Unsupervised Mesh Optimization for Improved Numerical Stability of Finite Volume Methods

Mohammad Zandsalimy and Carl Ollivier-Gooch

Mechanical Engineering Department, The University of British Columbia

Advanced Modeling & Simulation (AMS) Seminar Series
NASA Ames Research Center, August 17$^{\text{th}}$, 2023

mohammad.zandsalimy@ubc.ca

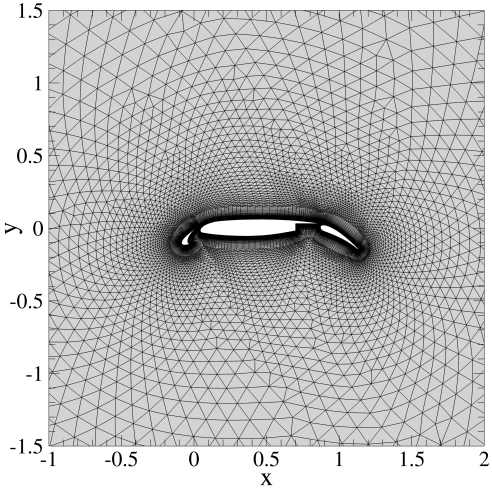
Content

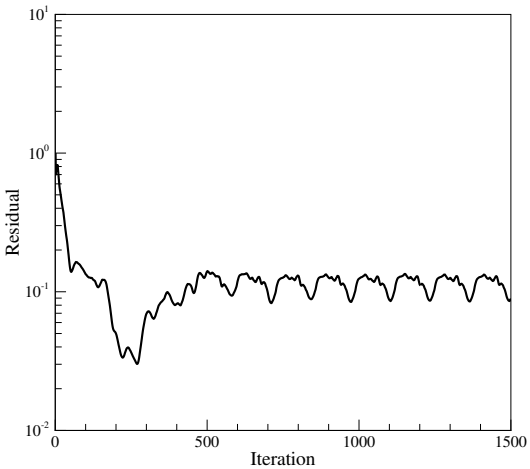
- Background
- Approach
- Primary Results
- Solution Mode Analysis
- Secondary Results
- Future Plans

Part I

Background

CFD Stability Issues





Numerical Solution Method

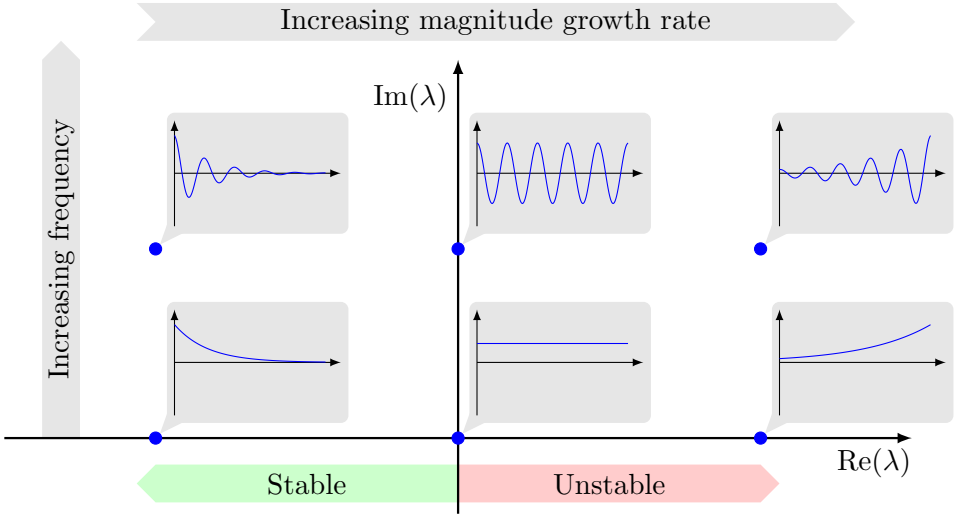
$$\frac{\partial U}{\partial t}(t,\vec{x}) + \vec{\nabla}\cdot\vec{F}(t,\vec{x}) = f(t,\vec{x}) \tag{1}$$

$$\frac{d\bar{U}}{dt} = -\frac{1}{|V_i|}\oint_{\partial V_i}\left(\vec{F}\cdot\vec{n}\right)dA + \frac{1}{|V_i|}\int_{V_i} f\ dV = R(\bar{U}) \tag{2}$$

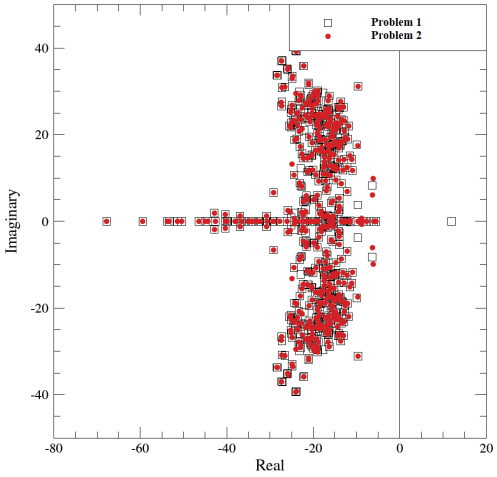
$$\frac{\delta\vec{U}}{\delta t} \coloneqq \frac{\vec{U}^{n+1}-\vec{U}^{n}}{\delta t} = \frac{1}{2}\left(\vec{R}(\vec{U}^{n+1}) + \vec{R}(\vec{U}^{n})\right) \tag{3}$$

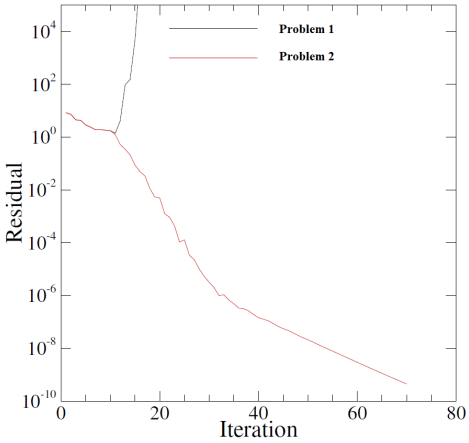
$$\left(\frac{1}{\delta t}I - \frac{1}{2}\frac{\partial\vec{R}}{\partial\vec{U}}\right)\delta\vec{U} = \vec{R}(\vec{U}^{n}) \tag{4}$$

Dynamic Response of Linear Systems



Burgers Problem, Crank-Nicolson Time Stepping





Part II

Approach

Eigenanalysis

- Right and left eigenvectors of $\boldsymbol{A} \in \mathbb{C}^{n \times n}$,

$$\boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{x}, \quad \boldsymbol{x} \in \mathbb{C}^n \tag{5}$$

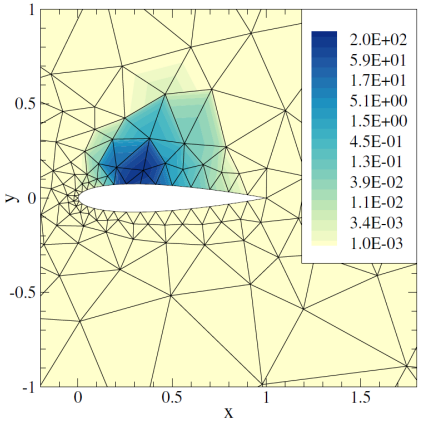
$$\boldsymbol{y}^H \boldsymbol{A} = \lambda \boldsymbol{y}^H, \quad \boldsymbol{y} \in \mathbb{C}^n \tag{6}$$

- **Krylov-Schur** methods in SLEPc[1]
- Spectral transformation to isolate the unstable modes

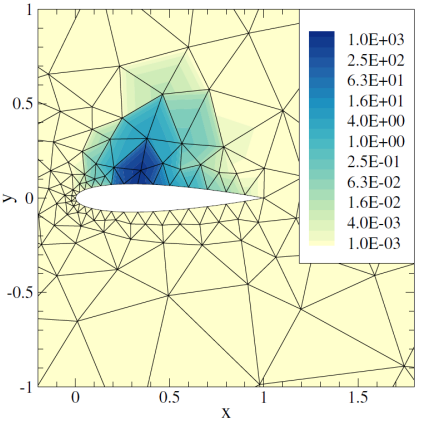
[1] Vicente Hernandez, Jose E. Roman, and Vicente Vidal. "SLEPc: A scalable and flexible toolkit for the solution of eigenvalue problems". In: *ACM Trans. Math. Software* 31.3 (2005), pp. 351–362.

Vertex Selection

- The gradient of $\lambda = 0.9633 + 0I$ w.r.t. the movement of each vertex in the mesh
- 2D Euler problem around the NACA-0015 with 2 unstable modes



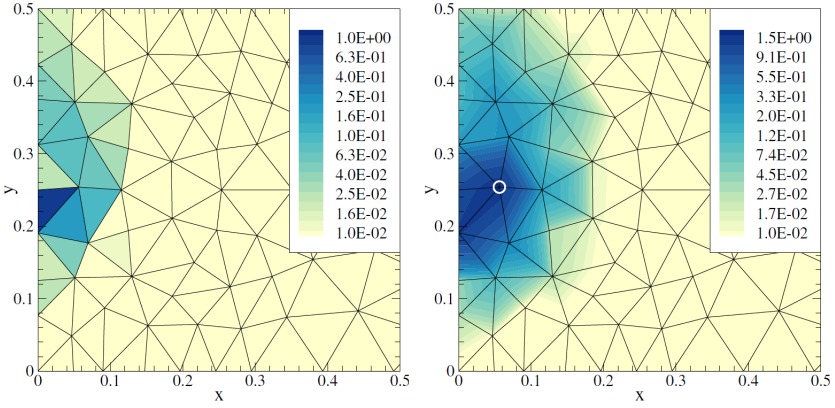
(a) The gradient with respect to x



(b) The gradient with respect to y

Vertex Selection (continued)

- Vertex selection weight in a Burgers problem



(a) The normalized absolute value of eigenvector components in the cells

(b) The summation of weights for each vertex

Eigenvalue Gradient

- To find the direction and magnitude of vertex perturbation
- **Direct method** using finite differences
- **Indirect method**,

$$\frac{d\lambda}{d\zeta} = \boldsymbol{y}^H \frac{d\boldsymbol{A}}{d\zeta}\boldsymbol{x} \tag{7}$$

$$\frac{d\boldsymbol{A}}{d\zeta} = \frac{\partial \boldsymbol{A}}{\partial U}\frac{\partial U}{\partial \zeta} + \frac{\partial \boldsymbol{A}}{\partial \zeta} \approx \frac{\partial \boldsymbol{A}}{\partial \zeta} \tag{8}$$

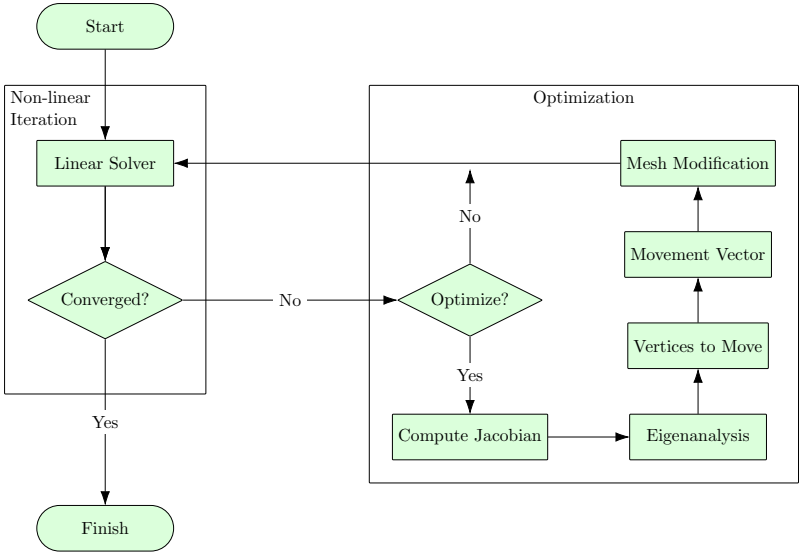
Mesh Modification

- Use Taylor series expansion,

$$\lambda_{\text{new}} = \lambda_{\text{old}} + \frac{d\lambda}{d\zeta}\delta\zeta \tag{9}$$

- Push the unstable eigenvalues to the left side of the spectrum
- Careful with high aspect-ratio cells

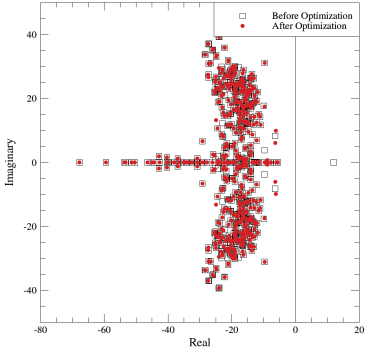
Mesh Optimization Approach

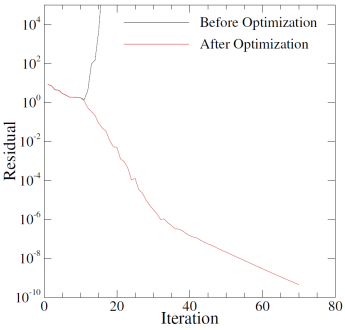


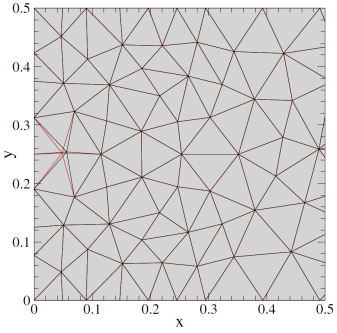
Part III

Primary Results

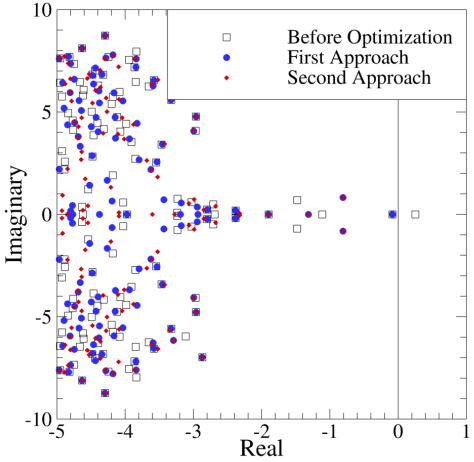
Burgers Problem, 500 Cells, Crank-Nicolson Time Stepping

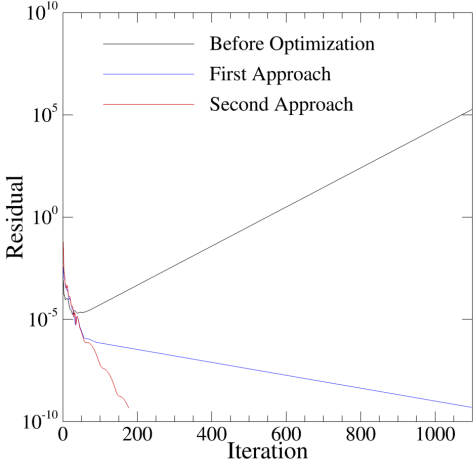




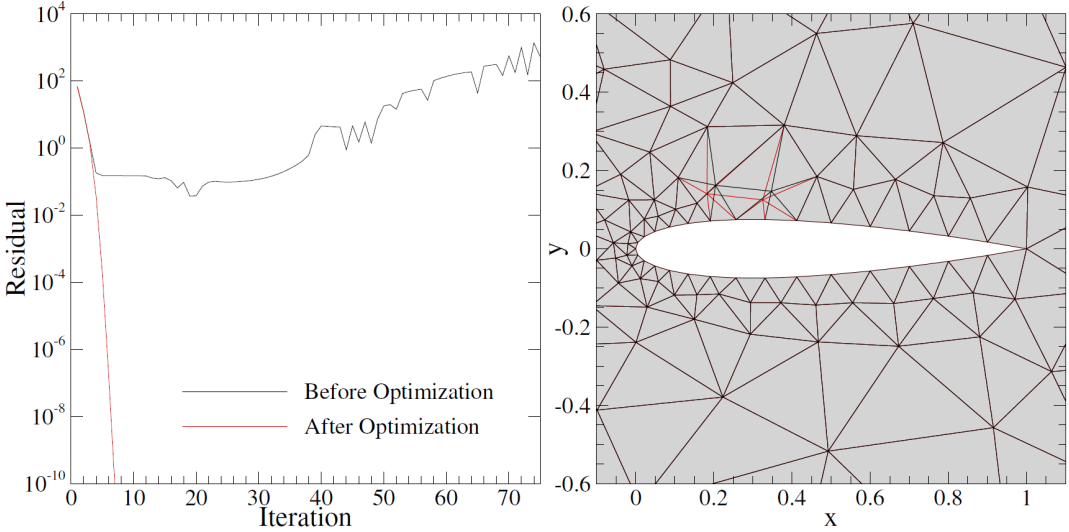


3D Advection Problem, 800 Cells, Crank-Nicolson Time Stepping

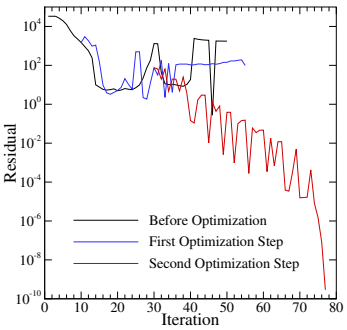


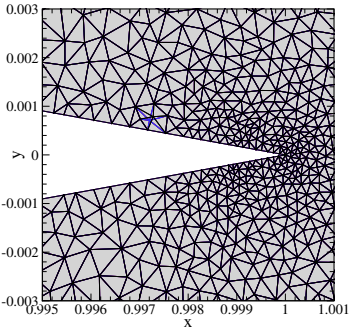


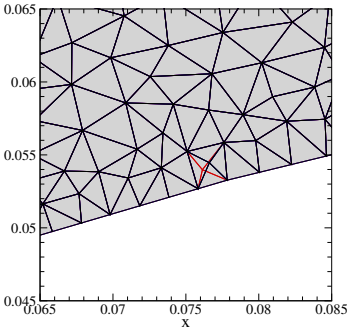
Euler Problem, 600 Cells, Implicit Euler Time Stepping



Euler Problem, 29000 Cells, Implicit Euler time stepping







Part IV

Solution Mode Analysis

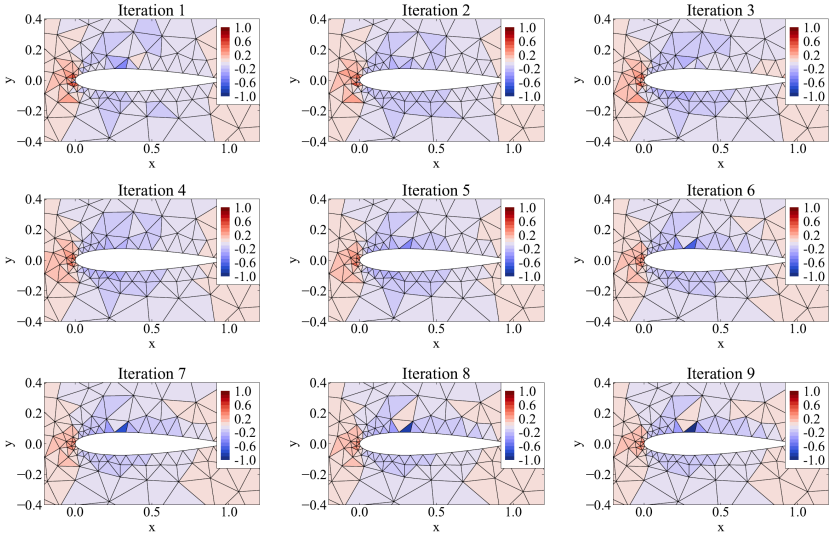
Solution Mode Analysis

- Automated optimization
- Computationally feasible
- Singular Value Decomposition

$$\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H \tag{10}$$

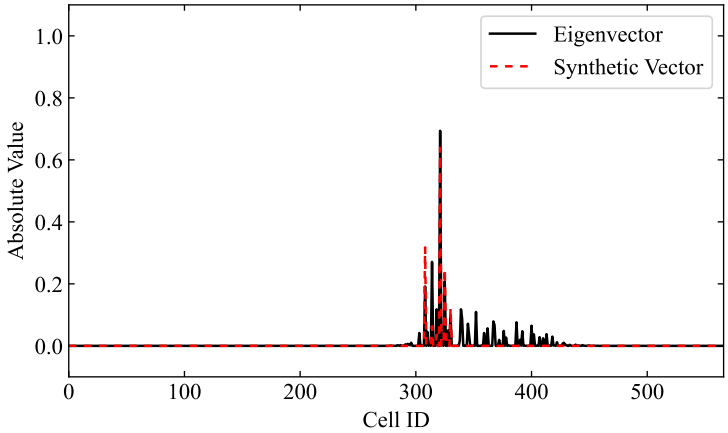
$$\boldsymbol{A} \in \mathbb{C}^{m\times n}, \quad \boldsymbol{U} \in \mathbb{C}^{m\times m}, \quad \boldsymbol{\Sigma} \in \mathbb{R}^{m\times n}, \quad \boldsymbol{V} \in \mathbb{C}^{n\times n}, \quad m > n$$

Dominant Solution Mode



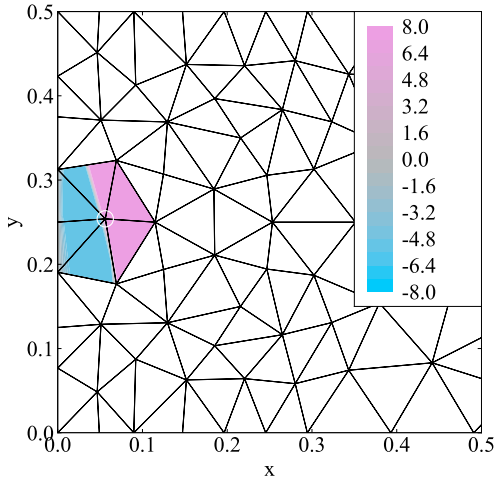
Synthetic Vectors

- Resembling the eigenvector
- Same vertex suggestion

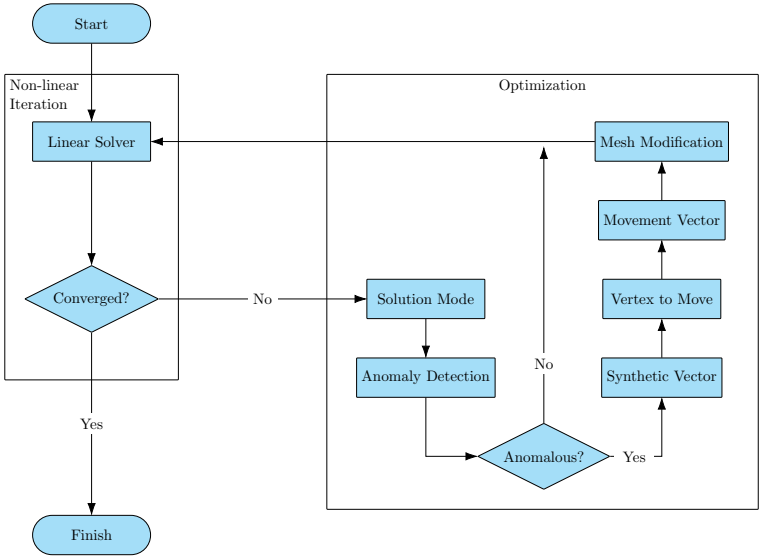


Vertex Movement Window

- The real part of the dominant eigenvalue versus vertex location in a Burgers problem



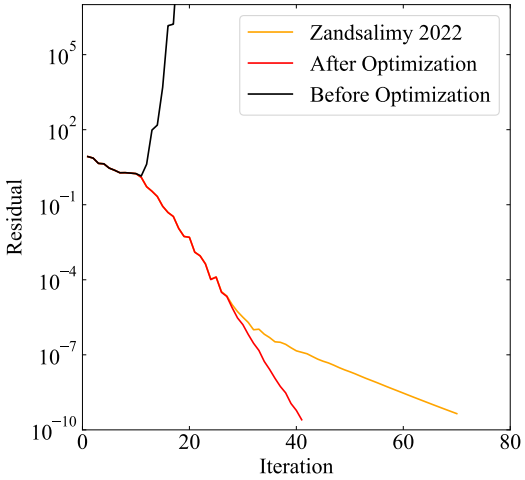
Improved Mesh Optimization Algorithm

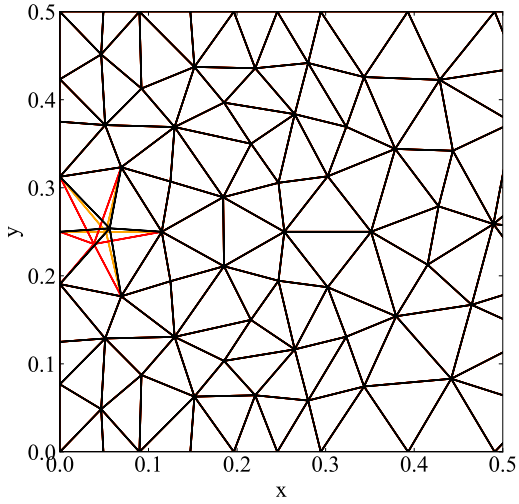


Part V

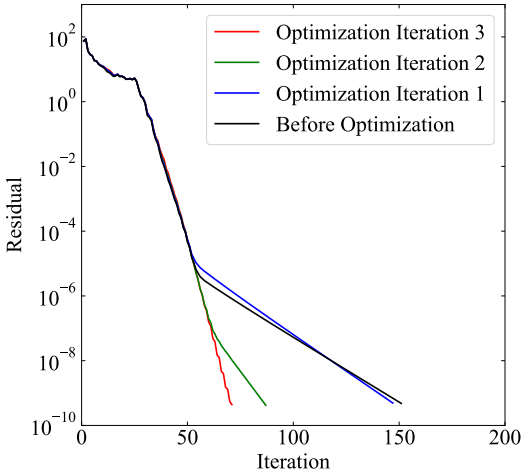
Secondary Results

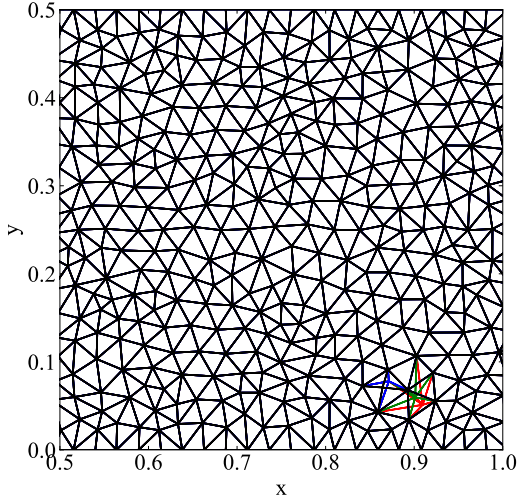
Burgers Problem, 500 Cells, Crank-Nicolson



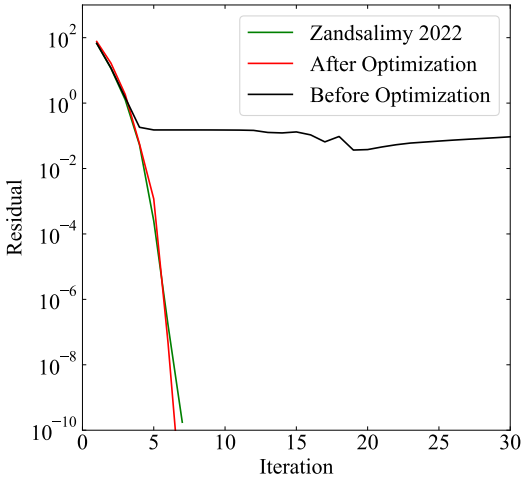


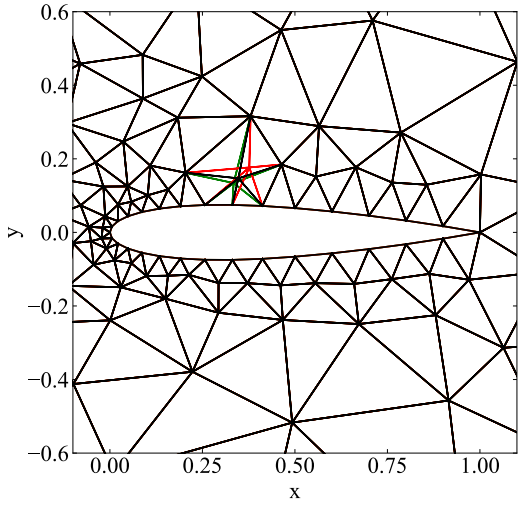
Burgers Problem, 4500 Cells, Crank-Nicolson



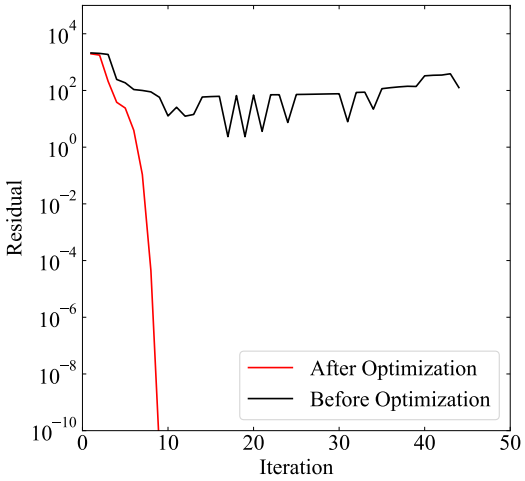


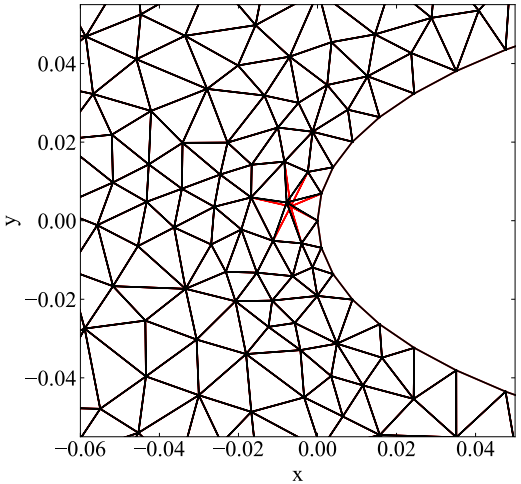
Euler Problem, 600 Cells, Implicit Euler





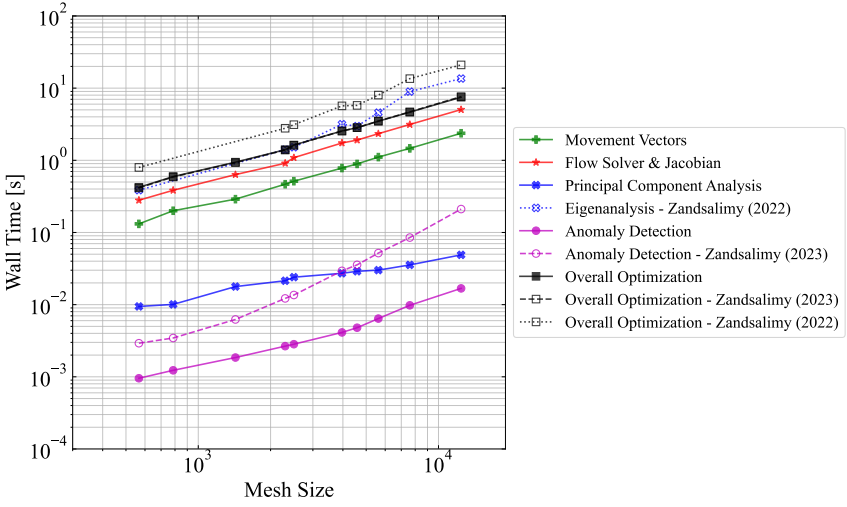
Euler Problem, 7600 Cells, Implicit Euler





Efficiency

- Optimization run-time of an Euler problem



Part VI

Contributions and Future Plans

Contributions

- Accelerate the mesh optimization procedure[2]
- A novel approach to remediate the opposing eigenmodes
- Extend the application to 3D problems
- Pair with Pseudo-Transient Continuation and CFL evolution strategies
- Eliminate eigenanalysis[3]
- Automation of optimization
- Residual vector analysis
- PCA solution mode analysis
- DMD solution mode analysis

[2] Mohammad Zandsalimy and Carl Ollivier-Gooch. "A novel approach to mesh optimization to stabilize unstructured finite volume simulations". In: *Journal of Computational Physics* 453 (2022), p. 110959. ISSN: 0021-9991. DOI: https://doi.org/10.1016/j.jcp.2022.110959.

[3] Mohammad Zandsalimy and Carl Ollivier-Gooch. "Unsupervised Residual Vector Analysis for Mesh Optimization". In: *AIAA SciTech 2023 Forum*. Presented in AIAA SciTech 2023 Forum. Jan. 2023.

Current and Future Goals

- Ansys Fluent simulations
- Anisotropic meshes and turbulent simulations
- Approximate eigenanalysis of large-sparse systems
- Novel guidelines for mesh generation

Acknowledgement







