Broadband Reconstruction of Inhomogeneous Turbulence using a Combination of Data and Physics Driven Modeling

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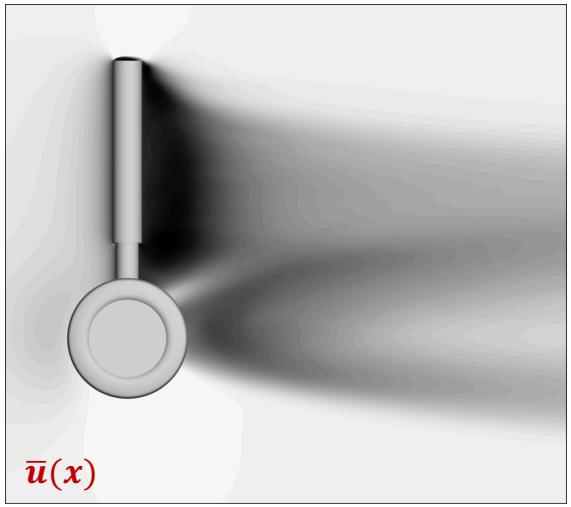
Computational Aerosciences Branch
NASA Ames Research Center

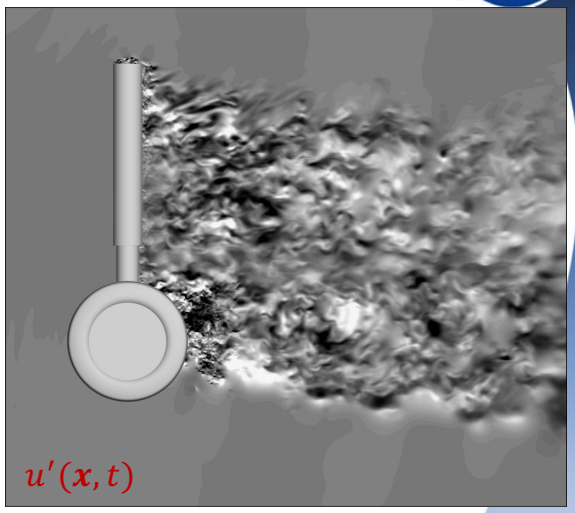
Stanford University

Advanced Modeling and Simulation Seminar
NASA Ames Research Center, Moffett Field, CA
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What do we mean by field reconstruction?







Immersed Boundary WMLES of LAGOON Landing Gear. See Wong et al., AIAA 2022-2850 for further details.

Consider a flow field decomposition:

$$\boldsymbol{u}(\boldsymbol{x},t) = \bar{\boldsymbol{u}}(\boldsymbol{x},t) + \boldsymbol{u}'(\boldsymbol{x},t)$$

TOTAL — MEAN — FLUCTUATION (focus of present work)

The problem definition



Problem Description: Given a field, $\boldsymbol{u}'(\boldsymbol{x},t)$ generate a field, $\tilde{\boldsymbol{u}}'(\boldsymbol{x},t)$ such that

1. It is solenoidal:

$$\nabla\cdot\tilde{\boldsymbol{u}}'=0;\qquad \tilde{\boldsymbol{u}}'(\boldsymbol{x},t)=\nabla\times\boldsymbol{A}_{\omega}+\nabla\phi_{BC};\qquad \nabla^2\phi_{BC}=0$$

2. Estimates the second order, two-point statistics accurately*

$$\left\langle u_i'(\boldsymbol{x}+\boldsymbol{y},t+\tau)u_j'(\boldsymbol{y},\tau)\right\rangle=\left\langle \tilde{u}_i'(\boldsymbol{x}+\boldsymbol{y},t+\tau)\tilde{u}_j'(\boldsymbol{y},\tau)\right\rangle$$

Important Note: Equivalence between original and reconstructed fields is required to be purely statistical and not pointwise (LES accuracy must be described statistically)

$$\boldsymbol{u}'(\boldsymbol{x},t)\neq\tilde{\boldsymbol{u}}'(\boldsymbol{x},t)$$

* Present work only focuses on temporally stationary cases.

Why is it relevant?



Reduced order model (sparse representation) for flows needed in many applications

1. Aeroacoustics: Air-frame noise, fan noise, jet noise
2. Geophysics: Synoptic (Days) + Meso (Hours) + Microscale (Seconds/Minutes) fluctuations – Atmospheric Science, Oceanography, Wind Energy, etc.
3. Fluid-solid interactions: Unsteady pressure loading and vibrations, solid motions within fluid (particle-laden flows, etc.)
4. Hybrid RANS-LES: fluctuation generation at interfaces

Bottom Line: Vast array of applications where fluctuations (as opposed to steady state) make up for nearly the entire figure of merit.

Outline

1. Two Problems:
   a. Actuator Disk wake + HIT co-flow
   b. LAGOON Landing Gear wake

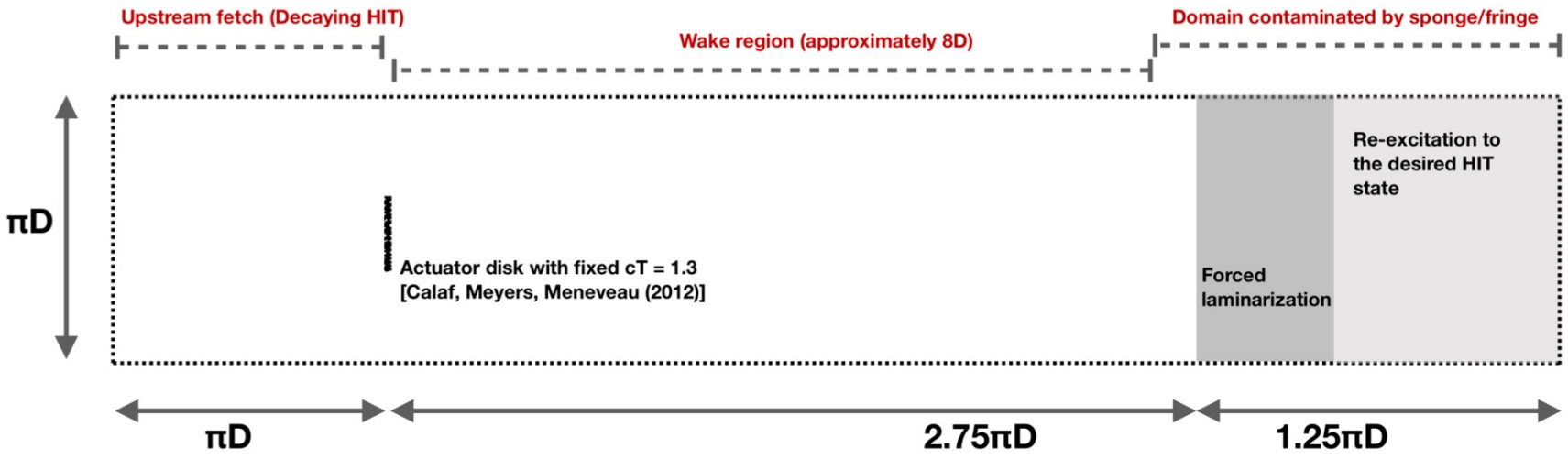
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   a. Global modes: SPOD with TKE norm
   b. The best-case scenario: Truncated Expansions
   c. What's missing? What's needed?

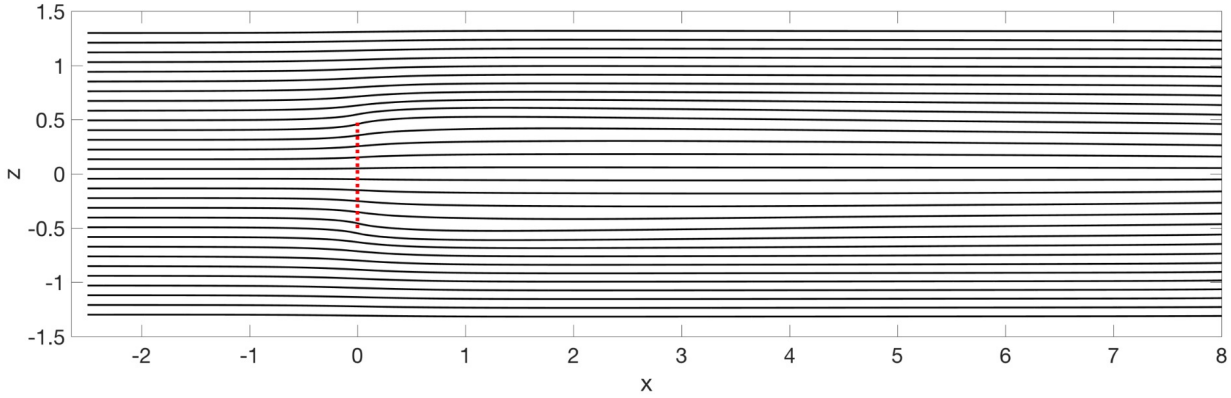
3. Physics driven model:
   a. Wavelets: optimizing spatio-spectral localization
   b. WKB-variant of Rapid-Distortion theory
   c. Benchmark validation: HIT and Stratified Ekman Layer

4. Results:
   a. Actuator Disk wake: Fields, Reynolds stresses and spectra
   b. LAGOON Landing Gear wake: Fields and Reynolds stresses

5. Outlook:
   a. Summary
   b. Is this just an academic exercise, or is any of this useful?

Problem 1: Actuator Disk with HIT co-flow/inflow





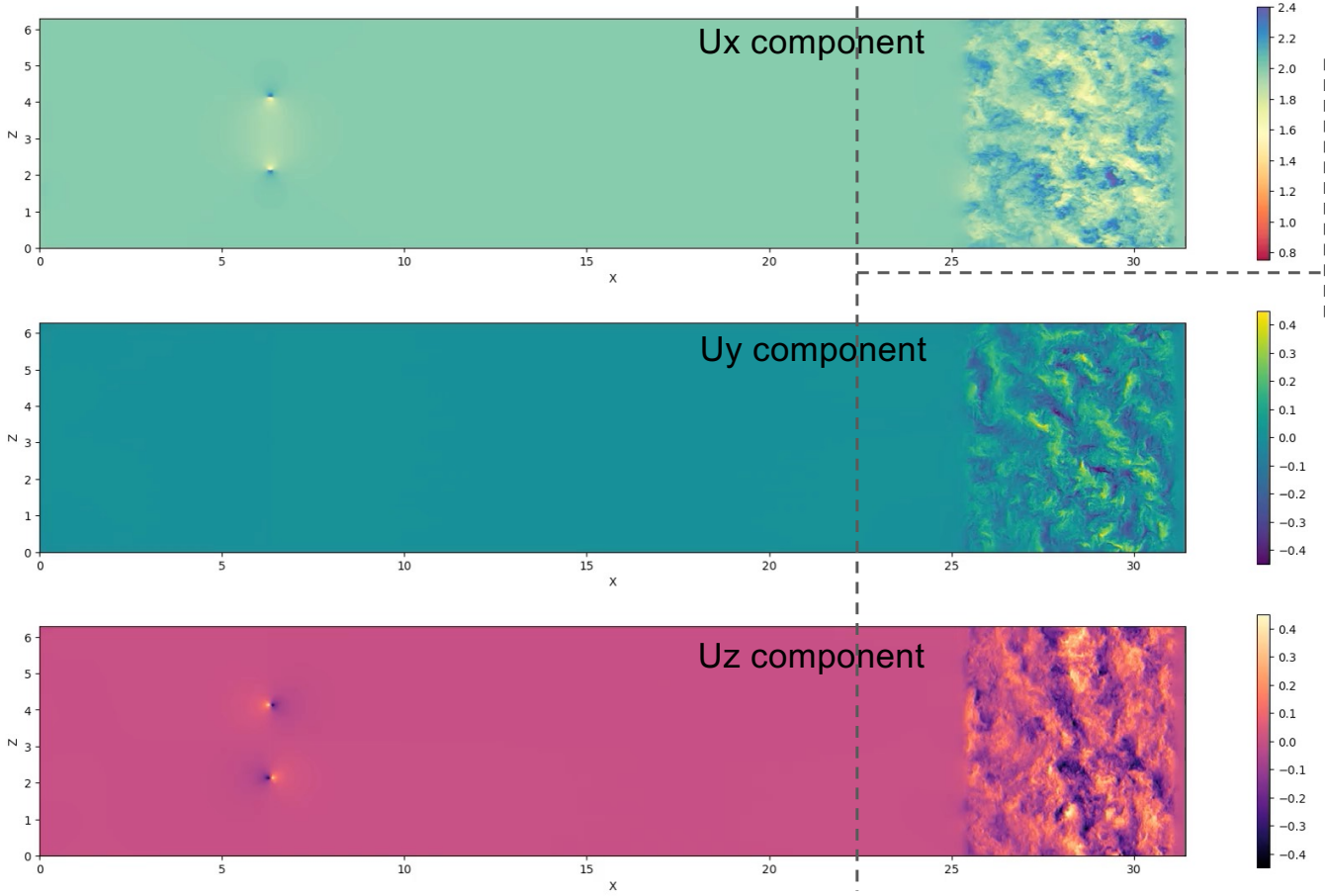
Incompressible Flow Formulation

Fully spectral spatial discretization

RK4 time discretization

Sigma SGS model; $Re \to \infty$

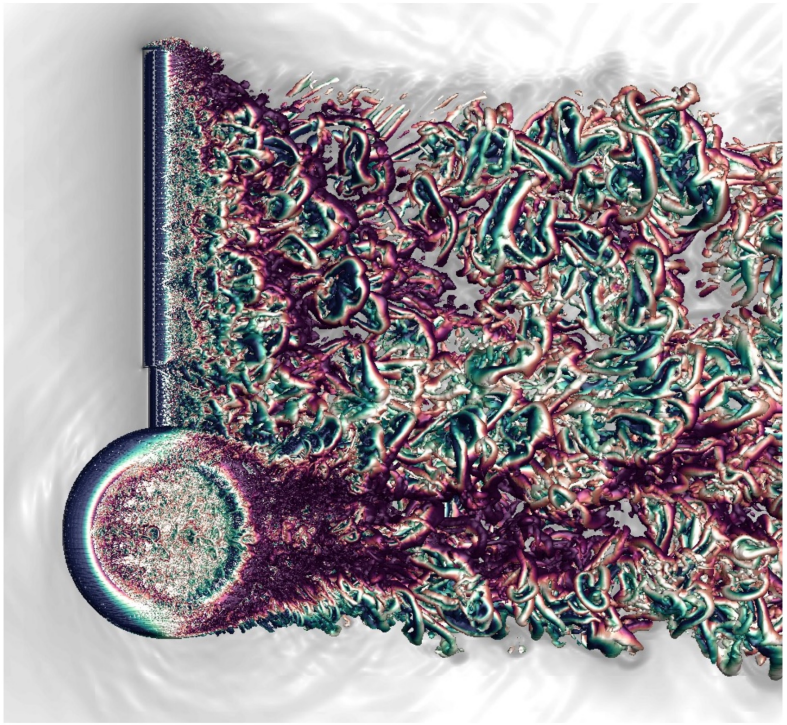
Problem 1: Actuator Disk with HIT co-flow/inflow



About 2.5TB of time-resolved data collected on a single plane (x/D=6.67) for processing and analysis.

Ghate, et al., J. Fluid Mech., 2019 for more information on the data

Problem 2: LAGOON Landing Gear Wake



Compressible Flow IB-WMLES Formulation (LAVA)

Problem Mach number is low; $Ma_\infty = 0.23$

4th order Finite Difference Discretization (with some upwinding)

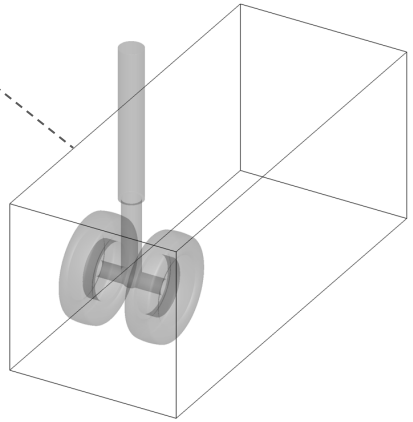
RK3 Time Discretization

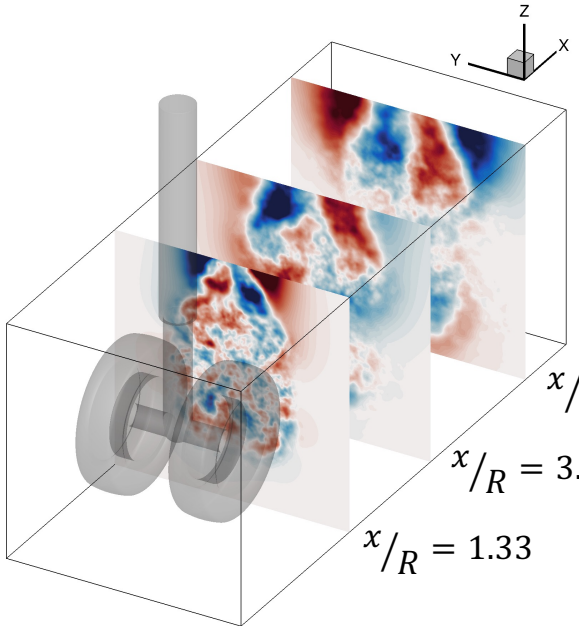
Vreman SGS model; $Re_D = 1.55 \times 10^6$

Wong, et al., AIAA 2022-2850 for more information on the data

Problem 2: LAGOON Landing Gear Wake

About 40TB of time-resolved data collected in a volume around the LG. Analysis performed for $\frac{x}{R} > 1$ (downstream of the wheel)





Full 3D volume modelled, and results visualized on 3 separate planes in the wake

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Data Driven Modeling: A Global Basis



We wish to express the velocity field as:

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{j=1}^{N} a_j \boldsymbol{\Psi}_j(\boldsymbol{x},t)$$

Such that the modes are mutually orthogonal, $\int_{x,t} \boldsymbol{\Psi_i}(x,t)\, \boldsymbol{\Psi_j^*}(x,t) dx dt = \delta_{i,j}$ with their energies given by $a_i a_i^* = \lambda_i$.

If the modes are optimally selected to maximize a norm (such as TKE), the decomposition is either:

a. Proper Orthogonal Decomposition (POD) (or SVD) for space-only case
b. Dynamic Mode Decomposition (DMD) for space-time case (Schmid, 2008); turbulence has space-time coherence

Following Lumley (1967), if a dimension (space or time) is homogeneous, POD modes are the Fourier modes in that dimension.

Temporally stationary phenomena – Temporal Fourier-Transform reduces dimensionality of the SVD/POD problem (Lumley's original formulation, further clarified by Towne et al., JFM, 2017); commonly referred to as "Spectral POD" (SPOD) since 2017.

Data Driven Modeling: SPOD

Hence, the global basis we seek is

$$\widehat{\boldsymbol{u}}(\boldsymbol{x}, f) = \sum_{j=1}^{N} a_j(f)\widehat{\boldsymbol{\Psi}}_{\boldsymbol{j}}(\boldsymbol{x}, f)$$

where $\widehat{\boldsymbol{u}}(\boldsymbol{x}, f) = \int_{-\infty}^{\infty} \boldsymbol{u}(\boldsymbol{x}, t)e^{-i2\pi ft}dt$

If other spatial dimensions are homogeneous (azimuthal symmetry, periodicity, etc.), dimensionality of $\Psi(x, t)$ is further reduced by taking additional Fourier transforms in those dimensions

Since the goal is to model 2nd order velocity correlations, the mode shapes $\boldsymbol{\Psi}_j(\boldsymbol{x}, f)$ are selected to maximize the TKE norm (appropriate for low-Mach number cases).

Relevance:

Once the mode shapes, $\boldsymbol{\Psi}_j(x, f)$ and their intensities $\lambda_j(f) = a_j(f)a_j^*(f)$ are determined; random ensemble of data can be generated

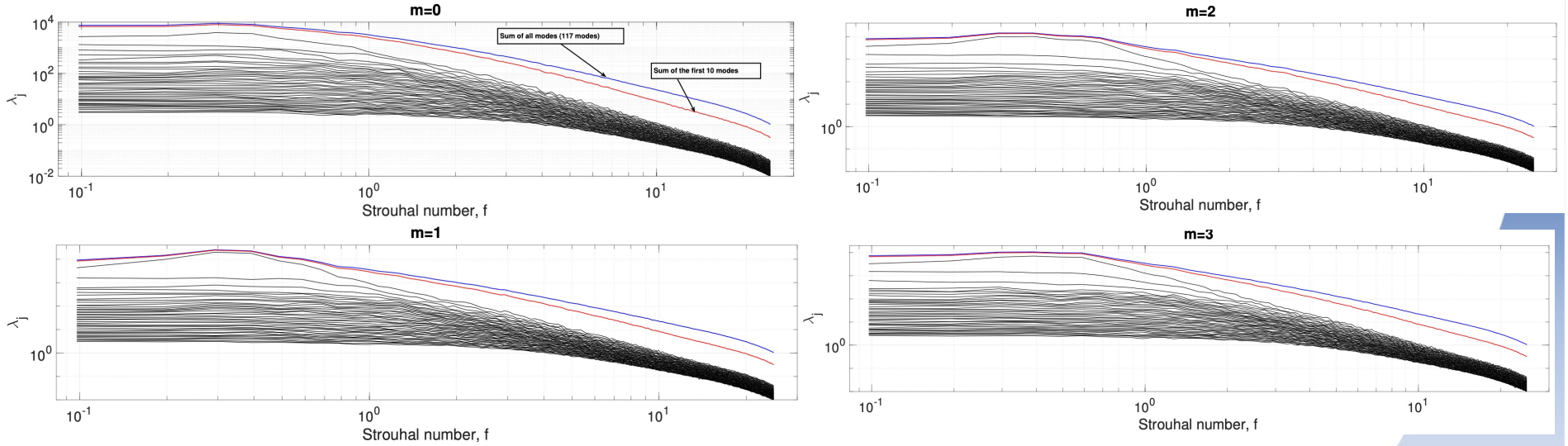
$$\widehat{\widetilde{\boldsymbol{u}}}(x, f) = \sum_{j=1}^{N} b_j(f)\widehat{\boldsymbol{\Psi}}_{\boldsymbol{j}}(\boldsymbol{x}, f)$$

where $b_j = \sqrt{\lambda_j}e^{i\theta}; \theta \in [0, 2pi)$ is a random number.

Main Question: How easily can $\boldsymbol{\Psi}_{\mathrm{j}}(x, f)$ and $\lambda_j(f)$ be determined ("learned") for a given problem?

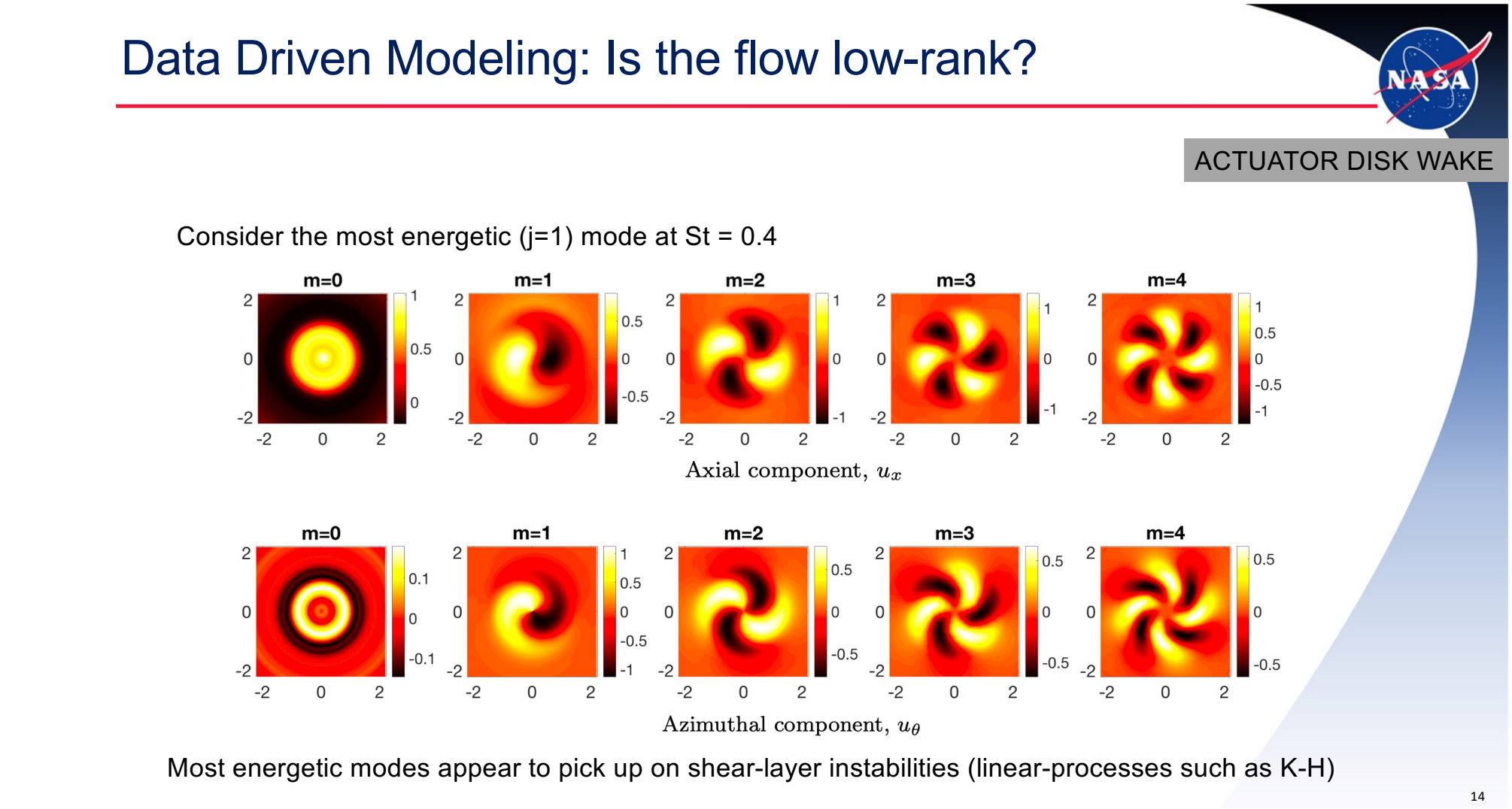
Data Driven Modeling: Is the flow low-rank?

ACTUATOR DISK WAKE



Low-rank expansions likely to work only for low Strouhal numbers, significant loss of energy would occur at high Strouhal numbers

Data Driven Modeling: Is the flow low-rank?

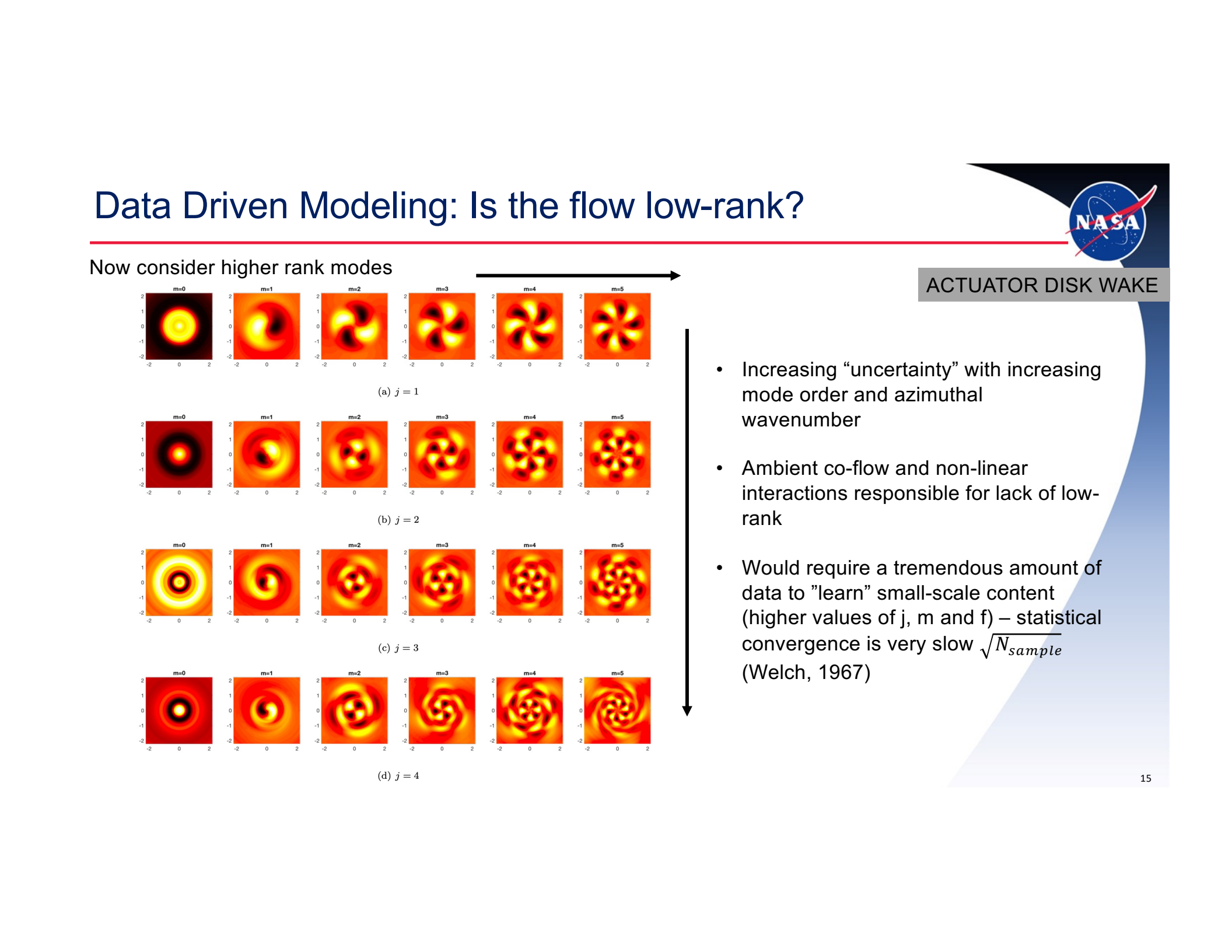


ACTUATOR DISK WAKE

Consider the most energetic (j=1) mode at St = 0.4

Most energetic modes appear to pick up on shear-layer instabilities (linear-processes such as K-H)

Data Driven Modeling: Is the flow low-rank?



Now consider higher rank modes

(a) $j = 1$

(b) $j = 2$

(c) $j = 3$

(d) $j = 4$

- Increasing “uncertainty” with increasing mode order and azimuthal wavenumber
- Ambient co-flow and non-linear interactions responsible for lack of low-rank
- Would require a tremendous amount of data to ”learn” small-scale content (higher values of j, m and f) – statistical convergence is very slow $\sqrt{N_{sample}}$ (Welch, 1967)

Data Driven Modeling: Using only “high-confidence” modes

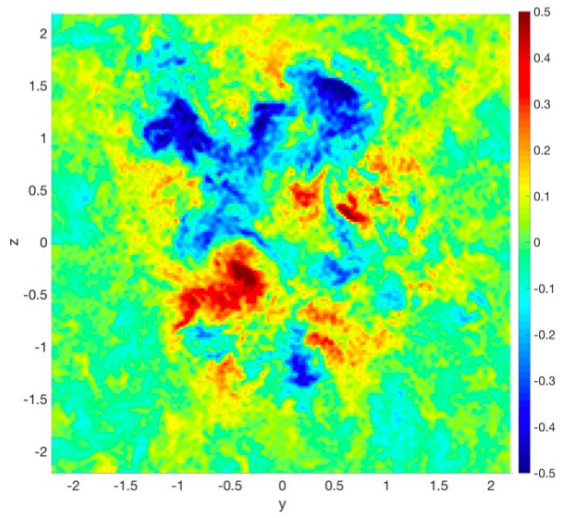
$$\boldsymbol{u}^{\text{trunc}}(y,z,t) = \boldsymbol{\mathcal{I}}_{(r,\theta)\to(y,z)} \left\{ \sum_{|f|<F_{co}} \left(\sum_{|m|<M_{co}} \left(\sum_{j<j_{max}} a_j(m,f)\boldsymbol{\Psi}_j(m,f,r)e^{-i(m\theta+ft)} \right) \right) \right\}$$

ACTUATOR DISK WAKE

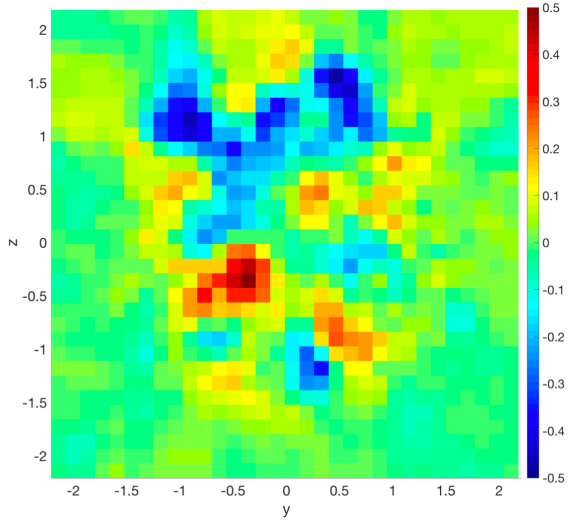
$$\boldsymbol{u}^{\text{res}}(y,z,t) = \boldsymbol{u}(y,z,t) - \boldsymbol{u}^{\text{trunc}}(y,z,t)$$

DATA – ONLY MODEL FEASIBLE

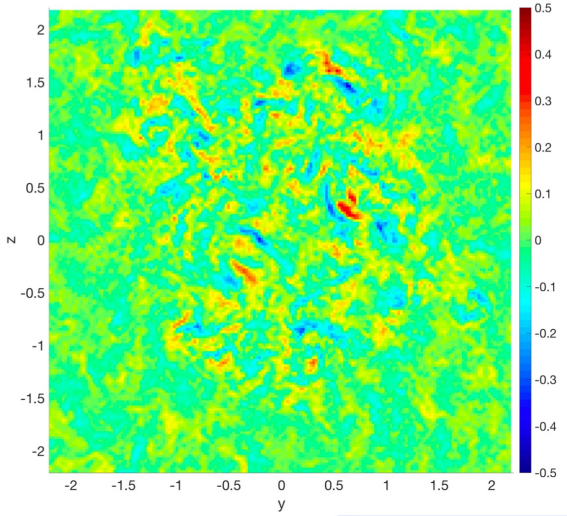
DATA-ONLY MODEL DIFFICULT



$u(y,z)$



$u^{\text{trunc}}(y,z)$



$u^{\text{res}}(y,z)$

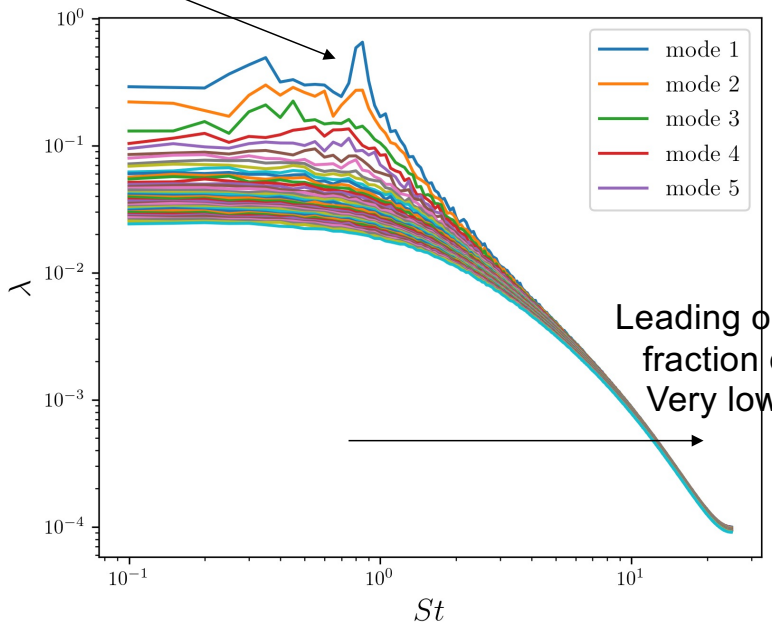
Data Driven Modeling: Is the flow low-rank?

LAGOON LANDING GEAR

Situation **substantially worse** for the LAGOON Landing Gear due to lack of homogeneous directions (only temporally stationary)

Large Tone at St = 0.85
(St=0.2 based on strut diameter)

Sufficient data for 40 total modes per St

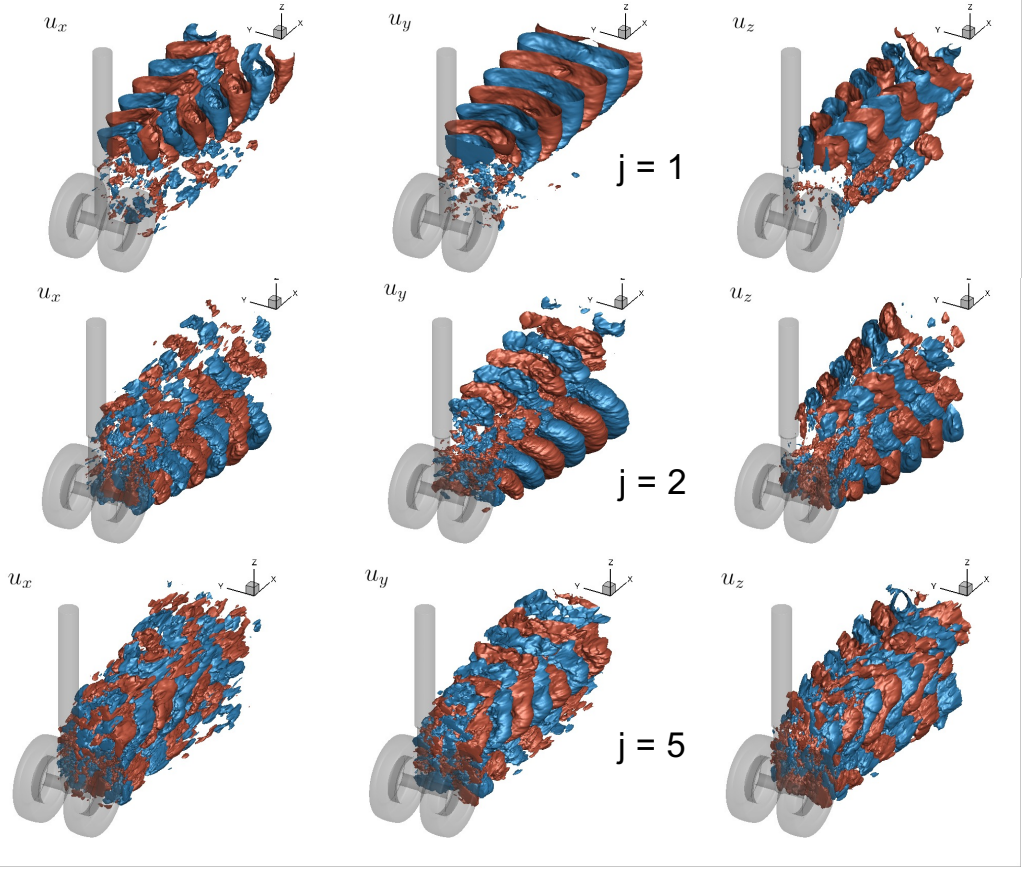


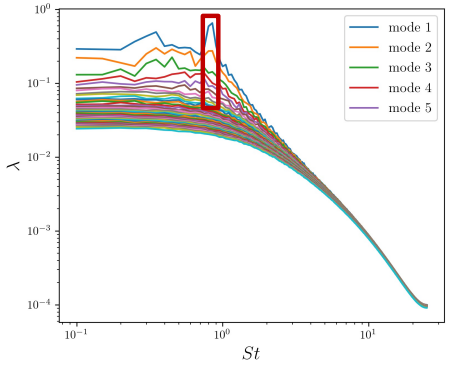
Leading order mode captures a smaller fraction of energy with increasing St
Very low statistical confidence in Ψ_j

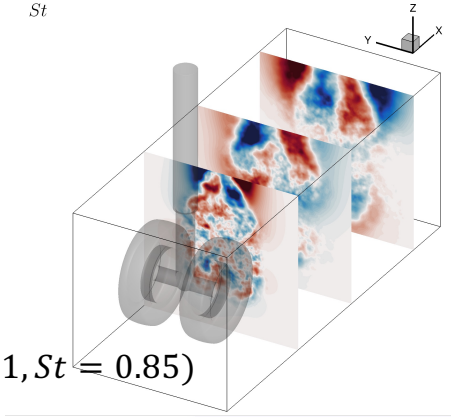
Data Driven Modeling: Is the flow low-rank?

LAGOON LANDING GEAR

"High-confidence modes": appear to capture linear-instabilities in the flow







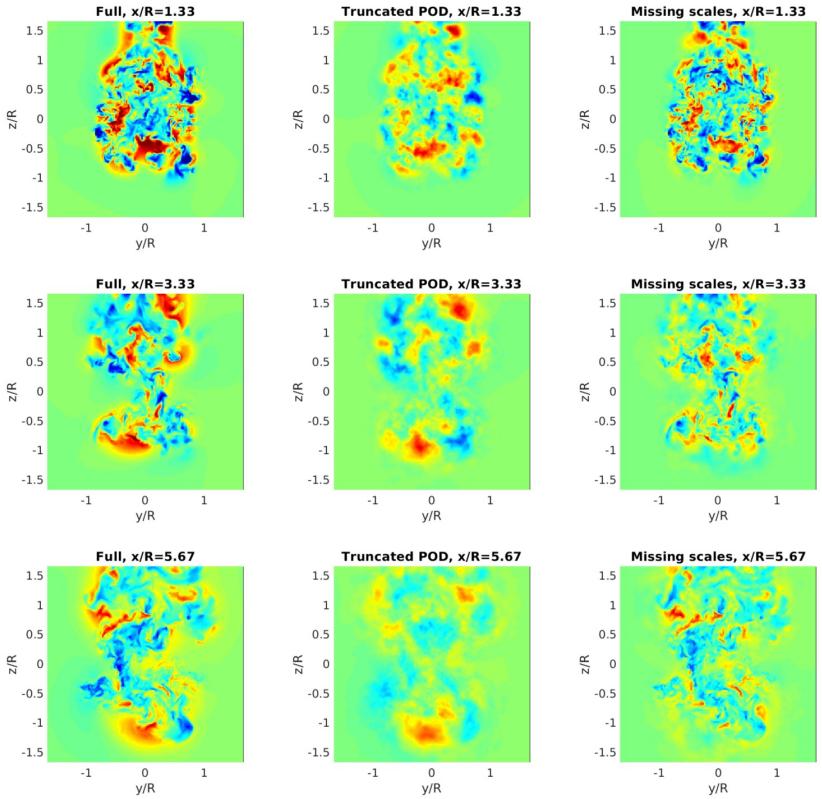
$\Psi_x(j = 1, St = 0.85)$

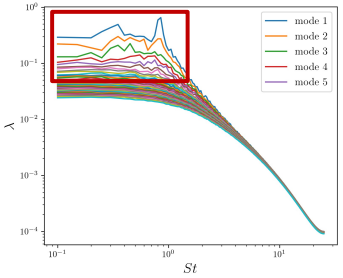
Data Driven Modeling: Using only “high-confidence” modes

LAGOON LANDING GEAR

Only using St<1.5 and j<20 (excluding St=0):

X-component of velocity fluctuations, u_x





1. Majority of the energy is missing from the truncated expansion
2. High-confidence POD modes appear to primarily capture the “largest scales” of motion
3. Even with >40TB of training data, finer scales cannot be represented using meaningful (high confidence) global modes
4. Need additional modeling to **“enrich”** missing scales back into the POD expansion
5. Typically, these low-St, low order modes can also be “learned” using coarse-grid LES

Can data-driven super-resolution solve the problem?



- A flurry of papers on “super-resolution using GANs” in the past 2 years
  - Google-scholar search for “super resolution” + “neural network” + “turbulence” returns **>100 relevant papers since 2021**
  - Smaller subset – limiting to JFM, Phys. Rev. Fluids and PoF still has **>10 papers in past 5 years**
  - Majority of publications try to reproduce “exact field” instead of “statistically equivalent” fields – not a bad idea if higher order moments are also needed
  - Papers by **Subramaniam et al., arxiv (2020)** (used 3D fields) and **Fukami et al., J. Fluid Mech (2021)** (used 3D time-resolved data) are more “complete” than the rest

- 2 main shortcomings:
  - Limited to low-Reynolds numbers – much of the work focuses on “enrichment” within dissipation range, not sufficient inertial subrange content
  - Non-general – have not shown that models trained on channel – work for flat plate, or models trained for $Re_\tau \approx 180$ works at $Re_\tau \approx 1000$

- This problem has a much richer history in the turbulence community (they just didn’t call it super-resolution) (dating back to “triple decomposition – Hussain & Reynolds 1970; Domaradski & Loh 1999 – LES “enrichment”, etc.)

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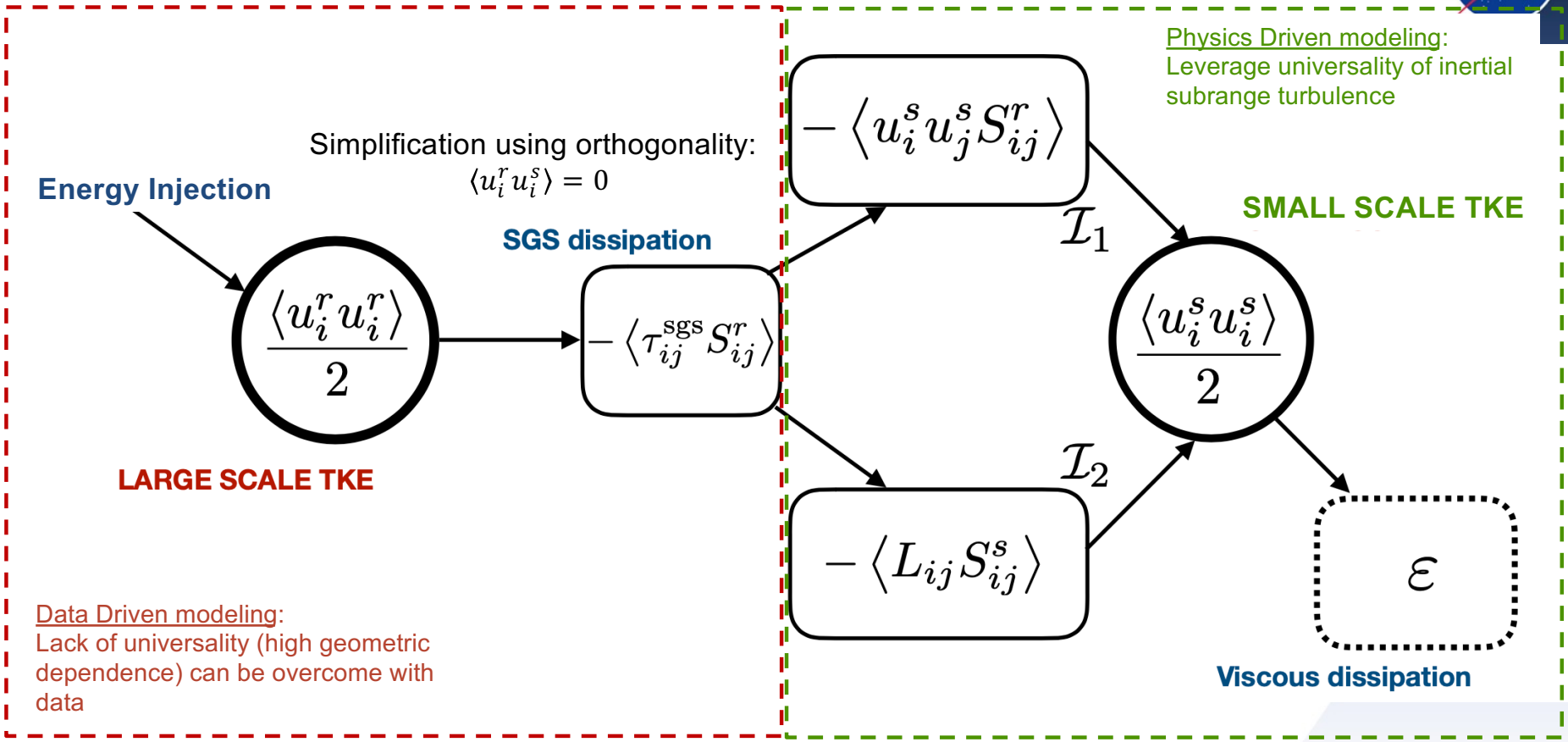
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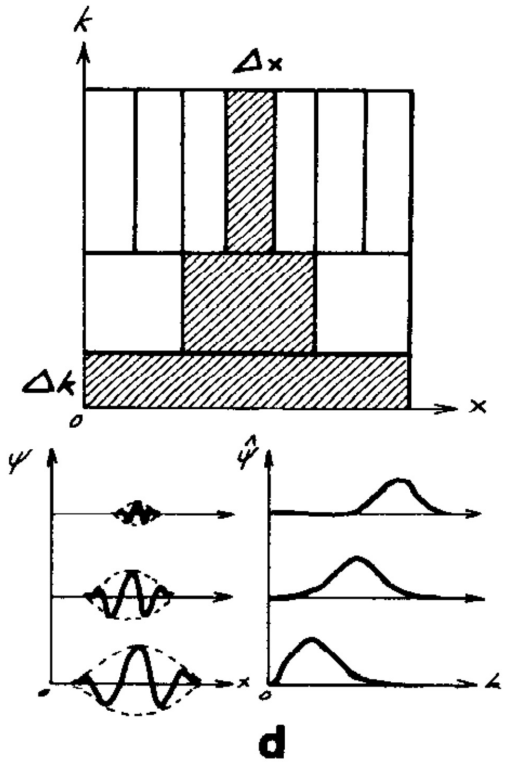
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Data Driven Modeling: What's missing?



Physics Driven Model: Wavelets to the rescue?



Uncertainty Principle
(Fourier Duality)
requires: $\Delta_X \Delta_k \geq 2\pi$

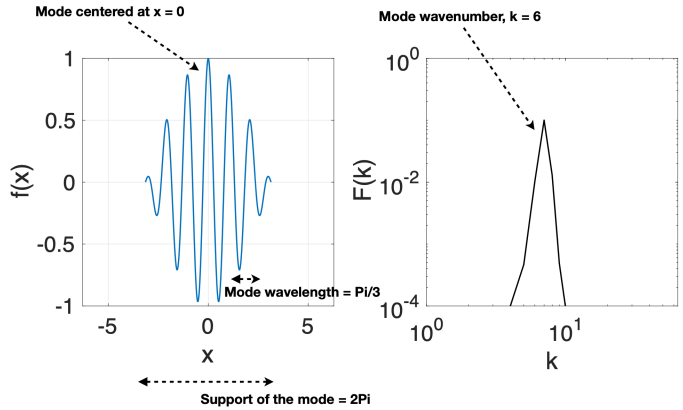
Wavelets can in principle provide **optimal spatio-spectral localization**

Could solve the representation issue – but most wavelet NS algorithms are **not much cheaper** than traditional CFD algorithms (See Farge, Annual Rev. Fluid Mech., 1992)

Special case: Gabor wavelet
$f_{supp}(\varepsilon(x - x_0))e^{ik\cdot(x-x_0)}$
has interesting properties applicable to quasi-homogeneous turbulence*

* See Ghate & Lele (J. Fluid Mech., 2020) for the math.

Physics Driven Model: Gabor modes

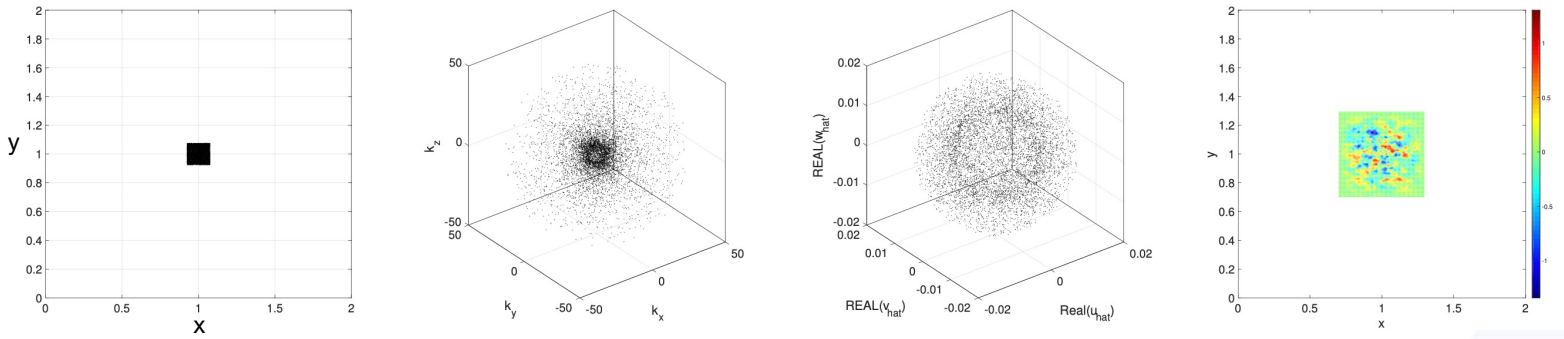


Typical compression in **Degrees of Freedom** > 95%

Physical space rendering can be performed using an $O\left(Nlog(N)\right)$ algorithm (See Ghate & Lele, J. Fluid Mech., 2017)

The scale separation parameter is defined as: $\varepsilon = \frac{\lambda}{\Delta}$

Multiple modes



Physics Driven Model: Governing equations (WKB-RDT)



Governing equations for Gabor modes

1. Motion described in a *sweeping* frame:

$$\partial_t x_j = U_j^0$$

2. An Eikonal equation for evolution of a wavenumber:

$$\partial_t k_j = -k_m \partial_j U_m^0$$

3. A WKB-RDT approximation for evolution of complex amplitude

$$\partial_t \hat{u}_i = \left(\frac{2k_i k_m}{k^2} - \delta_{im} \right) \hat{u}_j \partial_j U_m^0 + \left(\frac{k_i k_j}{k^2} - \delta_{ij} \right) g_j \beta \hat{\theta} - (\nu + \nu_t) k^2 \hat{a}_i + \hat{f}_i^{\perp} - 2\epsilon_{ijk} \Omega_j \hat{u}_k$$

$$\partial_t \hat{\theta} = -\hat{u}_j \partial_j \Theta^0 - (\kappa + \kappa_t) k^2 \hat{\theta} + \hat{f}_\theta$$

where, $\hat{f}_i$ and $\hat{f}_\theta$ are Gabor projections of the Leonard stress terms $\partial_j L_{ij}$ and $\partial_j q_j$ respectively.

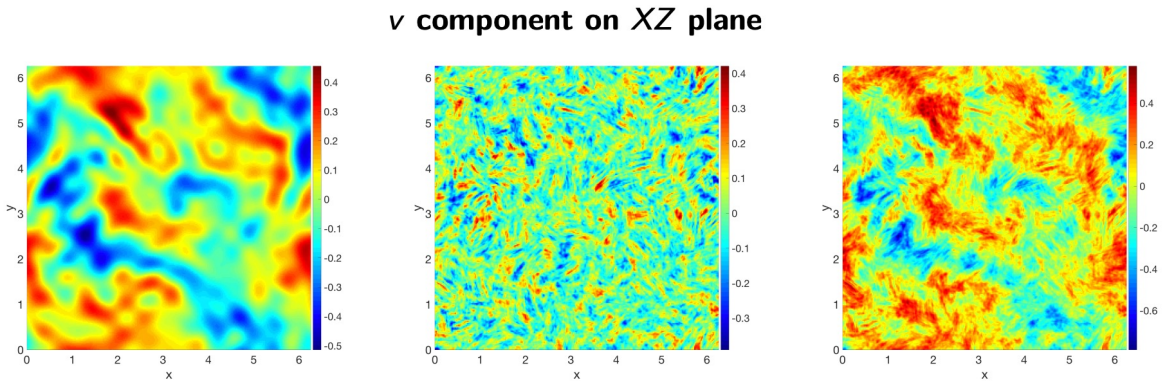
Important consideration: The ODEs governing evolution of the Gabor modes are only accurate up to leading order in ε; the proposed model is not a numerical method.

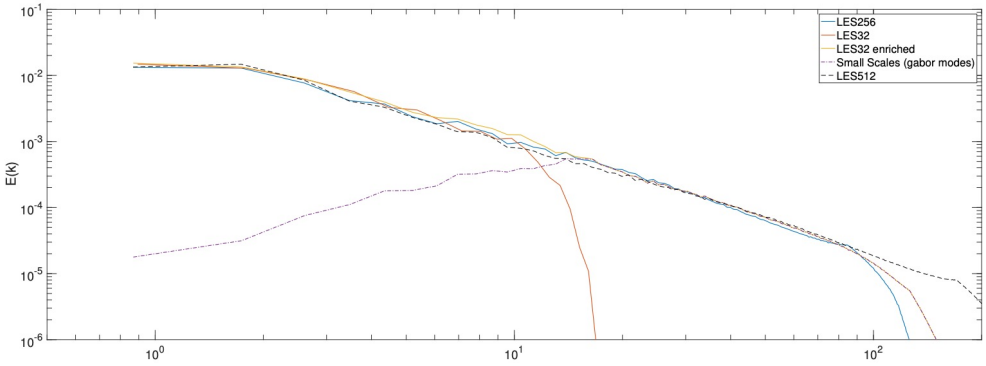
Model for the local (in scale space) convective non-linearity

The action of the convective non-linearity due to local triadic interactions will be modeled using a spectral viscosity based on Renormalization Group Theory (RNG) (see Canuto & Dubovikov, PoF, 1996)

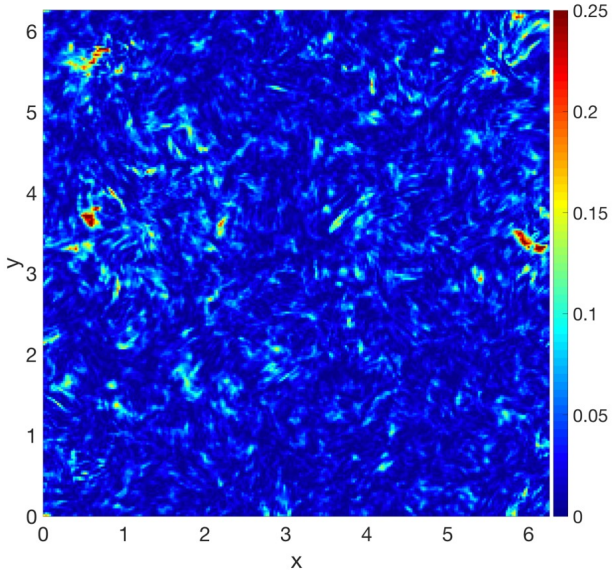
$$\widehat{\partial_j h_{ij}}^{\perp} = -\nu_t(k) k^2 \hat{u}_i \quad , \quad \nu_t(k) = \left(\nu^2 + c_\nu \int_k^\infty q^{-2} E(q) dq \right)^{1/2} - \nu$$

Physics Driven Model: Validation 1 – HIT

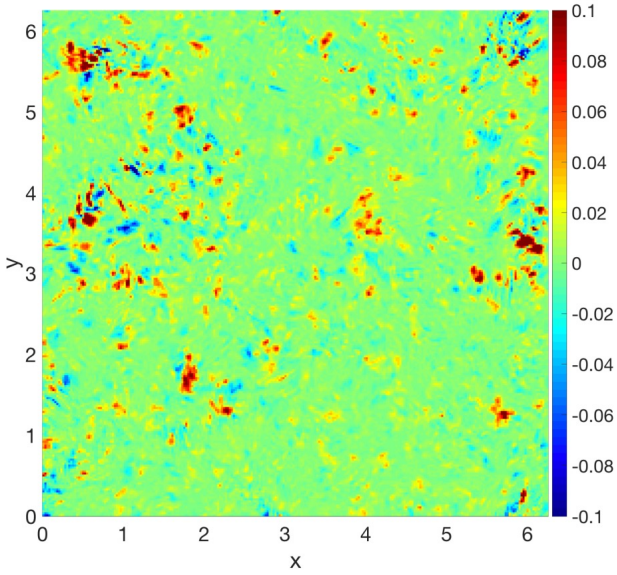




Physics Driven Model: Validation 1 – HIT



Small-scale Kinetic energy, $u_i^r u_i^r$



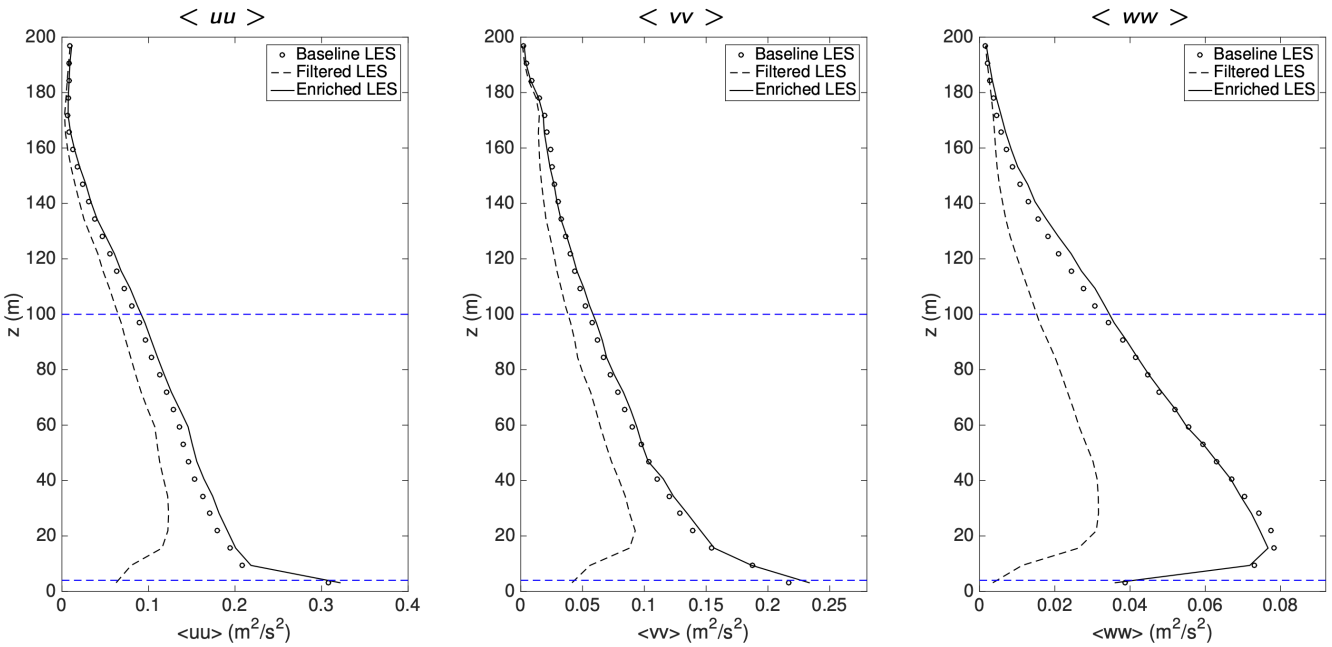
Inter-scale transfer, $\mathcal{I}_1 = -u_i^s u_j^s S_{ij}^r$

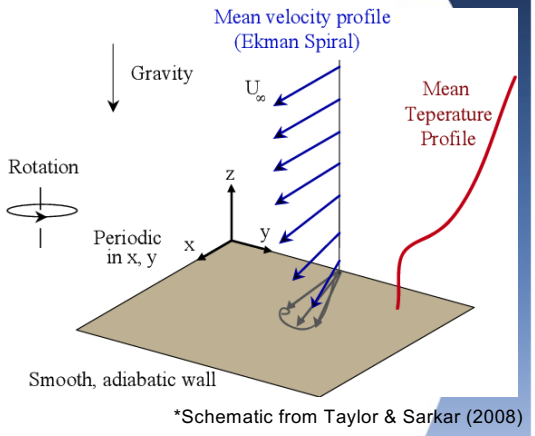
Global inter-scale energy transfers

1. $\mathcal{I}_1 = -\left\langle u_i^s u_j^s S_{ij}^r \right\rangle$: 0.025 (Gabor mode enrichment); 0.0255 (exact)
2. $\mathcal{I}_2 = -\left\langle L_{ij} S_{ij}^s \right\rangle$: 0.0018 (Gabor mode enrichment); 0.0016 (exact)

Physics Driven Model: Validation 2 – Stratified Ekman Layer

Single-point variances $< uu >$, $< vv >$ and $< ww >$ profiles in z





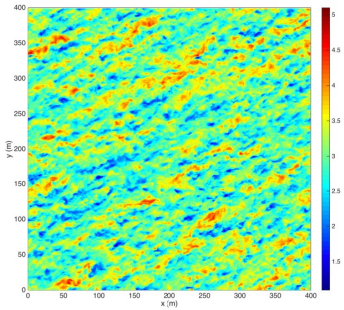
*Schematic from Taylor & Sarkar (2008)

*Recall that due to periodicity in wall-tangent directions, Fourier-filtering is same as truncated POD expansion

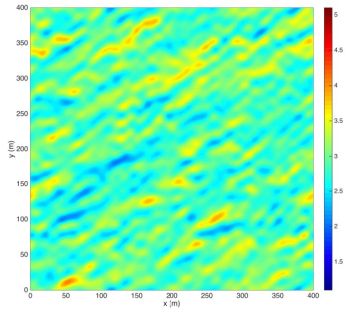
Physics Driven Model: Validation 2 – Stratified Ekman Layer

Contours of u on $z = 6$m plane

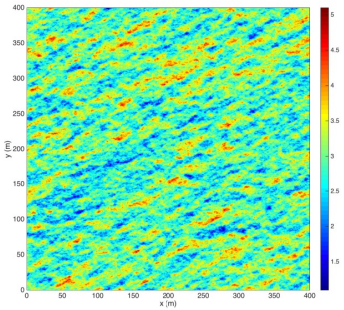
Baseline LES



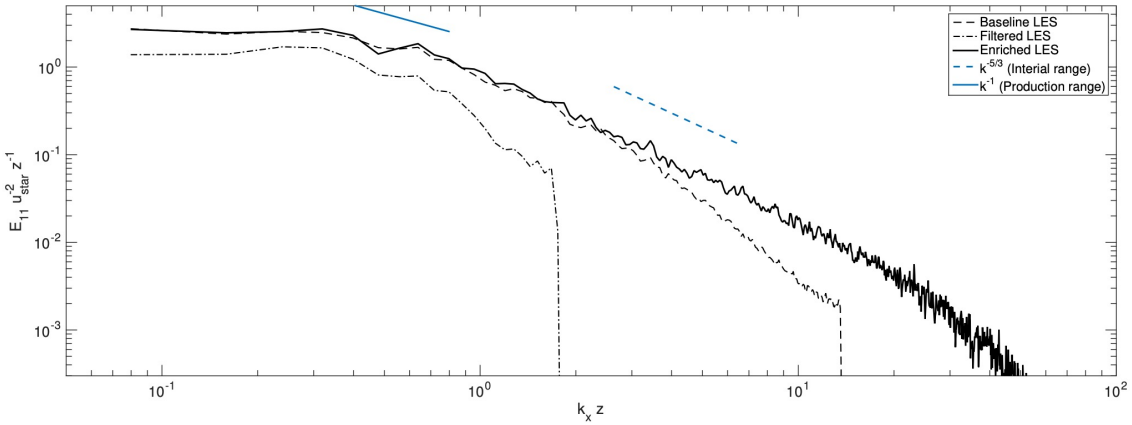
Coarse LES



Enriched LES



k_x spectra of u on $z = 6$m plane



*See Ghate & Lele (J. Fluid Mech., 2017) for additional details of this case.

Outline

1. Two Problems:
   a. Actuator Disk wake + HIT co-flow
   b. LAGOON Landing Gear wake

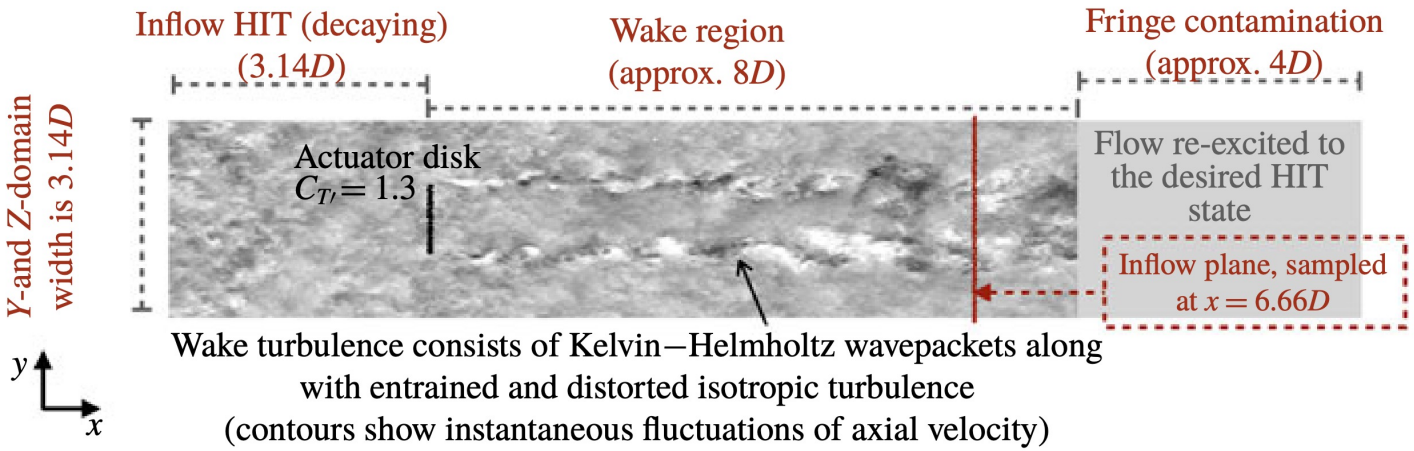
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   a. Global modes: SPOD with TKE norm
   b. The best-case scenario: Truncated Expansions
   c. What's missing? What's needed?

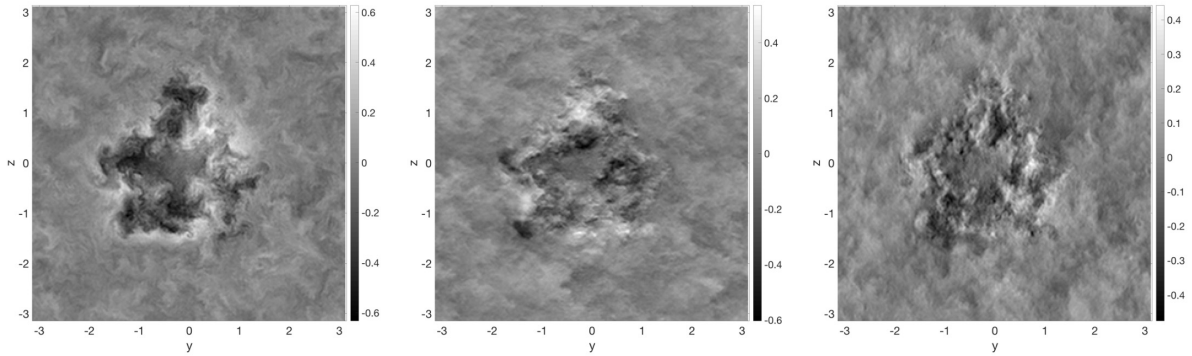
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   b. WKB-variant of Rapid-Distortion theory
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4. Results:
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   b. LAGOON Landing Gear wake: Fields and Reynolds stresses

5. Outlook:
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   b. Is this just an academic exercise, or is any of this useful?

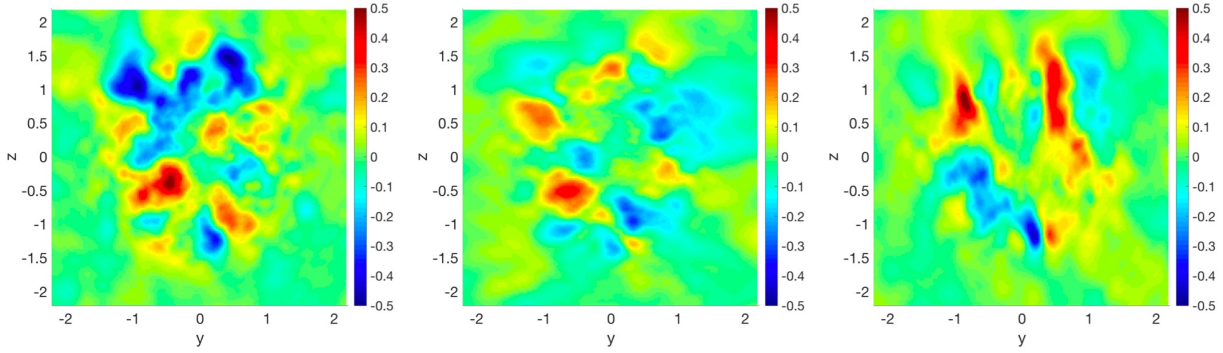
Actuator Disk Wake





(d) u (fluctuation) (e) v (fluctuation) (f) w (fluctuation)

The combined model: Actuator Disk wake



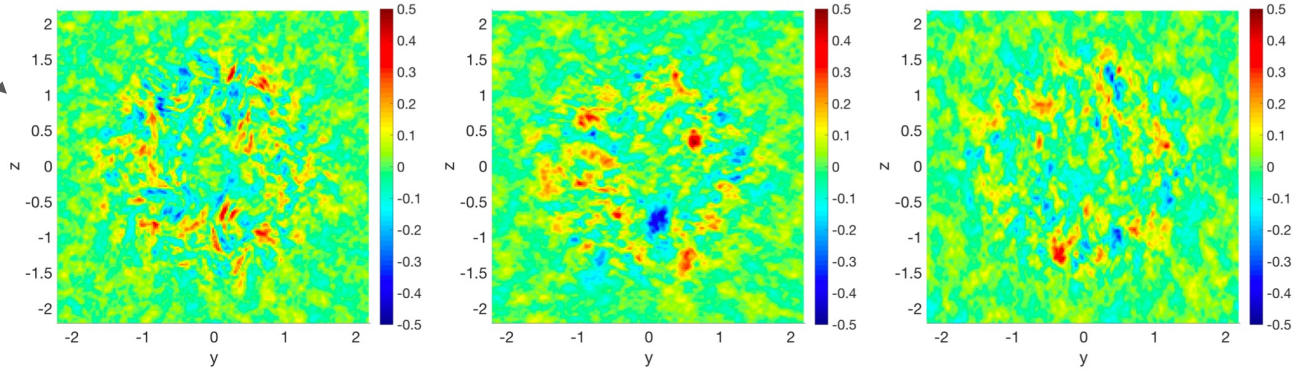
(a) $u^{\mathrm{trunc}}(y,z)$ (b) $v^{\mathrm{trunc}}(y,z)$ (c) $w^{\mathrm{trunc}}(y,z)$

ACTUATOR DISK WAKE

DATA DRIVEN
(Truncated SPOD)

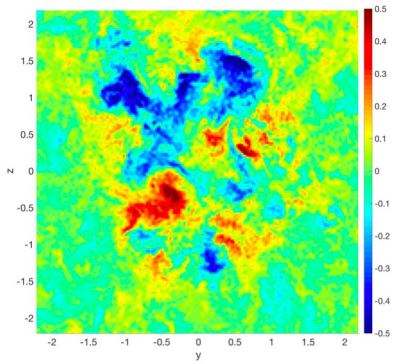
Inputs to the physics-driven model (along with gradients + mean flow)

PHYSICS DRIVEN
(Gabor Modes)

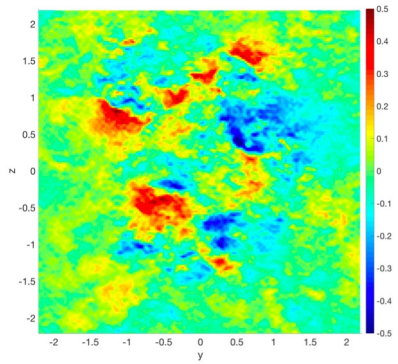


(d) $u^{\mathrm{gab}}(y,z)$ (e) $v^{\mathrm{gab}}(y,z)$ (f) $w^{\mathrm{gab}}(y,z)$

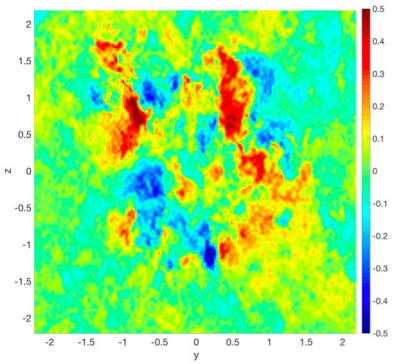
The combined model: Actuator Disk wake



$u(y,z)$

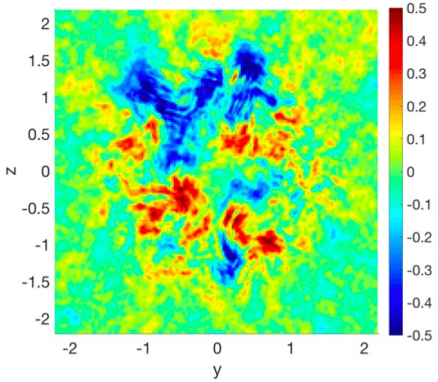


$v(y,z)$

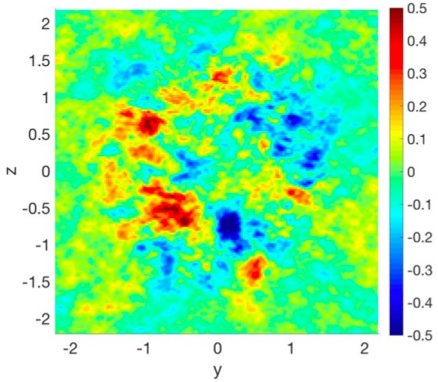


$w(y,z)$

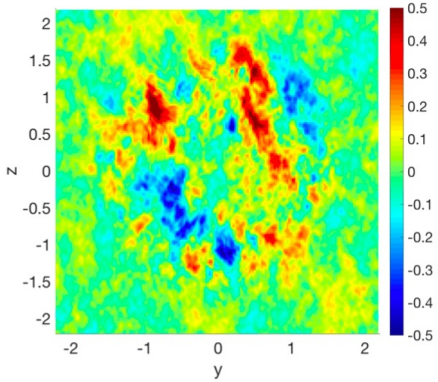
REFERENCE
(HIGH RESOLUTION LES)



$u^{\text{enrich}}(y,z)$



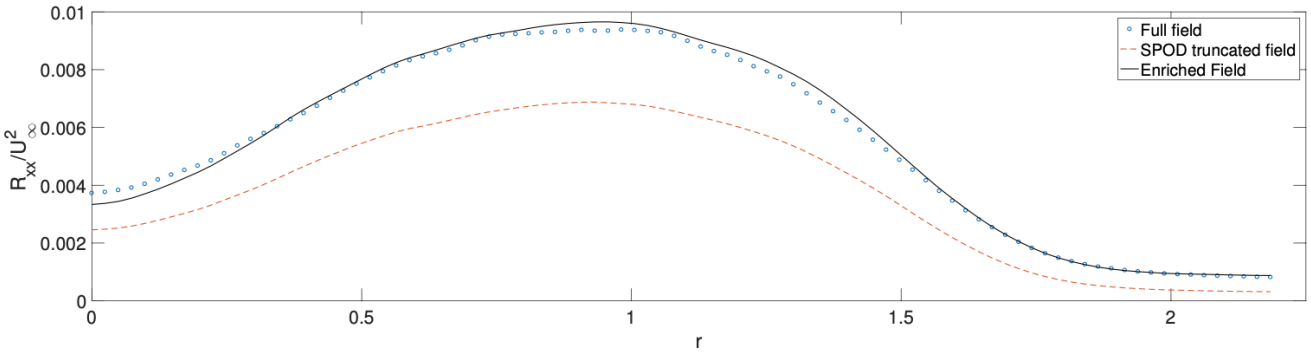
$v^{\text{enrich}}(y,z)$



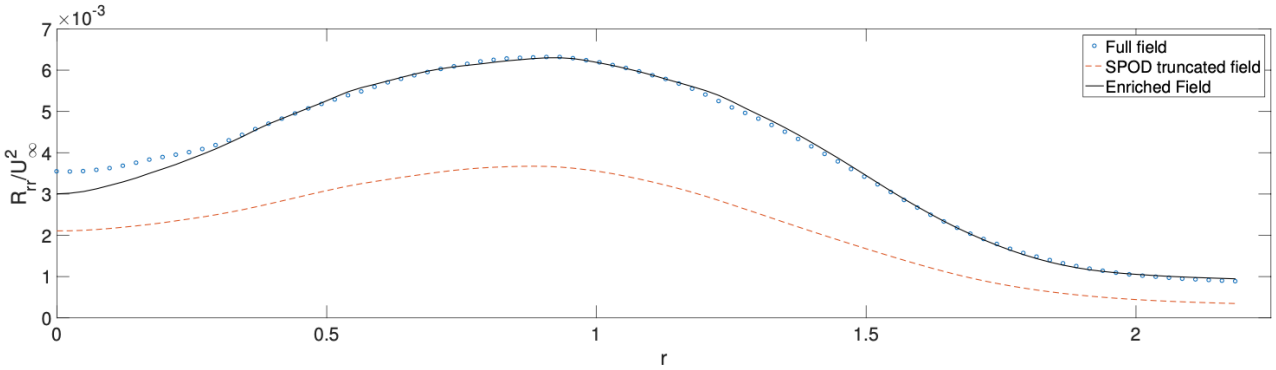
$w^{\text{enrich}}(y,z)$

MODELLED
(SPOD + GABOR
MODES)

The combined model: Actuator Disk wake

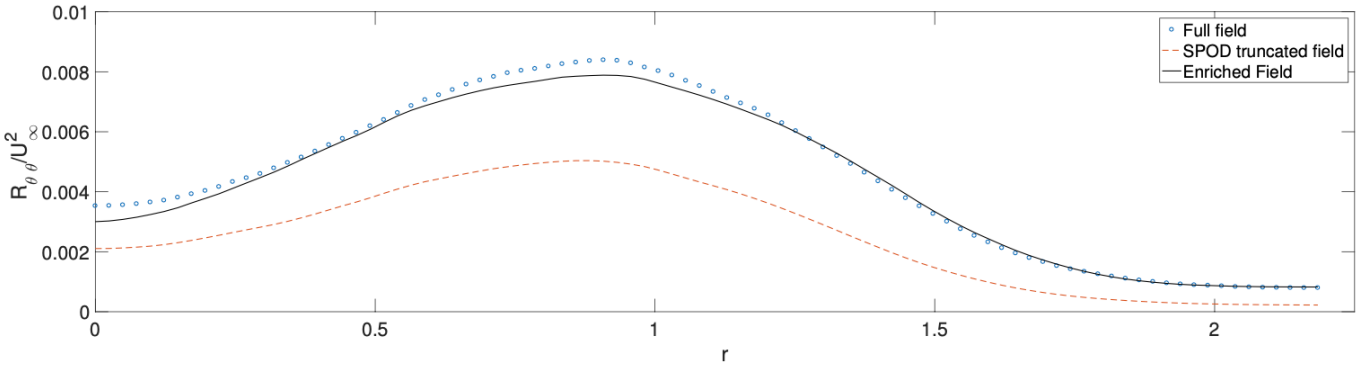


(a) $\langle u_x u_x \rangle$

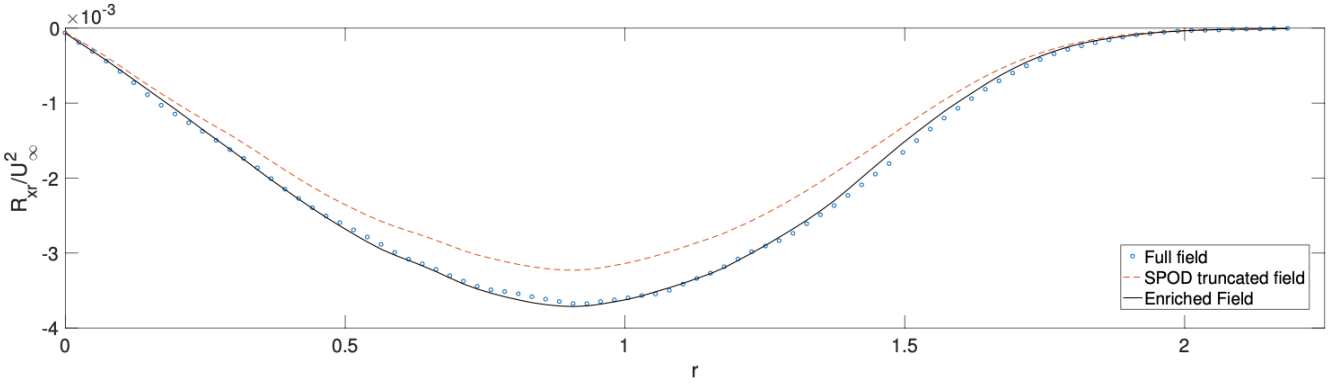


(b) $\langle u_r u_r \rangle$

The combined model: Actuator Disk wake

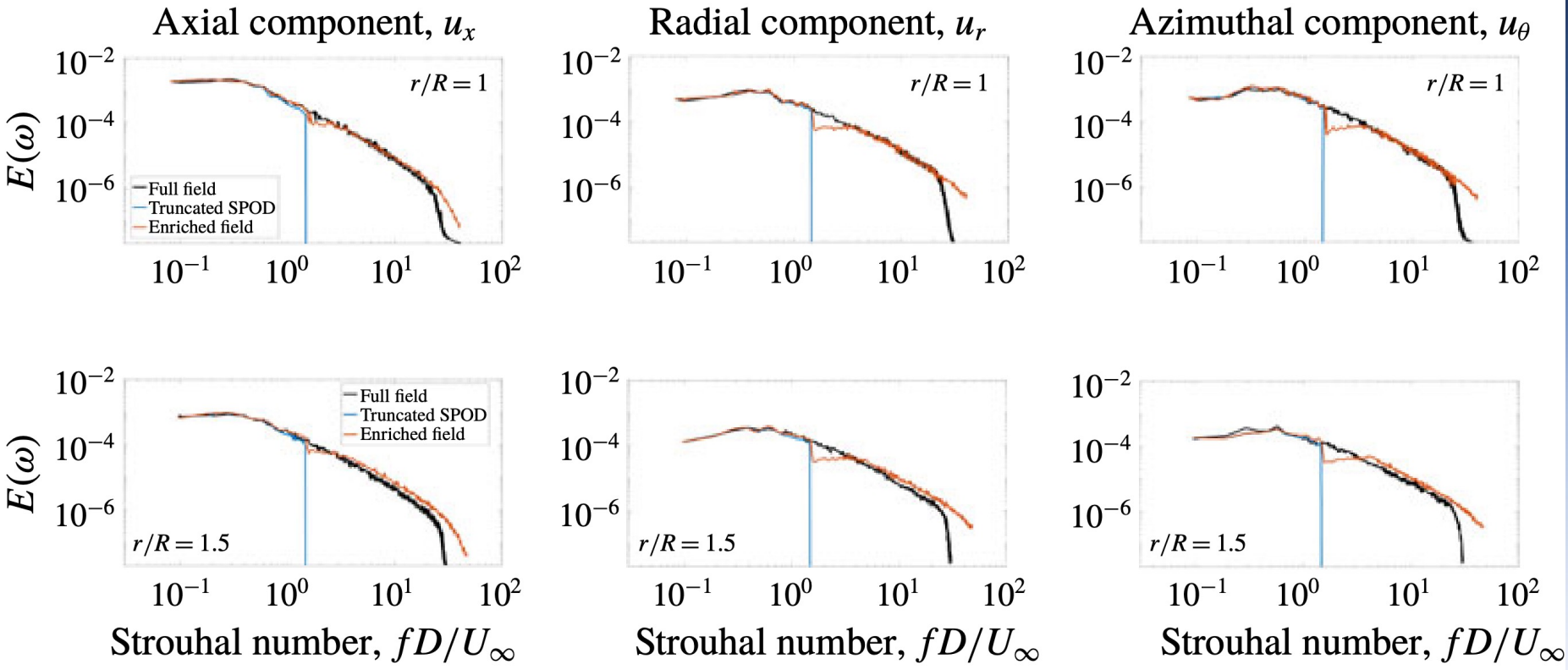


(c) $\langle u_\theta u_\theta \rangle$

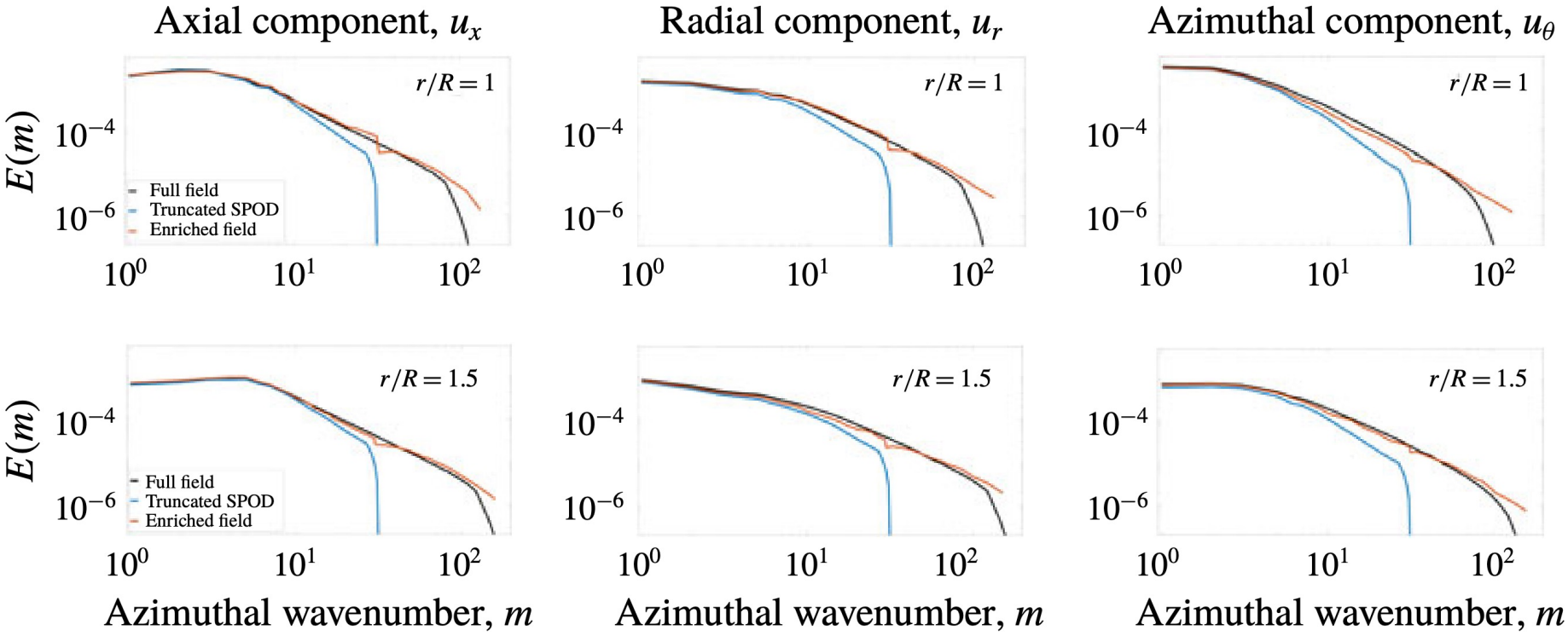


(d) $\langle u_x u_r \rangle$

The combined model: Actuator Disk wake

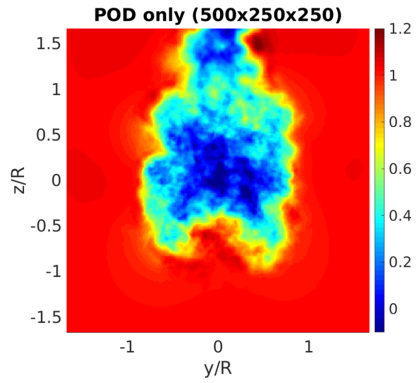


The combined model: Actuator Disk wake

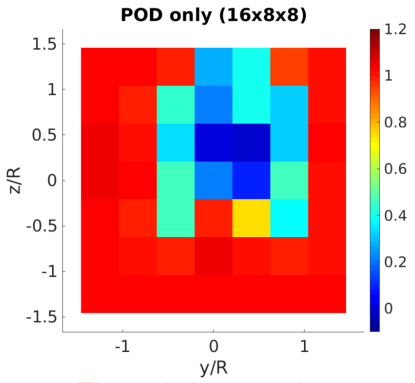


The combined model: LAGOON Landing Gear wake

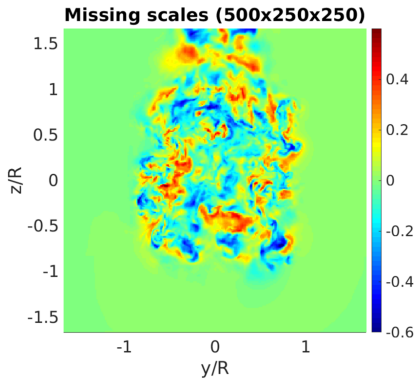
- Landing Gear case is **substantially more complicated**:
  - While SPOD-truncation is a low-pass filter in time; interpretation as a physical space filter is more complicated
  - **Massive inhomogeneity** in all spatial directions - presence of a **"turbulence interface"** separating vortical region from potential flow region
- Use a 2-level enrichment strategy
  - Step 1: Evaluate the truncated POD solution on a coarse-grid that can represent it
    SCALE 1: $16 \times 8 \times 8$ points
  - Step 2: Physics driven model generates SCALE 2 using SCALE 1
    SCALE 2: $64 \times 32 \times 32$ points
  - Step 3: Physics driven model generates SCALE 3 using SCALE 1 and SCALE 2
    SCALE 3: $500 \times 250 \times 250$ points



Best-case scenario using data-only model



The only input to the physics-driven model (including gradients)

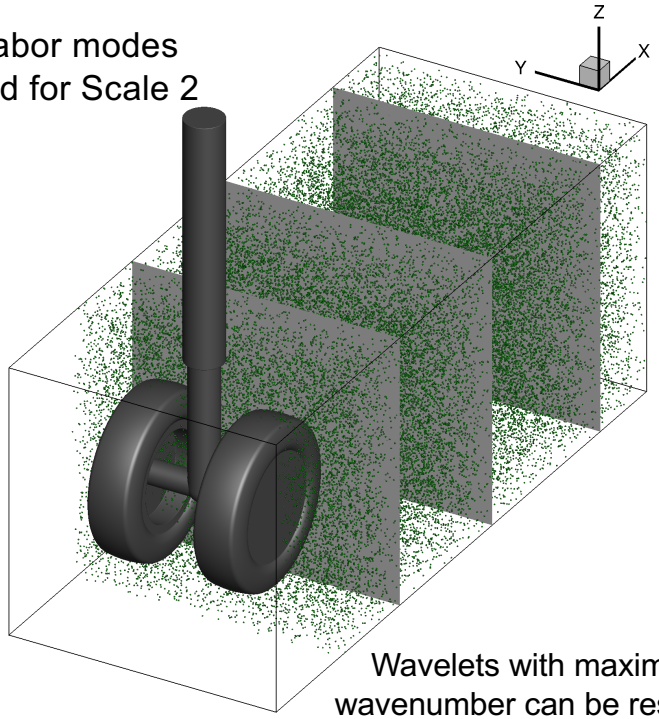


Expected output from physics-driven model should be statistically equivalent to this

u_x component shown on the y-z plane located at $x/R = 1.33$

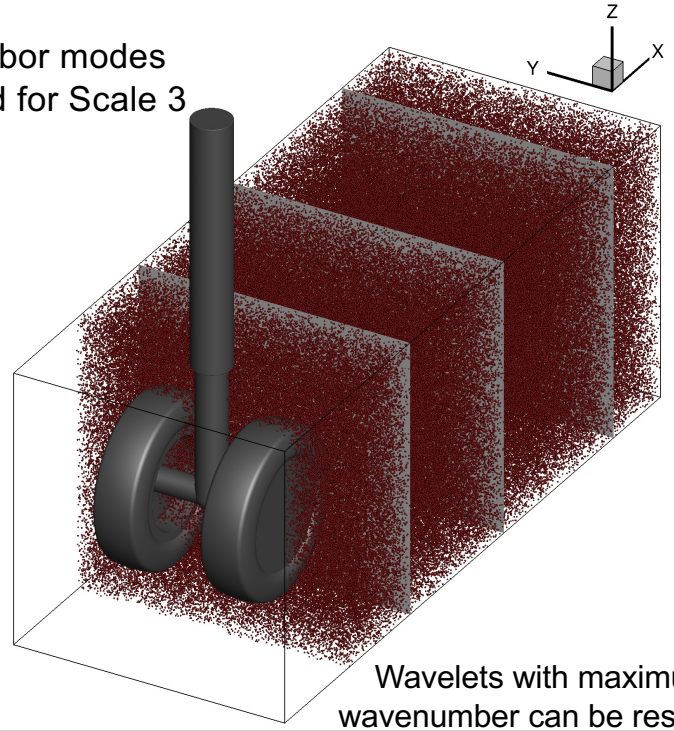
The combined model: LAGOON Landing Gear wake

Gabor modes used for Scale 2



Wavelets with maximum wavenumber can be resolved on a $64\times32\times32$ mesh

Gabor modes used for Scale 3

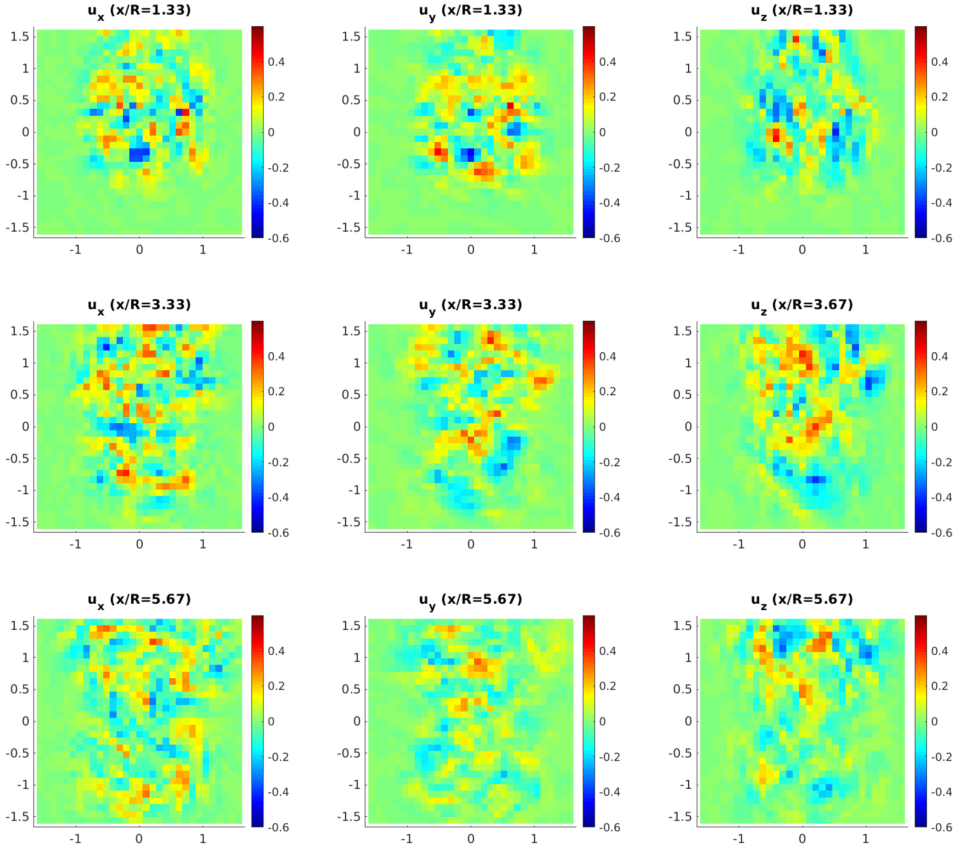


Wavelets with maximum wavenumber can be resolved on a $500\times250\times250$ mesh

Mode locations shown at an arbitrary time

The combined model: LAGOON Landing Gear wake

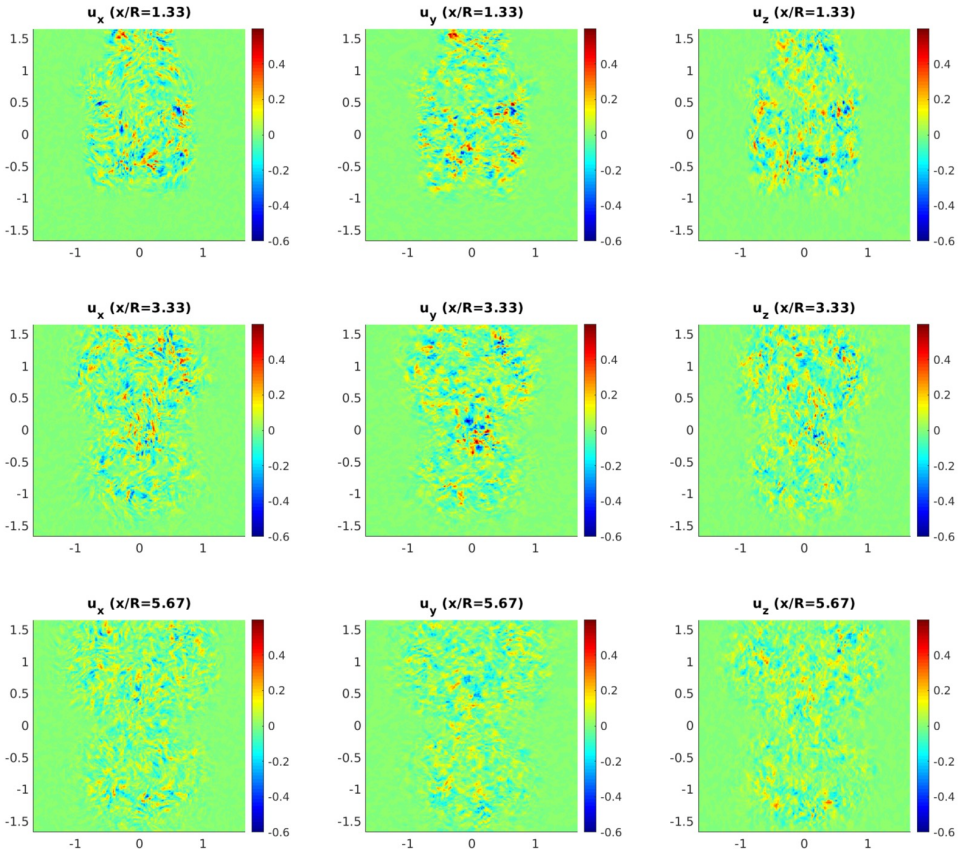
Model Output for Scale 2:



Data from Scale 2 is rendered on a $64 \times 32 \times 32$ grid

The combined model: LAGOON Landing Gear wake

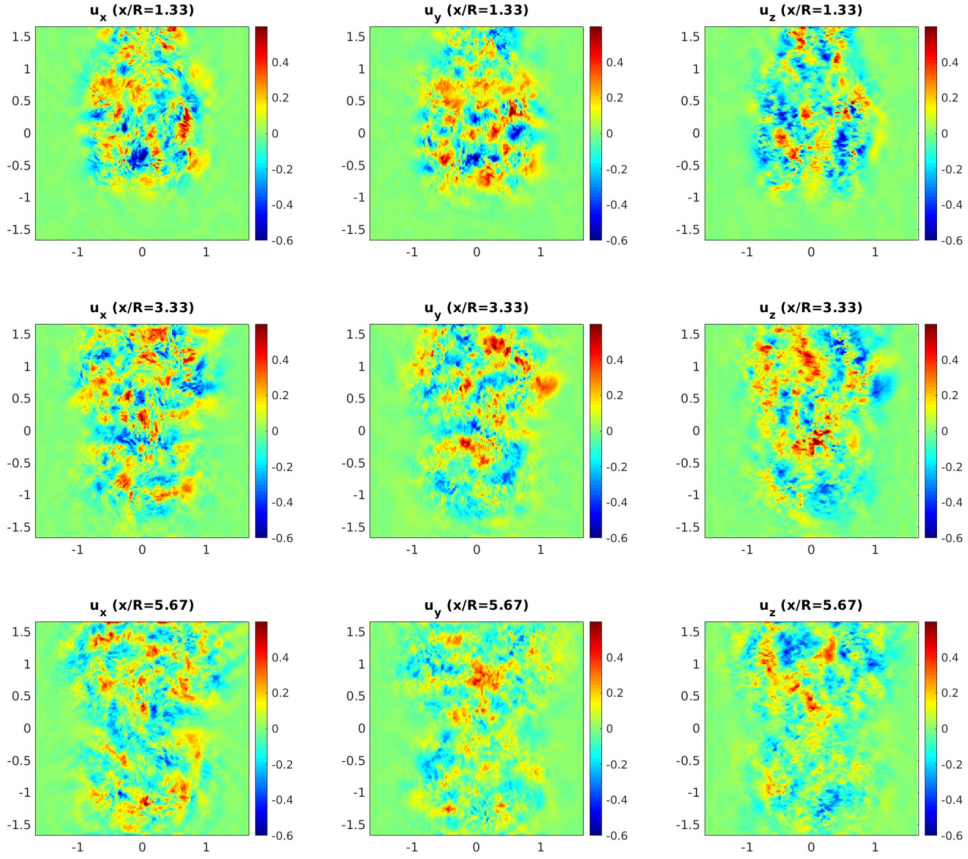
Model Output for Scale 3:



Data from Scale 3 is rendered on a 500×250×250 grid

The combined model: LAGOON Landing Gear wake

Combining Scales 2 and 3 recovers the “missing scales”!

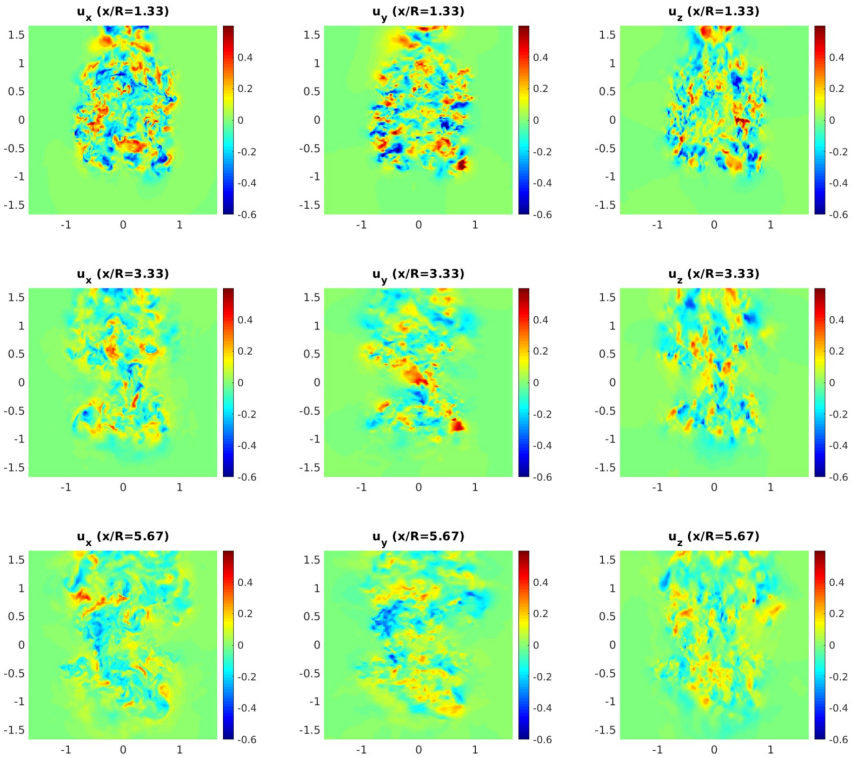


Data from Scale 2 is interpolated (cubic splines) on a $500\times250\times250$ and added to Scale 3 data.

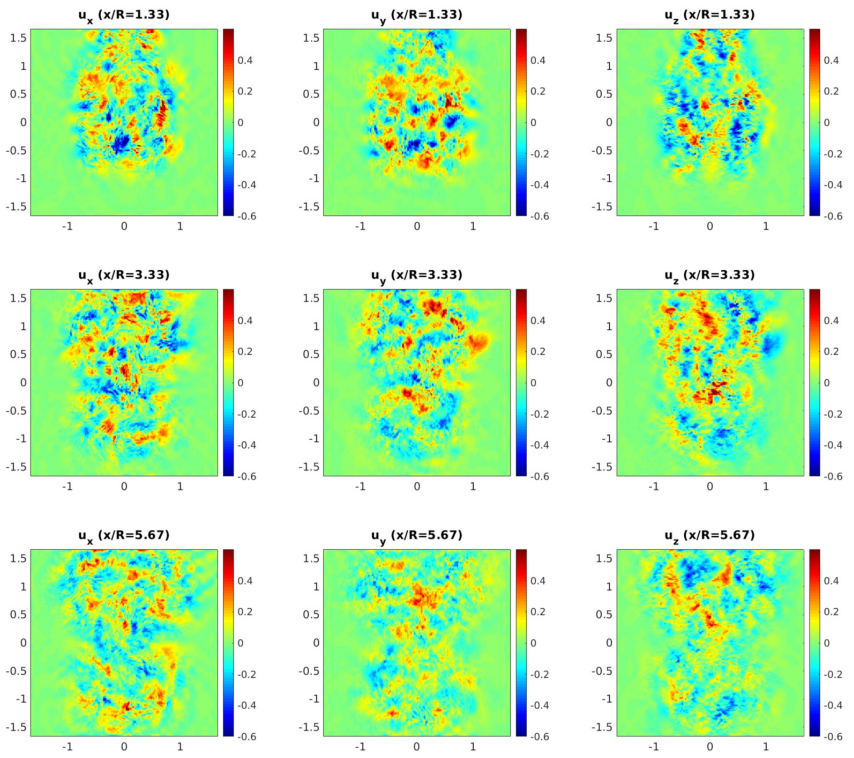
The combined model: LAGOON Landing Gear wake

Comparing with the “true” missing scales (recall that the **equivalence is statistical and NOT pointwise**)

Missing = Reference LES – Truncated POD



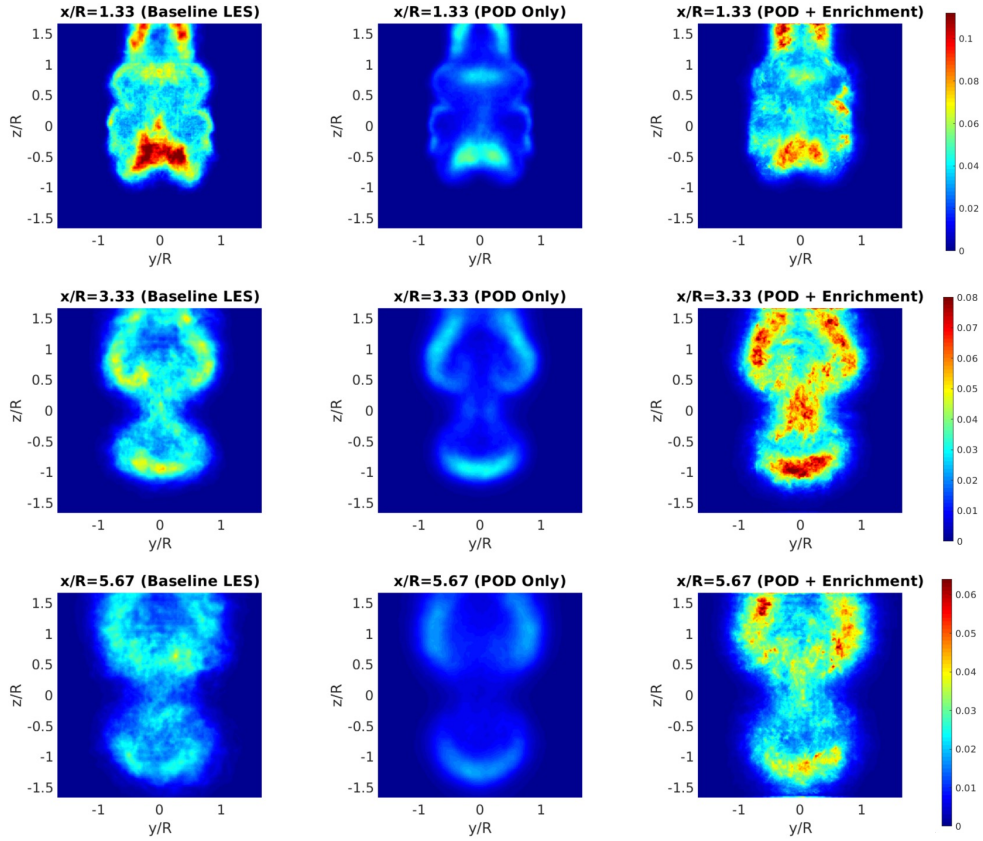
Model Generated



The combined model: LAGOON Landing Gear wake

Comparing single-point correlations (Reynolds stresses)

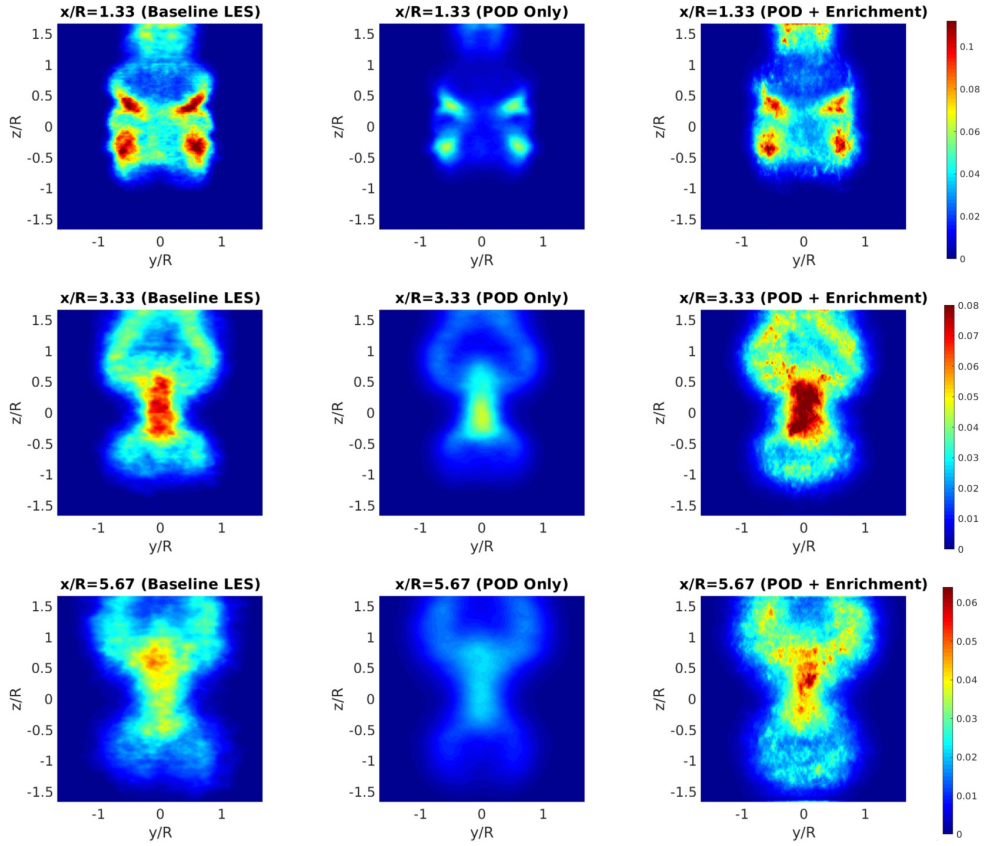
Normal-stress, R_{11}



The combined model: LAGOON Landing Gear wake

Comparing single-point correlations (Reynolds stresses)

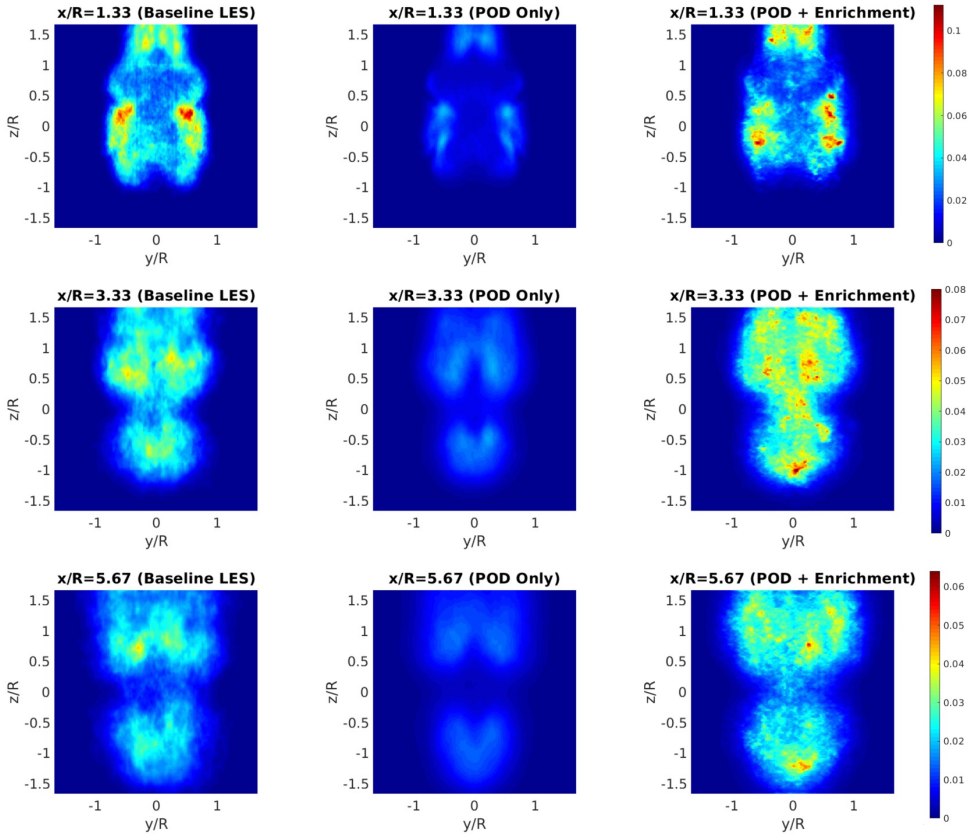
Normal-stress, R_{22}



The combined model: LAGOON Landing Gear wake

Comparing single-point correlations (Reynolds stresses)

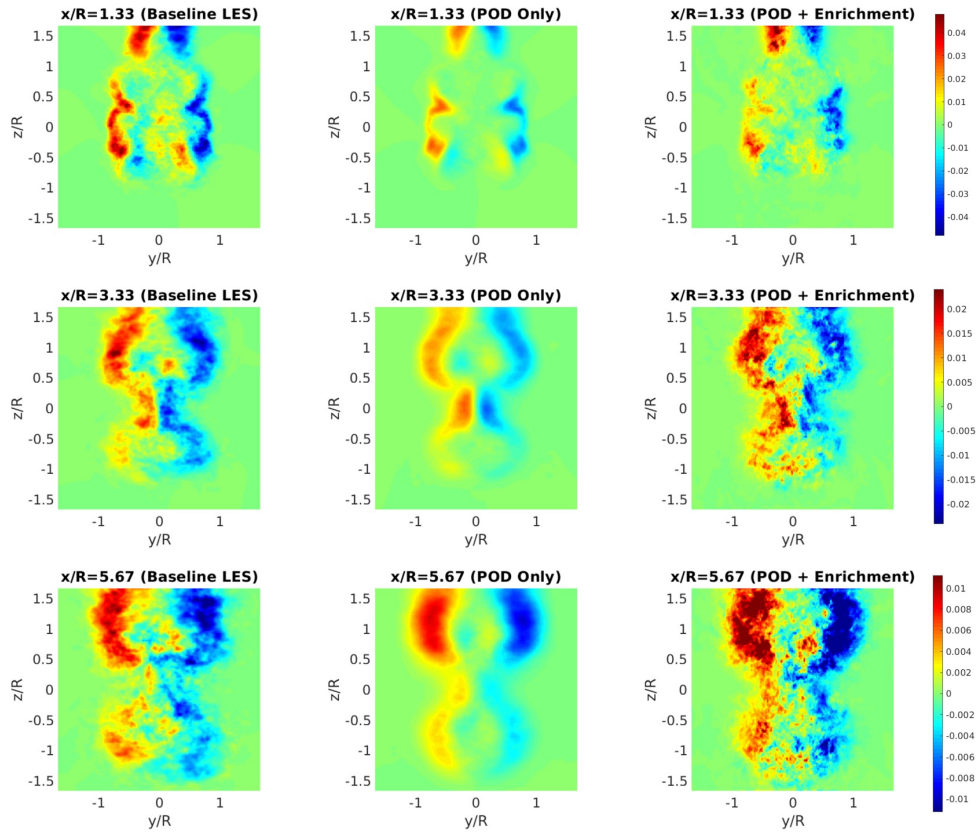
Normal-stress, R_{33}



The combined model: LAGOON Landing Gear wake

Comparing single-point correlations (Reynolds stresses)

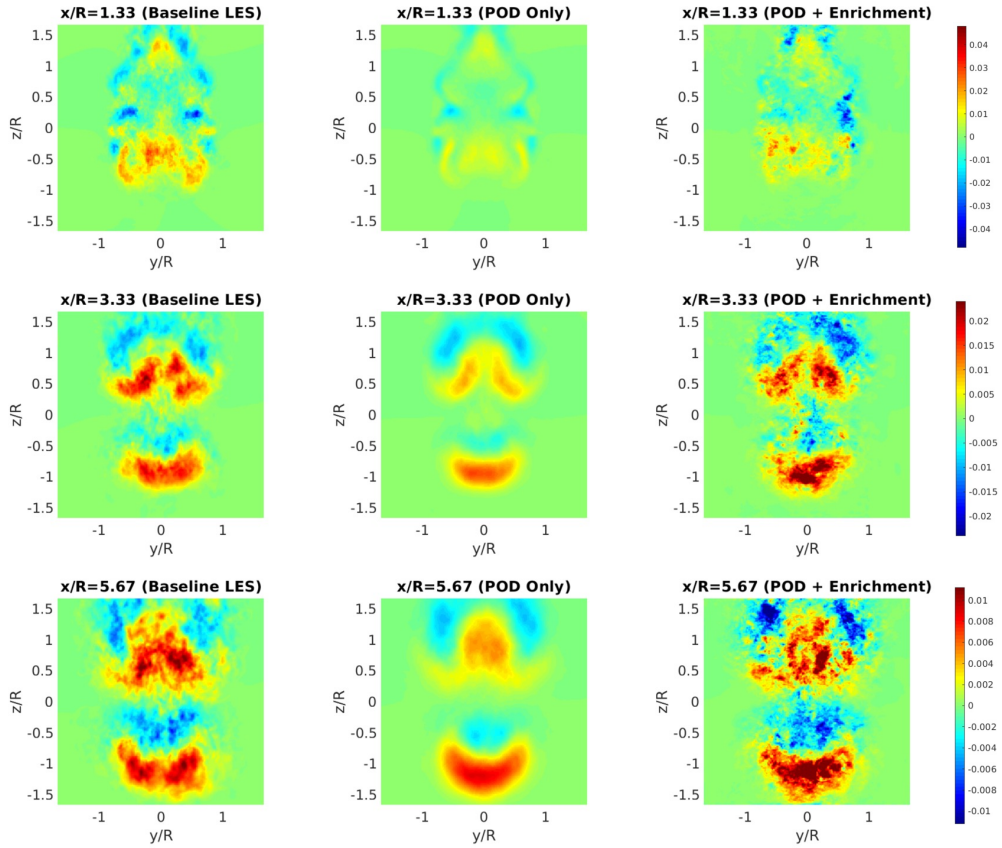
Cross-stress, R_{12}



The combined model: LAGOON Landing Gear wake

Comparing single-point correlations (Reynolds stresses)

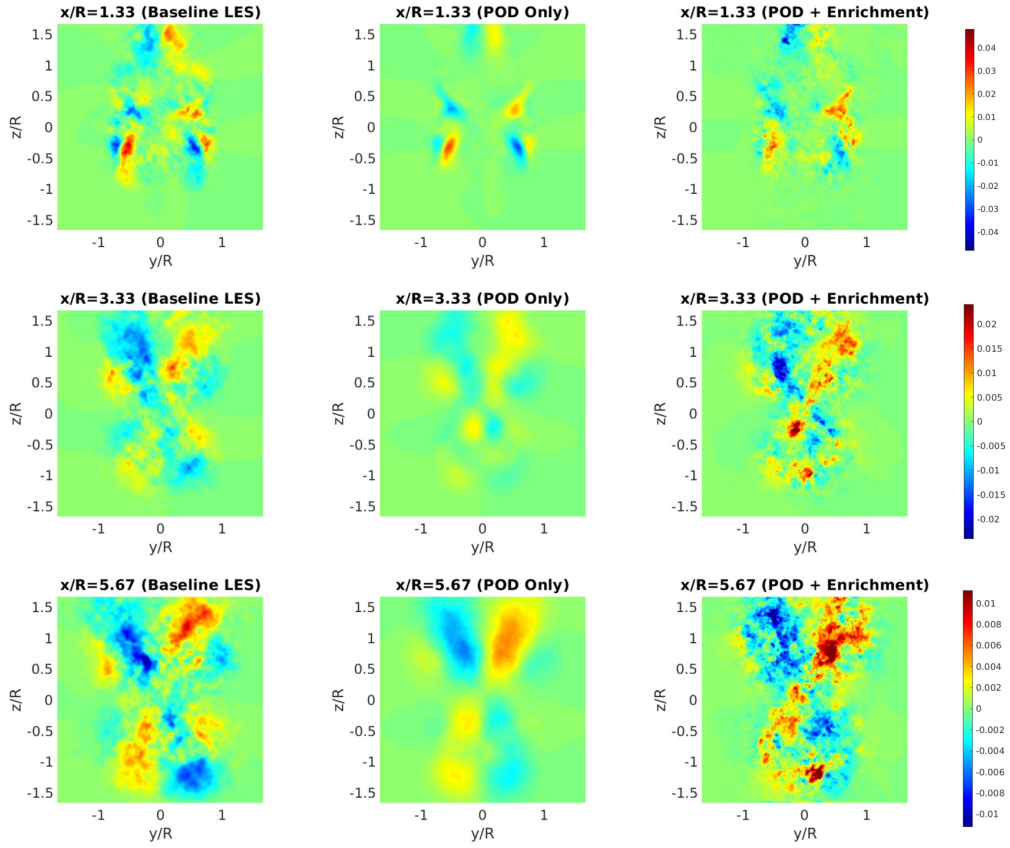
Cross-stress, R_{13}



The combined model: LAGOON Landing Gear wake

Comparing single-point correlations (Reynolds stresses)

Cross-stress, R_{23}



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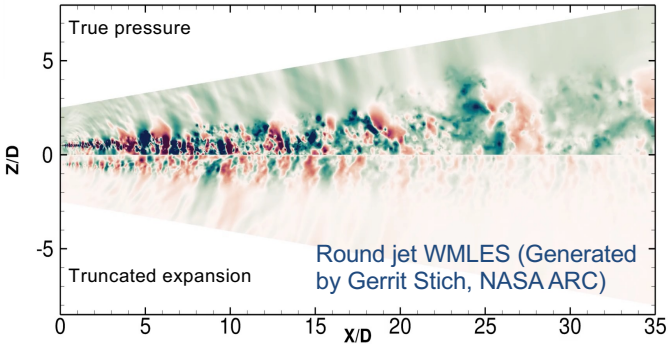
Summary

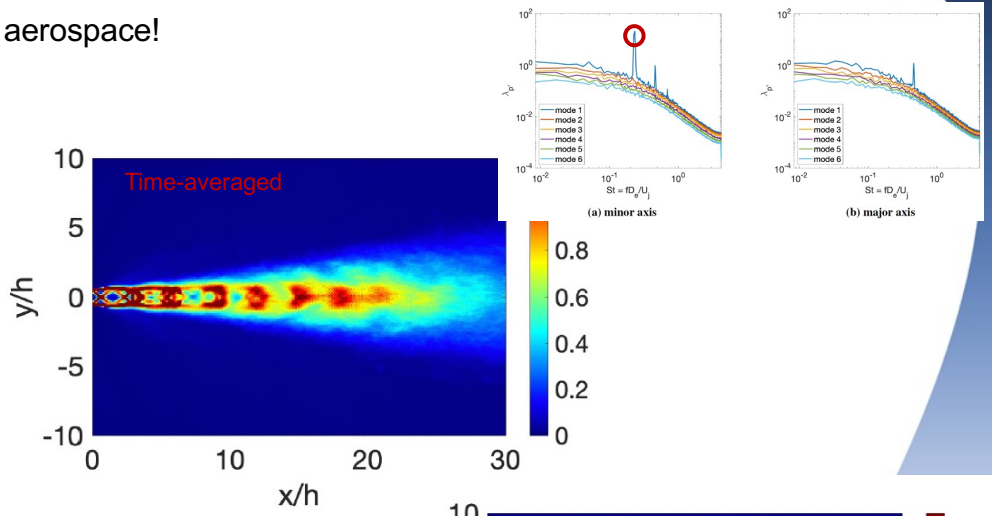


- Presented a strategy to model (synthesize) broadband turbulence based on a combination of data and physics-driven modeling
- Global-basis models based on data-driven “learning” are excellent at capturing flow-instability events (such as rapid growth caused by shear-layer instability) – highly geometry/boundary-condition dependent physics
- Problem of dimensionality persists in data-driven modeling: real world problems do not have homogeneous directions (would need unfeasible amounts of data)
- Plenty of room for physics-driven models to still thrive in the “age of big-data”:
  - Proposed model inherits all the invariants of incompressible Navier-Stokes (Galilean, rotational symmetries)
  - Easily generalizable for other physics: stratification and frame-rotation are naturally accounted
  - Same model work for a range of flows: HIT, Stratified Ekman boundary-layers, free-shear flows
- Ideal of using truncated POD reconstruction is useful – low-order modes can be generated using coarse-grid LES
- Linear projection based models may be highly limiting – lots of potential for NNs to obtain more generalized representations

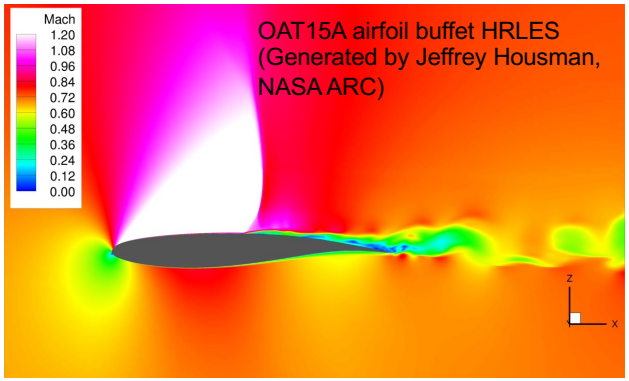
Is this model purely academic?

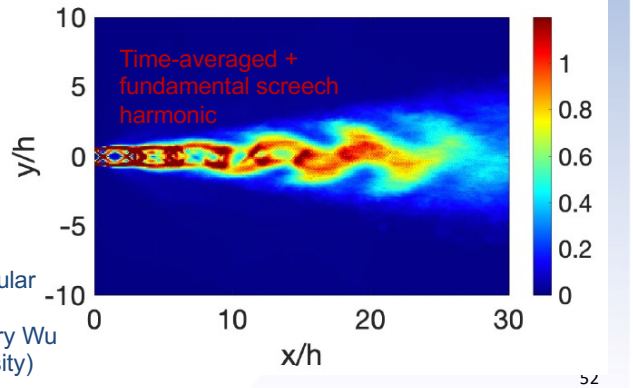
- Not necessarily. Many relevant applications just in aerospace!
- Problems with well-understood low-rank physics:





- Problems with well-defined scale-separation:





LES of under-expanded rectangular jet under screech conditions (Generated by Olivia Martin, Gary Wu & Sanjiva Lele, Stanford University)

Acknowledgements

- Financial support from ARMD T^3 project under an RCA task
- NSF support through CBET-1803378
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- Michael Barad and Jeffrey Housman for internal review and feedback