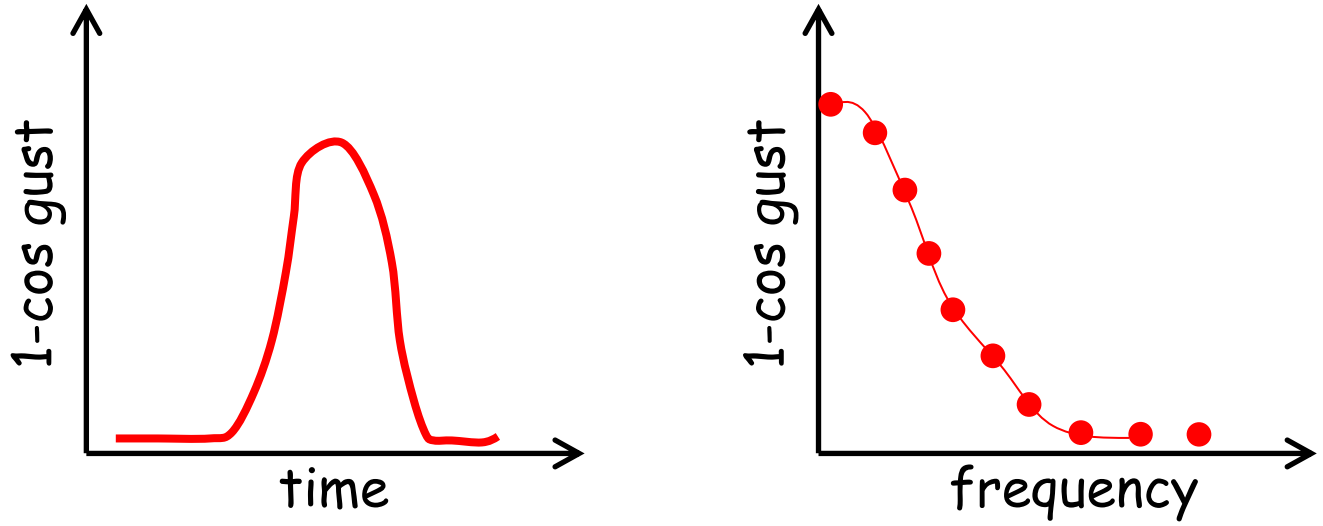




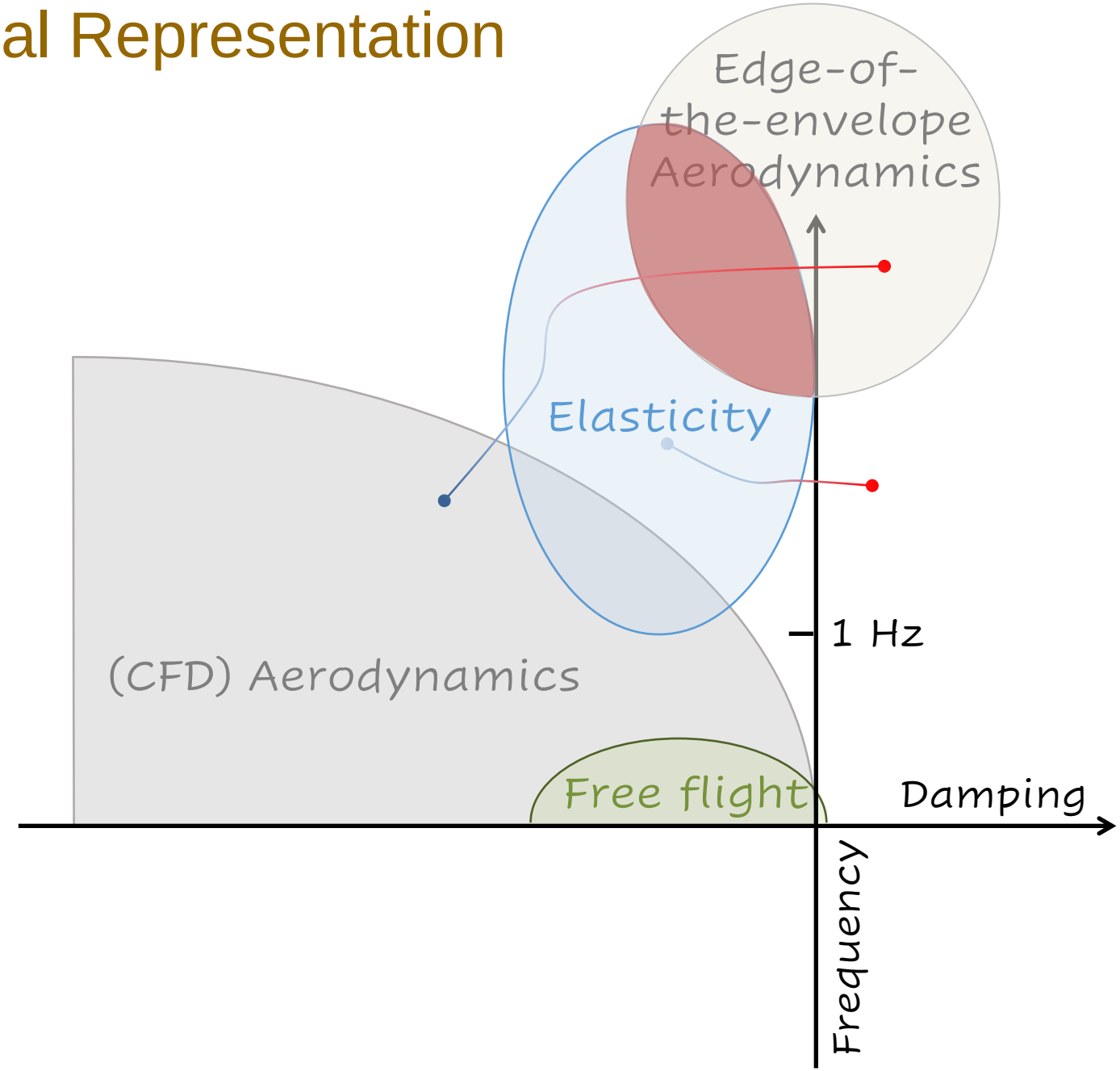


Basics

"… but practical signals are not sinusoidal."



Modal Representation



Schur Complement Approach for Flutter

$$M\ddot{\boldsymbol{x}} + C\dot{\boldsymbol{x}} + K\boldsymbol{x} = \boldsymbol{f}_a(\boldsymbol{u}, \boldsymbol{x}, \dot{\boldsymbol{x}}); \quad \dot{\boldsymbol{u}} = R(\boldsymbol{u}, \boldsymbol{x}, \dot{\boldsymbol{x}})$$

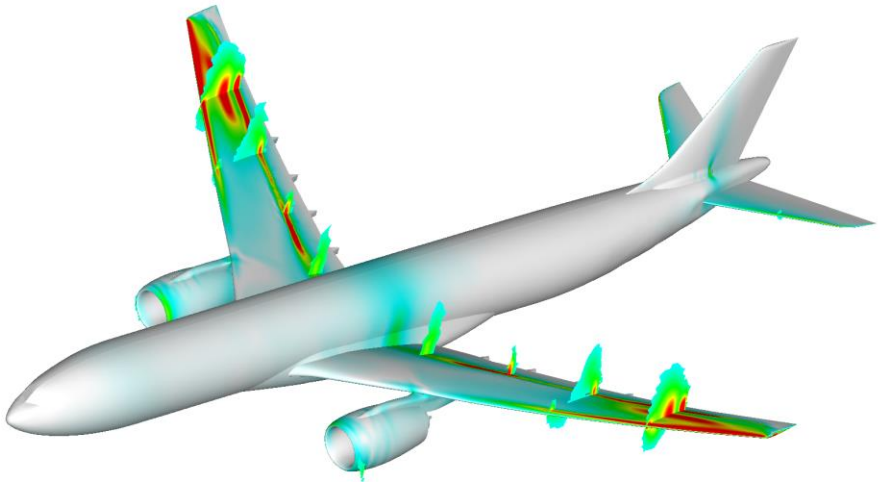
(mass, damping, stiffness)

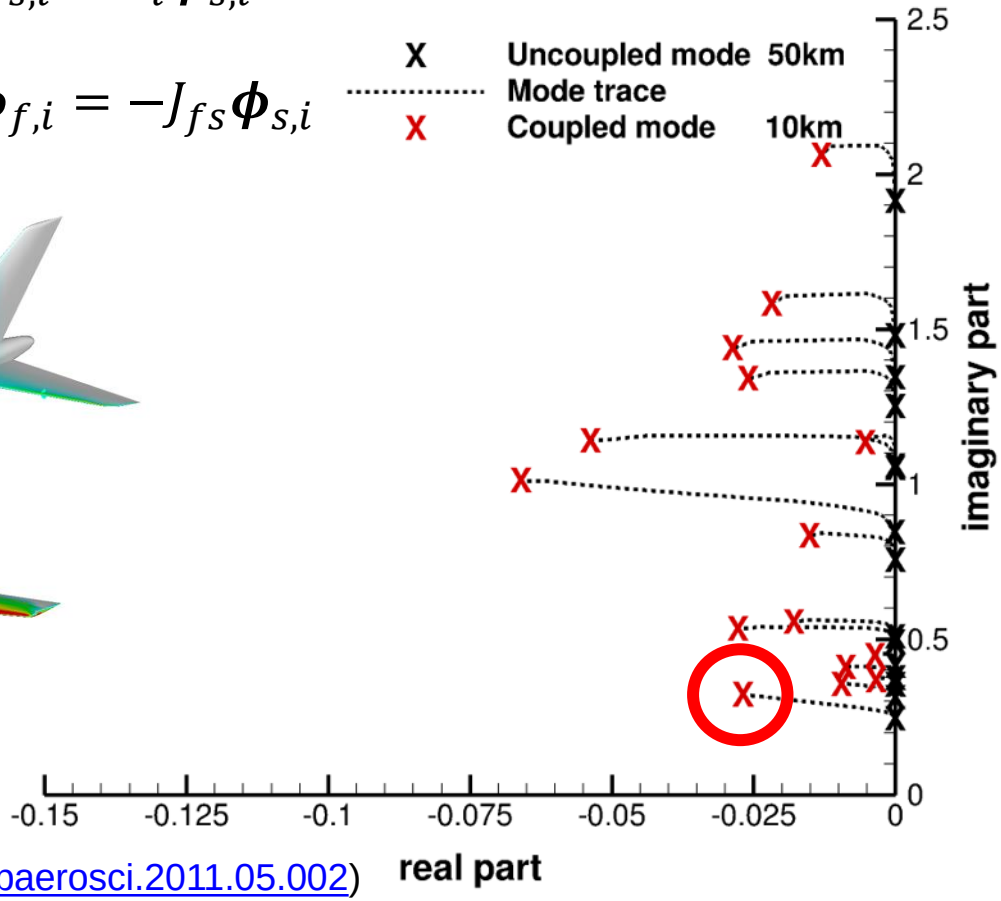
(unsteady aerodynamics)

$$\left(J_{ss} - J_{sf}\left(J_{ff} - \lambda_i I\right)^{-1} J_{fs}\right)\boldsymbol{\phi}_{s,i} = \lambda_i \boldsymbol{\phi}_{s,i}$$

$$\left(J_{ff} - \lambda_i I\right)\boldsymbol{\phi}_{f,i} = -J_{fs}\boldsymbol{\phi}_{s,i}$$

p- or pk-flutter method





Badcock et al. (2011, PAS, https://doi.org/10.1016/j.paerosci.2011.05.002)

Schur Complement Approach for Flutter

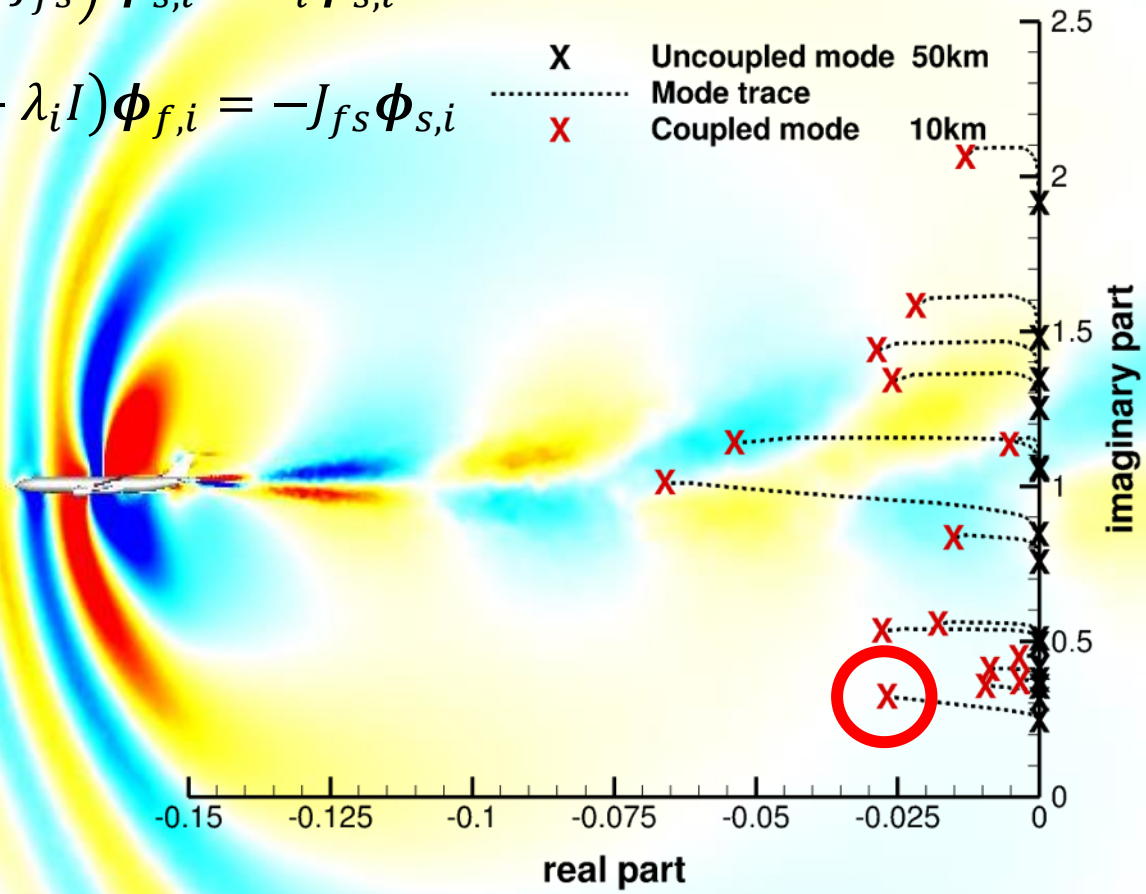
$$M\ddot{\boldsymbol{x}} + C\dot{\boldsymbol{x}} + K\boldsymbol{x} = \boldsymbol{f}_a(\boldsymbol{u}, \boldsymbol{x}, \dot{\boldsymbol{x}}); \quad \dot{\boldsymbol{u}} = R(\boldsymbol{u}, \boldsymbol{x}, \dot{\boldsymbol{x}})$$

(mass, damping, stiffness)

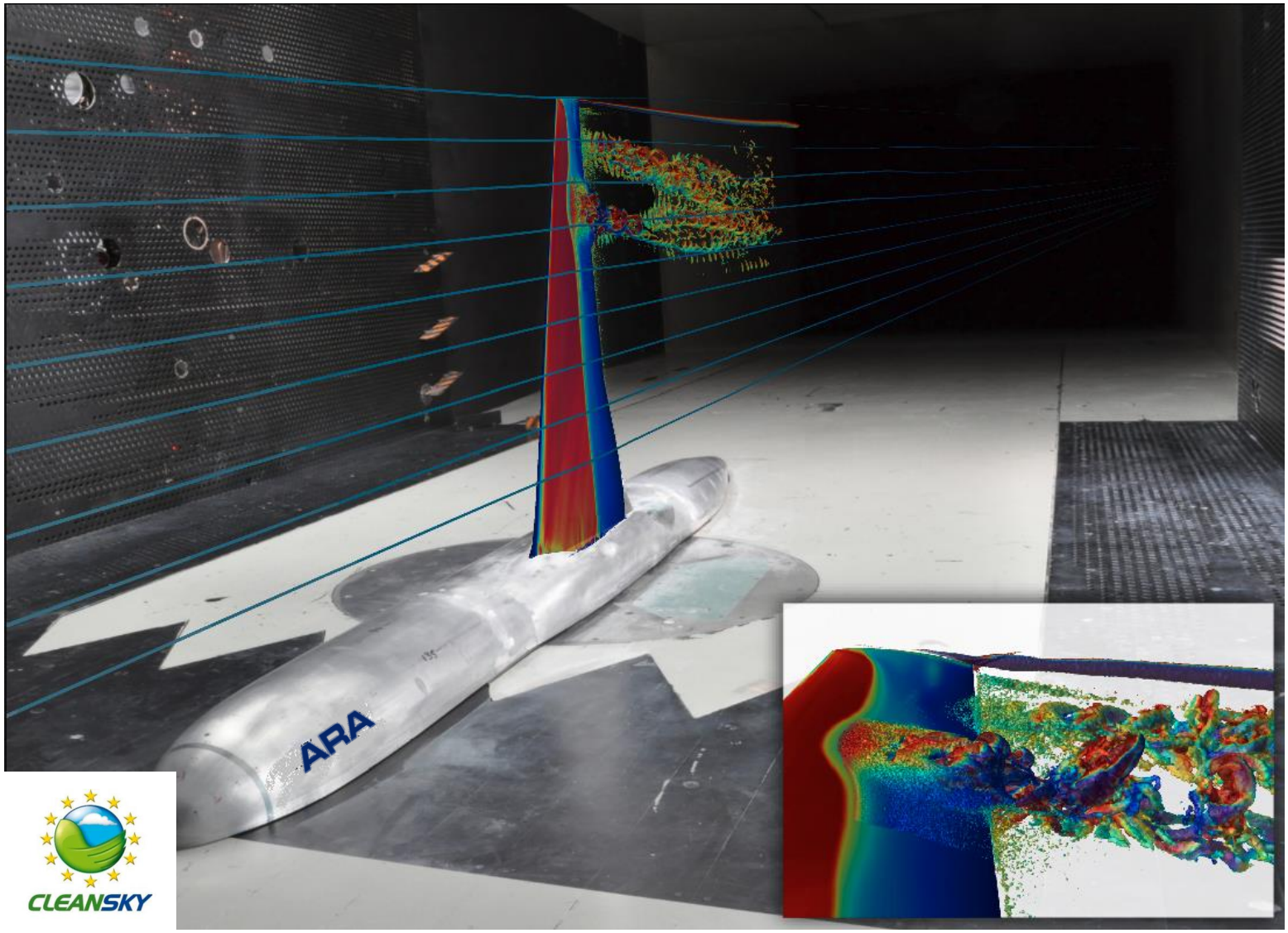
(unsteady aerodynamics)

$$\left(J_{ss} - J_{sf}\left(J_{ff} - \lambda_i I\right)^{-1} J_{fs}\right)\boldsymbol{\phi}_{s,i} = \lambda_i \boldsymbol{\phi}_{s,i}$$

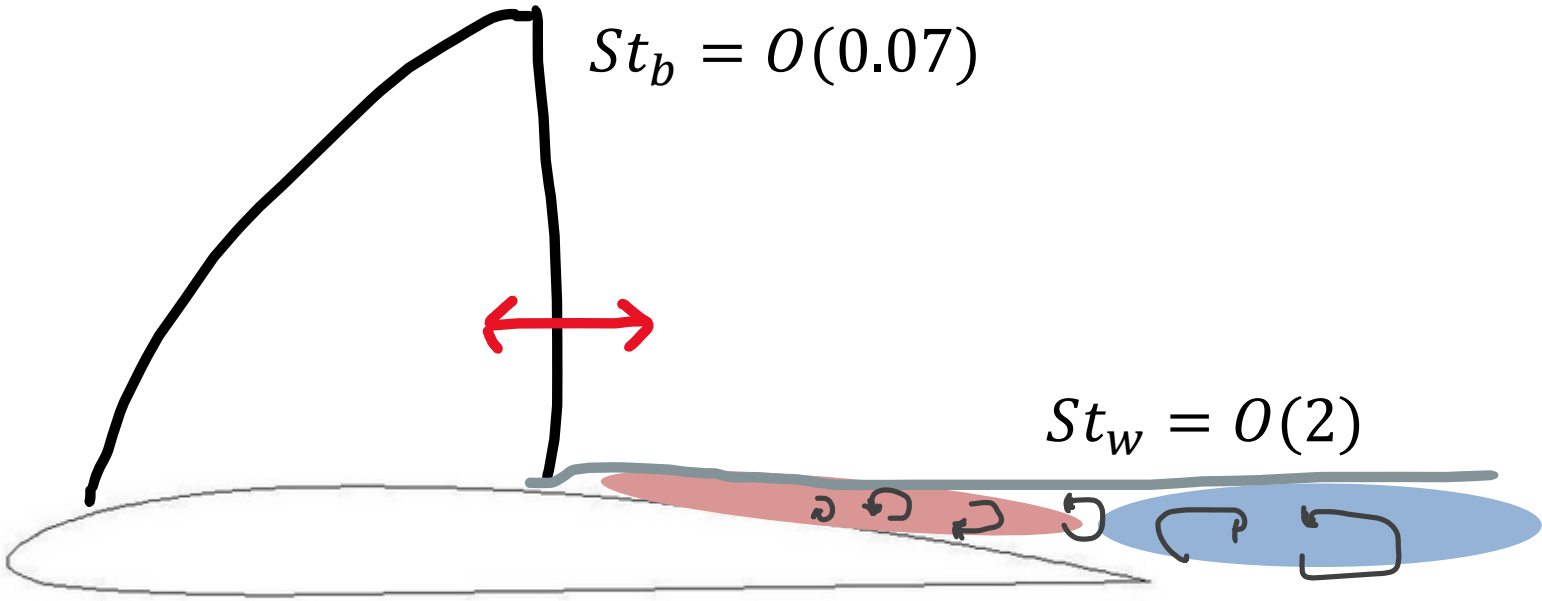
$$\left(J_{ff} - \lambda_i I\right)\boldsymbol{\phi}_{f,i} = -J_{fs}\boldsymbol{\phi}_{s,i}$$



Edge-of-the-envelope Aerodynamics



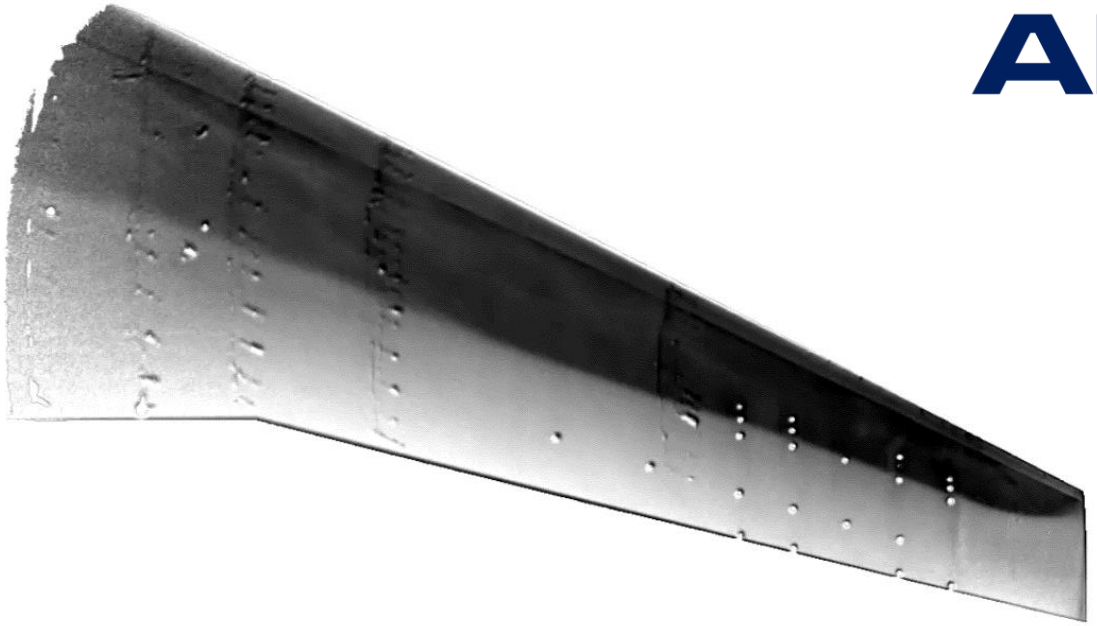
Shock Buffet and Modes – 2D Aerofoil



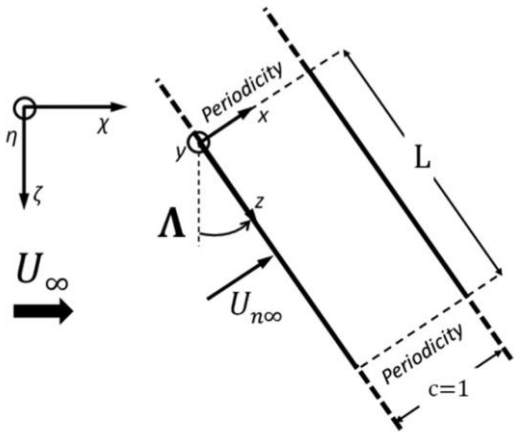
Crouch et al (2007, 2009), Sartor et al (2015), Moise et al (2023), and many more

Shock Buffet and Modes – 3D Wing

Courtesy of ARA
(https://doi.org/10.1017/jfm.2019.906)

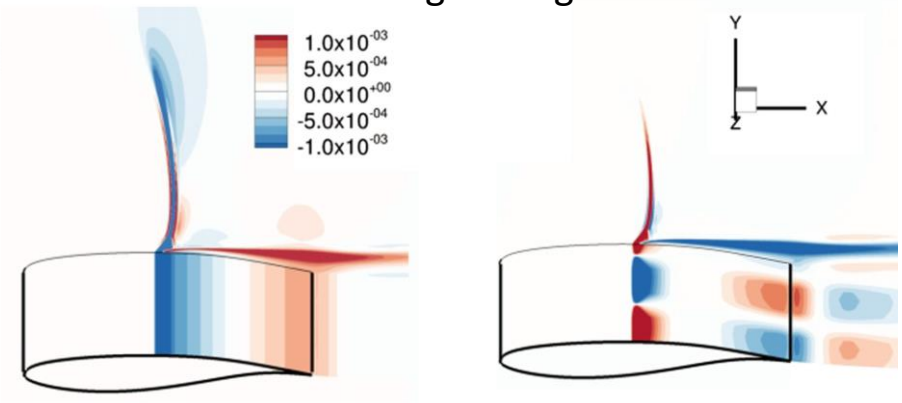


Shock Buffet and Modes – 2.5D Aerofoil/Wing

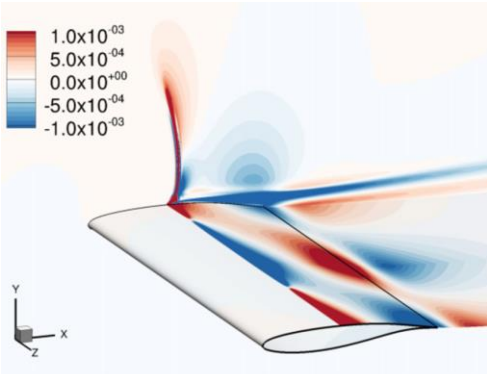


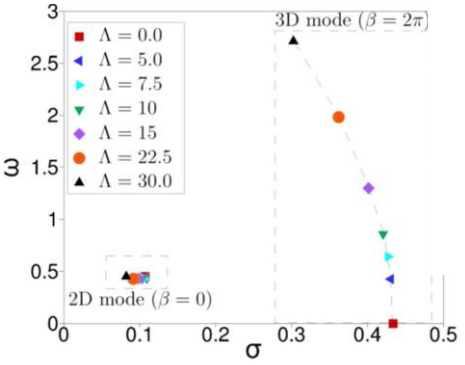
Crouch et al. 2019, JFM

Straight wing

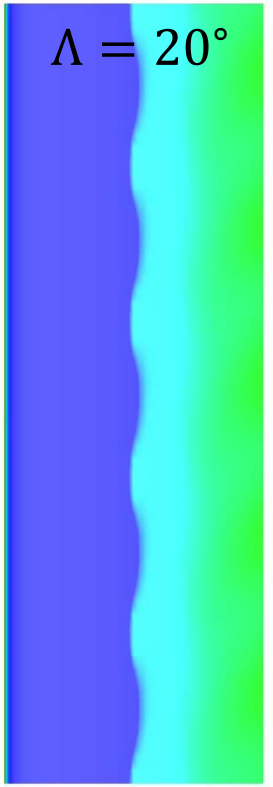


Swept wing



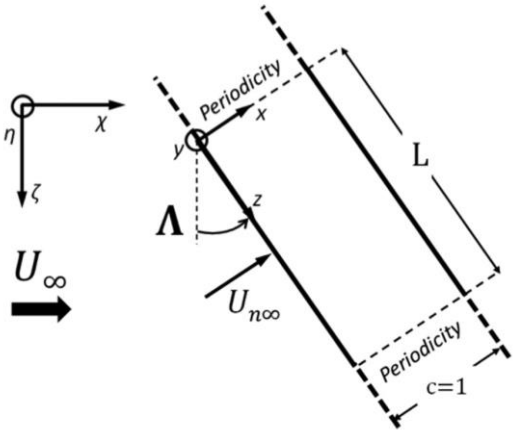


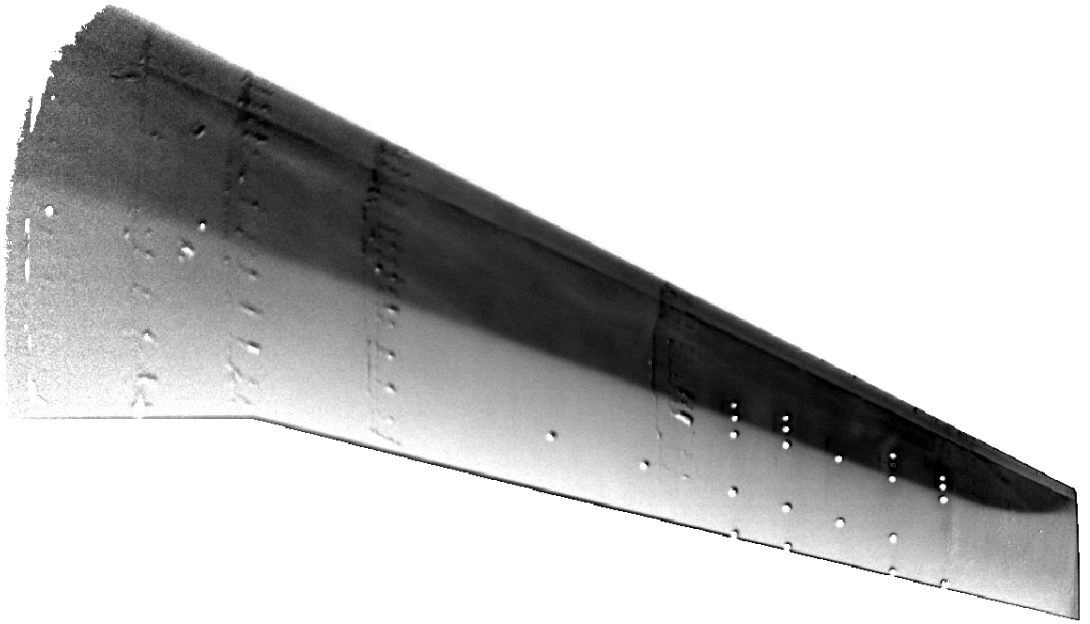
Paladini et al. 2019, PRF
Plante et al. 2020, JFM

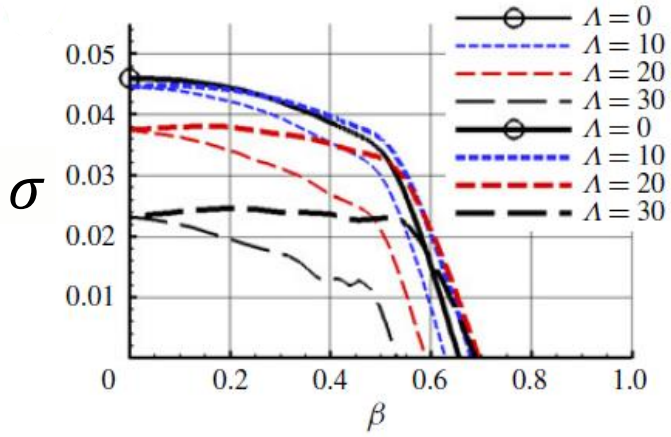


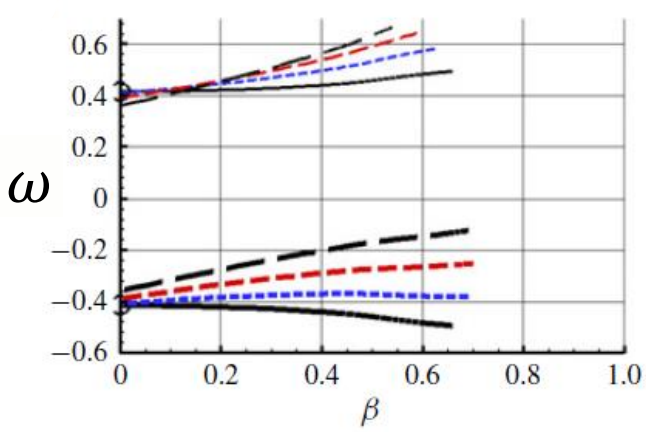
He & Timme 2021, JFM

Shock Buffet and Modes – The Missing Link









(adapted from Crouch et al. 2019, JFM)

Global Stability Methodology

- RANS (plus turb model) equations in semi-discrete form

$$R(\boldsymbol{u}) = \dot{\boldsymbol{u}}$$

- First-discretise-then-linearise, full linearisation, ansatz $\boldsymbol{u} = \hat{\boldsymbol{u}} e^{\lambda t}$

$$J_{ff}\hat{\boldsymbol{u}} = \lambda\hat{\boldsymbol{u}}$$

- (Outer) iterative implicitly restarted Arnoldi (or Krylov-Schur) method with shift-inverse transformation

$$\left(J_{ff} - \zeta I\right)^{-1}\hat{\boldsymbol{u}} = \frac{1}{\lambda - \zeta}\hat{\boldsymbol{u}}$$

- (Inner) iterative sparse linear solver

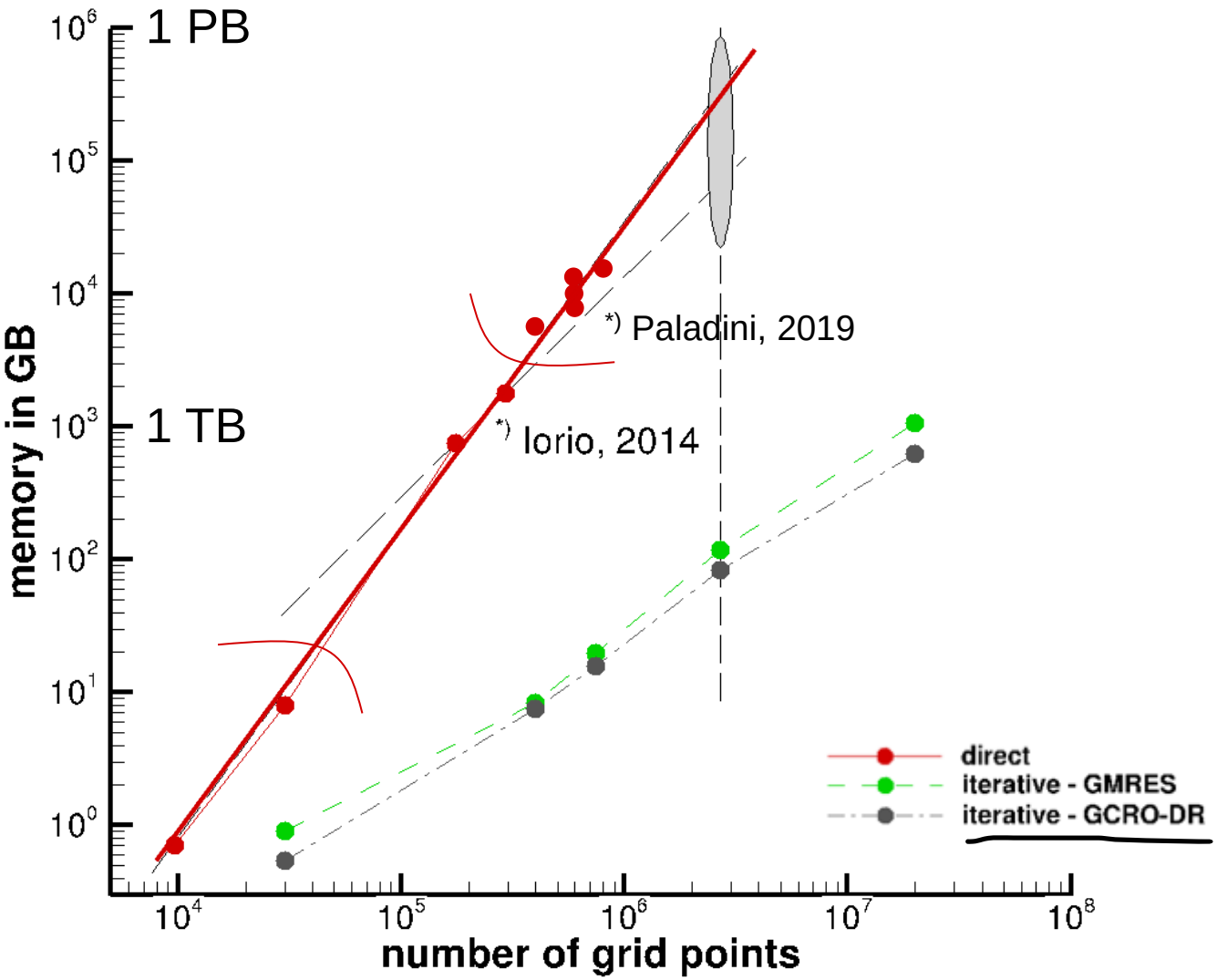
$$AP^{-1}(P\boldsymbol{x}) = \boldsymbol{b}$$

Matrix-free vs Jacobian-free

U S Vevek et al. (2023, AeroJ, https://doi.org/10.1017/aer.2023.108)

Memory of Direct vs Iterative

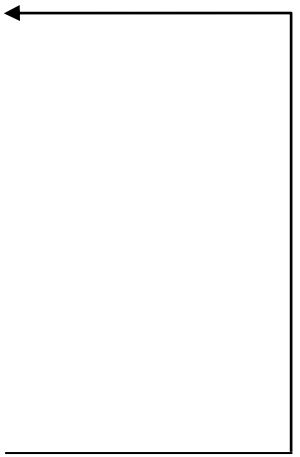
or Why we need iterative methods?



The Algorithm – GCRO-DR

Preconditioned Krylov Subspace Iterative Solver

1: $\boldsymbol{r} = \boldsymbol{b} - A\boldsymbol{x}$, $\boldsymbol{v}_0 = \frac{\boldsymbol{r}}{||\boldsymbol{r}||}$
2: **for** $i = 0 \rightarrow m$ **do**
3: $\quad \boldsymbol{v}_{i+1} = A\boldsymbol{v}_i$
4: $\quad$ **for** $j = 0 \rightarrow i$ **do**
5: $\quad\quad h_{j,i} = \boldsymbol{v}_j^H \boldsymbol{v}_{i+1}$, $\boldsymbol{v}_{i+1} = \boldsymbol{v}_{i+1} - h_{j,i}\boldsymbol{v}_j$
6: $\quad\quad h_{i+1,i} = ||\boldsymbol{v}_{i+1}||$, $\boldsymbol{v}_{i+1} = \frac{\boldsymbol{v}_{i+1}}{||\boldsymbol{v}_{i+1}||}$
7: $\quad$ **end for**
8: **end for**
9: $min||\beta\boldsymbol{e}_1 - \bar{H}_m\boldsymbol{y}_m|| \Longrightarrow$ **Givens rotation**
10: $\boldsymbol{x} = \boldsymbol{x} + V_m\boldsymbol{y}_m$



The Algorithm – GCRO-DR

Preconditioned Krylov Subspace Iterative Solver

$\boldsymbol{r} = \boldsymbol{b} - A\boldsymbol{x}$, $\boldsymbol{v}_0 = \frac{\boldsymbol{r}}{||\boldsymbol{r}||}$
for $i = 0 \rightarrow m$ **do**
  $\boldsymbol{v}_{i+1} = A\boldsymbol{v}_i$
  for $j = 0 \rightarrow i$ **do**
    $h_{j,i} = \boldsymbol{v}_j^H \boldsymbol{v}_{i+1}$, $\boldsymbol{v}_{i+1} = \boldsymbol{v}_{i+1} - h_{j,i}\boldsymbol{v}_j$
    $h_{i+1,i} = ||\boldsymbol{v}_{i+1}||$, $\boldsymbol{v}_{i+1} = \frac{\boldsymbol{v}_{i+1}}{||\boldsymbol{v}_{i+1}||}$
  end for
end for
$min||\beta \boldsymbol{e}_1 - \bar{H}_m \boldsymbol{y}_m|| \Longrightarrow$ **Givens rotation**
$\boldsymbol{x} = \boldsymbol{x} + V_m \boldsymbol{y}_m$
Compute k smallest eigenvectors $\boldsymbol{p}$ of $H_m + h_{m+1,m}^2 H^{-H} e_m e_m^H$
$[Q, R] = QR(\bar{H}_m P_k)$
Compute $C_k = V_{m+1} Q, U_k = (V_m P_k) R^{-1}$
$\boldsymbol{r} = \boldsymbol{b} - A\boldsymbol{x}$, $\boldsymbol{v}_0 = \frac{\boldsymbol{r}}{||\boldsymbol{r}||}$
Orthogonalise with $(I - C_k C_k^H)A$, $m - k$ iterations
$D_k = diag(1/||u_0||, ..., 1/||u_k||), B_k = C_k^H A V_{m-k}$
$\bar{G}_m = [D_k, B_k; 0, \bar{H}_{m-k}]$
$\hat{V}_m = [U_k D_k, V_{m-k}], \hat{W}_{m+1} = [C_k, V_{m-k+1}]$
$min||\hat{W}_{m+1}^H \boldsymbol{r} - \bar{G}_m \boldsymbol{y}_m|| \Longrightarrow$ **Givens rotation**
$\boldsymbol{x} = \boldsymbol{x} + \hat{V}_m \boldsymbol{y}_m$
Compute k smallest eigenvectors $\boldsymbol{p}$ of $\bar{G}_m^H \bar{G}_m \boldsymbol{p}_i = \lambda_i \bar{G}_m^H \hat{W}_{m+1}^H V_m \boldsymbol{p}_i$
$[Q, R] = QR(\bar{G}_m P_k)$
Compute $C_k = \hat{W}_{m+1} Q, U_k = (\hat{V}_m P_k) R^{-1}$

(Xu et al., 2016, https://doi.org/10.1016/j.compfluid.2016.10.018)

Bucolic RBC12 Test Case

Timme & Thormann (2016, https://doi.org/10.2514/6.2016-3848)

$AR = 7.78$

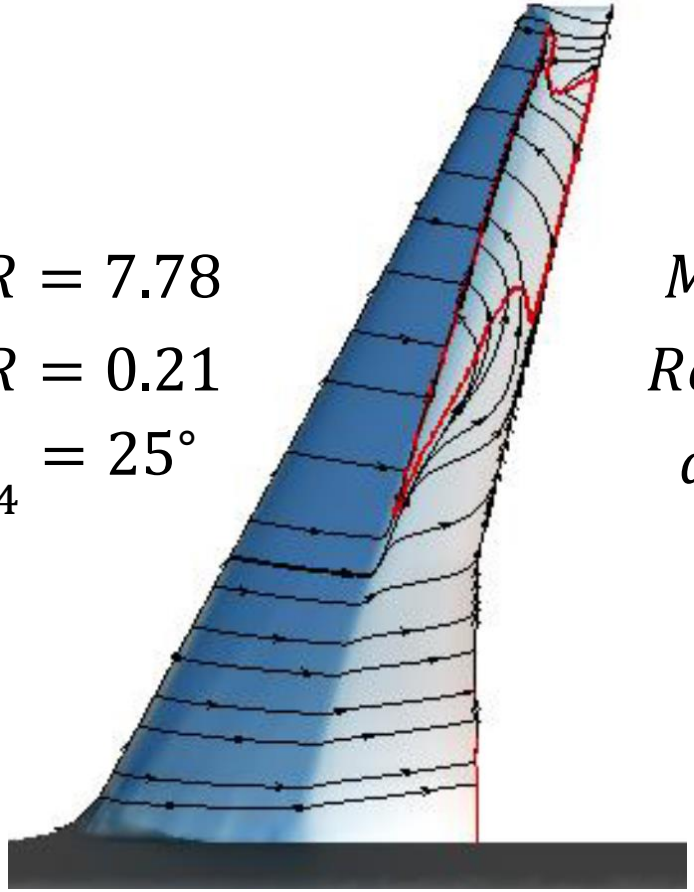
$TR = 0.21$

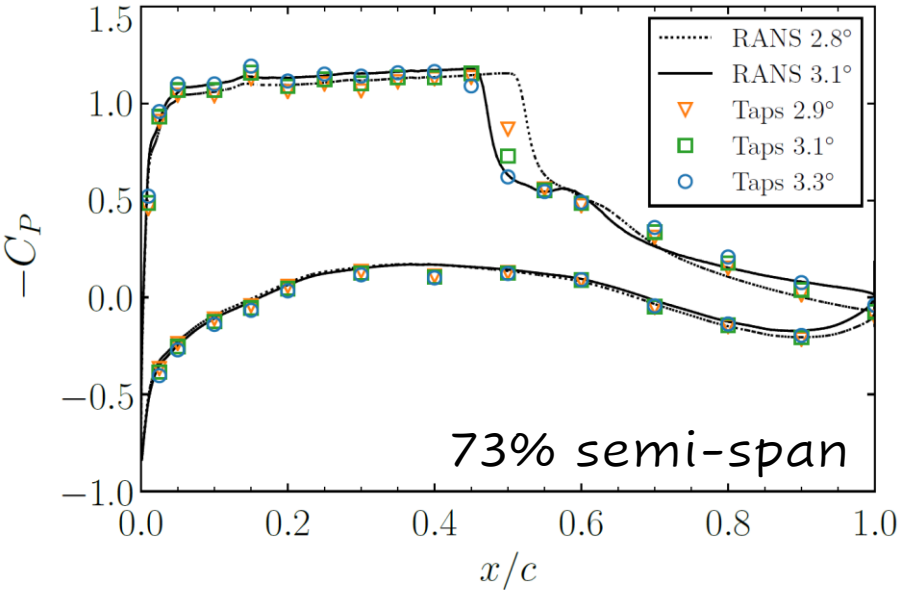
$\Lambda_{c/4} = 25°$

$M = 0.8$

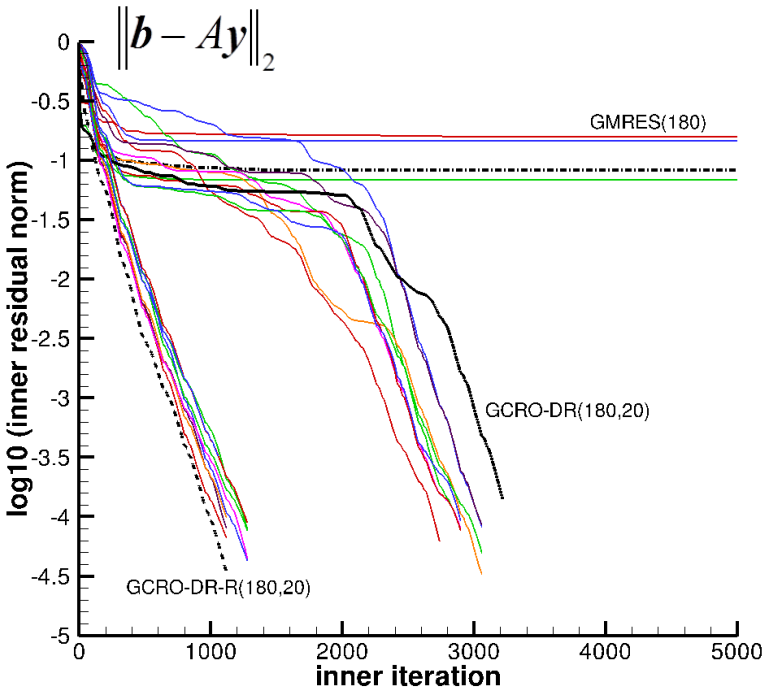
$Re = 3.75 \times 10^6$

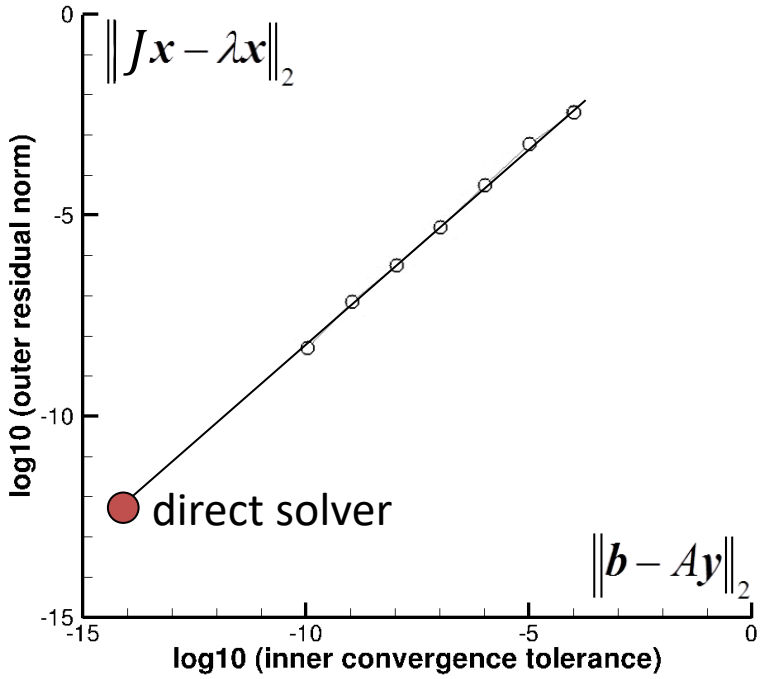
$\alpha = 3.0°$

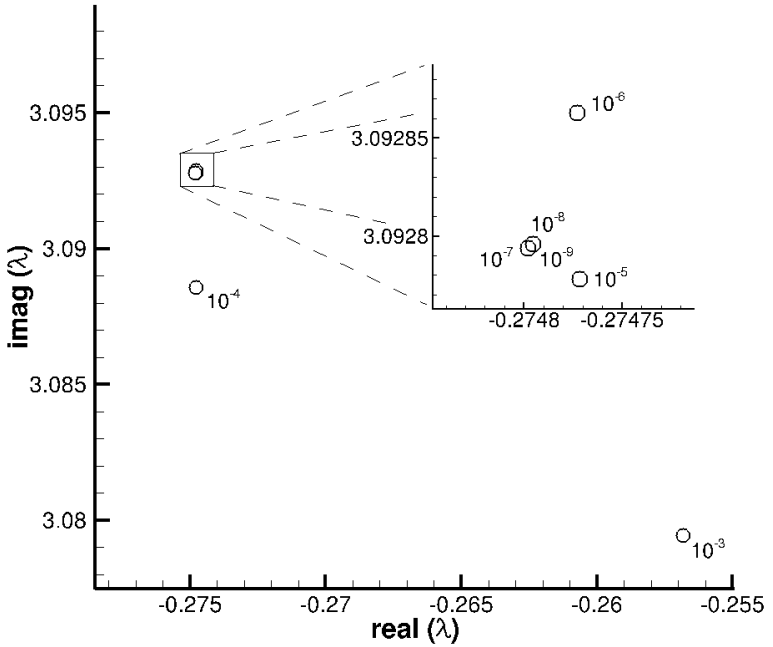




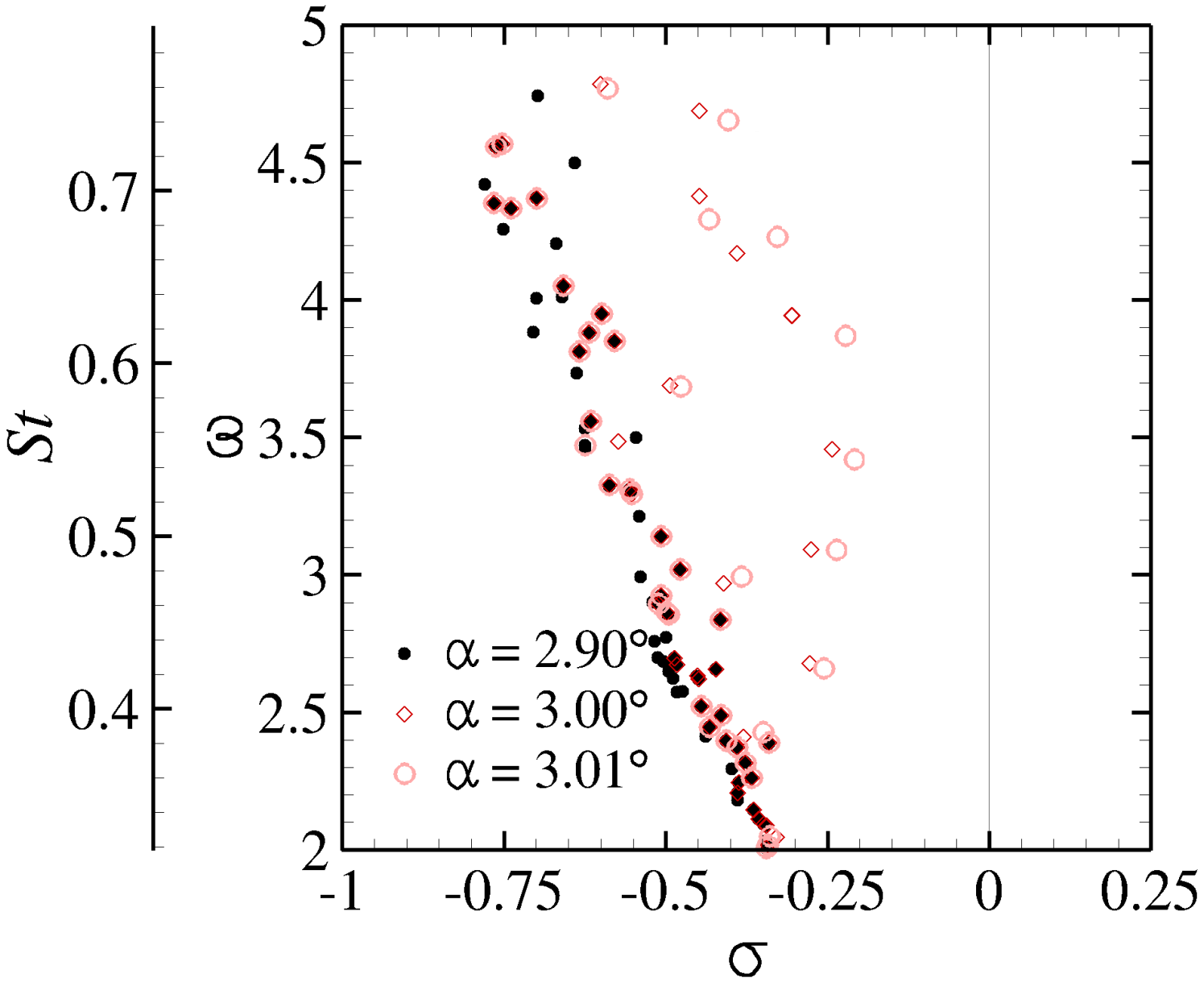
Inner-Outer Iterations

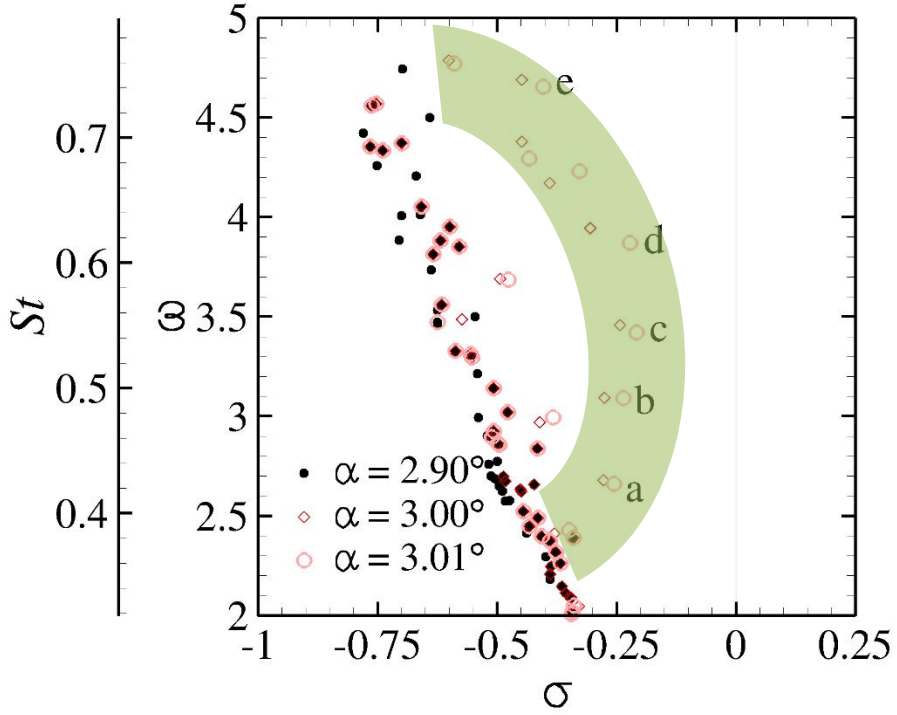


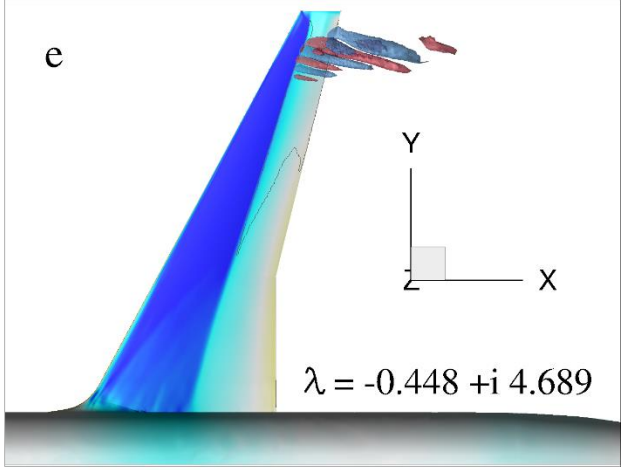


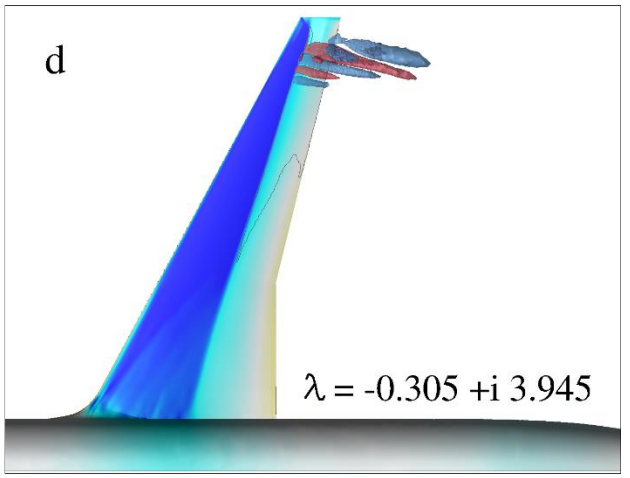


Eigenspectrum

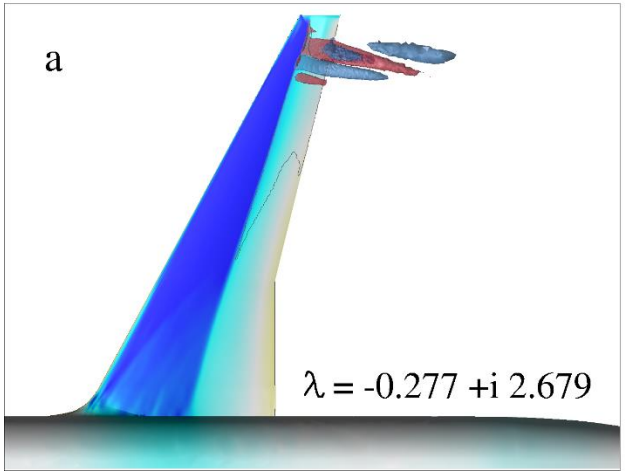


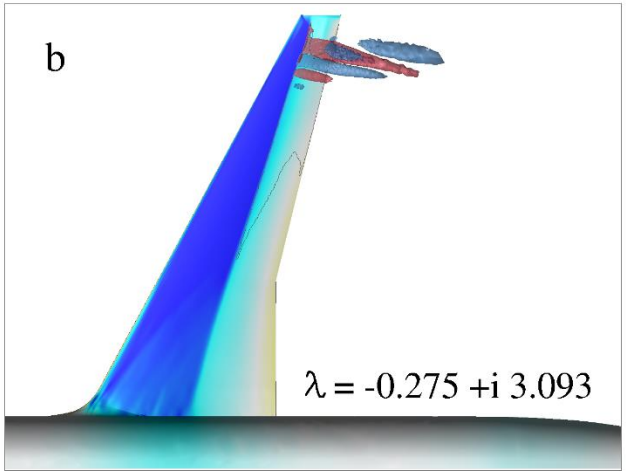


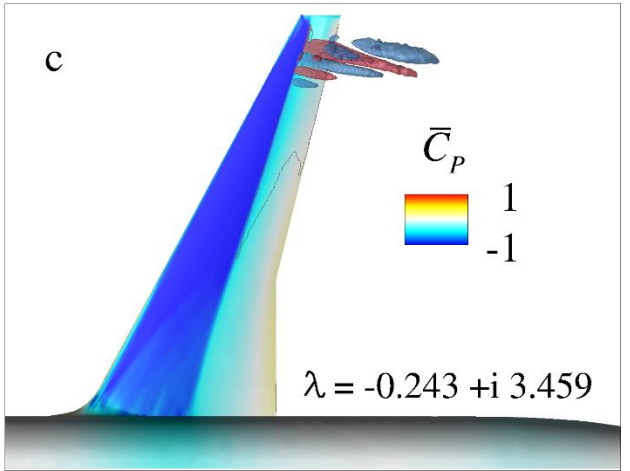




- Modes not shown in https://doi.org/10.2514/6.2016-3848 but later in https://doi.org/10.2514/6.2019-0037
- Instability (mode crossing) not computed

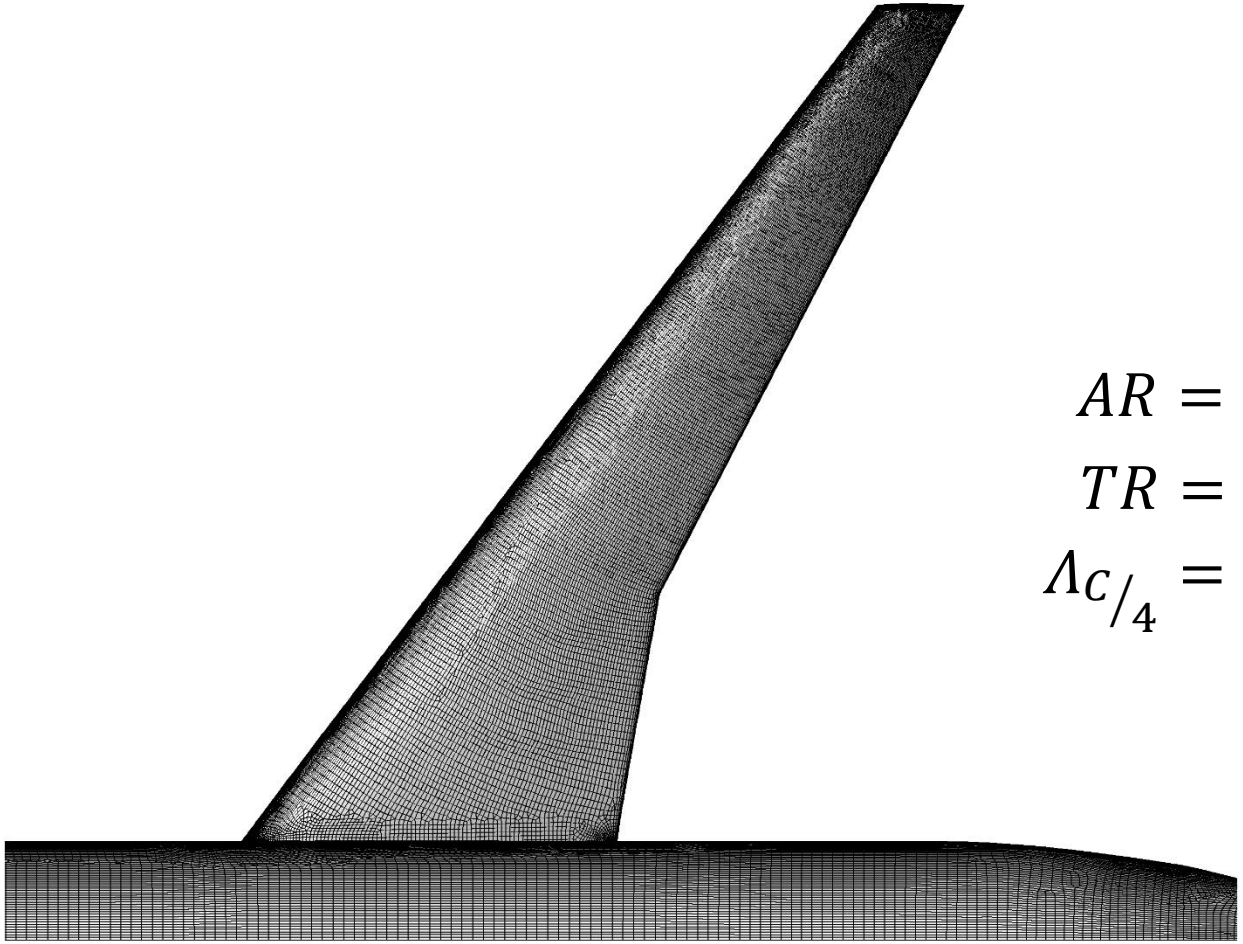






NASA Common Research Model

Timme (2020, JFM, https://doi.org/10.1017/jfm.2019.1001)



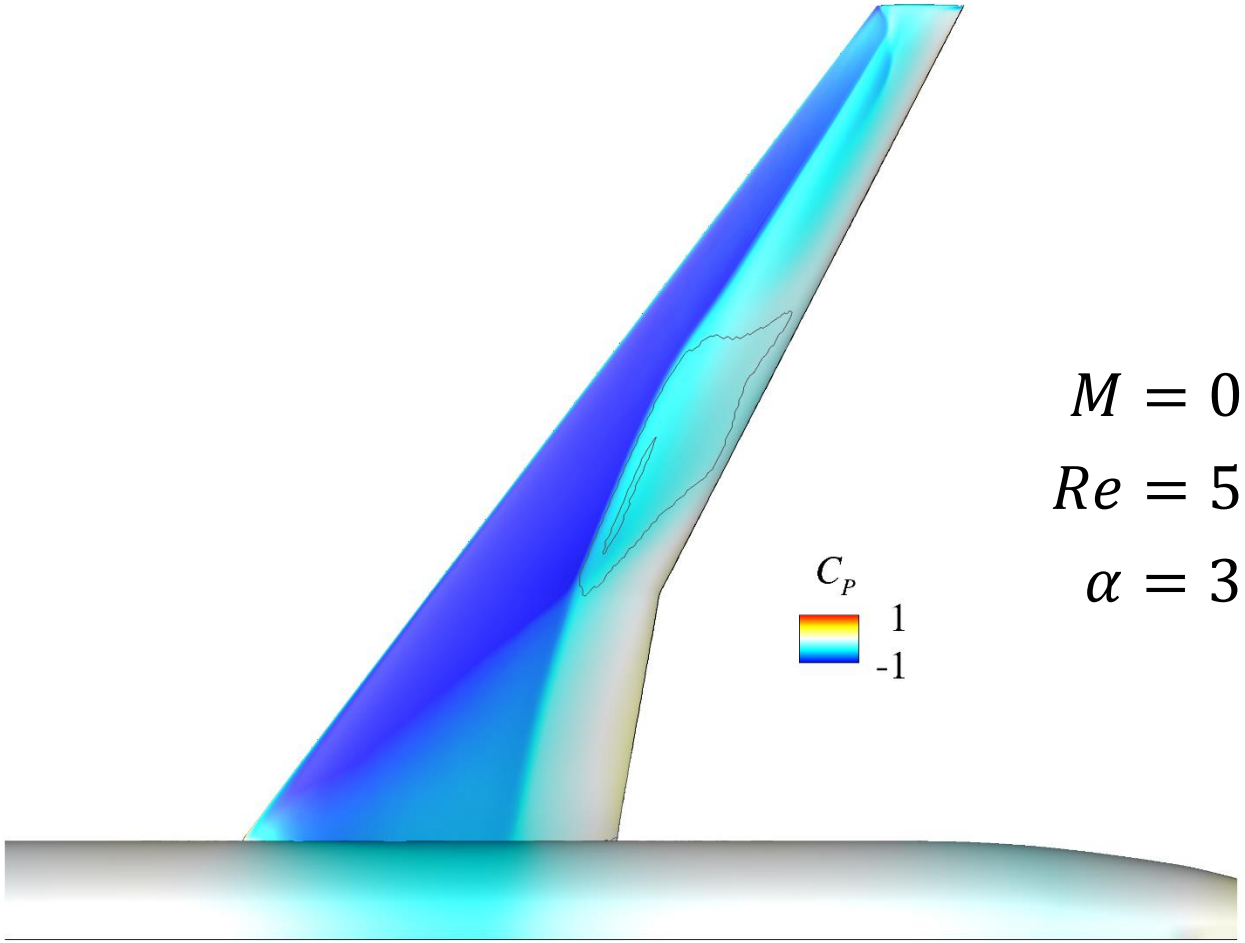
$$AR = 9$$

$$TR = 0.275$$

$$\Lambda_{C/4} = 35^\circ$$

3.1 million – **6.2 million** – 8.4 million

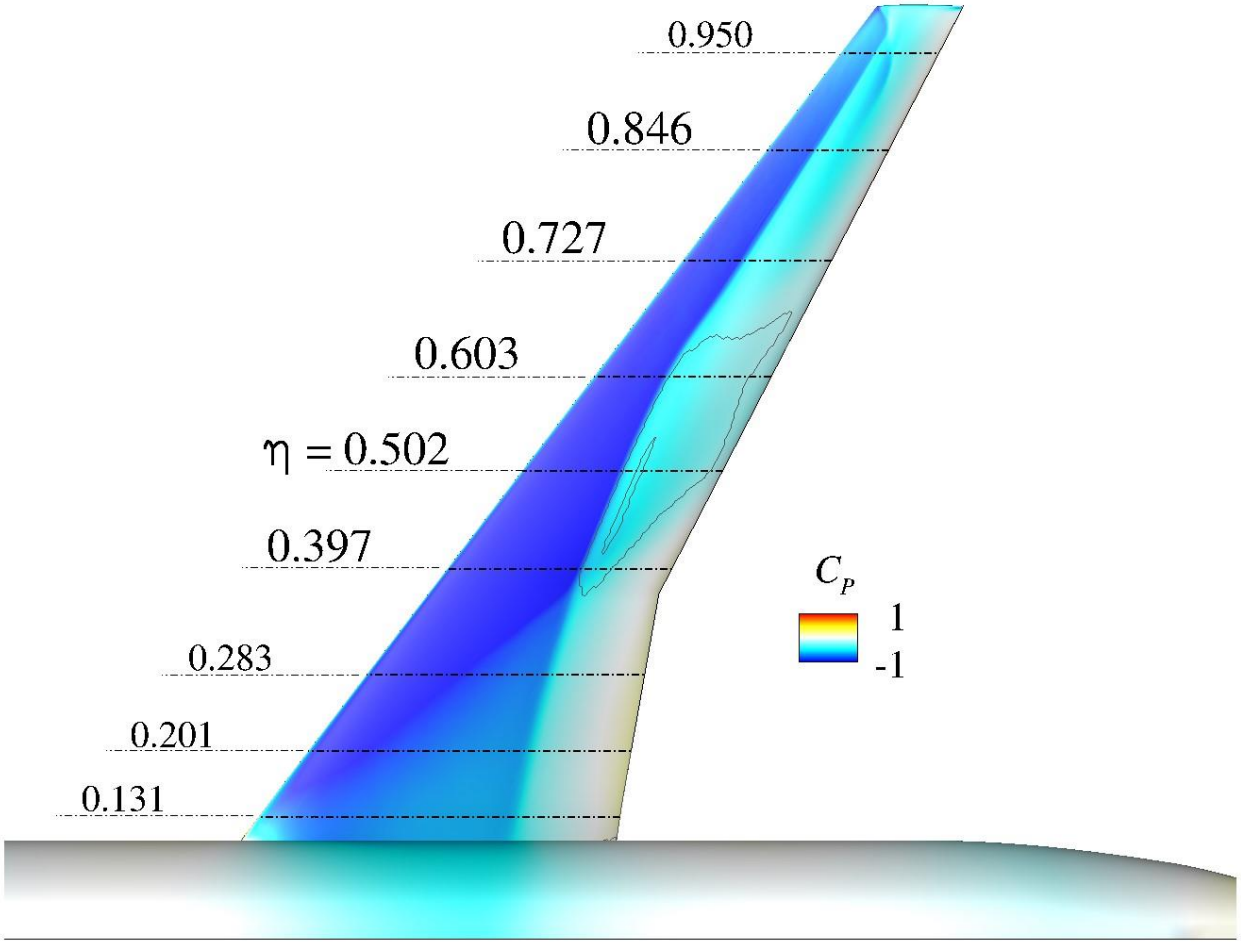
NASA Common Research Model

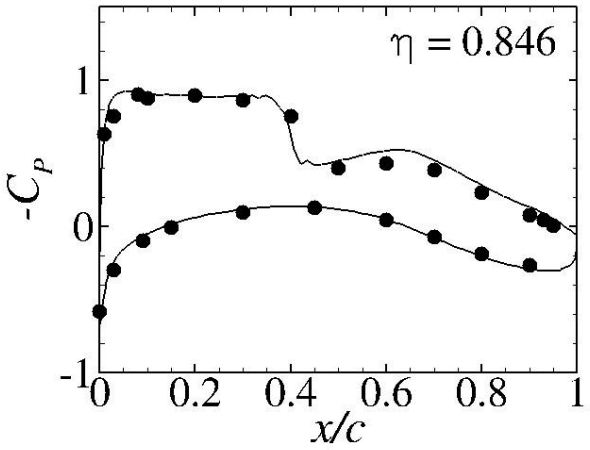


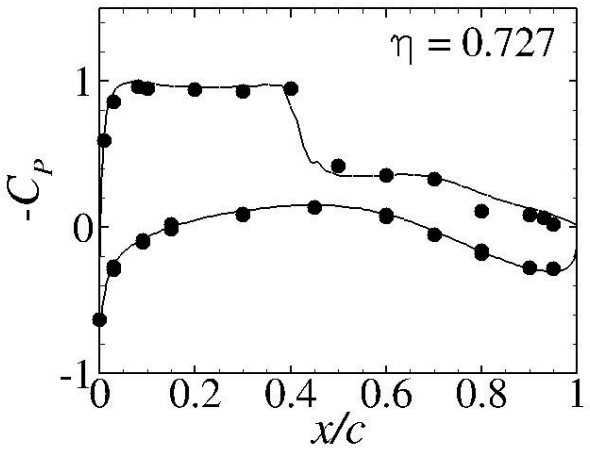
$$M = 0.85$$

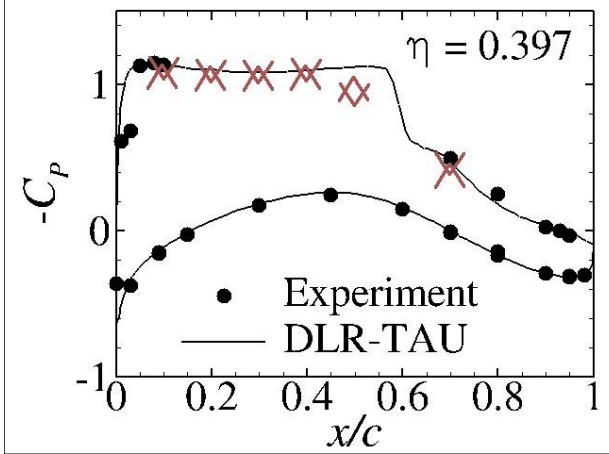
$$Re = 5.0 \times 10^6$$

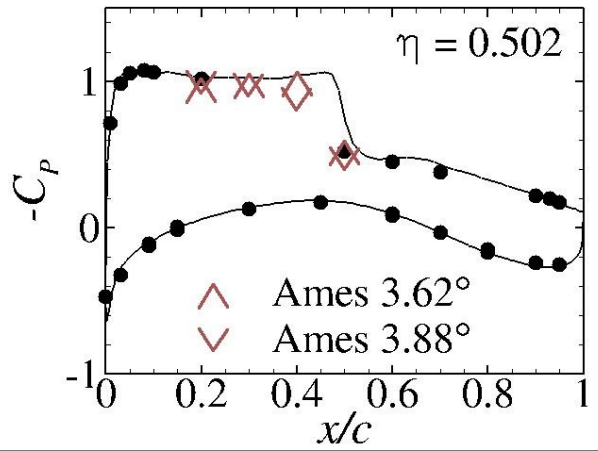
$$\alpha = 3.75^\circ$$

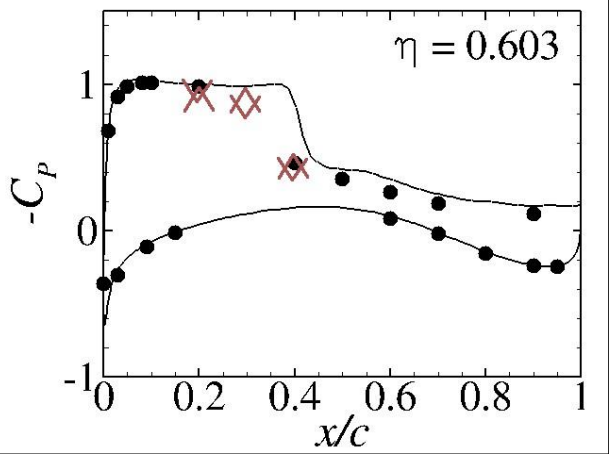

0.950
0.846
0.727
0.603
η = 0.502
0.397
0.283
0.201
0.131
C_P
1
-1


η = 0.846
$-C_P$
x/c

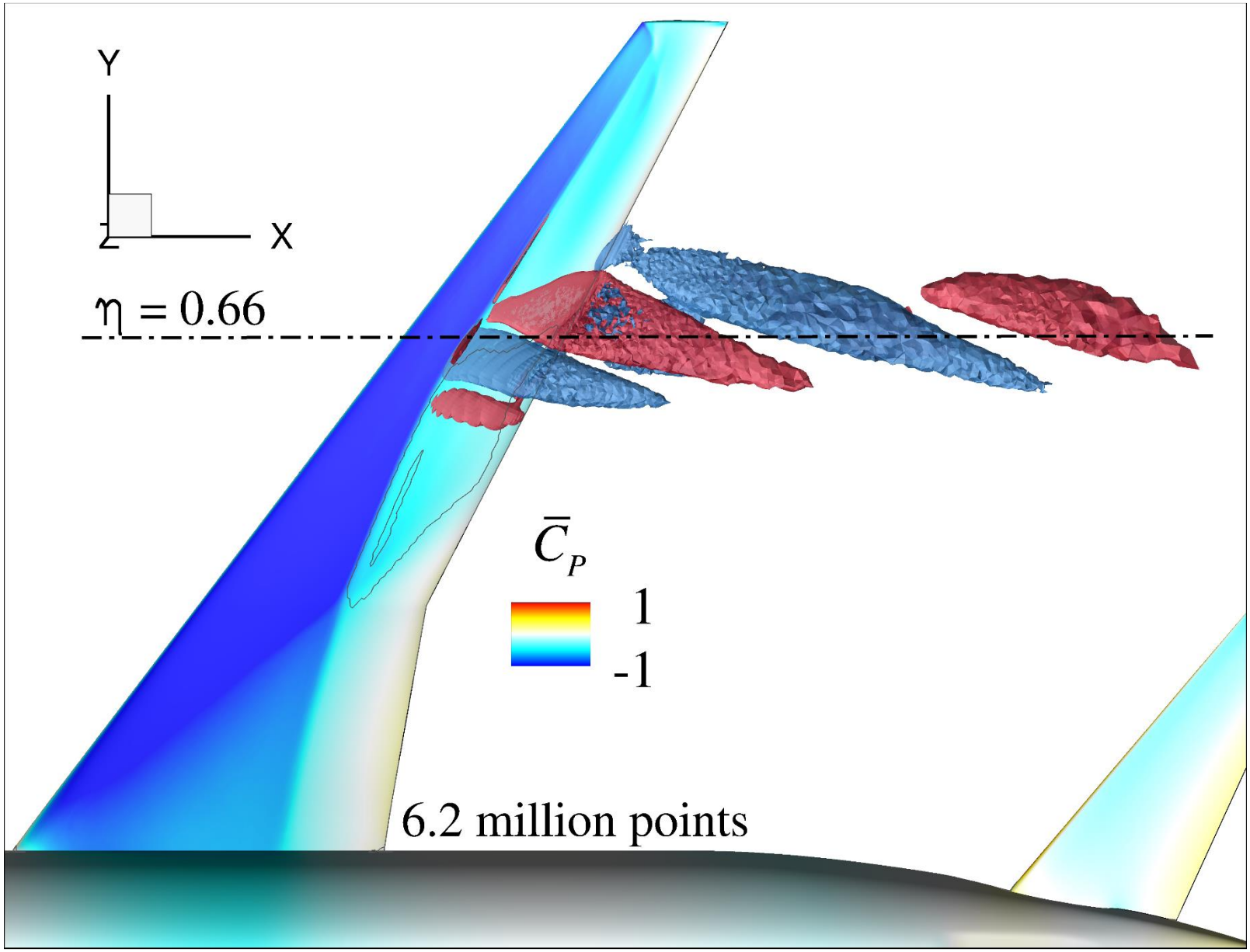

η = 0.727
$-C_P$
x/c


η = 0.397
$-C_P$
Experiment
DLR-TAU
x/c


η = 0.502
$-C_P$
Ames 3.62°
Ames 3.88°
x/c


η = 0.603
$-C_P$
x/c

Shock-Buffet Mode – Spatial Structure



Shock-Buffet Mode – Cell Propagation

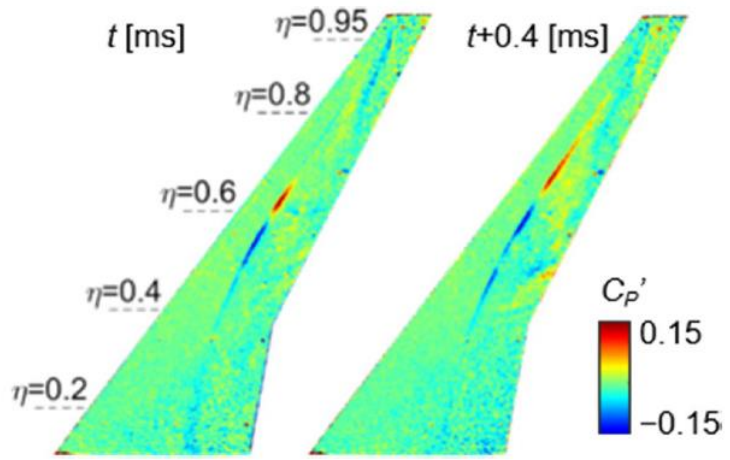
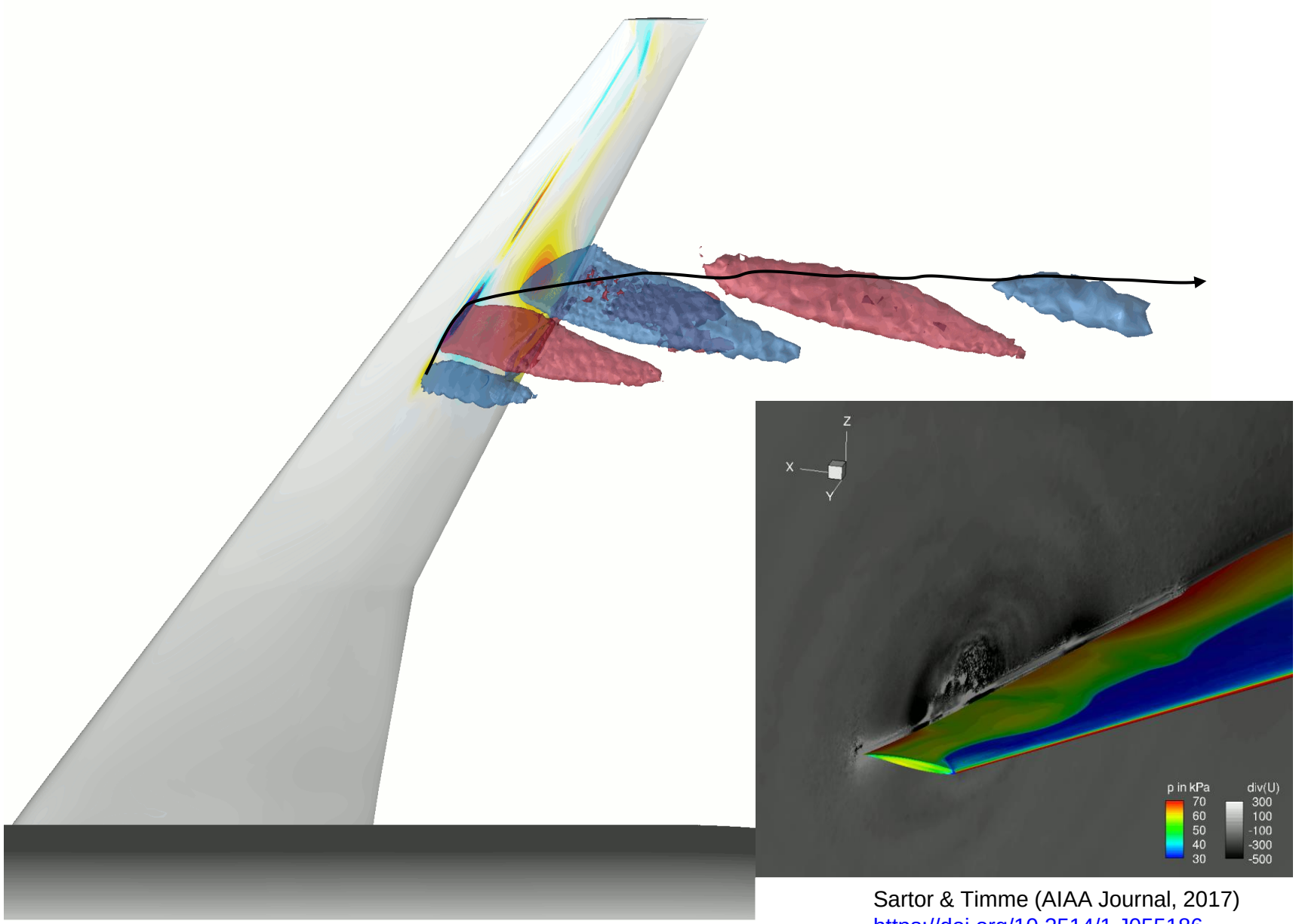


Fig. 14 Instantaneous C_P' distributions at $\alpha = 3.71$ deg ($M_\infty = 0.85$, $Re_C = 2.27 \times 10^6$)

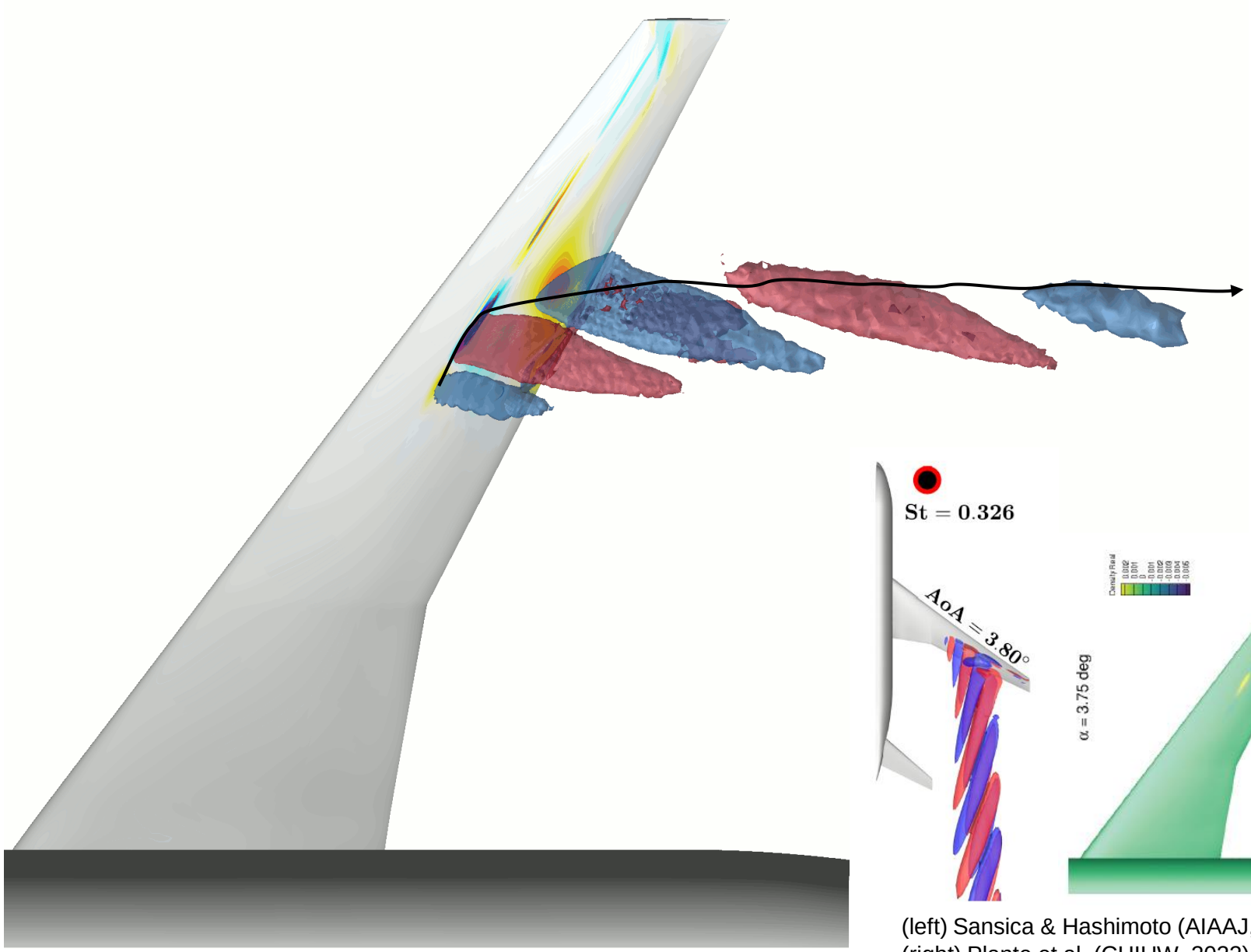
Sugioka et al. (Exp in Fluids, 2021)
https://doi.org/10.1007/s00348-020-03118-y

Shock-Buffet Mode – Cell Propagation



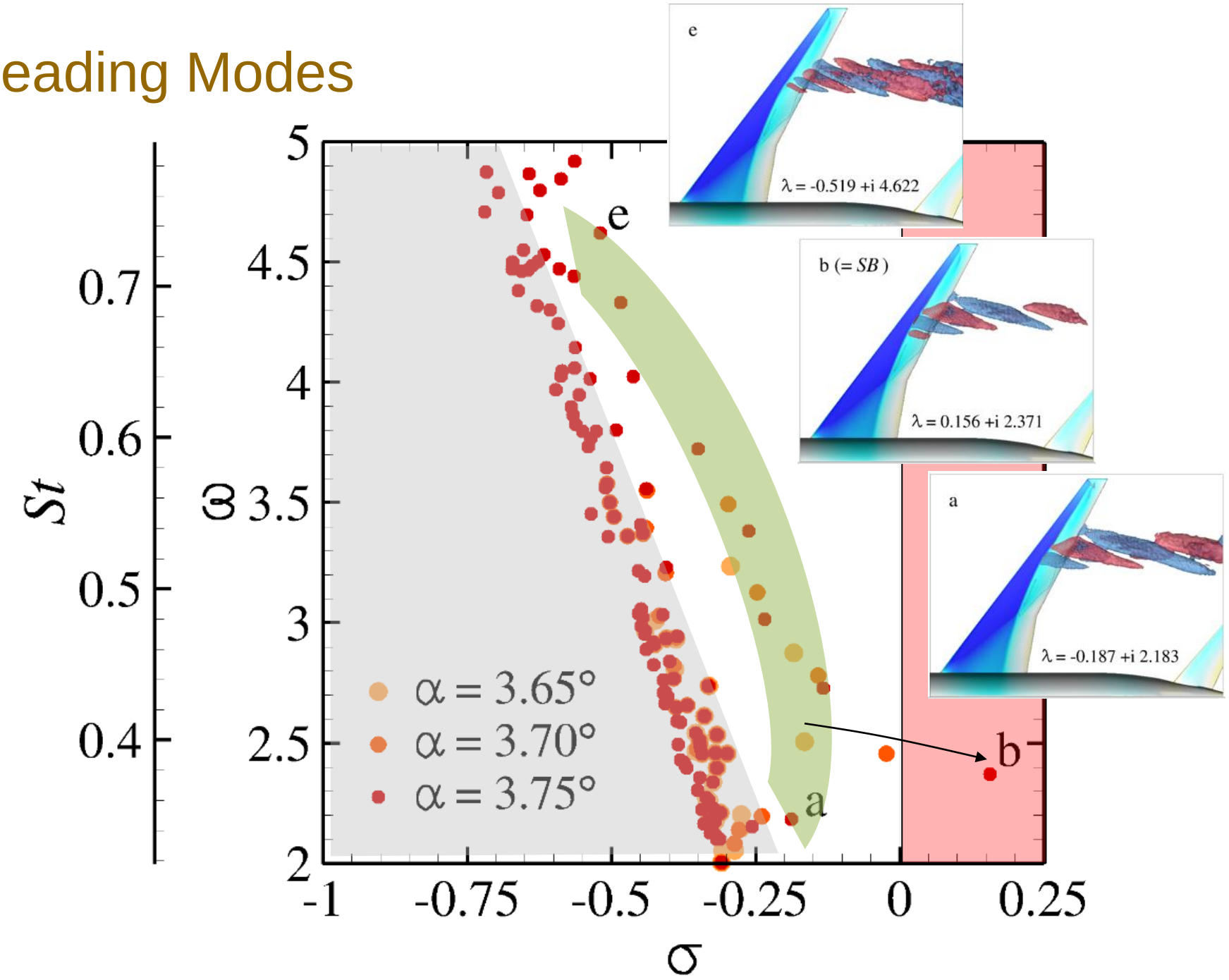
Sartor & Timme (AIAA Journal, 2017)
https://doi.org/10.2514/1.J055186

Shock-Buffet Mode – Cell Propagation

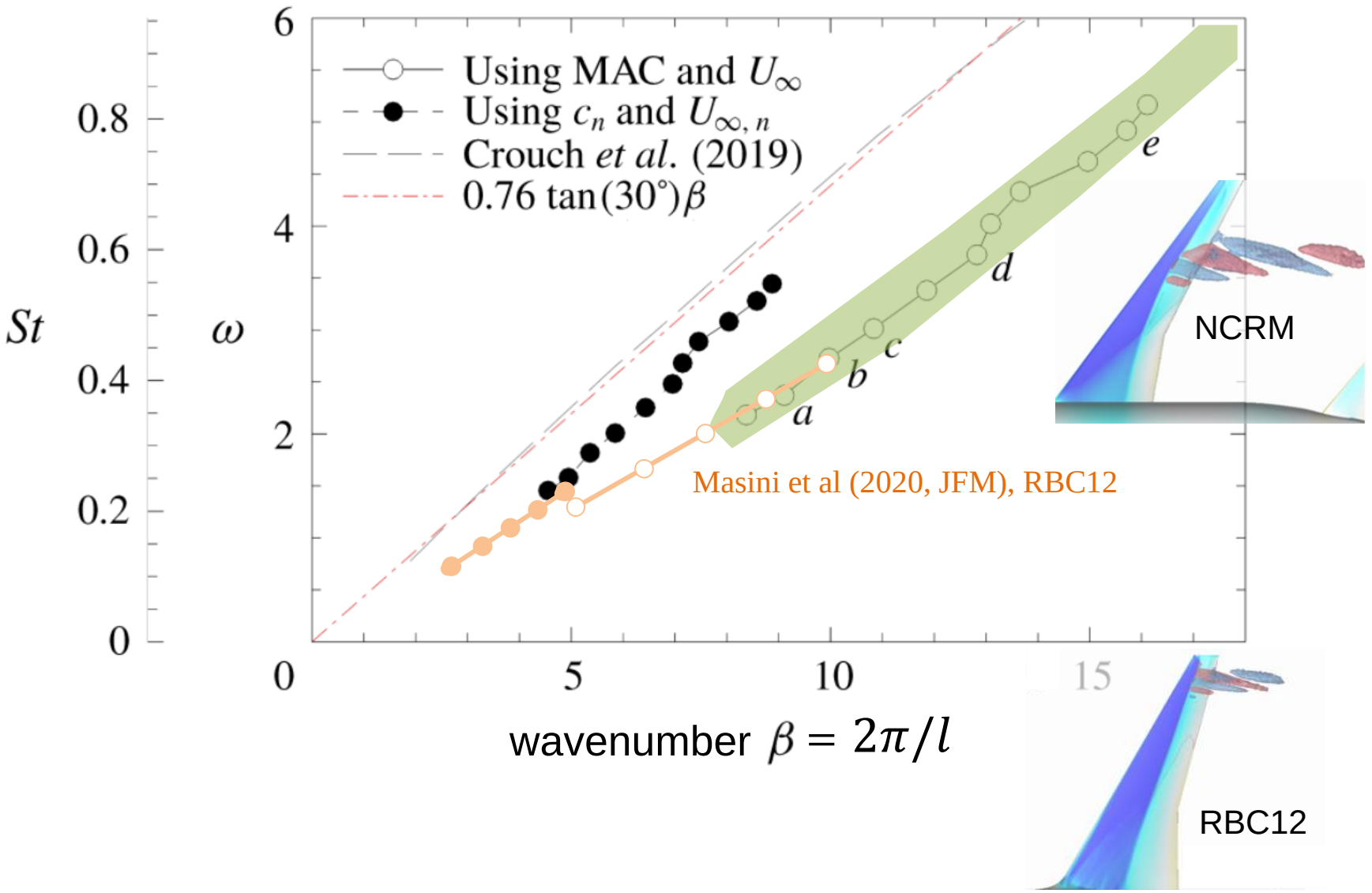


(left) Sansica & Hashimoto (AIAAJ, 2023)
(right) Plante et al. (CHIUW, 2022)

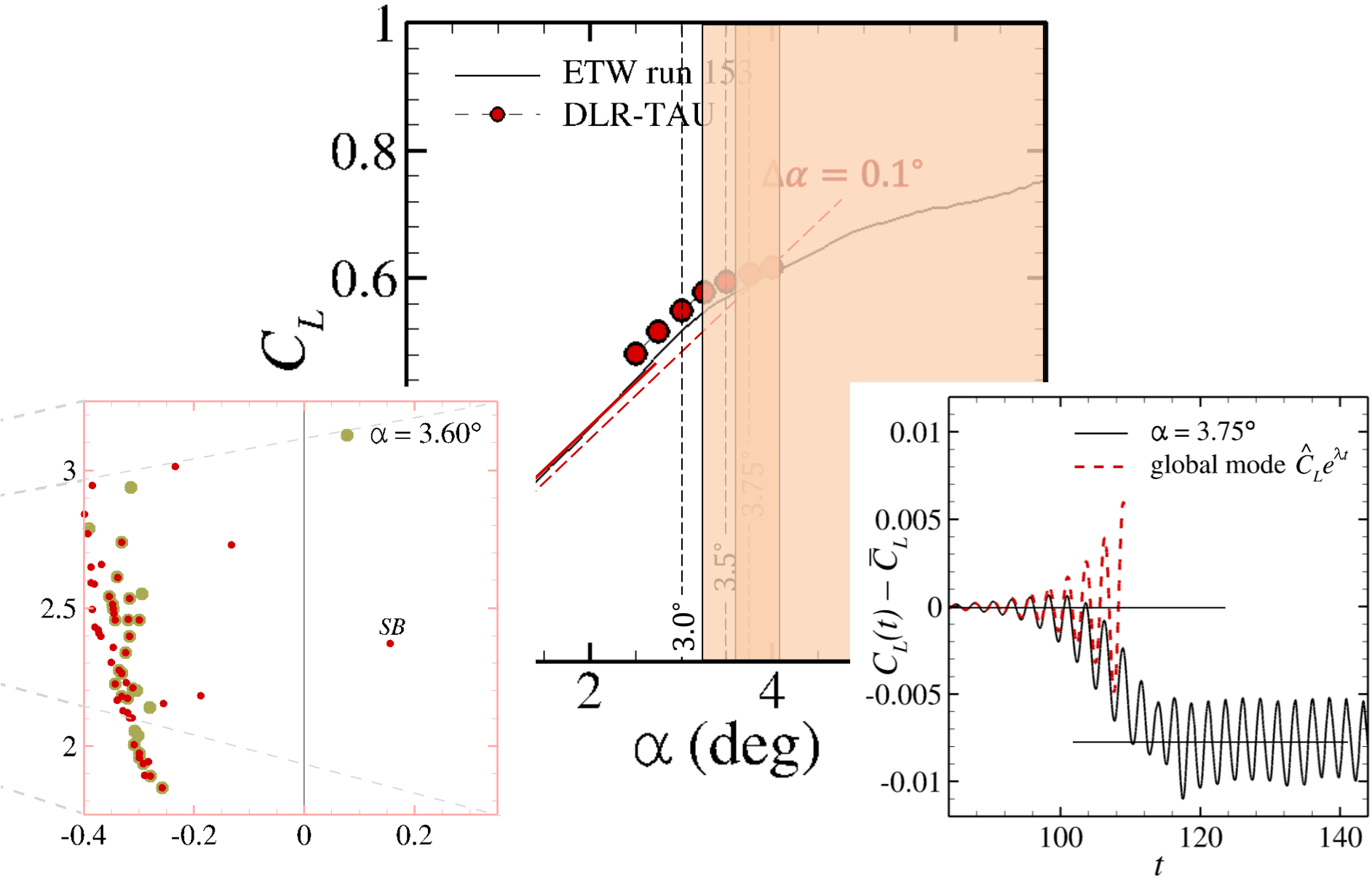
Leading Modes



Characteristic Band of Modes



A Tool for Design? Limitations & Opportunities



Resolvent Methodology

- RANS (plus turb model) equations in semi-discrete form

$$\dot{\boldsymbol{u}} = R(\boldsymbol{u})$$

- Linearise and introduce a time-dependent forcing, ansatz $\boldsymbol{u} = \hat{\boldsymbol{u}} e^{i\omega t}$

$$i\omega \hat{\boldsymbol{u}} = J_{ff}\hat{\boldsymbol{u}} + \hat{\boldsymbol{f}}$$

- Define resolvent operator H

$$\hat{\boldsymbol{u}} = -\left(J_{ff} - i\omega I\right)^{-1}\hat{\boldsymbol{f}} \text{ with } H = \left(J_{ff} - i\omega I\right)^{-1}$$

- Optimal response-gain-forcing triplets

$$G(\omega) = \max_{\hat{\boldsymbol{f}}} \frac{\langle \hat{\boldsymbol{u}}, \hat{\boldsymbol{u}} \rangle}{\langle \hat{\boldsymbol{f}}, \hat{\boldsymbol{f}} \rangle} = \max_{\hat{\boldsymbol{f}}} \frac{\langle H^{\dagger} H \hat{\boldsymbol{f}}, \hat{\boldsymbol{f}} \rangle}{\langle \hat{\boldsymbol{f}}, \hat{\boldsymbol{f}} \rangle}$$

  equivalent to SVD of $H = U\Sigma V^{H}$ or eigenvalue problem of $HH^{\dagger}$ (or $H^{\dagger}H$)

- Again (inner) iterative sparse linear solutions

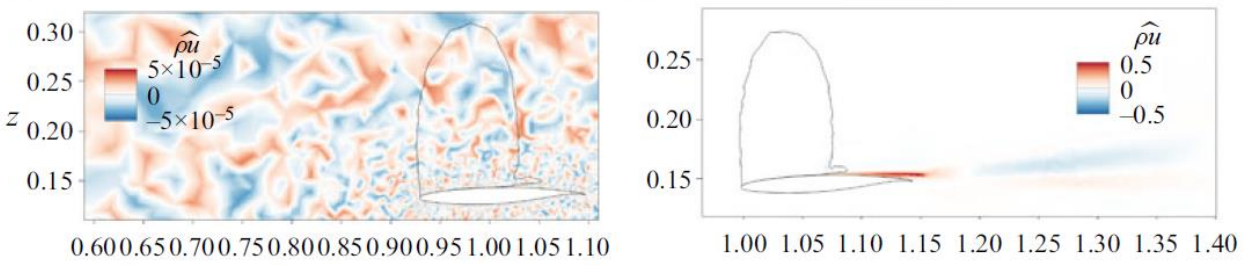
$$H^{-1}P^{-1}(P\hat{\boldsymbol{u}}) = \hat{\boldsymbol{f}} \text{ and } H^{-\dagger}P^{-1}(P\hat{\boldsymbol{f}}) = \hat{\boldsymbol{u}}$$

Iterative Resolvent Algorithm

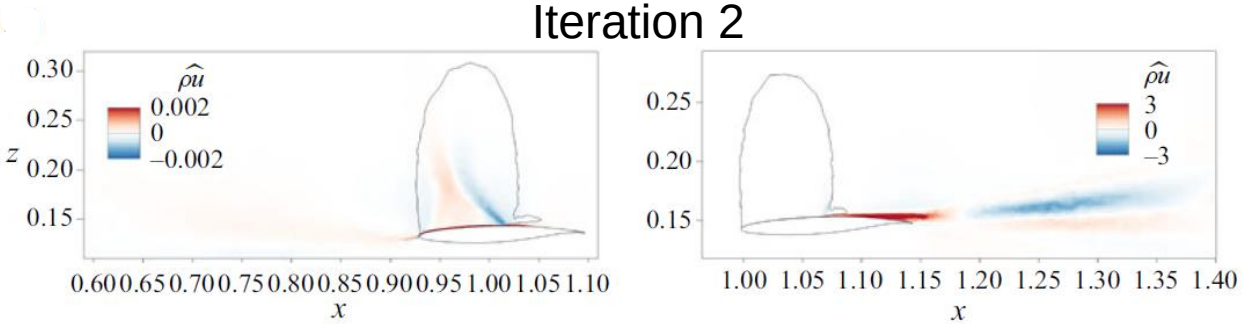
Houtman, Timme and Sharma (2023, Flow, https://doi.org/10.1017/flo.2023.8)

1: **for** $k = 1, \ldots, n$ **do**
2: Set a random initial forcing with $\langle \hat{\boldsymbol{f}}_k, \hat{\boldsymbol{f}}_k \rangle = 1$
3: **do**
4: $\hat{\boldsymbol{f}}_k = \hat{\boldsymbol{f}}_k - \sum_{l=1}^{k-1} \langle \hat{\boldsymbol{f}}_k, \hat{\boldsymbol{f}}_l \rangle \hat{\boldsymbol{f}}_l$
5: $\hat{\boldsymbol{f}}_{\text{old}} = \hat{\boldsymbol{f}}_k$
6: Solve $(\boldsymbol{J} - \mathrm{i}\omega \boldsymbol{I})\hat{\boldsymbol{w}}_k = \hat{\boldsymbol{f}}_k$ iteratively and approximately
7: $\sigma_k^2 = \langle \hat{\boldsymbol{w}}_k, \hat{\boldsymbol{w}}_k \rangle$ and $\hat{\boldsymbol{w}}_k = \hat{\boldsymbol{w}}_k / \sigma_k$
8: Solve $(\boldsymbol{J}^\dagger + \mathrm{i}\omega \boldsymbol{I})\hat{\boldsymbol{f}}_k = \hat{\boldsymbol{w}}_k$ iteratively and approximately
9: $\sigma_k^2 = \langle \hat{\boldsymbol{f}}_k, \hat{\boldsymbol{f}}_k \rangle$ and $\hat{\boldsymbol{f}}_k = \hat{\boldsymbol{f}}_k / \sigma_k$
10: **while** $\|\hat{\boldsymbol{f}}_k - \hat{\boldsymbol{f}}_{\text{old}}\| > c$
11: **end for**

Iteration 1



Iteration 2

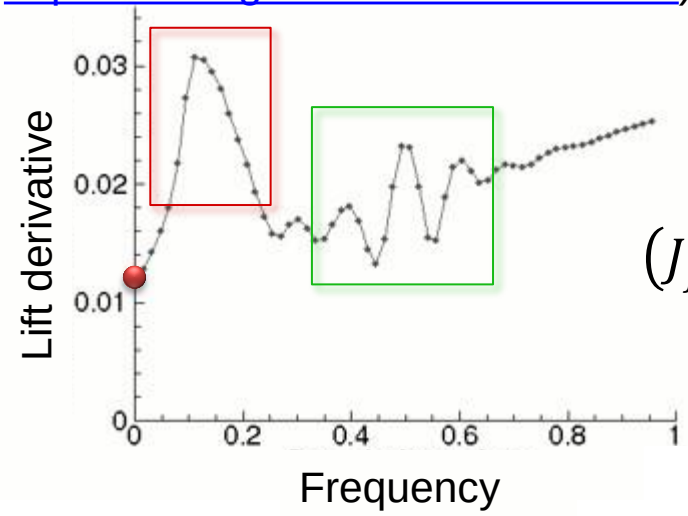


Little Detour: Linear Frequency Domain

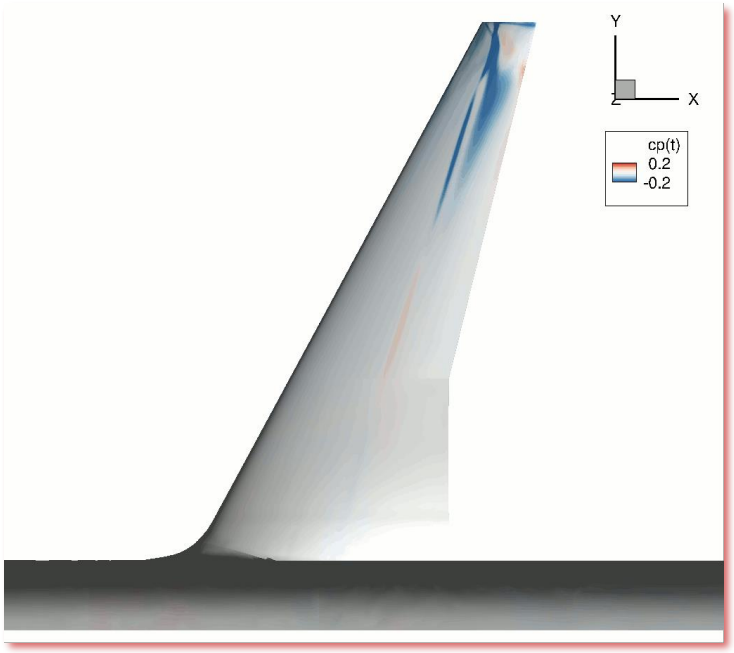
Timme & Thormann (2016, https://doi.org/10.2514/6.2016-3848)

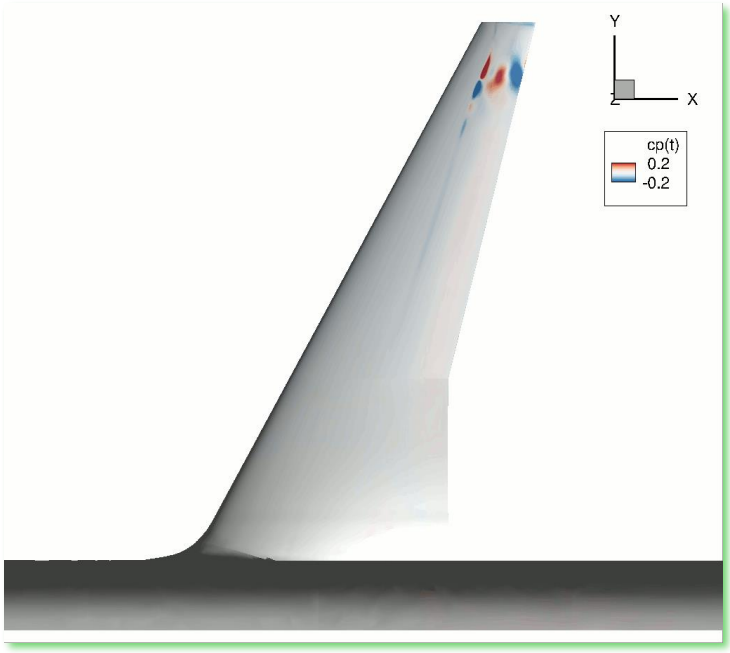


RBC12



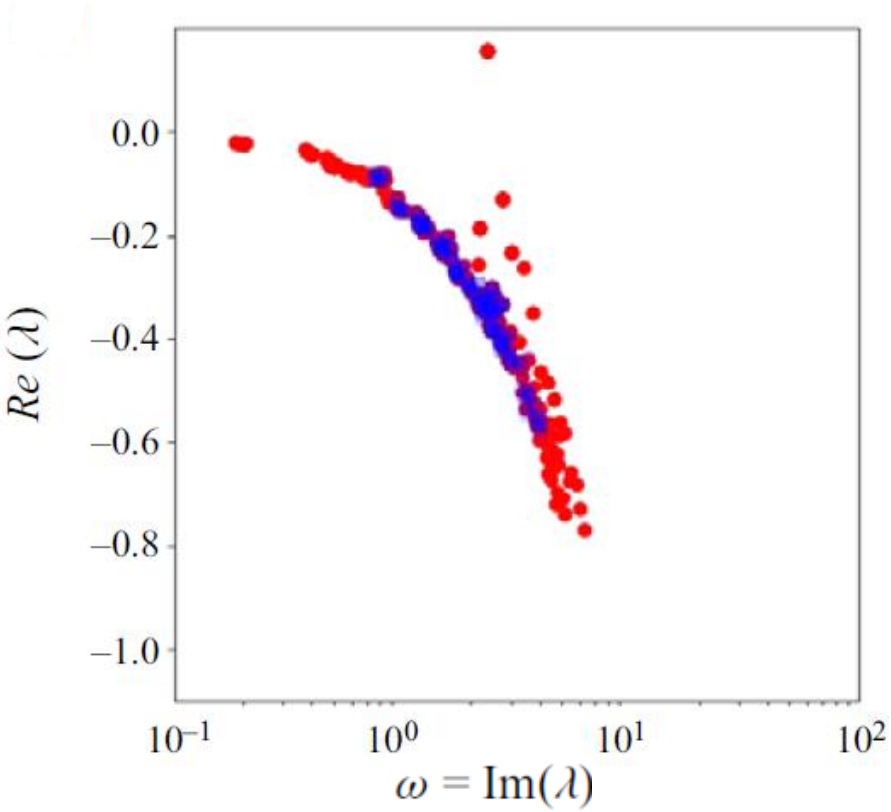
$$\left(J_{ff} - i\omega I\right)\hat{\boldsymbol{u}}_j = -J_{fs}\boldsymbol{e}_j$$

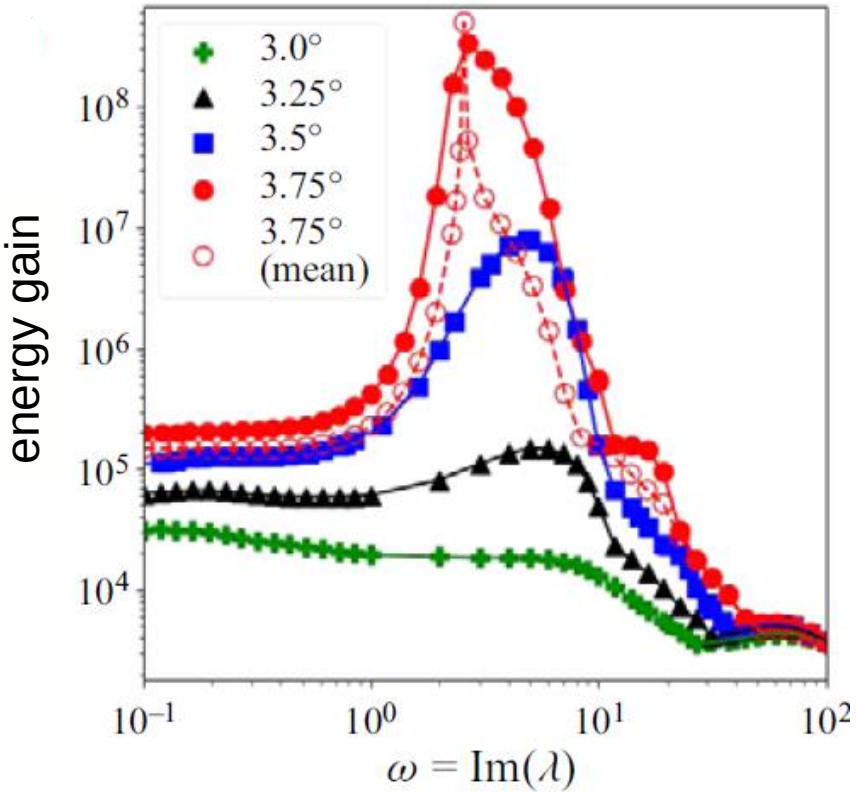




Global Stability vs Resolvent

Houtman, Timme and Sharma (2023, Flow, https://doi.org/10.1017/flo.2023.8)

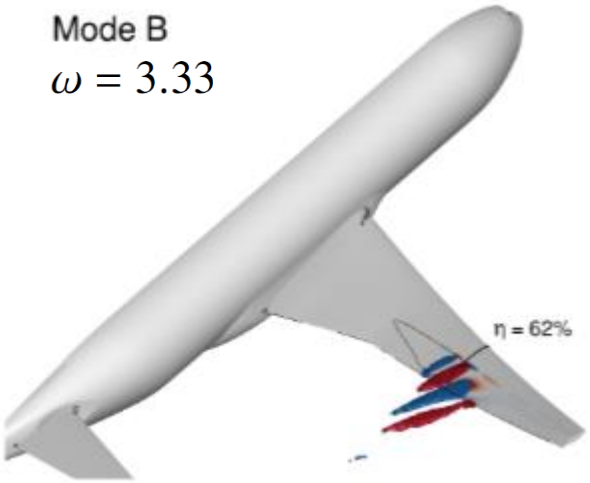


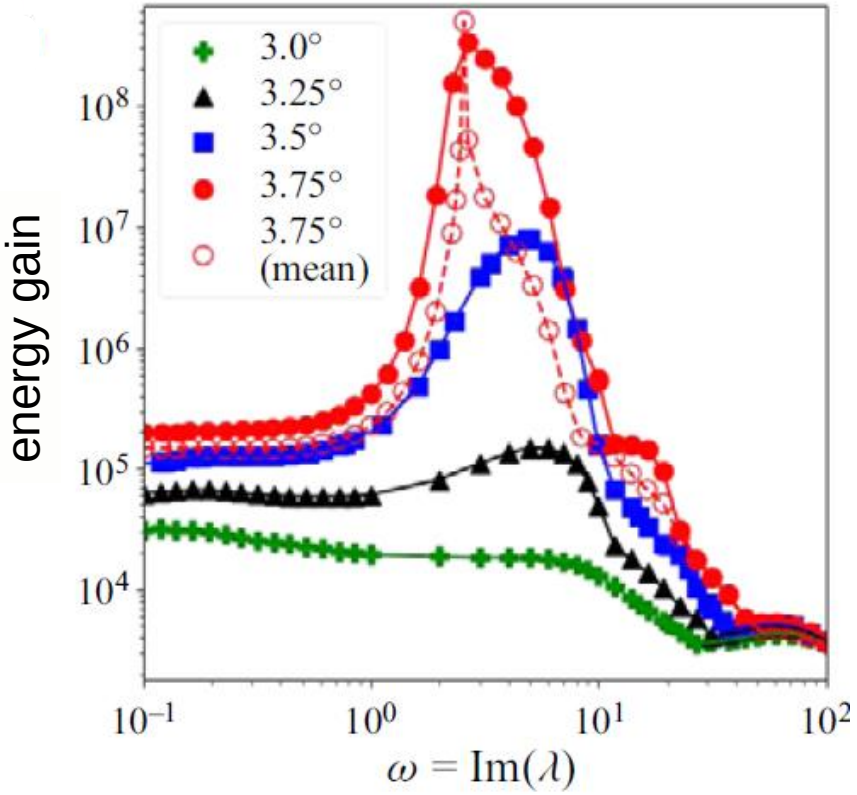


Resolvent Modes

Mode B

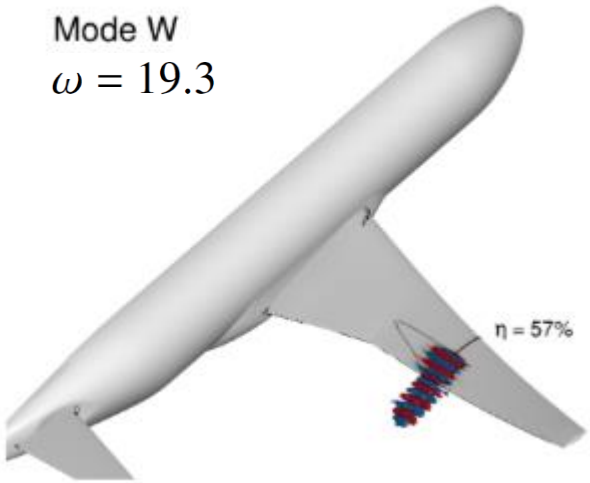
$\omega = 3.33$





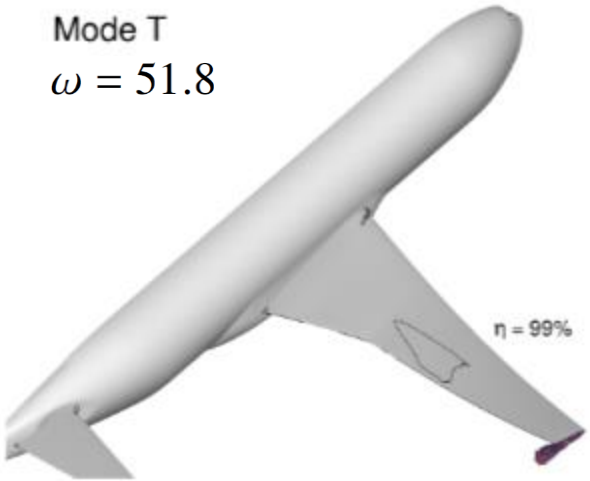
Mode W

$\omega = 19.3$

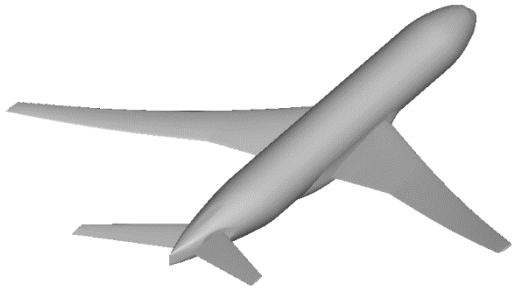


Mode T

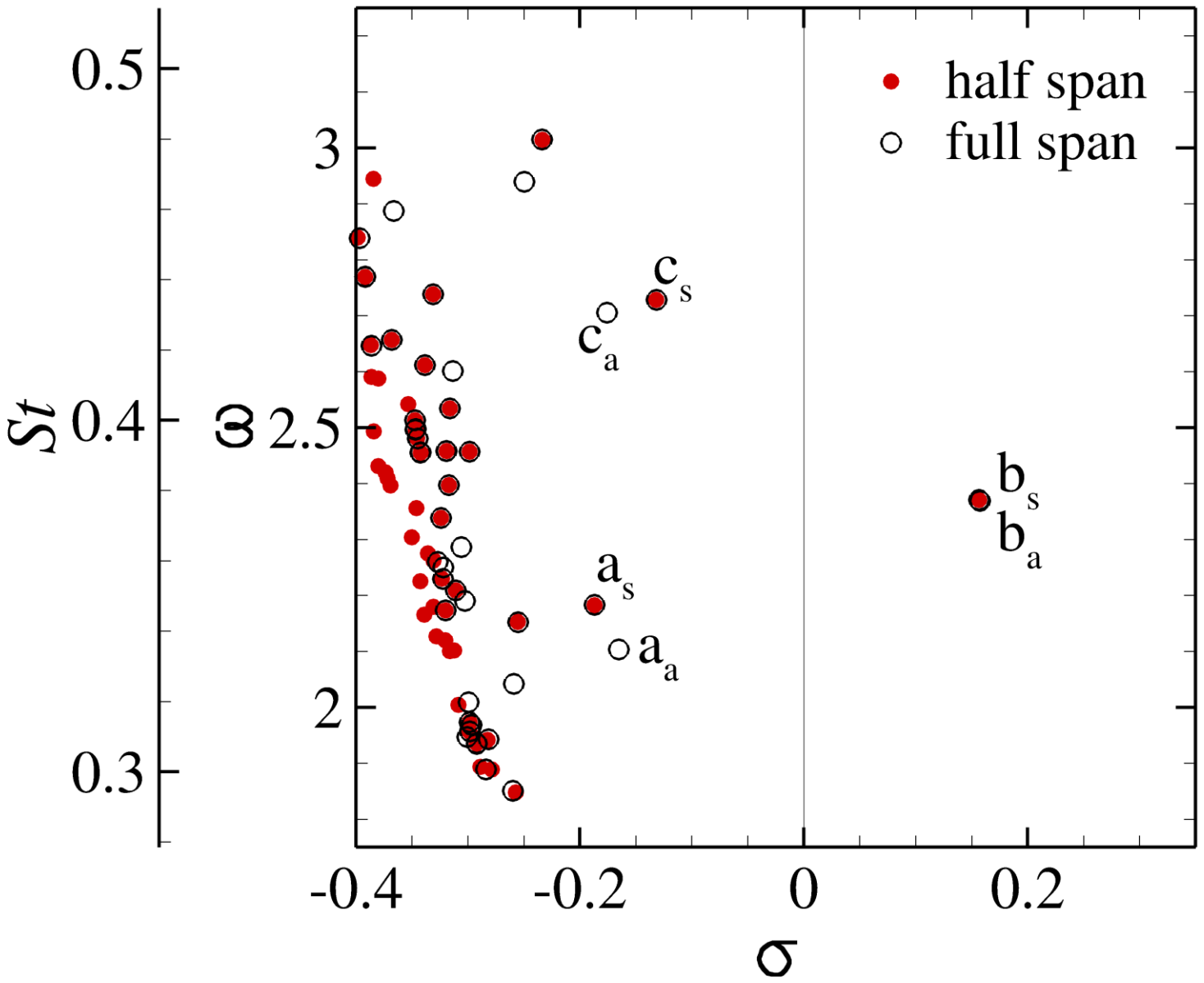
$\omega = 51.8$

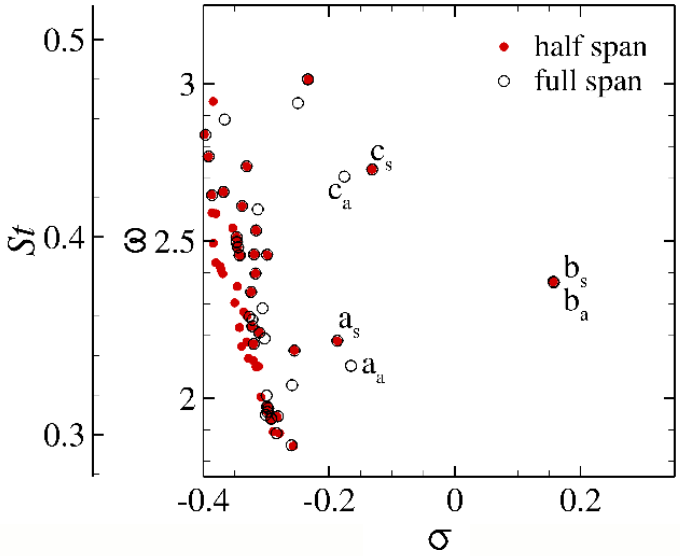


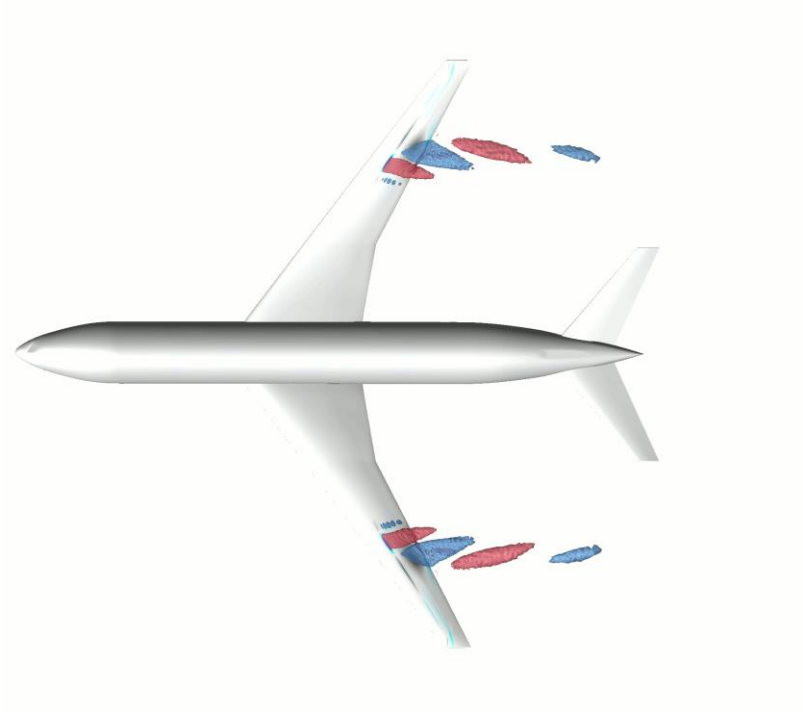
Full-Span Case



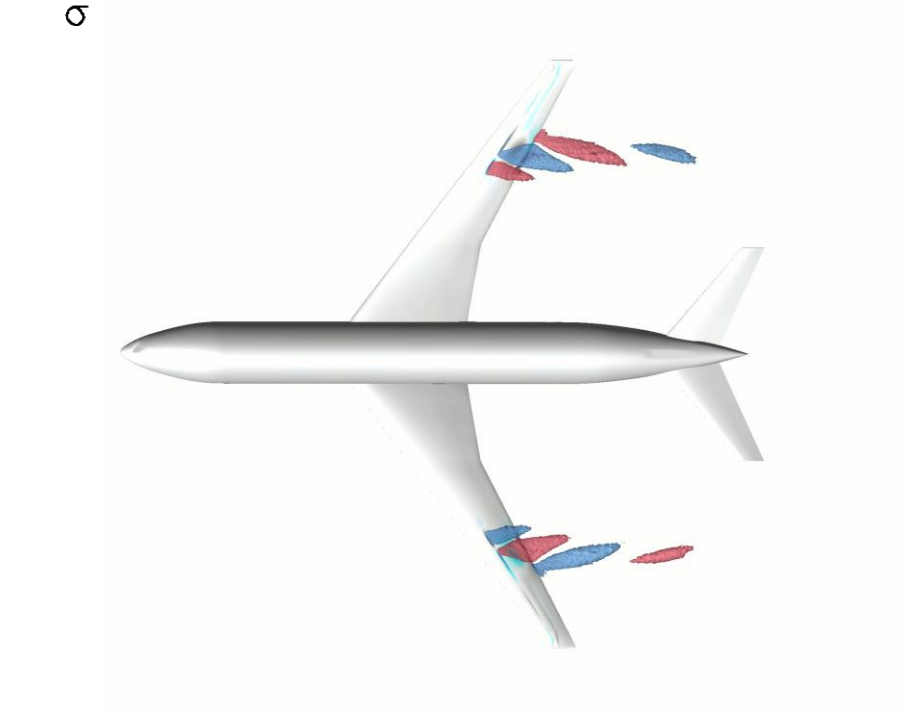
Full-Span Case







Symmetric



Anti-symmetric

Elastic Aircraft Case – Asymmetric

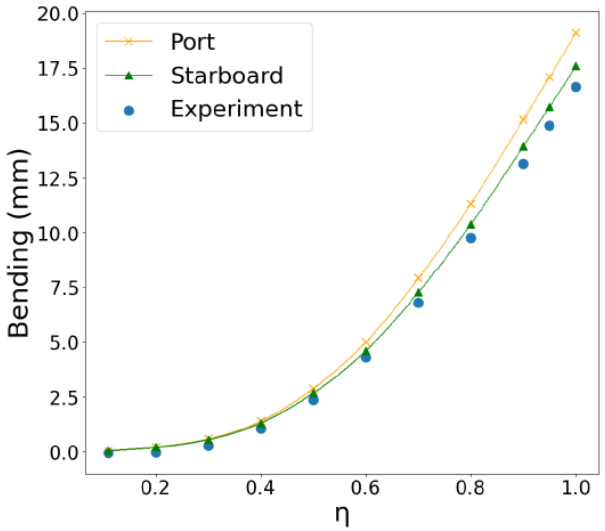
Houtman & Timme (2023, JFM, https://doi.org/10.1017/jfm.2023.413)

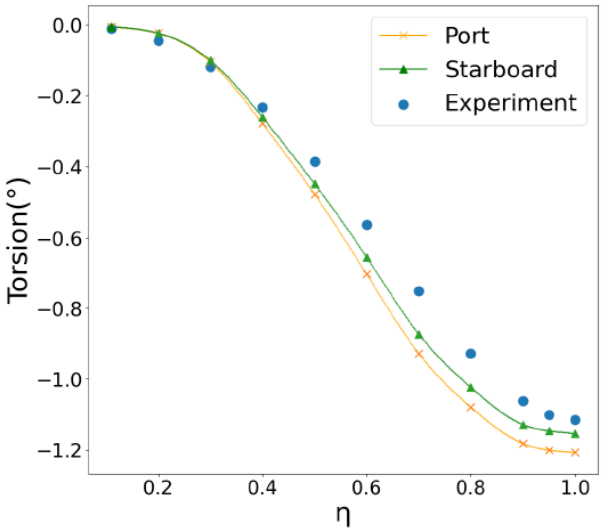
Structural mode shapes



Mode 2, 40.94 Hz Mode 20, 426.74 Hz Mode 25, 526.93 Hz

Static aeroelastic deformation





Coupled Eigenmode Solver

Recall the flutter method

(unsteady aerodynamics)

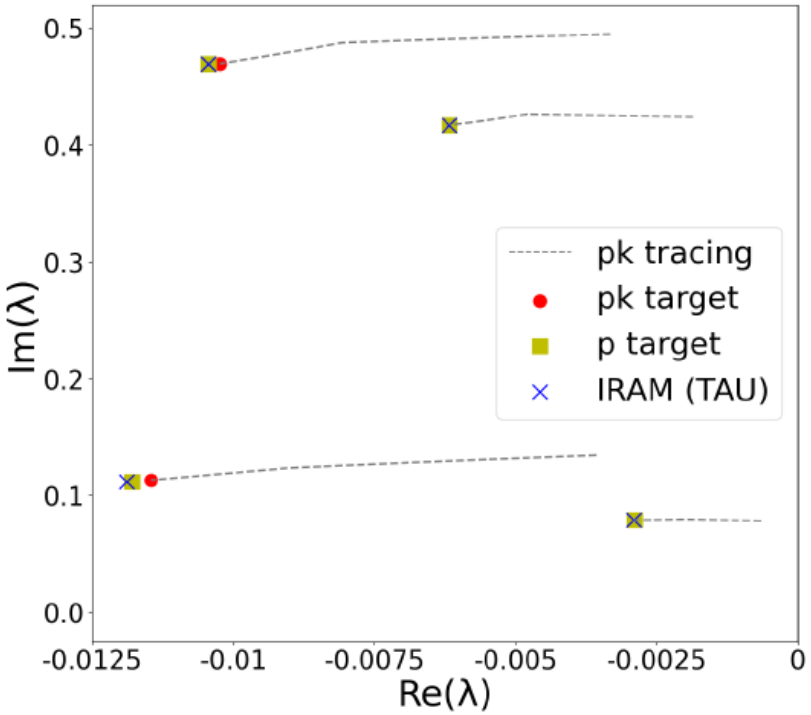
$$\begin{cases} \left(J_{ss} - J_{sf}\left(J_{ff} - \lambda_i I\right)^{-1} J_{fs}\right)\boldsymbol{\phi}_{s,i} = \lambda_i \boldsymbol{\phi}_{s,i} \\ \left(J_{ff} - \lambda_i I\right)\boldsymbol{\phi}_{f,i} = -J_{fs}\boldsymbol{\phi}_{s,i} \end{cases} \qquad \boldsymbol{\phi}_i = \begin{bmatrix} \boldsymbol{\phi}_{f,i} \\ \boldsymbol{\phi}_{s,i} \end{bmatrix}$$

(mass, damping, stiffness)

Requires a suitable shift and block matrix for Schur complement.

Instead use coupled approach (IRAM/KS)

$$\begin{pmatrix} J_{ff} & J_{fs} \\ J_{sf} & J_{ss} \end{pmatrix} \boldsymbol{\phi}_i = \lambda_i \boldsymbol{\phi}_i$$



Coupled Eigenmode Solver

Recall the flutter method

(unsteady aerodynamics)

$$\left(J_{ss} - J_{sf}\left(J_{ff} - \lambda_i I\right)^{-1} J_{fs}\right)\boldsymbol{\phi}_{s,i} = \lambda_i \boldsymbol{\phi}_{s,i}$$

$$\left(J_{ff} - \lambda_i I\right)\boldsymbol{\phi}_{f,i} = -J_{fs}\boldsymbol{\phi}_{s,i}$$

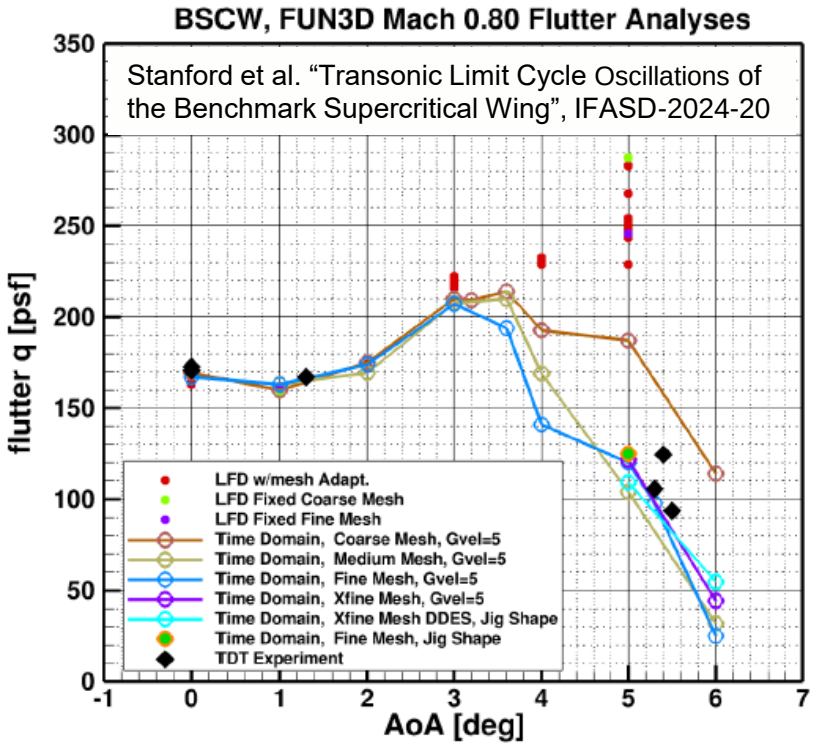
(mass, damping, stiffness)

$$\boldsymbol{\phi}_i = \begin{bmatrix} \boldsymbol{\phi}_{f,i} \\ \boldsymbol{\phi}_{s,i} \end{bmatrix}$$

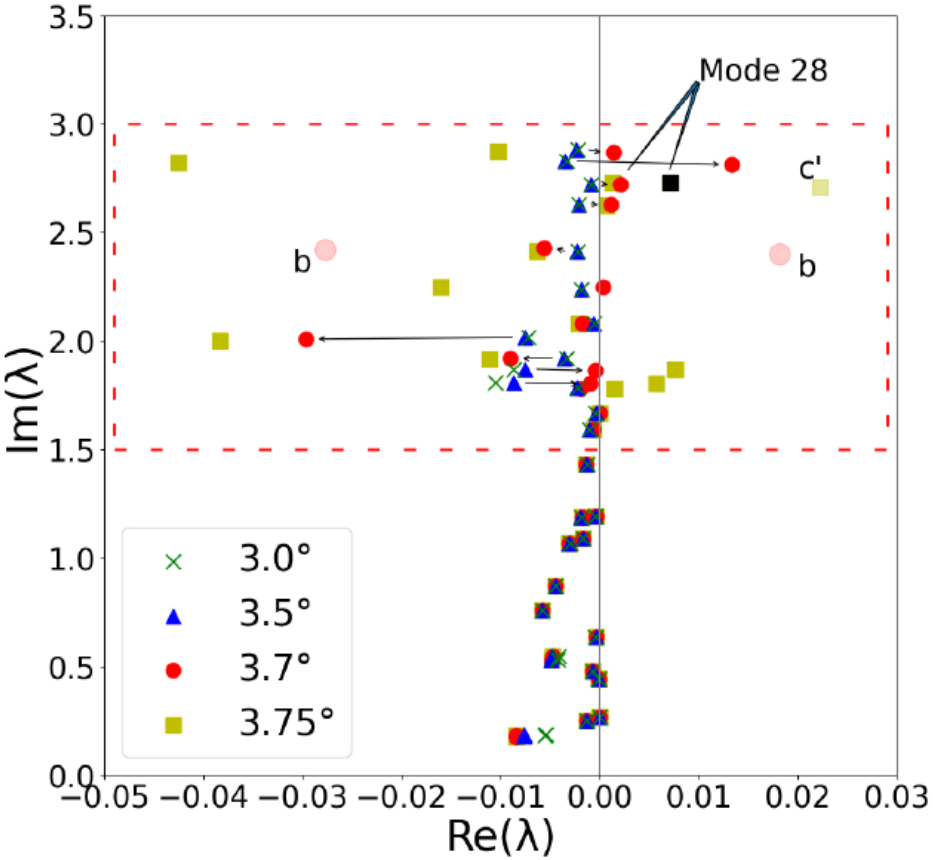
Requires a suitable shift and block matrix for Schur complement.

Instead use coupled approach (IRAM/KS)

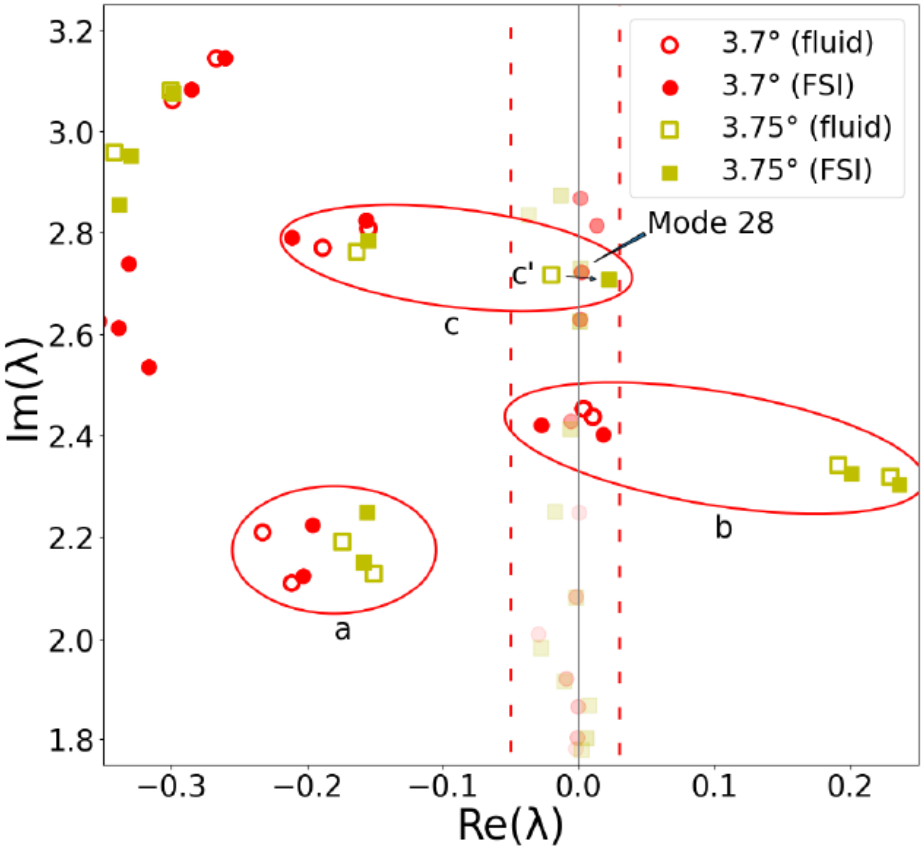
$$\begin{pmatrix} J_{ff} & J_{fs} \\ J_{sf} & J_{ss} \end{pmatrix}\boldsymbol{\phi}_i = \lambda_i \boldsymbol{\phi}_i$$



Coupled Eigenspectrum



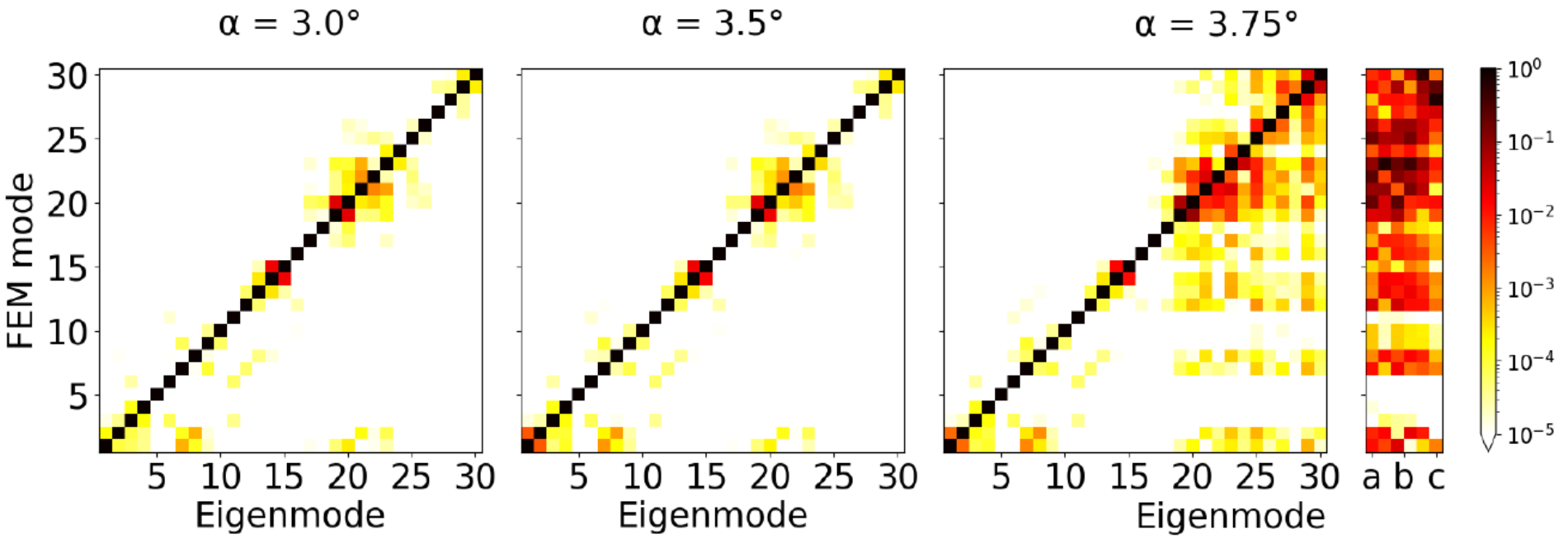
Focus on structural modes



Focus on fluid modes

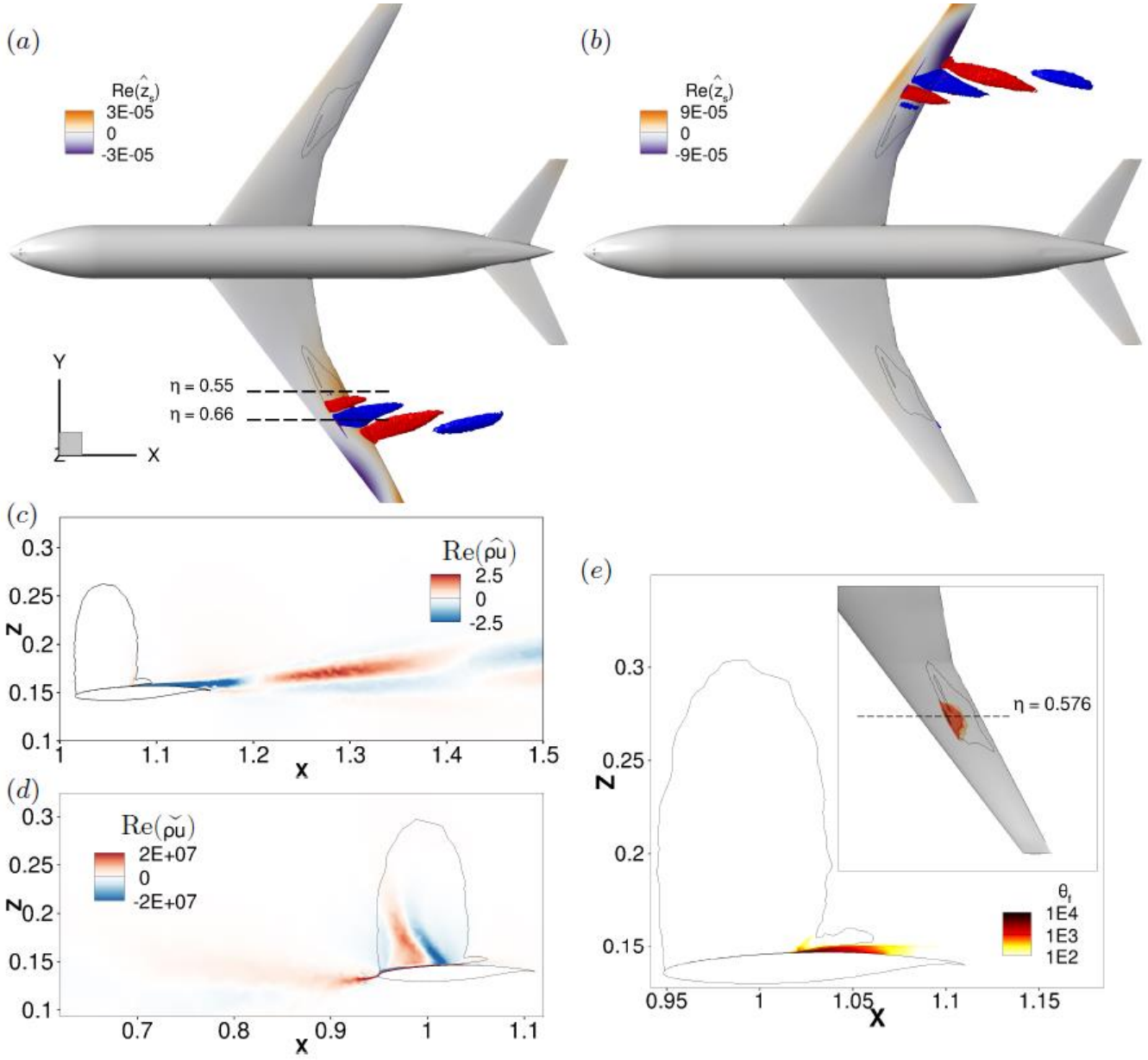
Modal Assurance Criterion

$$\boldsymbol{\phi}_i = \begin{bmatrix} \boldsymbol{\phi}_{f,i} \\ \boldsymbol{\phi}_{s,i} \end{bmatrix}$$



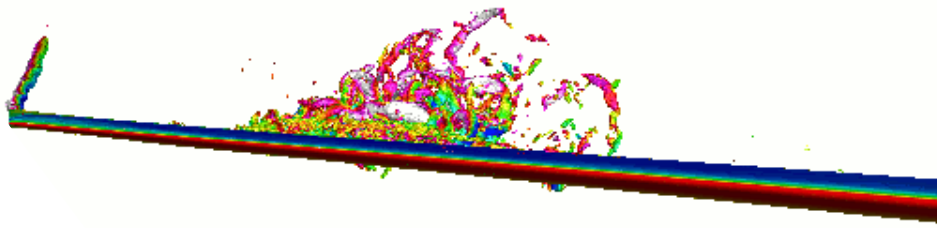
FSI Shock Buffet Modes

$$\boldsymbol{\phi}_i = \begin{bmatrix} \boldsymbol{\phi}_{f,i} \\ \boldsymbol{\phi}_{s,i} \end{bmatrix}$$



Summary

- Modal analysis of transonic flow features very popular currently
- Great for academic research and makes nice papers
- Growing evidence that operator-based modal methods are viable for industrial problems and describe pertinent physics also found with data-based approaches using experiments and unsteady CFD (DNS/LES)
- Ongoing efforts to demonstrate beyond well-defined academic problems



Questions?

