Advanced Modeling & Simulation (AMS) Seminar Series

NASA Ames Research Center, January 25th, 2024



Statistically Consisten Dispersion of Lin Loads to Uncertai Integrated Forces an Momen

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NASA Ames Computation

Outline

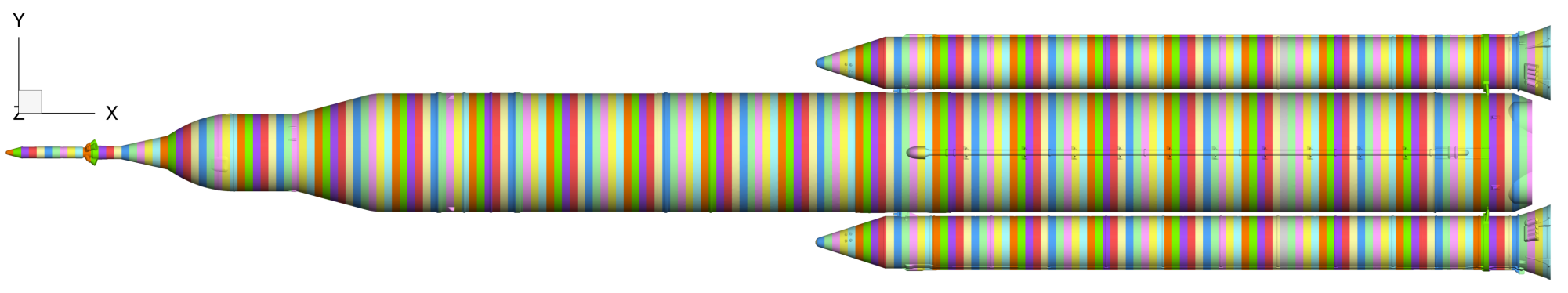
- Motivation
- Background
  - Force & Moment and Line Load Databases
  - Validation
- Method
  - Data Set
  - Dispersion Procedure
- Results
  - Validation: Force Coefficient
  - Validation: Force & Moment Coefficients
  - Simulated Uncertainty
- Extension to Surface Pressures
- Conclusion

Motivation

- This work addresses a specific problem that can occur during analysis of an ascent vehicle
- Problem essentially involves making the uncertain outputs of two aerodynamic databases consistent
- Two database types of interest here are integrated force and moment (FM), and “line load” (LL) databases
- These databases are constructed from data pulled from CFD simulations
- CFD done at many points along the vehicle’s potential trajectory (Mach, angle of attack, angle of sideslip, etc…)
- A database user can query the database at a given trajectory condition and recieve interpolated aerodynamic data output

Aerodynamic Database Background

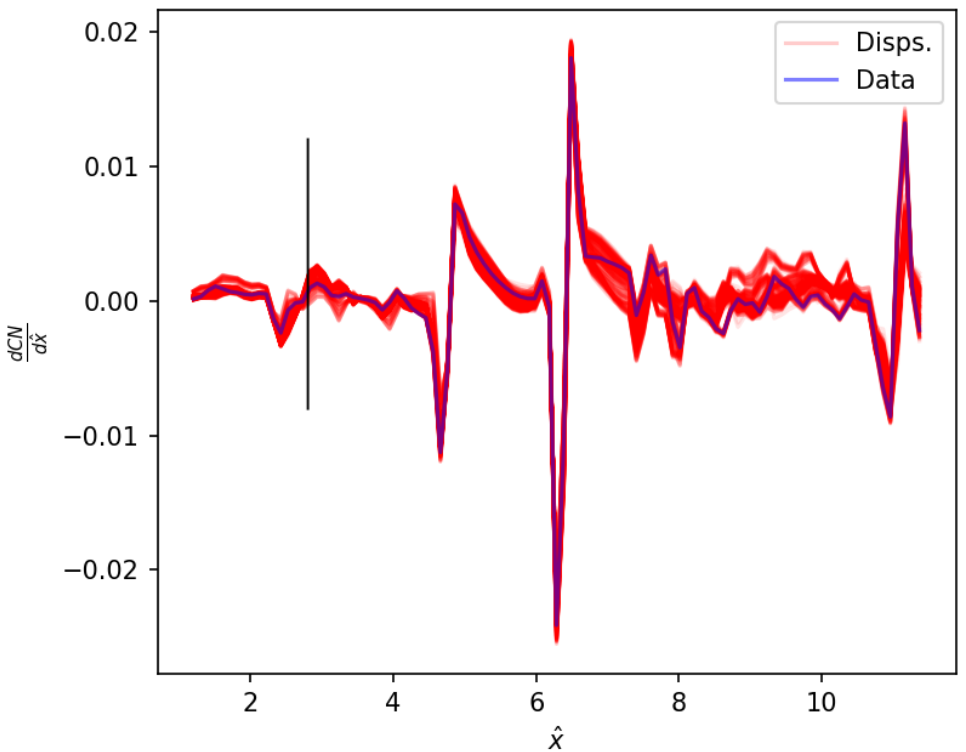
- FM database
  - Outputs integrated force & moment coefficients
  - Constructed from time-averaged static experimental data and CFD
  - Uncertainty model
- LL database
  - Outputs “line loads”
  - Constructed from the same CFD data
  - Uncertainty model?

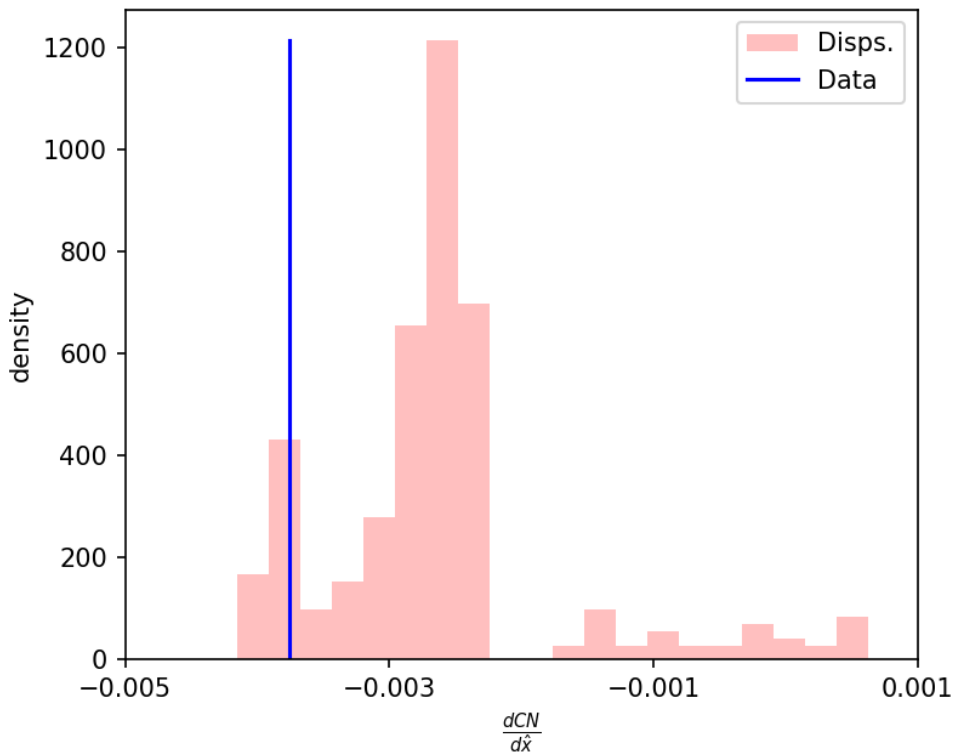


Line loads are the integrated loading on axial slices of the vehicle

LL Database Uncertainty

- LL database should also output distribution of line loads (dispersion)





- The FM database has valuable additional experimental data
- Can this valuable data (and corresponding uncertainty) be included in the uncertain line load output?

Consistent FM & LL Database Uncertainty

- FM and LL database outputs are related, need to be consistent

$$C_N = \int_{x_1}^{x_2} \mathbf{c}_N dx$$

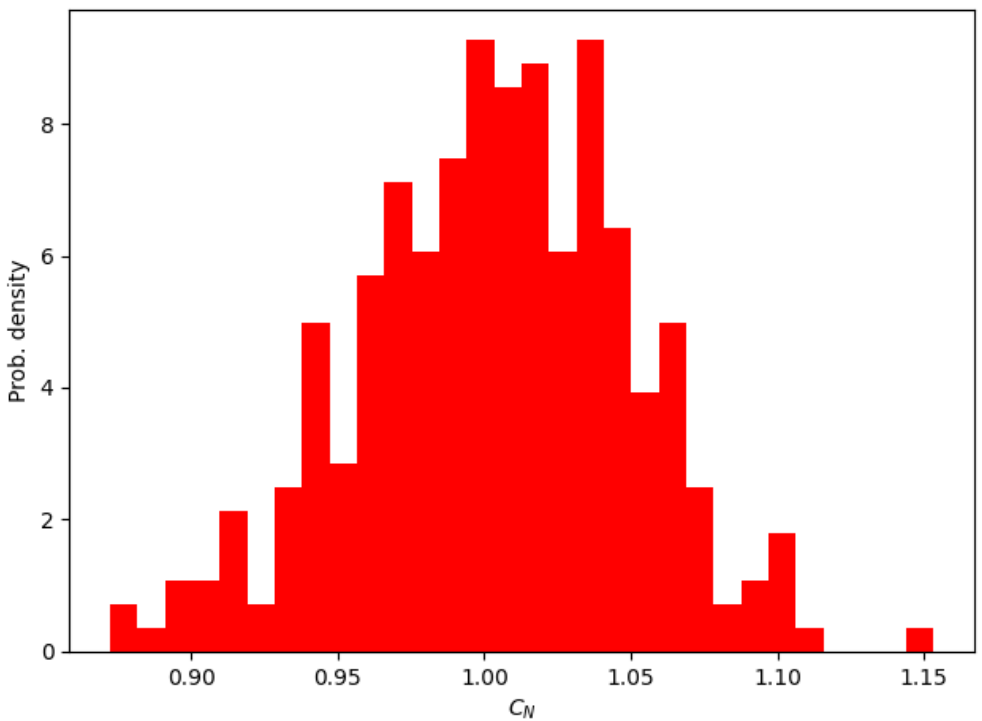
$$\mathbf{c}_m = \frac{x_{MRP} - x}{L_{ref}} \mathbf{c}_N$$

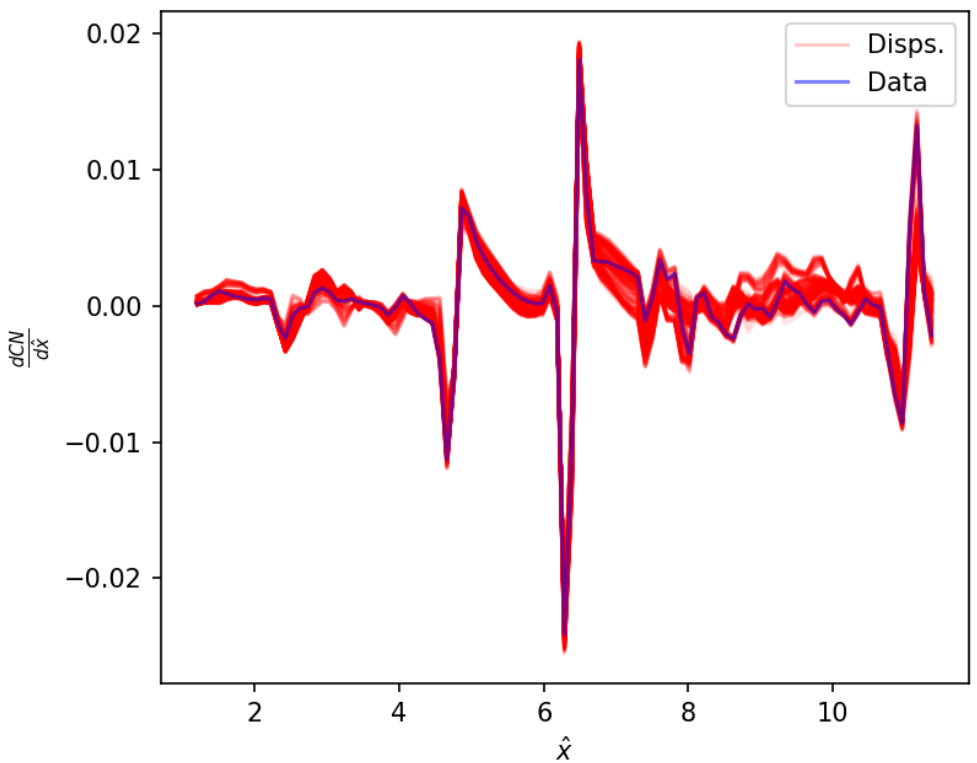
$$C_{LM} = \int_{x_1}^{x_2} \mathbf{c}_m dx$$

- The uncertain line load output should also follow these

FM & LL Database Dispersion Consistency

- Dalle et al. [1] successfully gave line load dispersion method
- Method based on adjusting existing line loads to integrated targets
- However, it didn't attempt statistical consistency
- Obeying known line load distribution critical for realistic dispersion
- Goal of current work is to create LL dispersion method that obeys known LL distribution





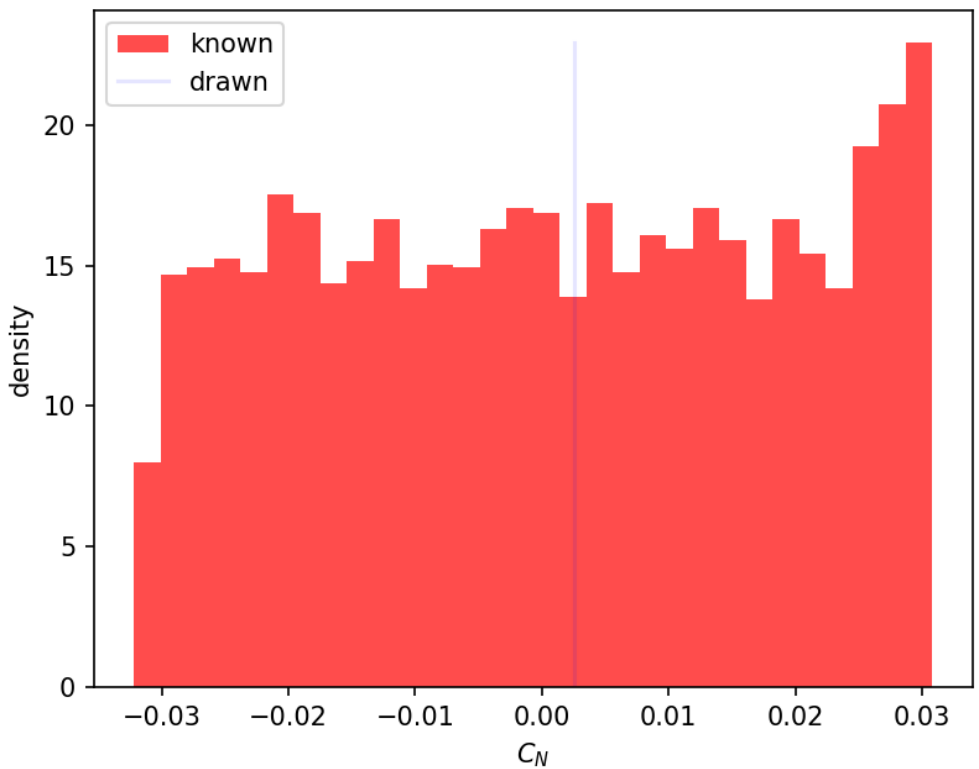
1. Dalle, Rogers, Lee, and Meeroff. AIAA 2018-3640

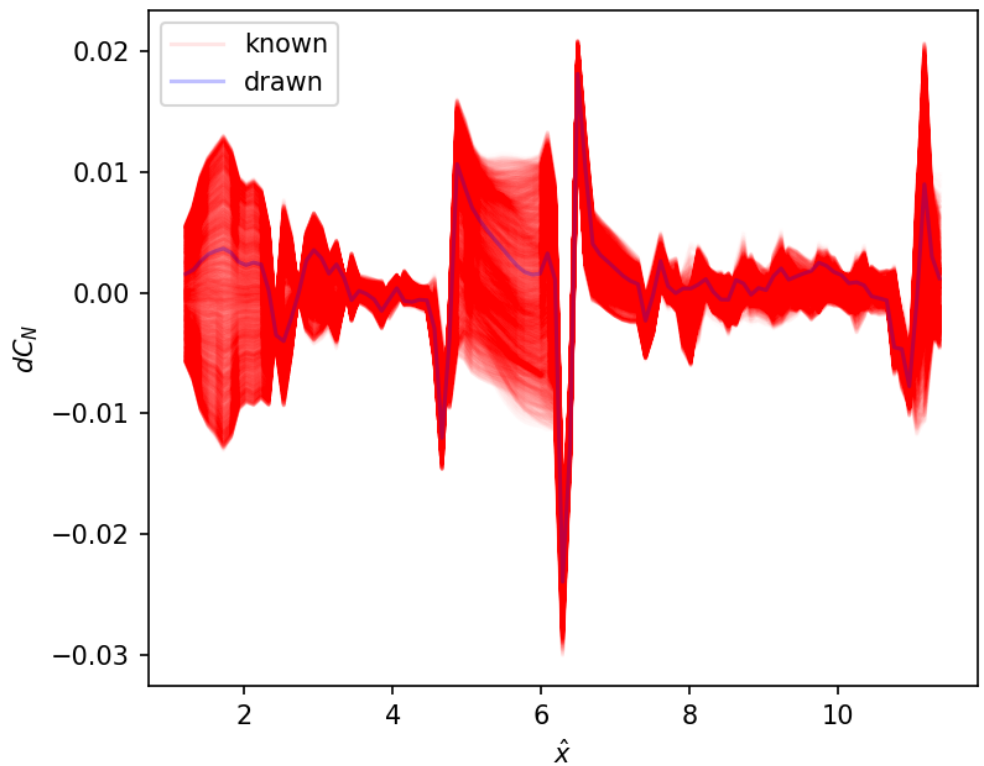
Validation Background

- Validation test used to compare known line loads and dispersion
- Distribution testing is challenging with high-dimensional data
- Maximum Mean Discrepancy [1] 2-sample test chosen to compare distributions
- Validation uses line load dispersion for 300 known integrated values

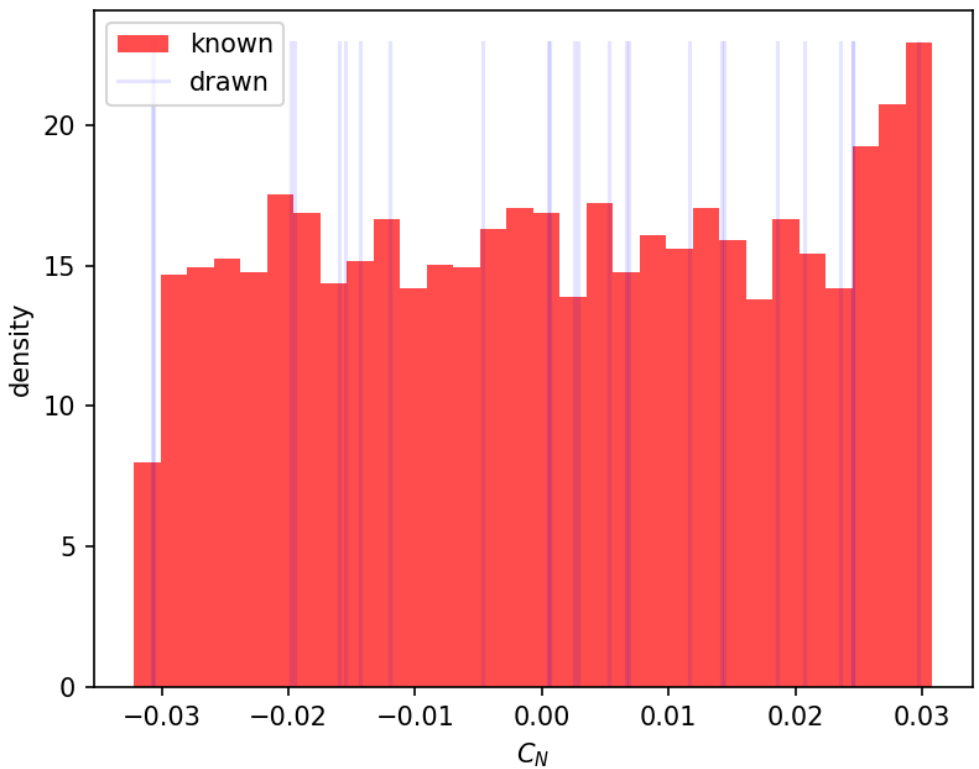
1. Gretton et al., A Kernel Two-Sample Test (2012)

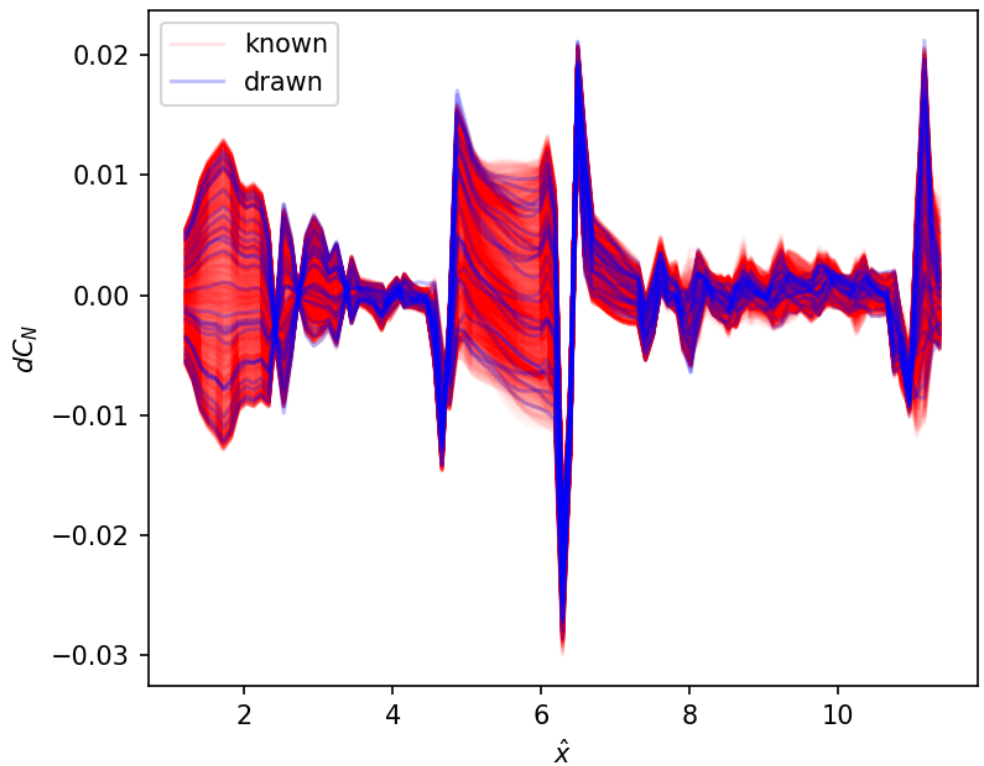
Validation Background



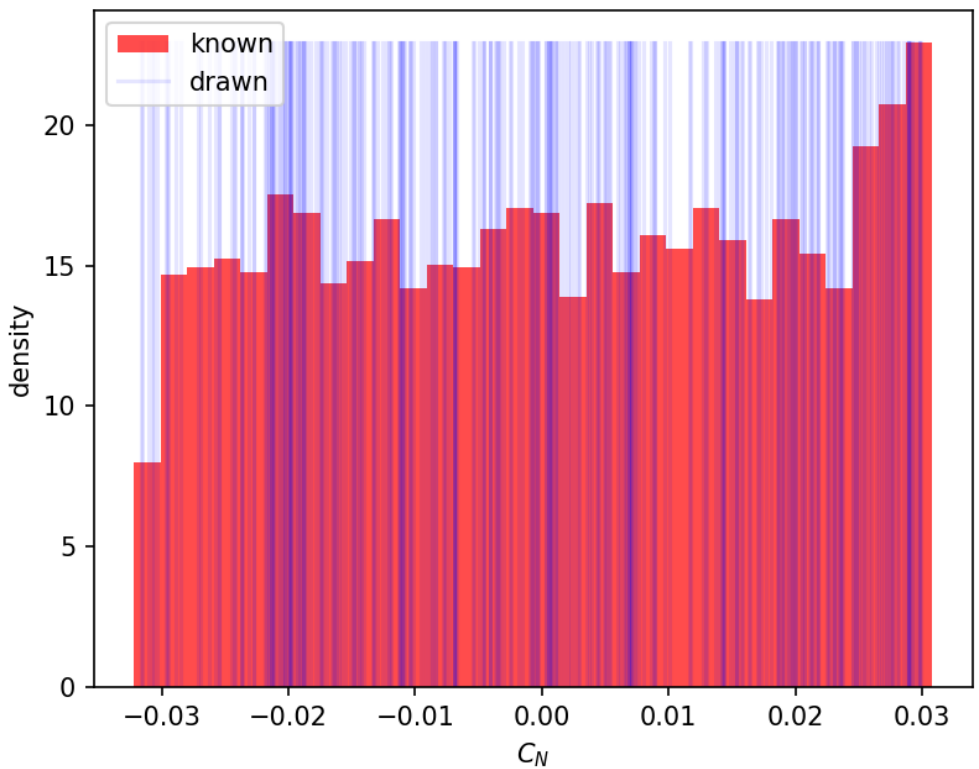


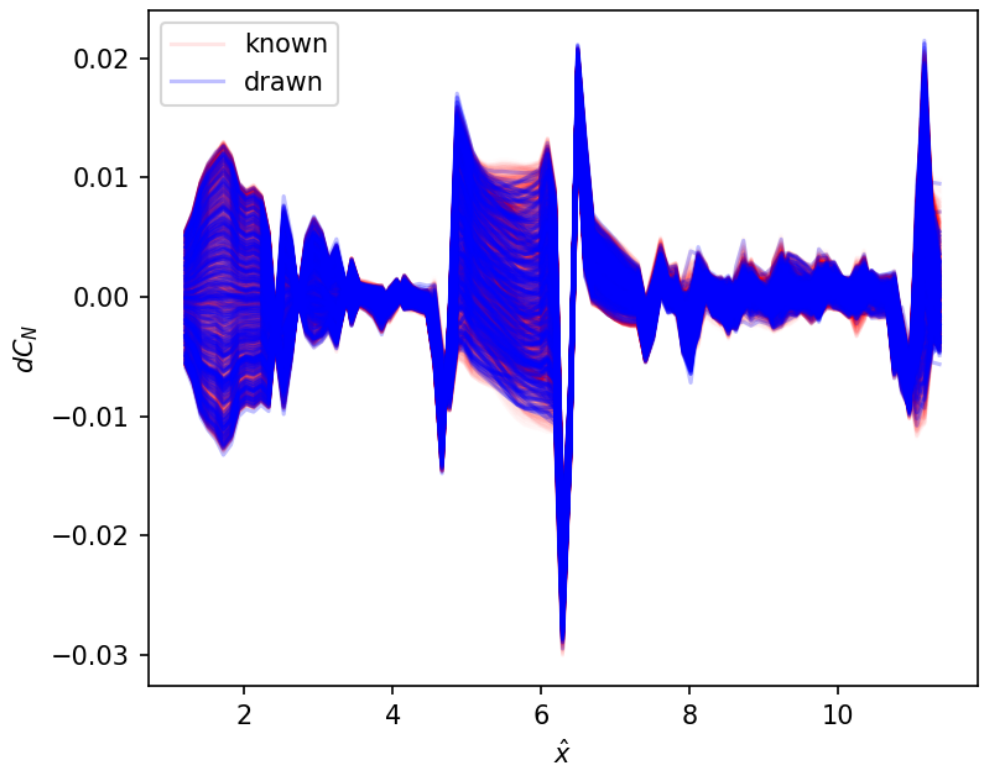
Validation Background





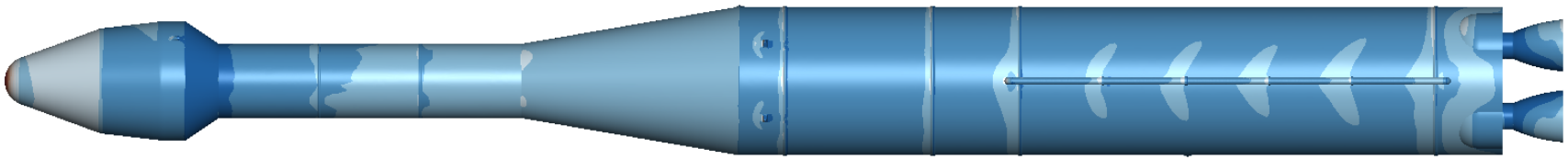
Validation Background



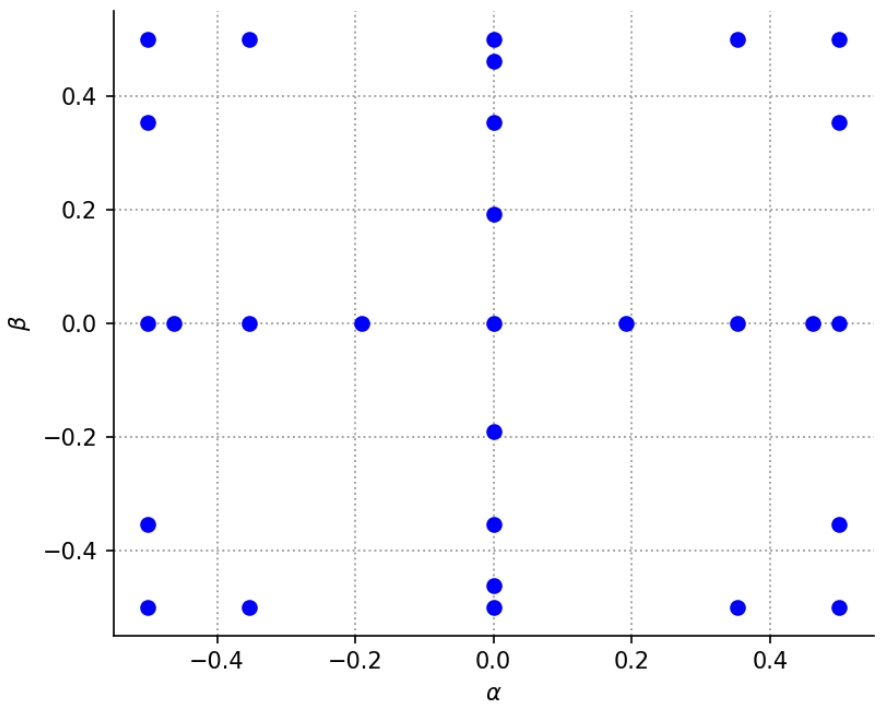


Data Set

- The raw data for the data set was from 29 FUN3D CFD cases on a generic Hammerhead style vehicle



- Cases varied between -0.5 to 0.5 in angle of attack (α) and angle of sideslip (β)
- A Gaussian Radial Basis Function response surface was built from the raw data
    - Surface sampled at 5000 points within the CFD bounds



Method

- Start by collecting line load deltas into matrix C'_N

$$C'_N = [\mathbf{c}'_{N,1}, \mathbf{c}'_{N,2}, \ldots, \mathbf{c}'_{N,n_c}]$$

- Perform Principal Component Analysis (PCA) on C'_N

$$SVD(C'_N) = \mathbf{U}_N \mathbf{\Sigma}_N \mathbf{V}_N^T$$

$$\tilde{\mathbf{U}}_N = \mathbf{U}_{N,[1:\kappa]}$$

$$\tilde{\mathbf{U}}_N \omega_j = \mathbf{c}'_{N,j}, \;\; q_{j \in [1,n_c]}$$

- Matrix Ω is built from all ω vectors
- Goal is now to generate line loads that follow $P(\Omega)$
- Generated line loads constructed by:

$$\tilde{\mathbf{U}}_N \omega^* = \mathbf{c}^*_N$$

$$\omega^* \sim P(\Omega)$$

Method

- Take a target integrated force &/or moment value(s), $C_{N,tar}$
- The process developed to generate a line load is as follows:

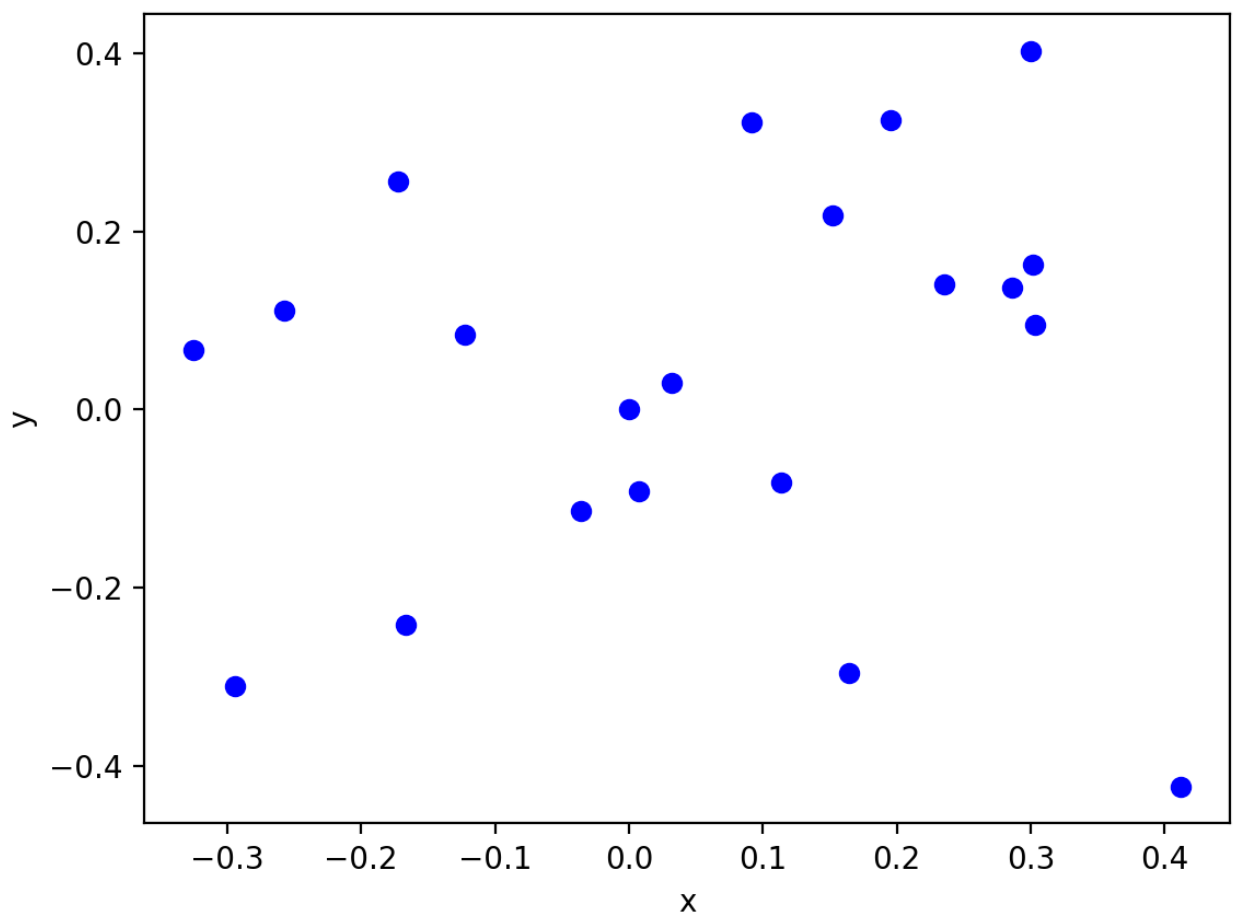
1. $\omega^* = \omega_j + \delta\omega$

   $\Omega = [\omega_0,\ \omega_1,\ \ldots \omega_j,\ \ldots]$

   $\delta\omega \sim \mathcal{N}(0, \boldsymbol{\Upsilon})$
2.
3.
4.
5.
6.
7.

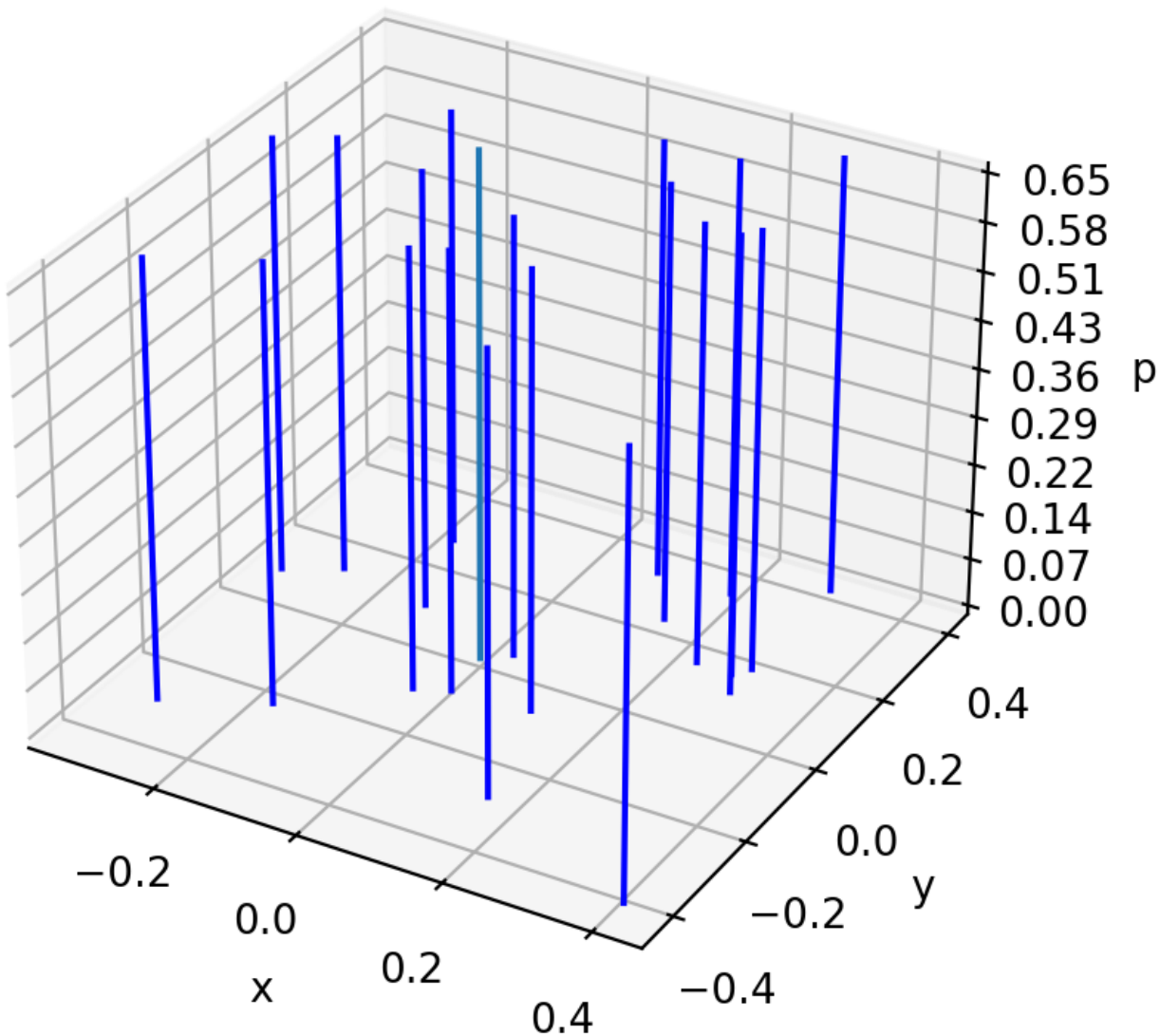
Sampling method

- Start with known 2-d example data
- Want to draw new sample that follows the data's distribution



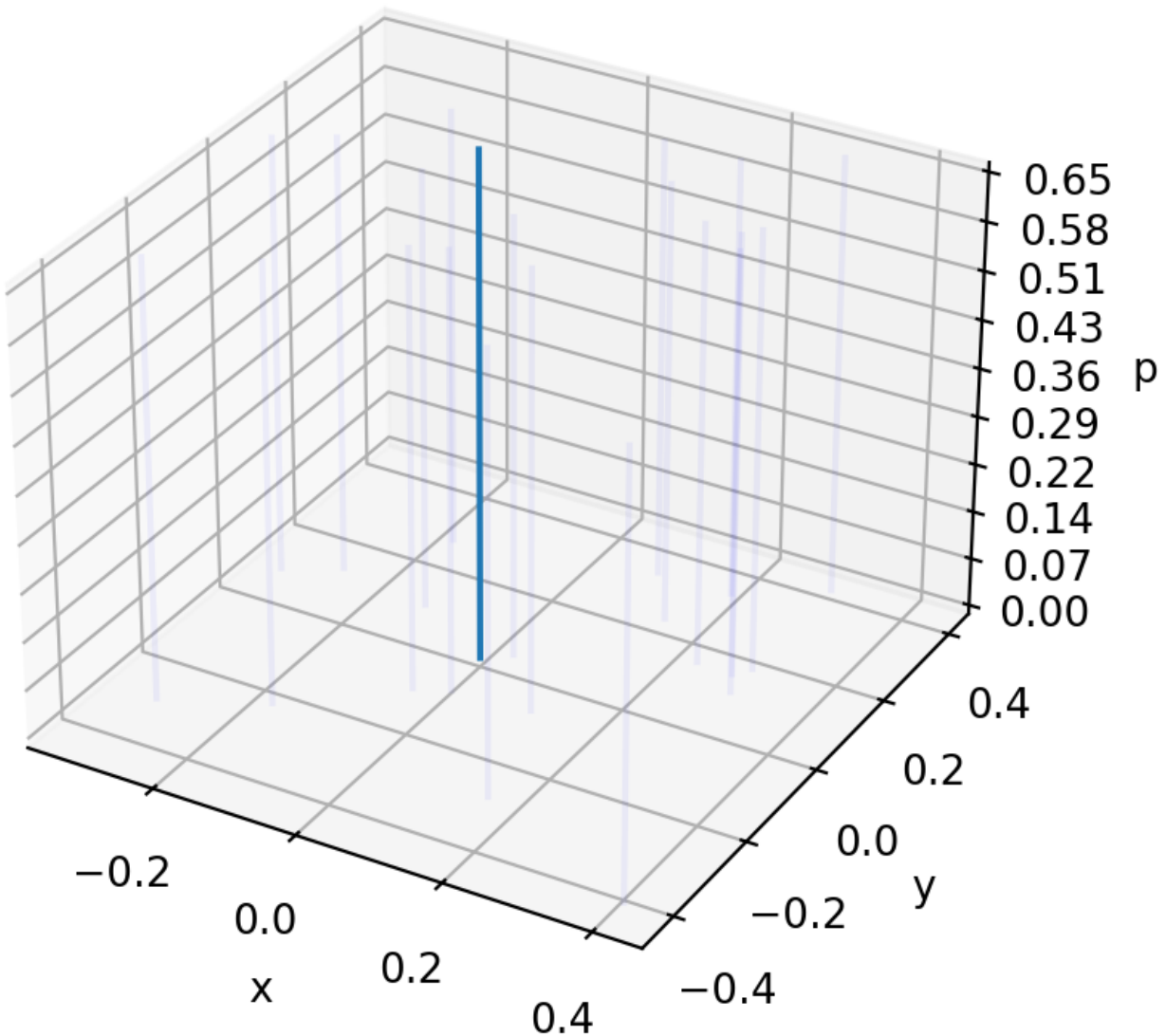
Sampling method

- Assign equal probability to each known data point



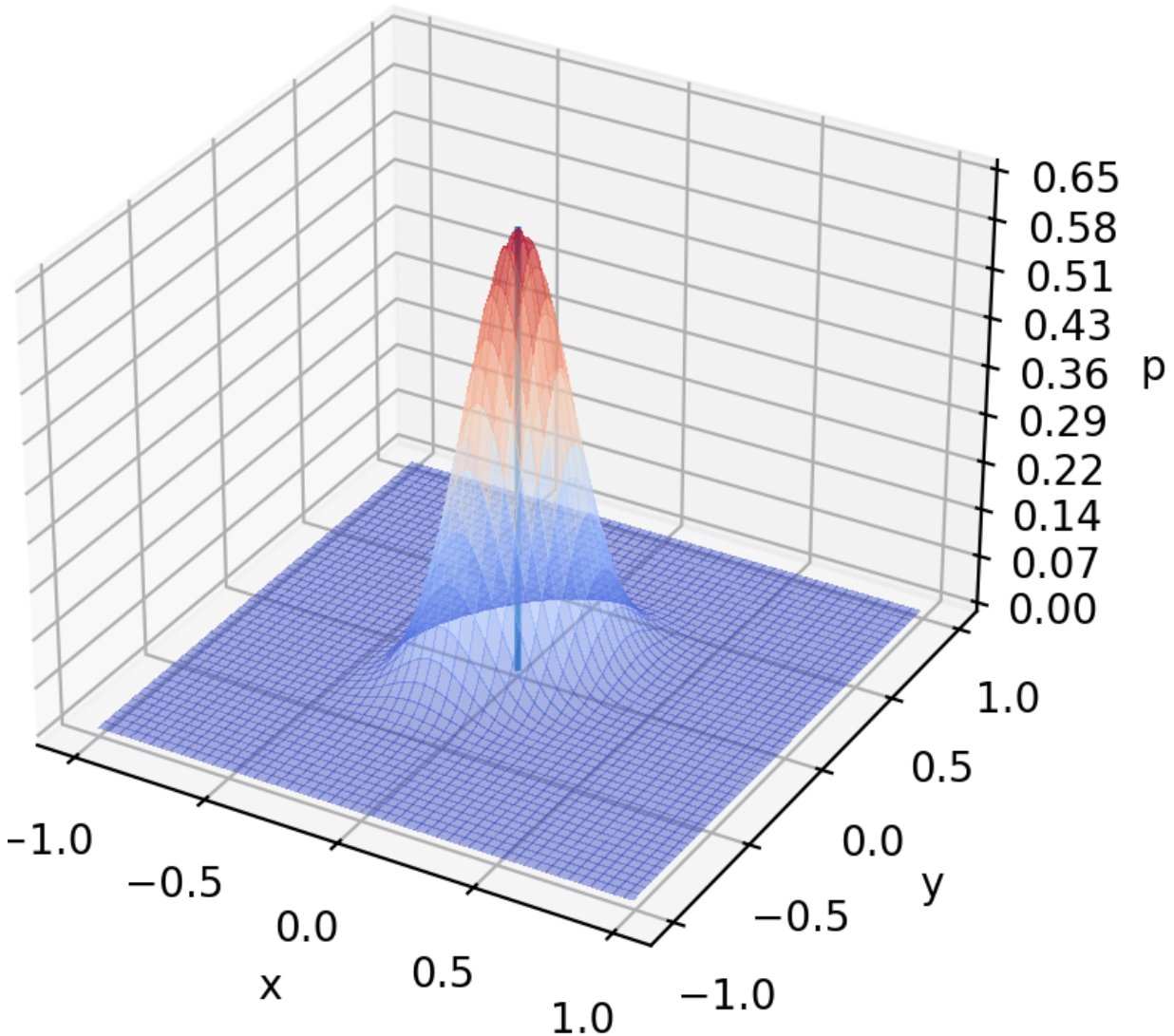
Sampling method

- Choose a known data point



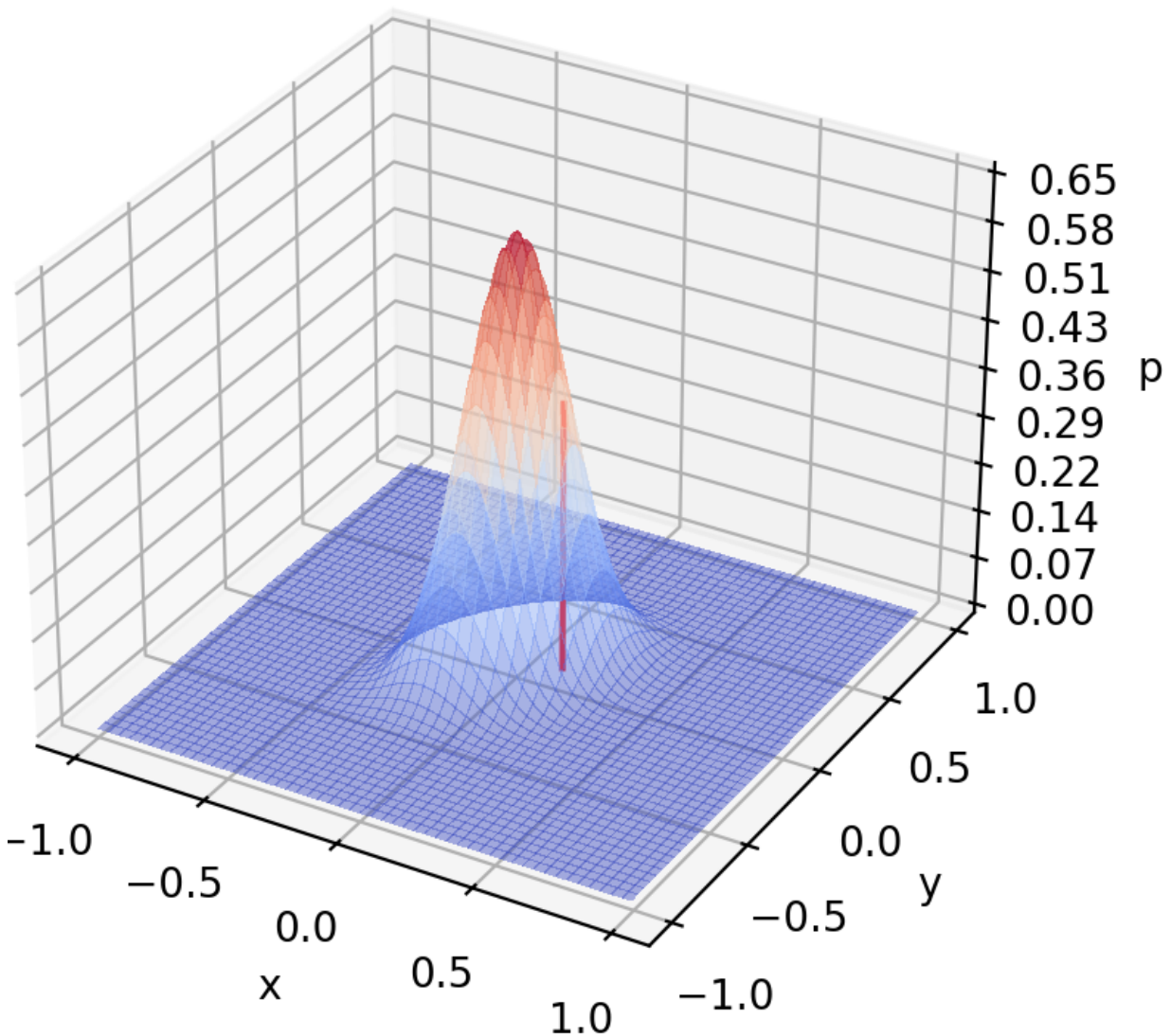
Sampling method

- Insert skewed Gaussian distribution centered on chosen point



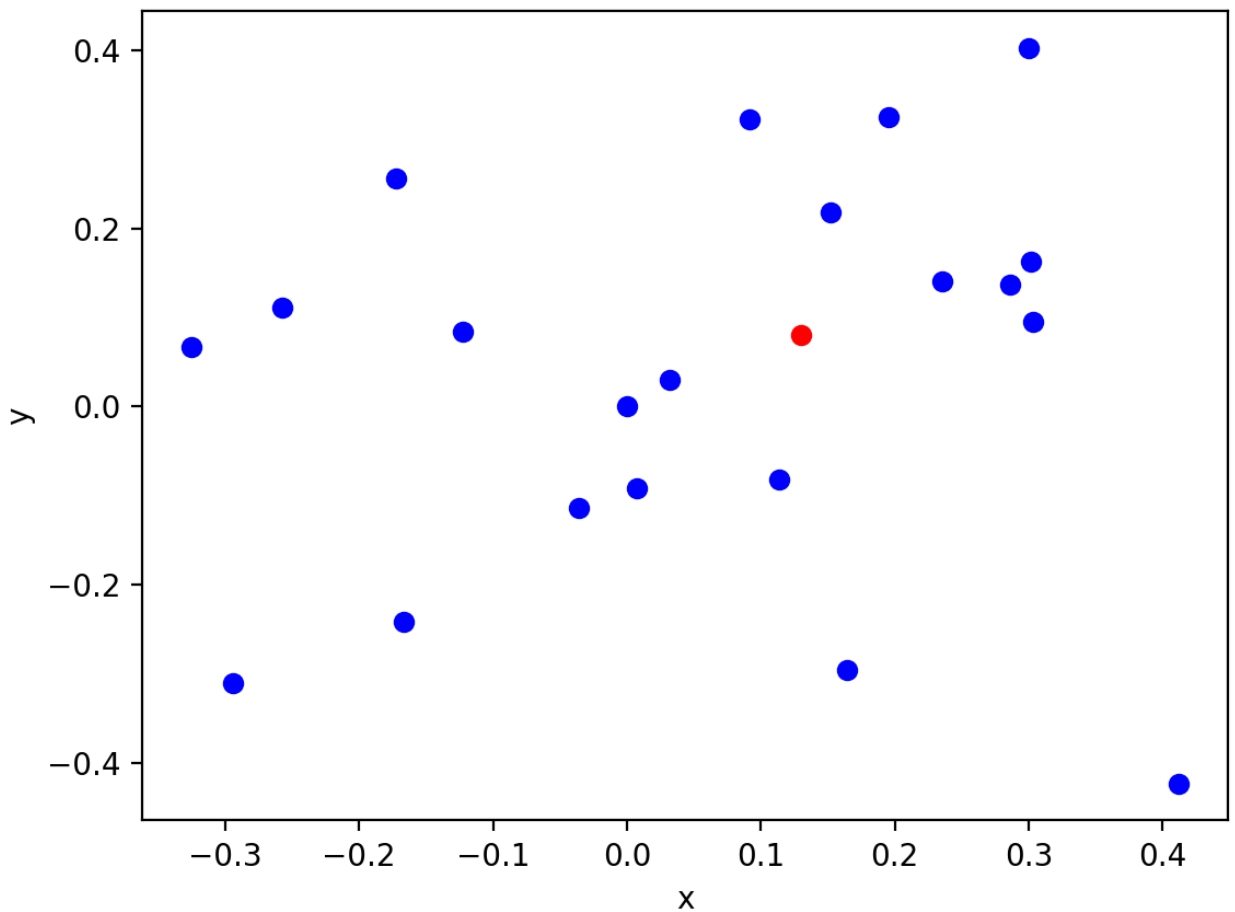
Sampling method

- Draw new point within Gaussian distribution



Sampling method

- New point approximately follows known distribution



Method

- Take a target integrated force &/or moment value(s), $C_{N,tar}$
- The process developed to generate a line load is as follows:

1. $\omega^* = \omega_j + \delta\omega$

   $\Omega = [\omega_0,\ \omega_1,\ \ldots \omega_j,\ \ldots]$

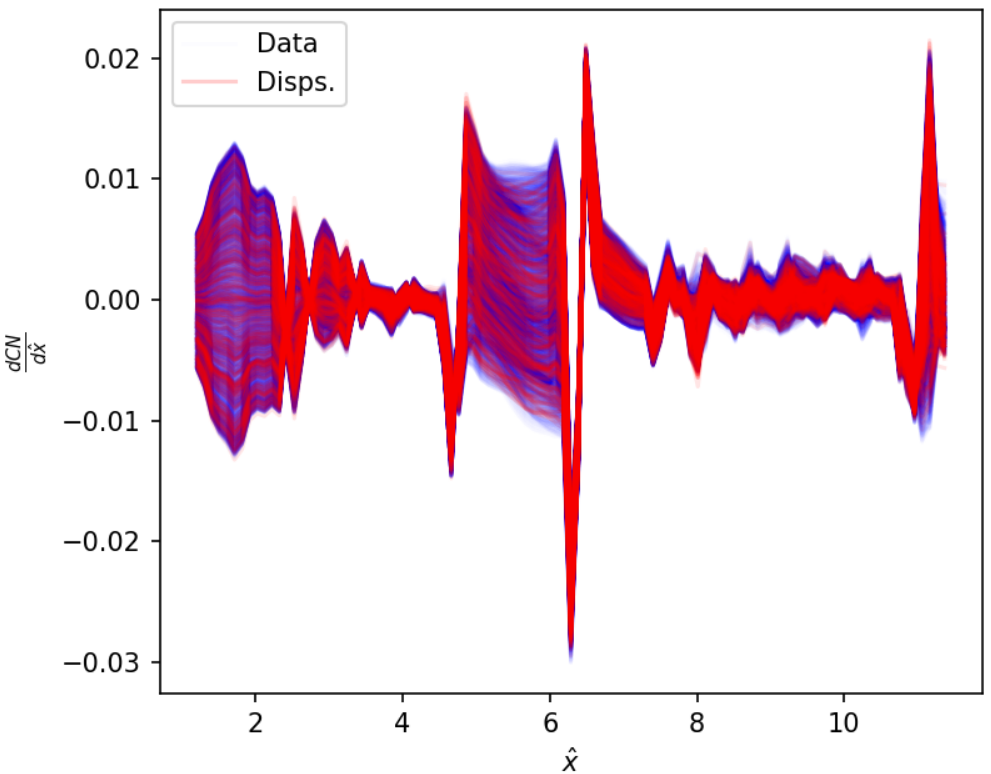
   $\delta\omega \sim \mathcal{N}(0, \boldsymbol{\Upsilon})$
2. Ignore 1st value in ω^*
3. Estimate $\tilde{\mathbf{U}}^*_{N,[2:\kappa]}\omega^*_{[2:\kappa]} = \mathbf{c}^*_N$
4. $\delta C_N = C_{N,tar} - \int_{x_1}^{x_2} \mathbf{c}^*_N dx$
5. Solve for $\omega^*_{[1]} = \frac{\delta C_N}{\int_{x_1}^{x_2} \tilde{\mathbf{U}}^*_{N,[1]} dx}$
6. Check $\omega^*_{[1]}$ within 2-D conditioned joint distributions $P_{\omega_1|\omega_2}(\omega^*_{[1]})$, $P_{\omega_1|\omega_3}(\omega^*_{[1]})$, etc.
7. If not, repeat from 1.

Results

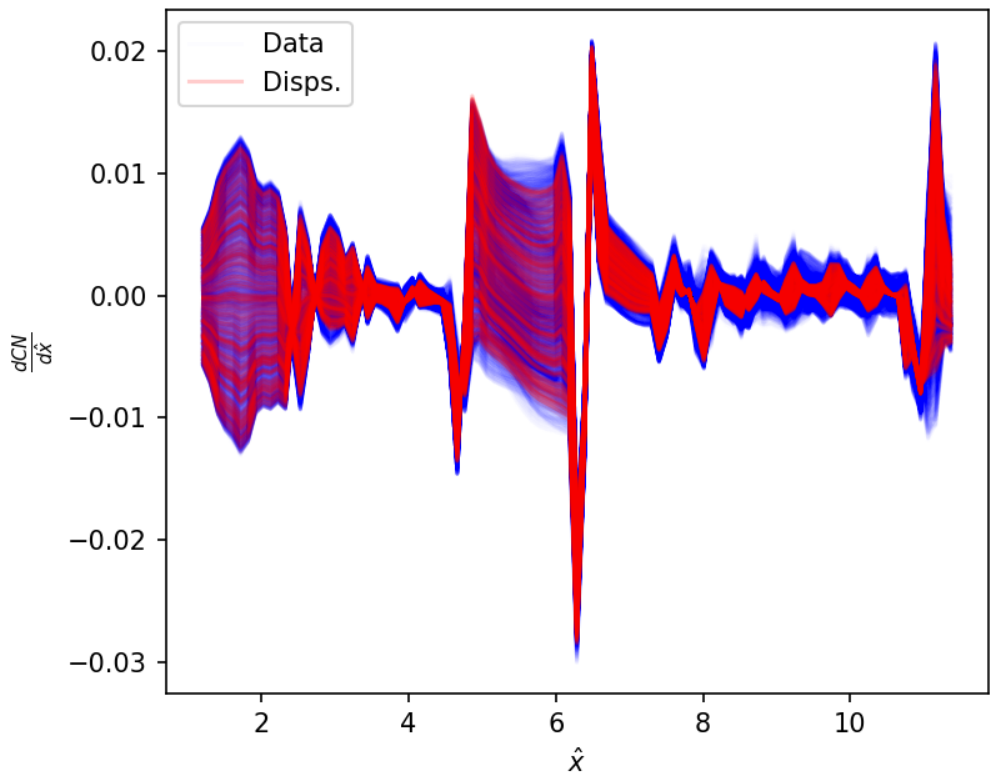
- Validation test for normal force coefficients, comparing this method to older approach
- Validation test for normal force and pitching moment coefficients
- Simulated uncertainty normal force coefficient dispersion

Validation: Normal Force Coefficient Dispersion

- 300 integrated normal force coefficients randomly pulled from the known data
- Line load generated corresponding to each of these drawn values, using new approach and old approach

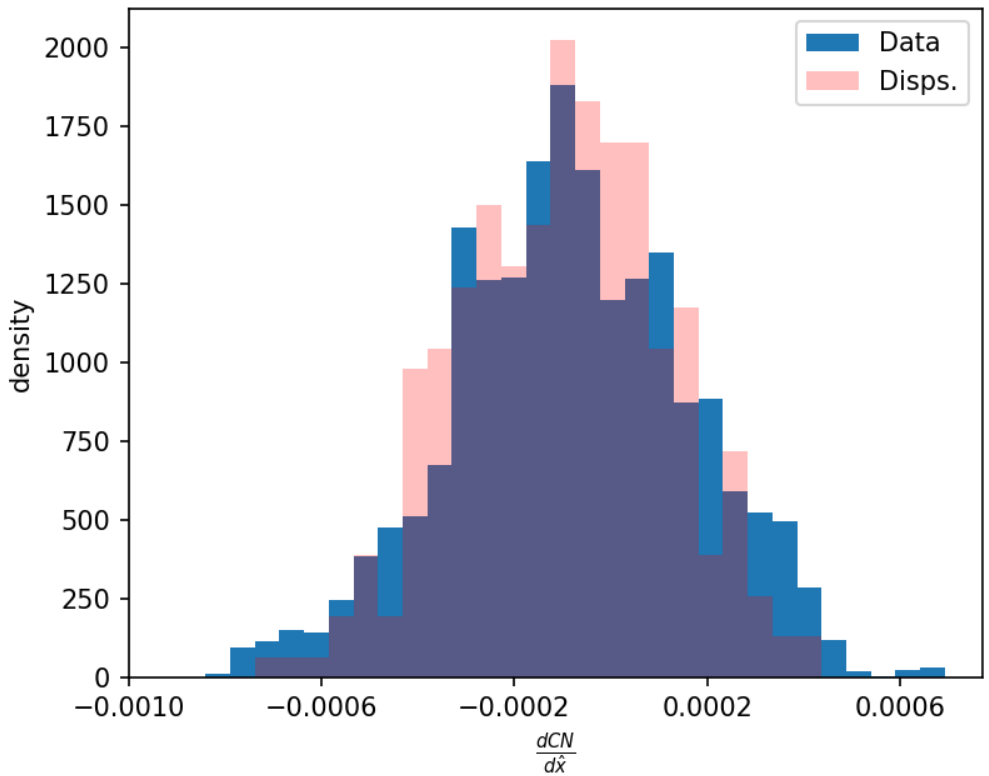


New Method

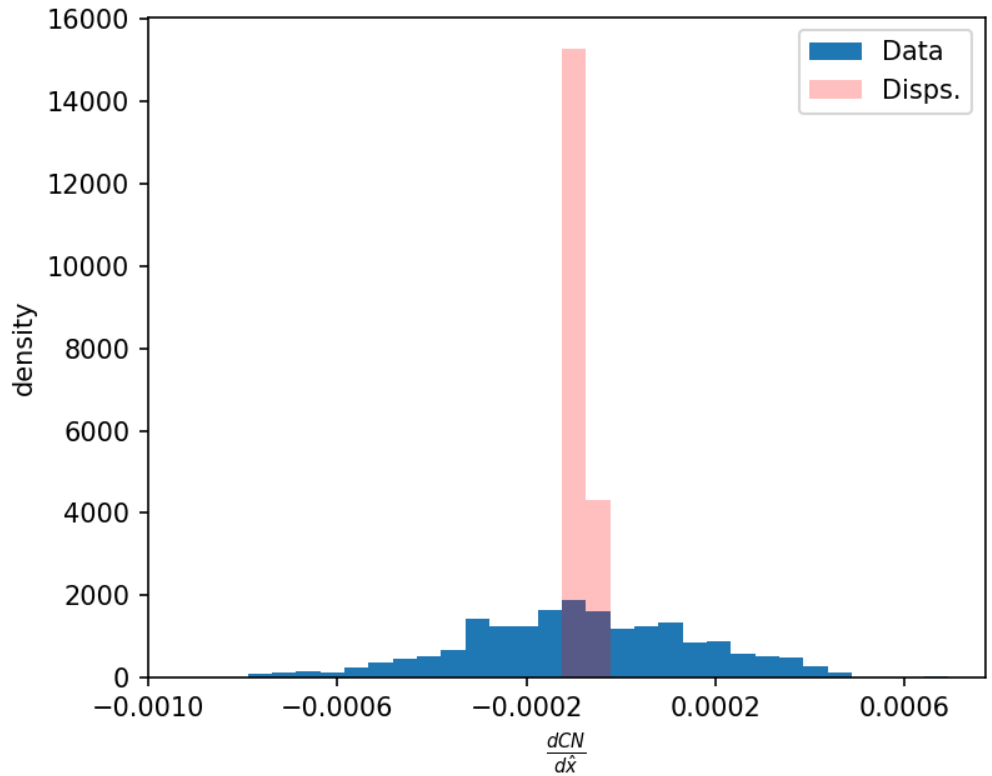


Old Method [1]

Validation: Normal Force Coefficient Dispersion

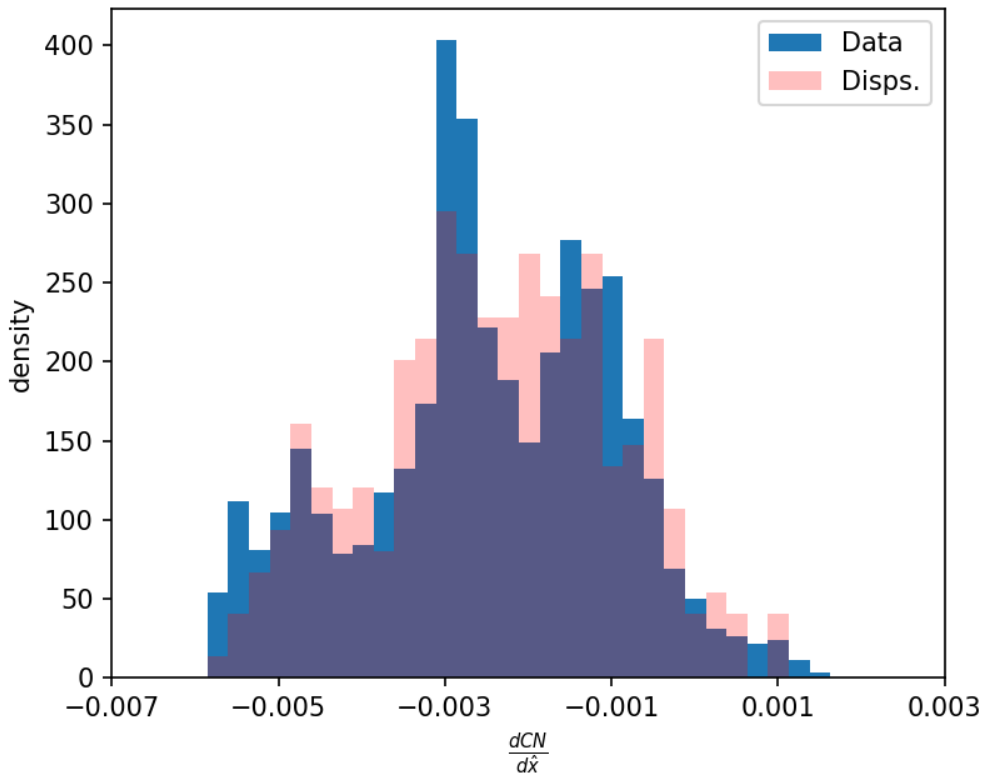


New Method, $\hat{x}_{15}$

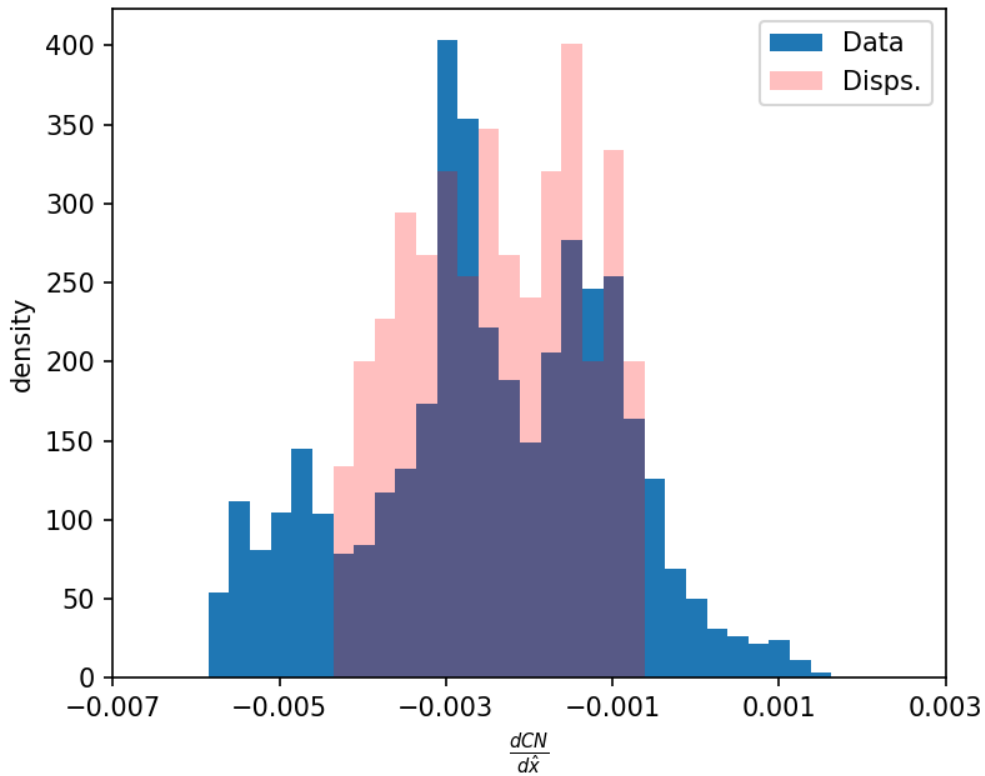


Old Method[1], $\hat{x}_{15}$

Validation: Normal Force Coefficient Dispersion

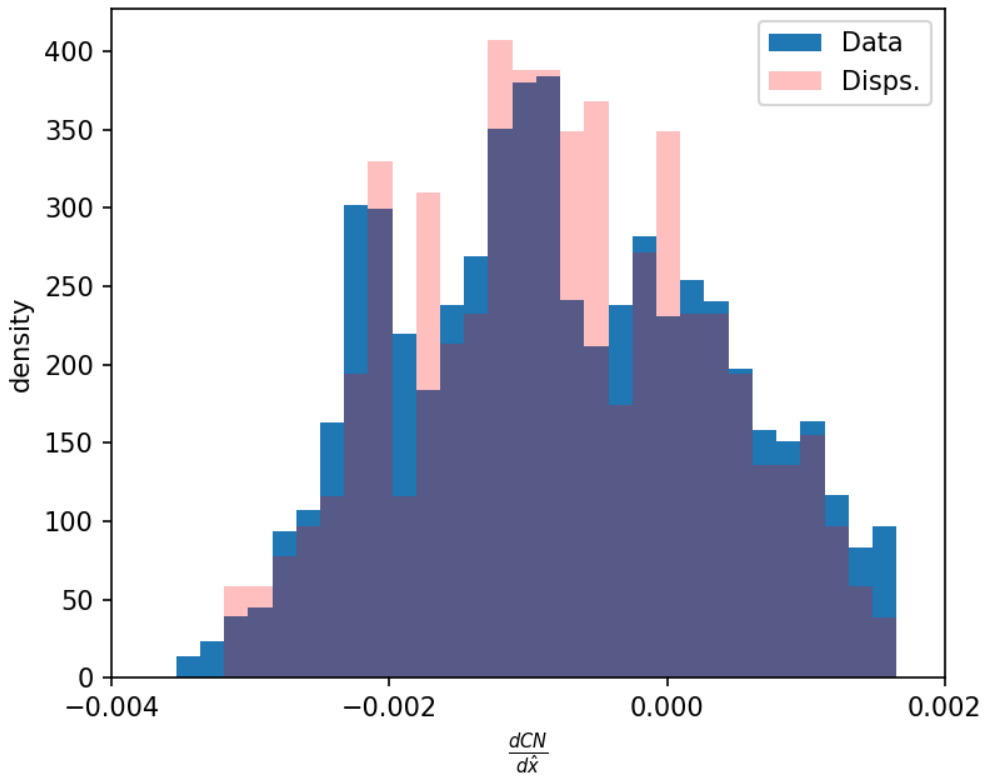


New Method, $\hat{x}_{33}$

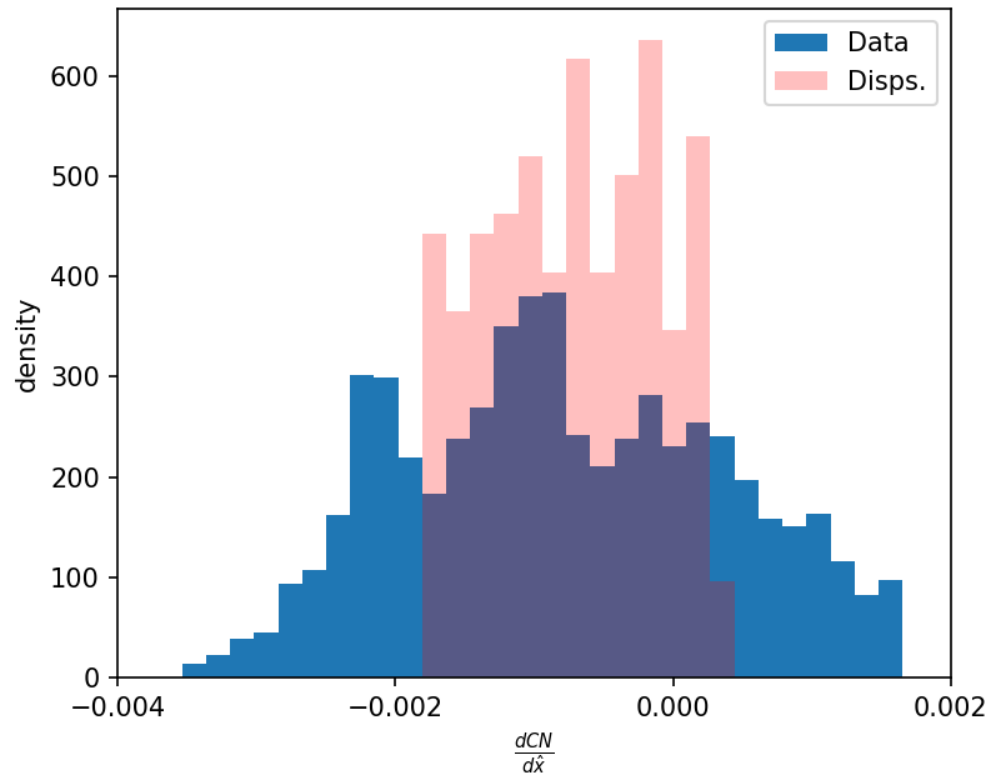


Old Method[1], $\hat{x}_{33}$

Validation: Normal Force Coefficient Dispersion

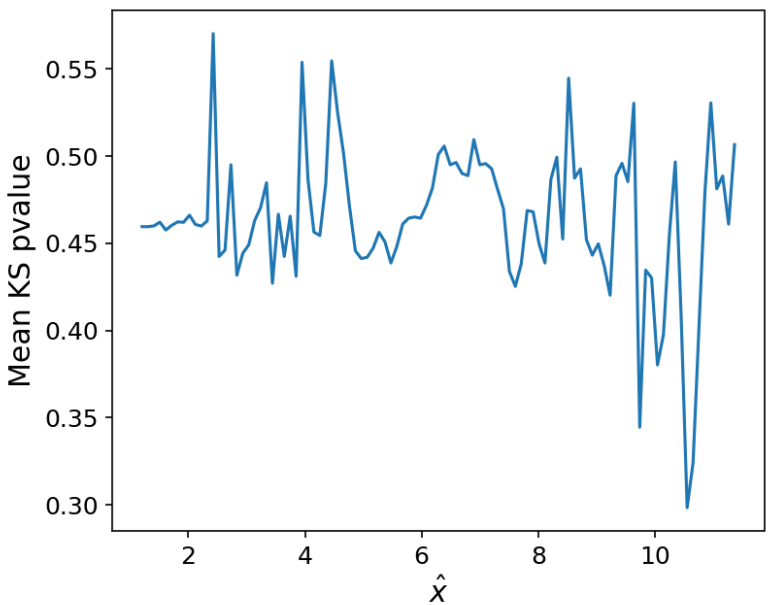


New Method, $\hat{x}_{72}$

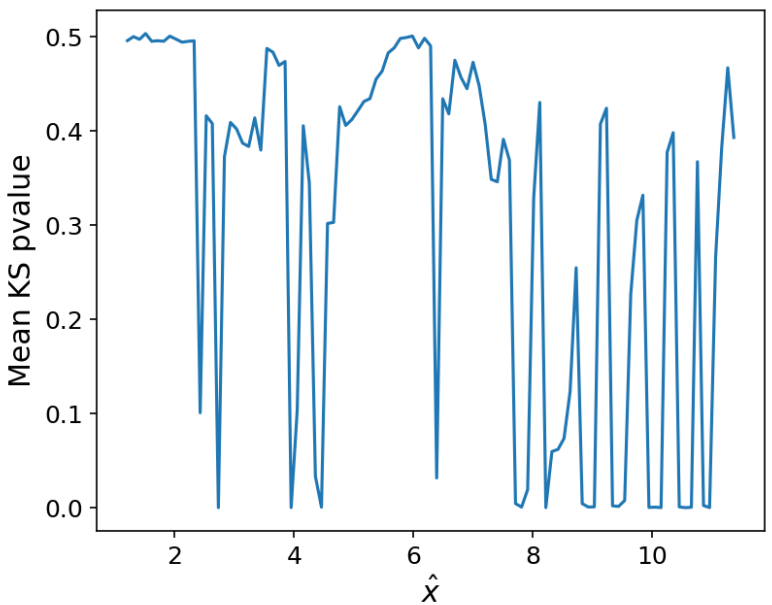


Old Method[1], $\hat{x}_{72}$

Validation: Normal Force Coefficient Dispersion



New Method, mean Kolmogorov-Smirnov Test p-val



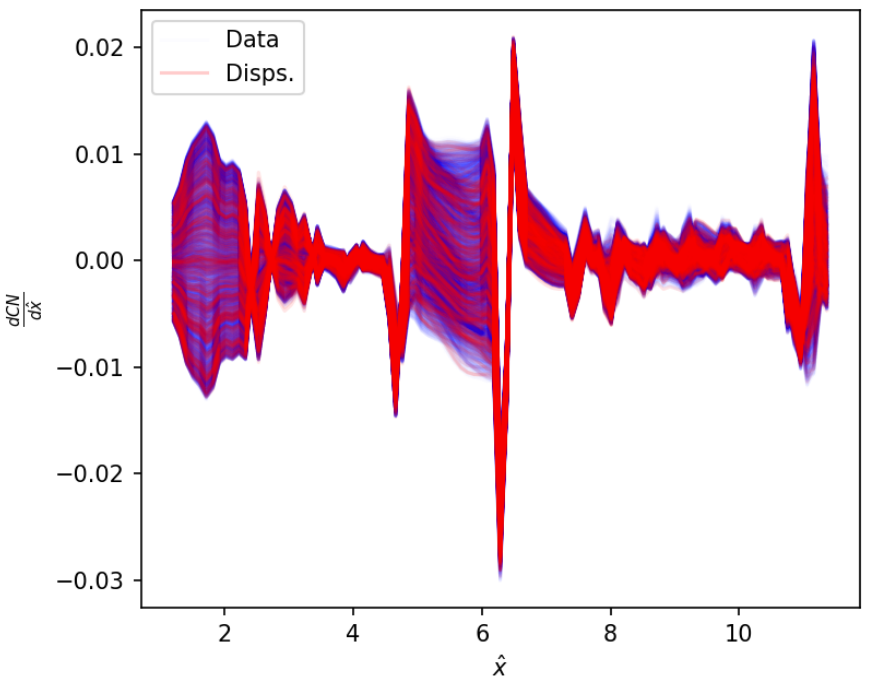
Old Method, mean Kolmogorov-Smirnov Test p-val

MMD 2-sample test {.striped .hover}

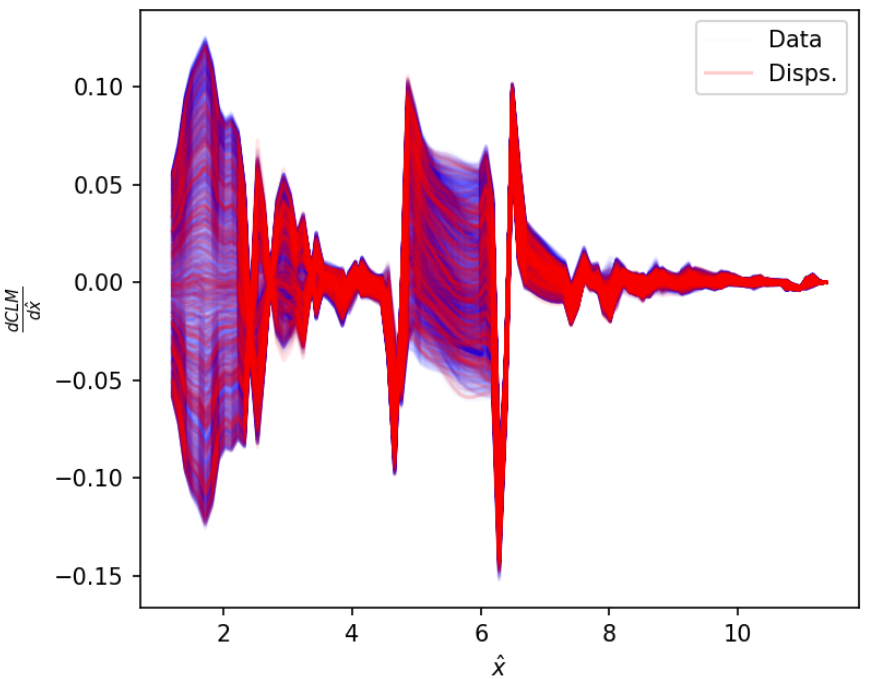
Method	Critical Value	MMD Score
Old	0.006983	0.012781
New	0.006597	0.006461

Validation: Normal Force and Pitching Moment Coefficient Dispersions

- 300 integrated normal force and pitching moment coefficients randomly pulled from the known data
- Line load generated corresponding to each pair of integrated coefficients

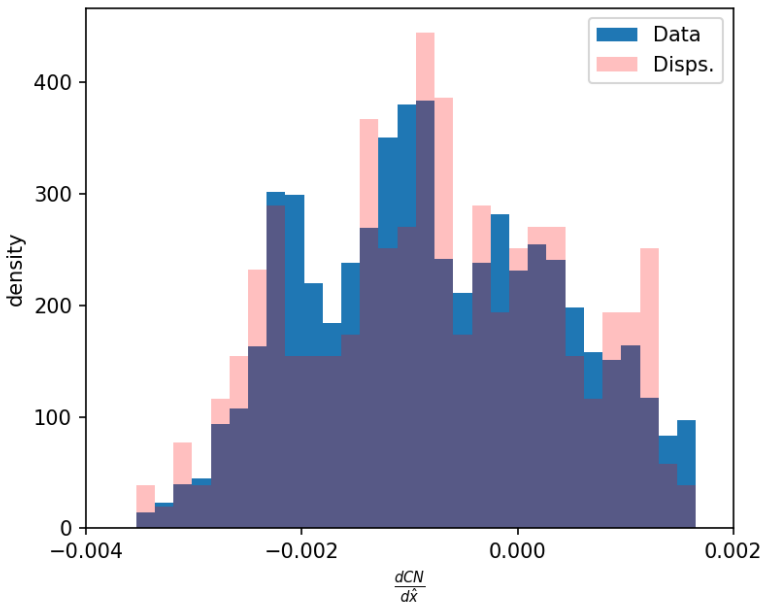


$\mathbf{c}_N$

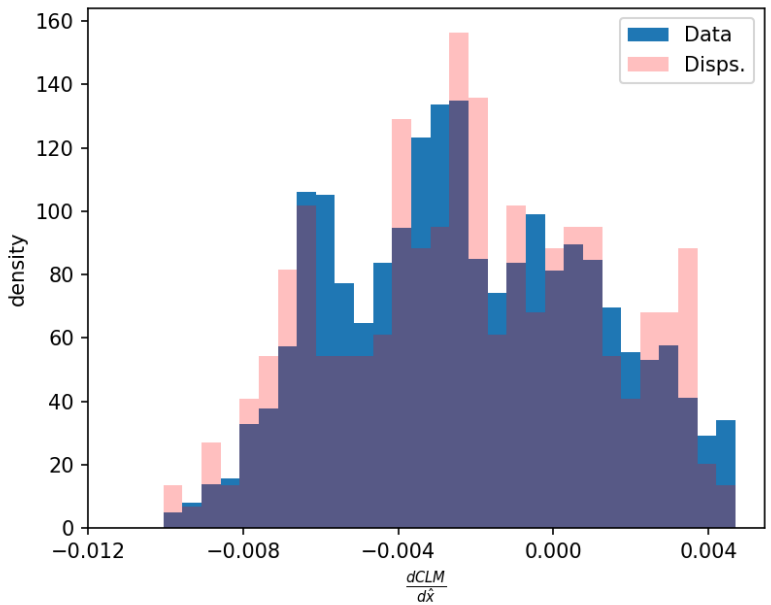


$\mathbf{c}_{LM}$

Validation: Normal Force and Pitching Moment Coefficient Dispersions



$\mathbf{c}_N, \hat{x}_{72}$



$\mathbf{c}_{LM}, \hat{x}_{72}$

MMD 2-sample test {.striped .hover}

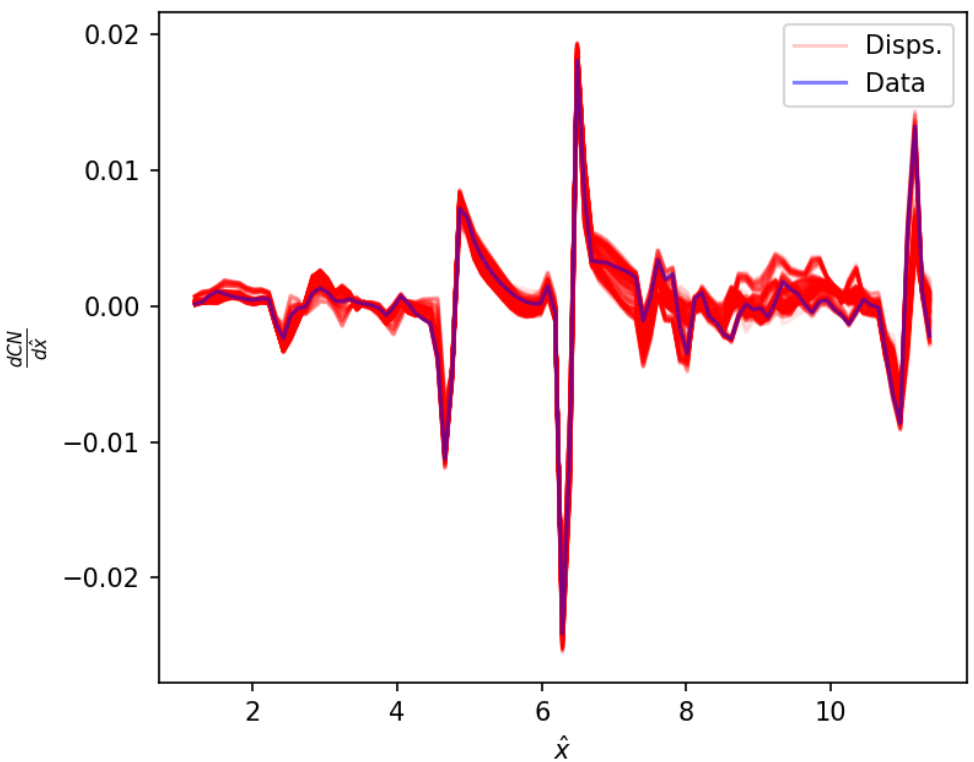
Coefficient	Critical Value	MMD Score
$\mathbf{c}_N$	0.007809	0.005829
$\mathbf{c}_{LM}$	0.007291	0.005436

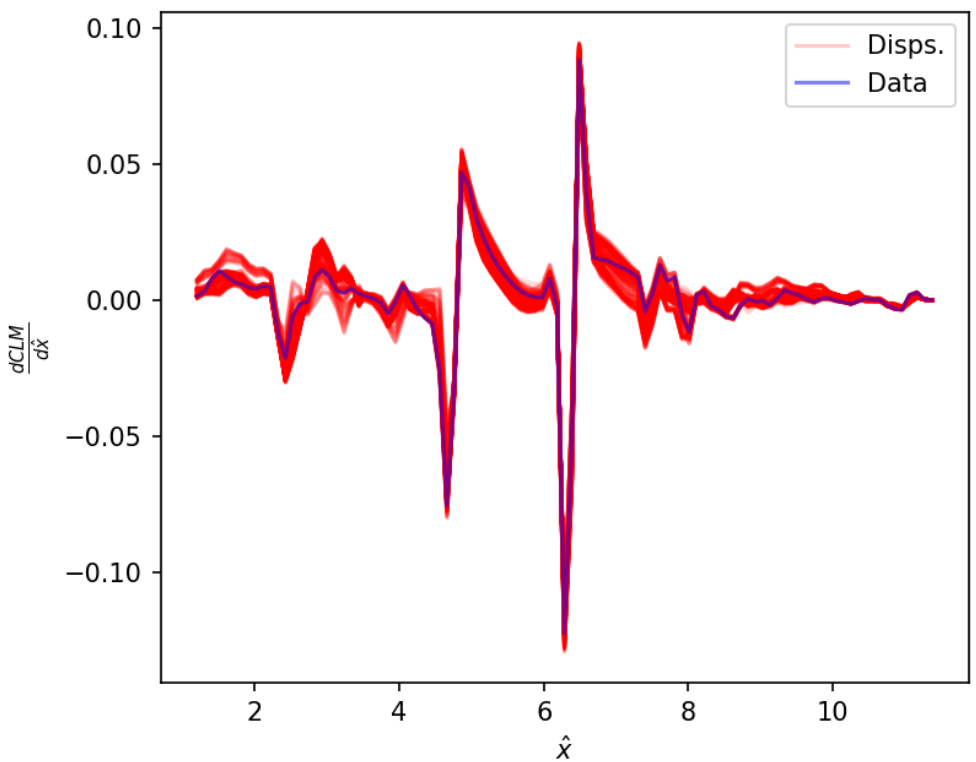
Simulated Uncertainty: Normal Force and Pitching Moment Coefficient Dispersion

- Simulated uncertainty at a known normal force (C_N) and pitching moment (C_{LM})
- Uncertainty modelled as $\epsilon \sim \mathcal{N}(0, 0.05)$

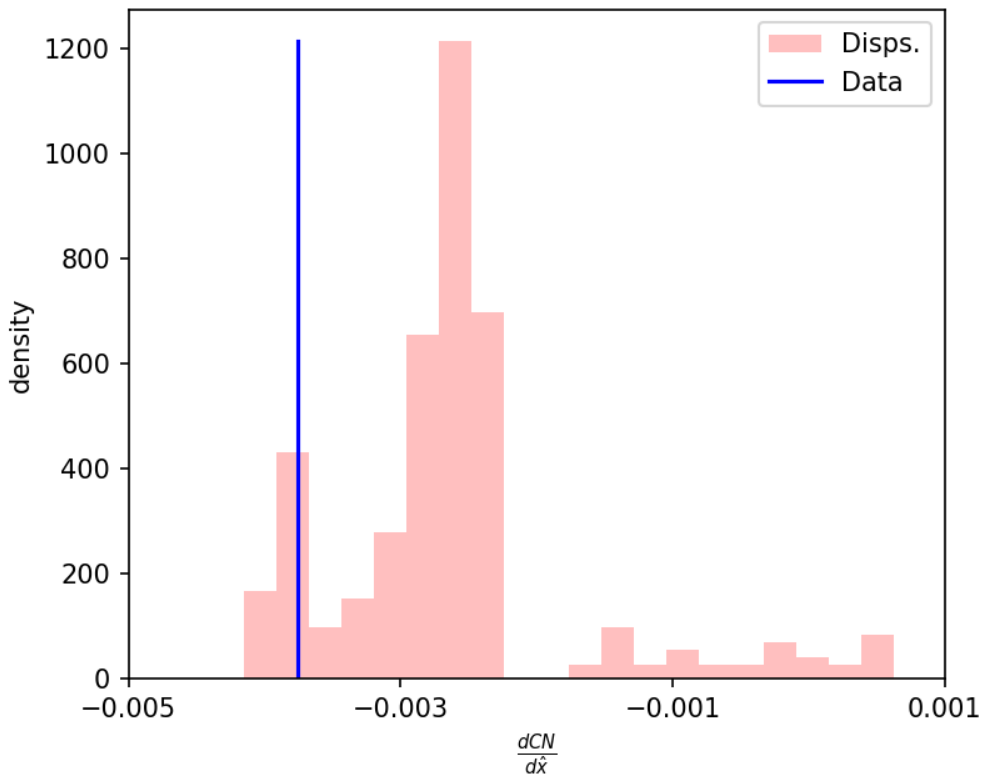
$$C^*_{N,i} = C_N(1 + \epsilon_i)$$

$$C^*_{LM,i} = C_{LM}(1 + \epsilon_i)$$

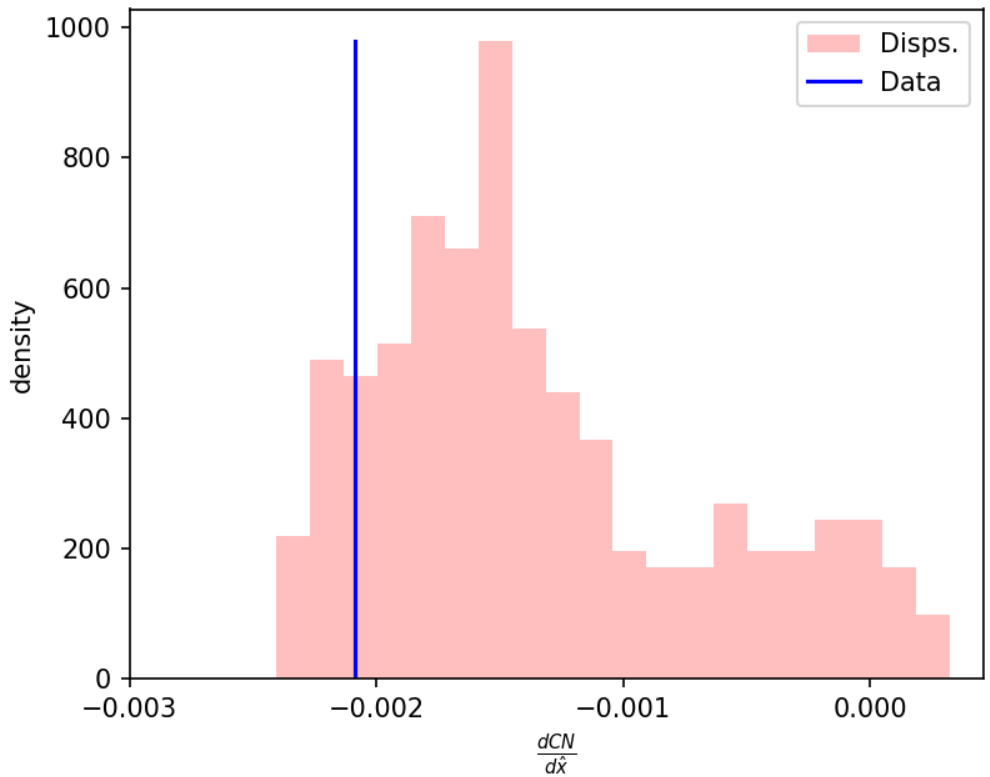




Simulated Uncertainty: Normal Force and Pitching Moment Coefficient Dispersion



$\mathbf{c}_N, \hat{x}_{33}$



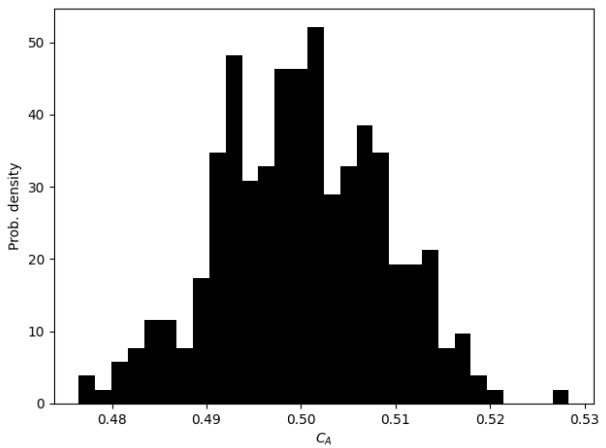
$\mathbf{c}_N, \hat{x}_{72}$

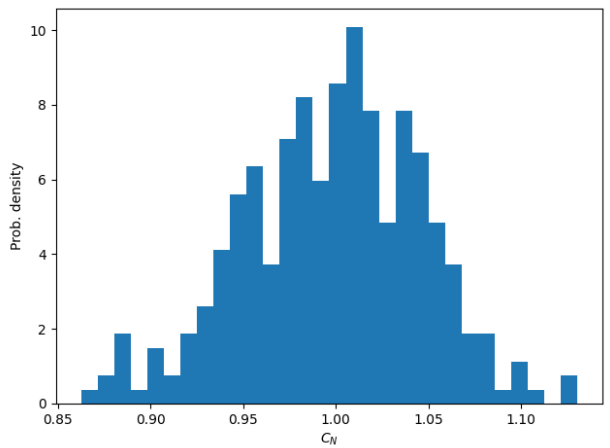
Extending Method to Surface Pressure

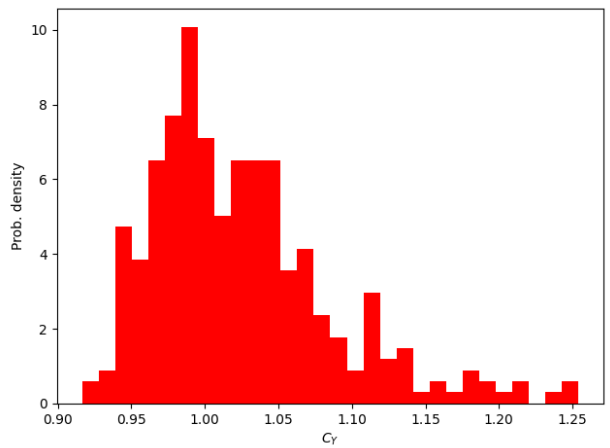
- Applying this method to surface pressure data not likely successfully
- Surface pressure data requires simultaneous matching 6 forces & moments
- Problematic step is conditional distribution checks
- Success probability diminishes quickly with dimensionality

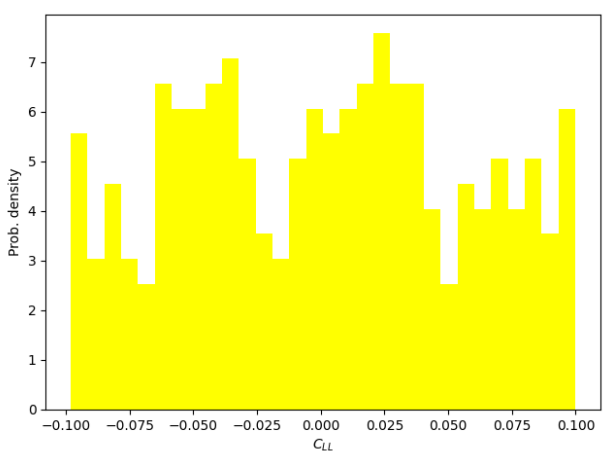
Extending Method to Surface Pressure

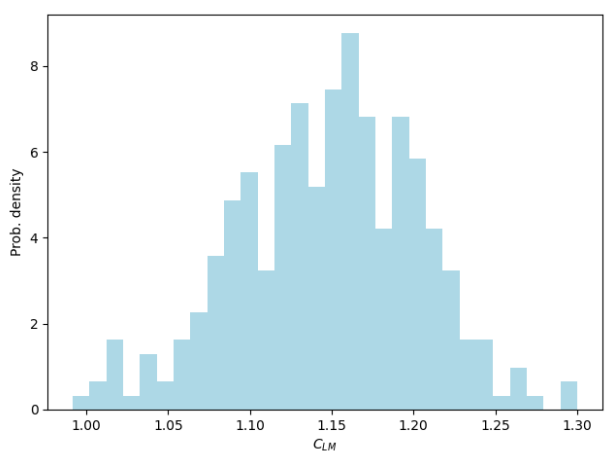
- Working on new approach based on statistical idea of a copula
- Currently integrated coefficient targets are loosely enforced
- Promising initial results, passes MMD test on basis coefficients

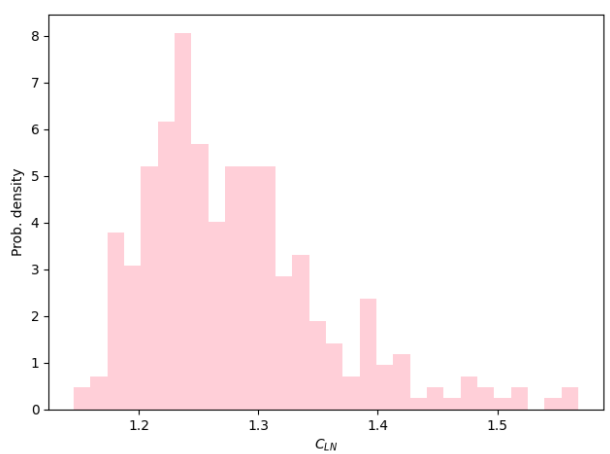












Extending Method to Surface Pressure

- Variance of surface pressure dispersion for 100 target integrated force and moments

  Light Red: Less c_p Variance

  Dark Red: More c_p Variance



Variance of surface pressure dispersion (x-y plane)



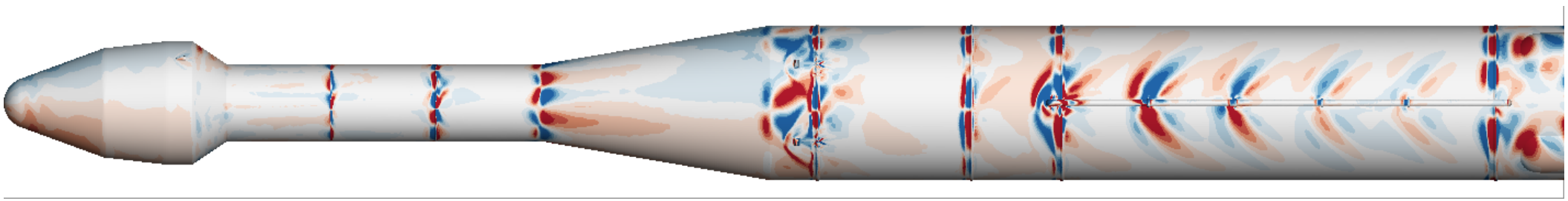
Variance of surface pressure dispersion (x-z plane)

Extending Method to Surface Pressure

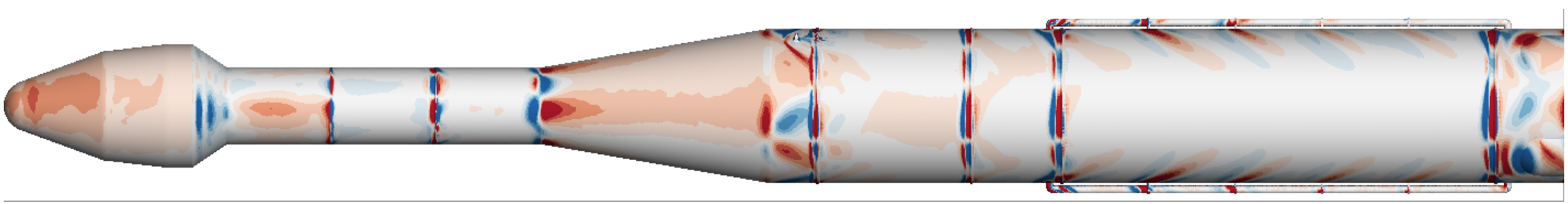
- Novel surface pressure generation for given integrated FM targets

  Blue: $-\Delta c_p$

  Red: $+\Delta c_p$



Dispersed and known surface pressure Δ (x-y plane)



Dispersed and known surface pressure Δ (x-z plane)

Conclusion

- Motive was reconciling uncertain outputs from FM and LL databases
- New method can generate line loads with integration consistency
- New method also shows large improvement in statistical consistency
- Success not expected with direct application to surface pressures (6DoF)
- Continue developing method with surface pressures in mind

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