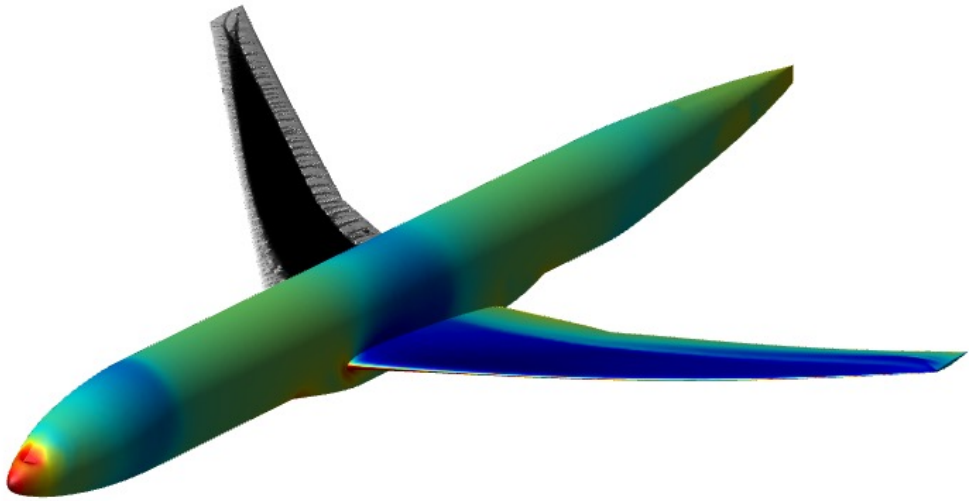
Aero-structural Topology Optimization Framework for Transonic Aircraft Wing box Design



Prateek Ranjan

Advanced Modeling & Simulation (AMS) Seminar Series

NASA Ames Research Center, December 9th, 2021

ILLINOIS

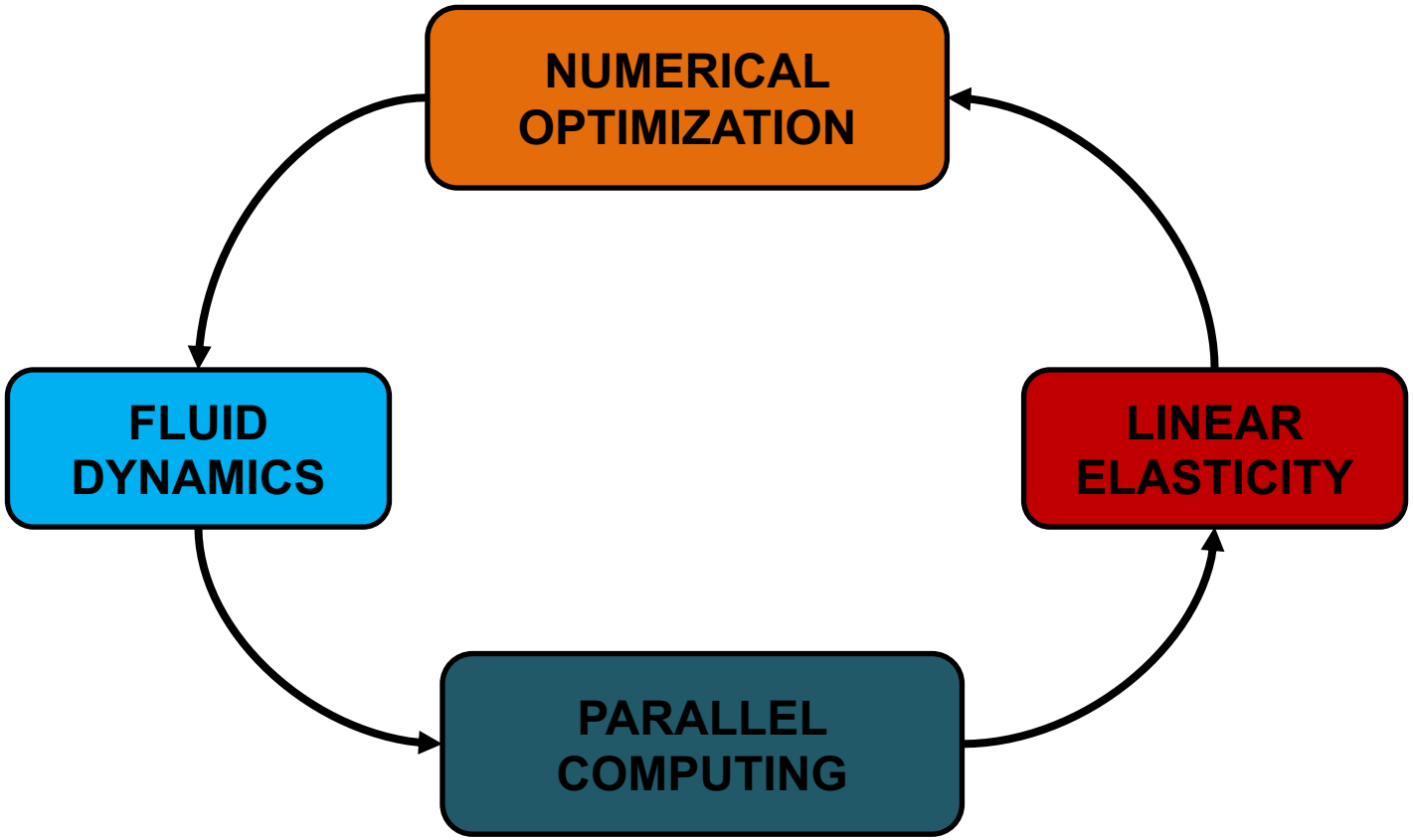
CDiLAB
Computational•Design•Innovation•Lab

NASA

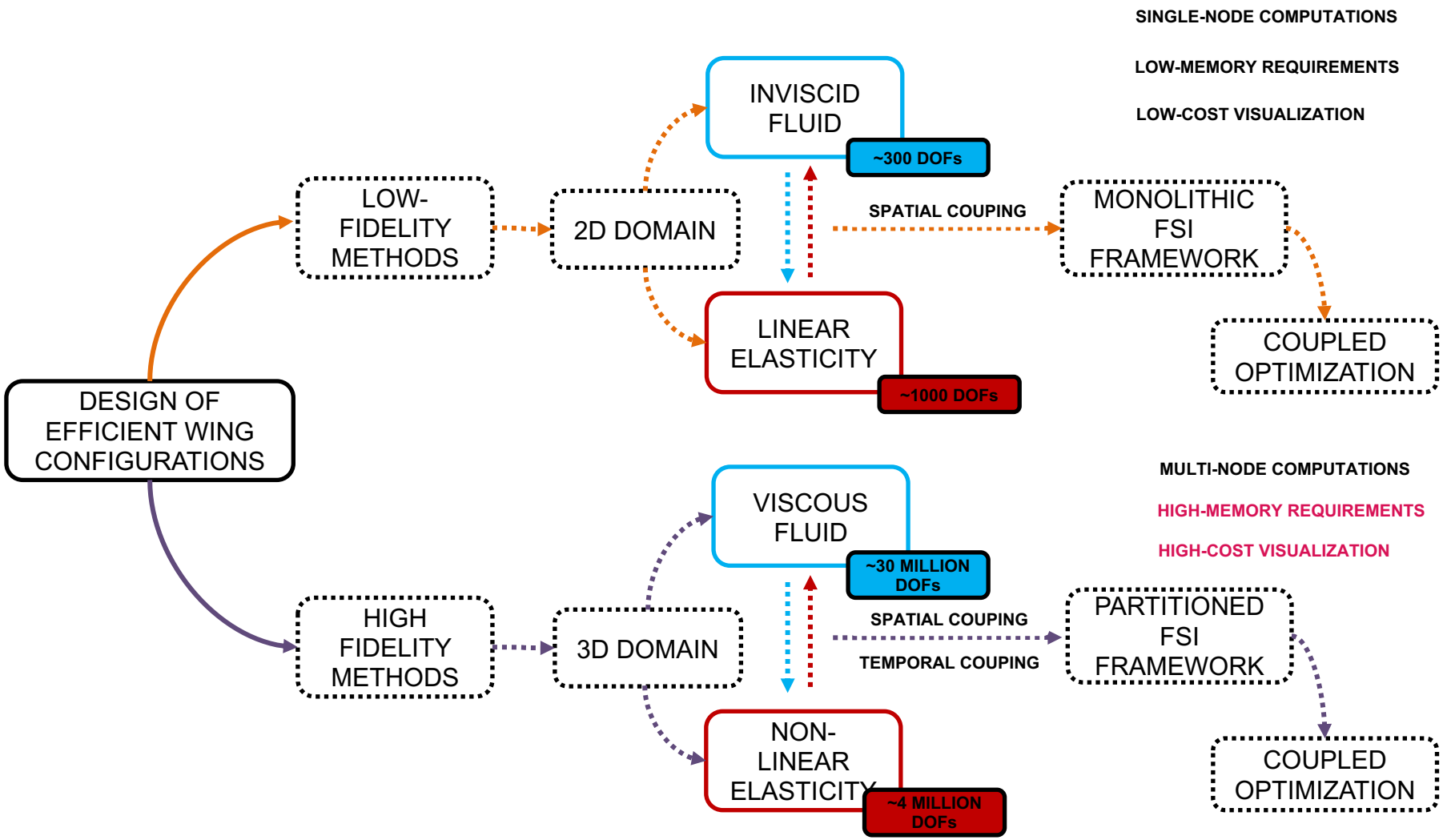
Outline

- Research Trajectory
- Motivation
- Structural Topology Optimization
- MDA Framework
- Adjoint Sensitivity Calculation
- Preliminary Results

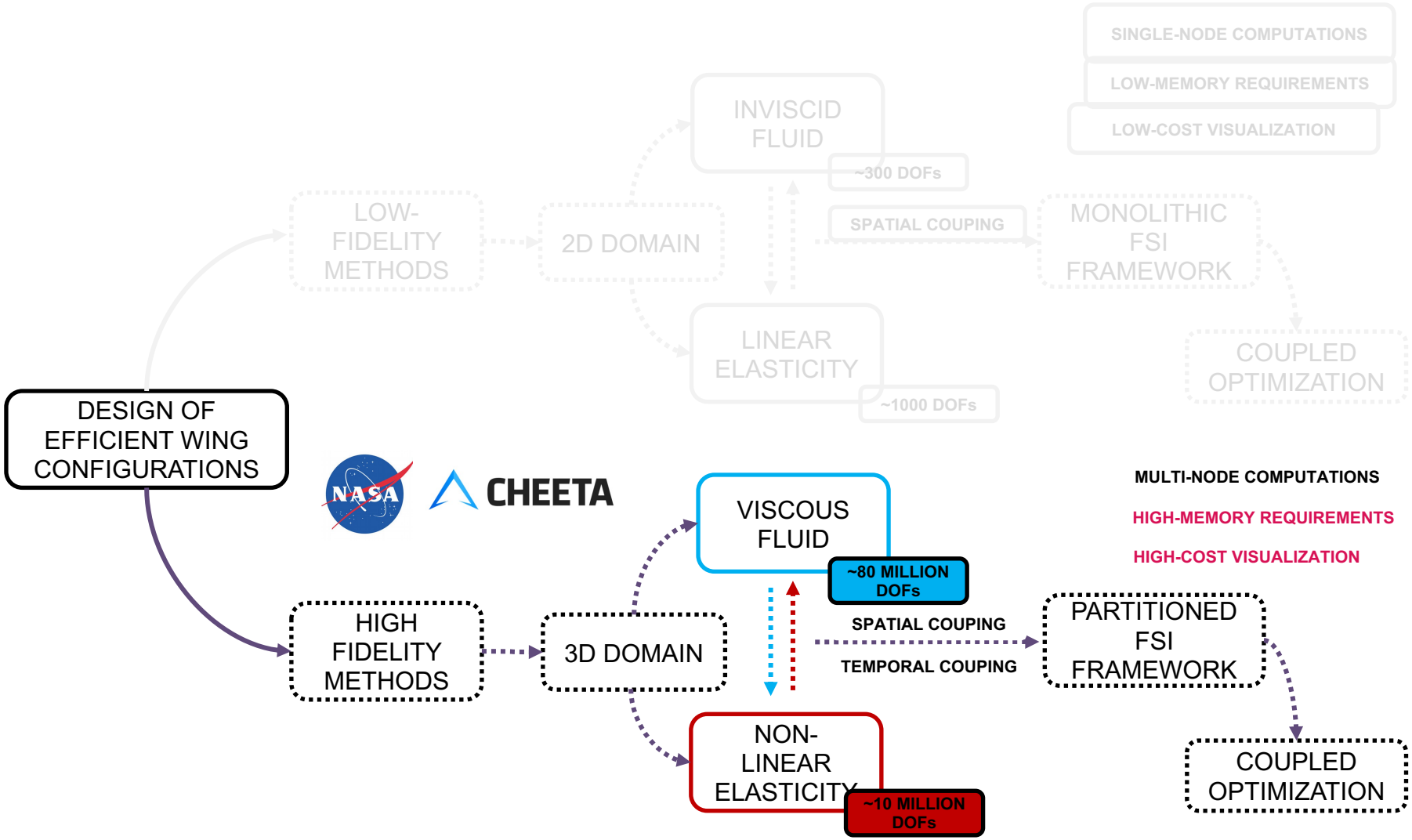
Broad Research Principle



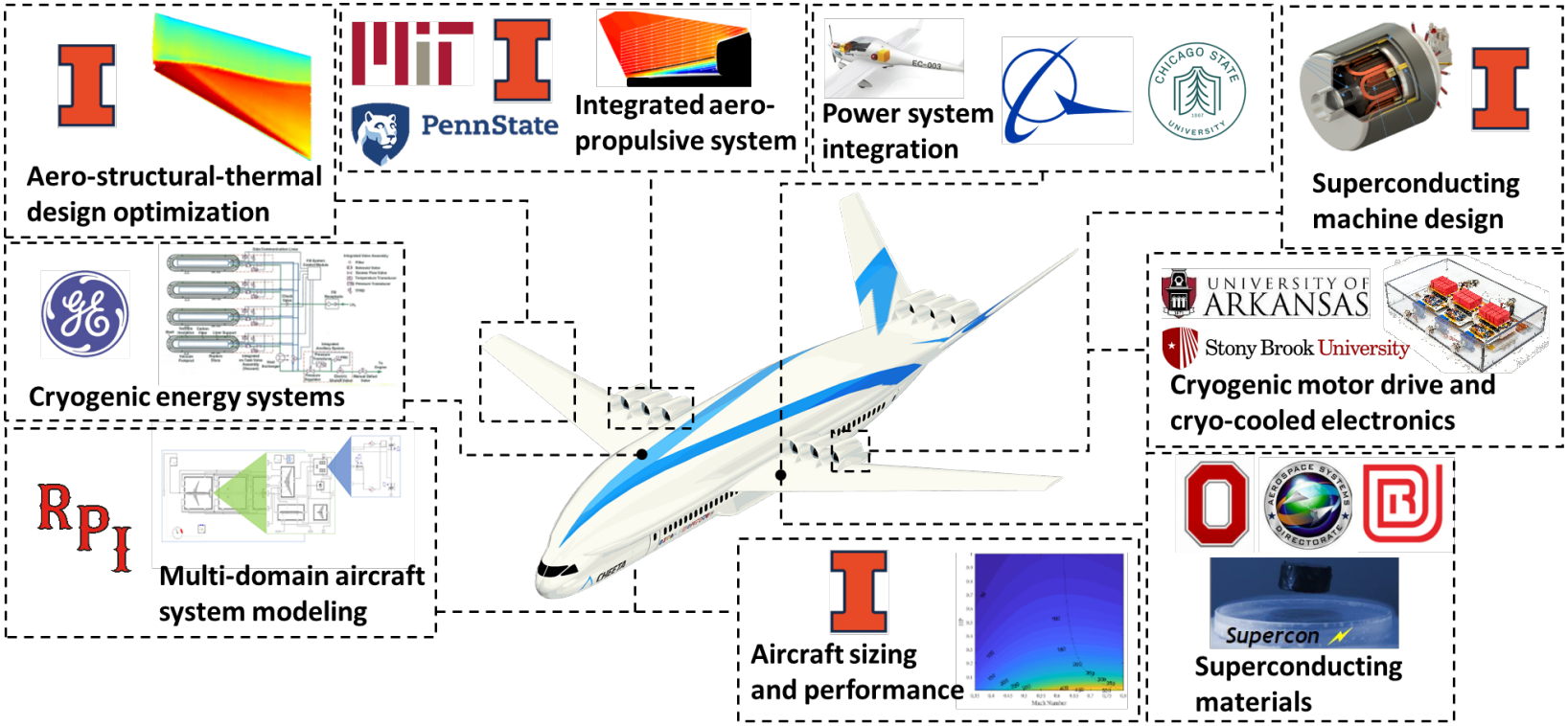
Research trajectory



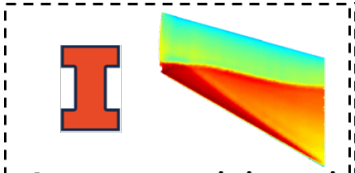
Research trajectory



CHEETA Research Program Overview

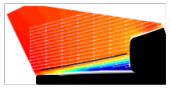


CHEETA Research Program Overview



Aero-structural-thermal design optimization

PennState



Integrated aero-propulsive system

Power system integration

Superconducting machine design

Cryogenic energy systems

Cryogenic motor drive and cryo-cooled electronics

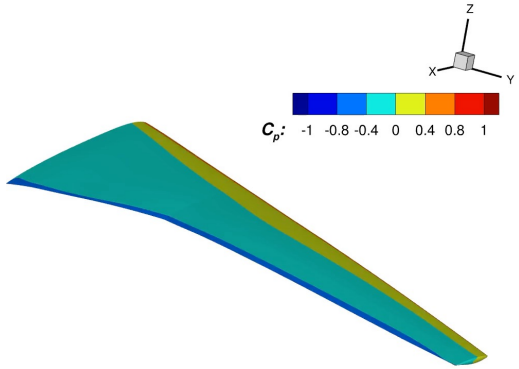
Multi-domain aircraft system modeling

Aircraft sizing and performance

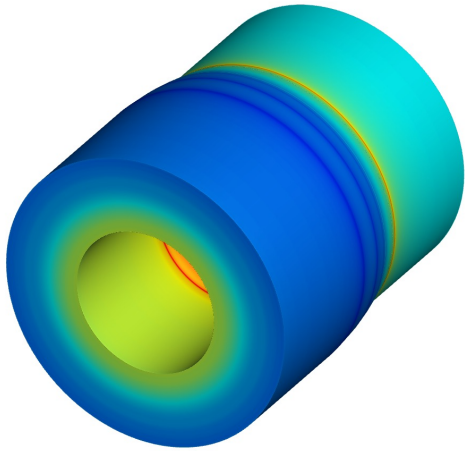
Superconducting materials

Key contributions

- Development of large-scale aircraft **Multi-Disciplinary Analysis** Framework
- Coupled Adjoint sensitivity analysis using **Analytic and AD methods**
- Co-Development of large-scale **structural optimization framework**
- Extension to aero-structural-propulsion integration : **Boundary layer Ingestion**

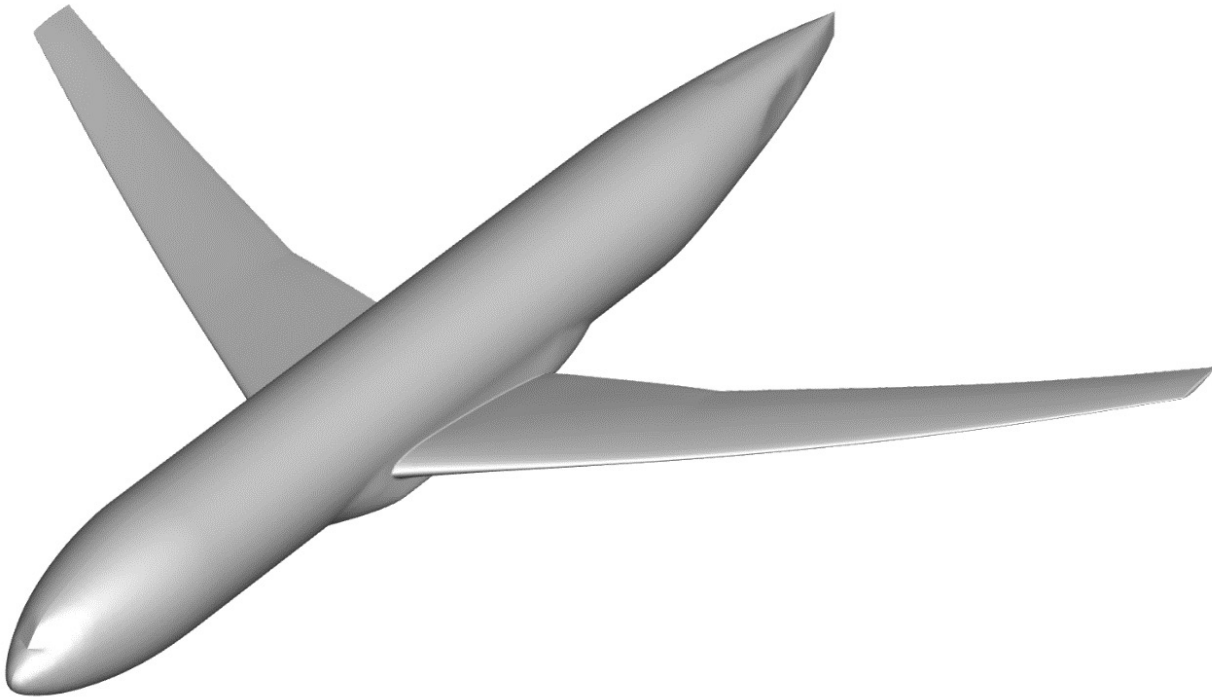


TIME-ACCURATE AERO-ELASTIC RESPONSE OF LOW-DENSITY CRM WINBOX



BOUNDARY LAYER INGESTION: WHITTLE FAN-RIG COMPUTATIONAL BENCHMARK

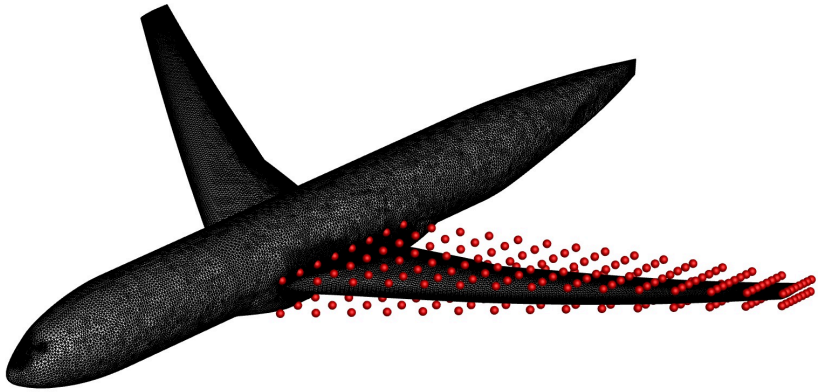
Broad objective



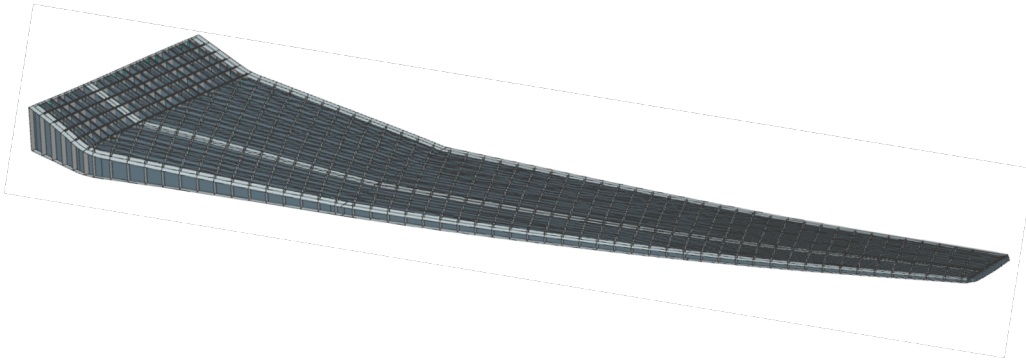
How we go beyond the conventional aircraft design approach of aero-structural optimization ?

Conventional Design Approach

- **OBJECTIVE:** Minimize aerodynamic drag and structural weight
- Sequential or coupled approach
- Coupled shape optimization for reduced drag
- Coupled structural shape, sizing and thickness optimization
- Fixed Structural topology for wing box design



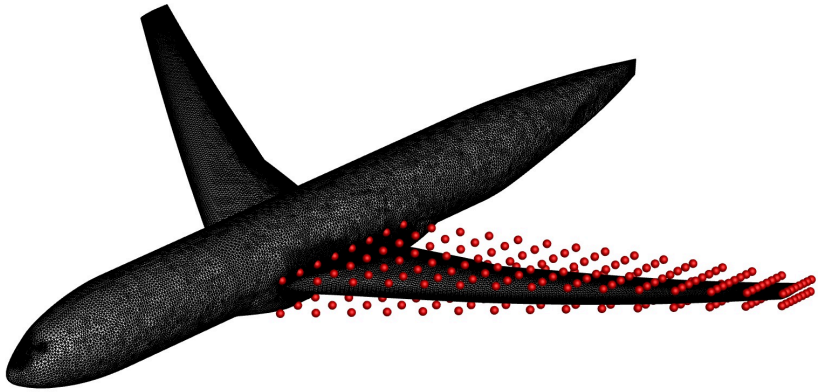
AERODYNAMIC DESIGN SPACE



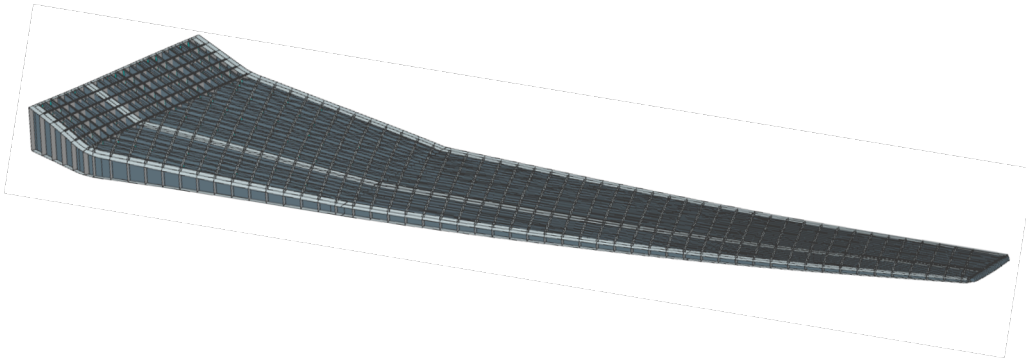
STRUCTURAL DESIGN SPACE

Conventional Design Approach

- **OBJECTIVE:** Minimize aerodynamic drag and structural weight
- Sequential or coupled approach
- Coupled shape optimization for reduced drag
- Coupled structural shape, sizing and thickness optimization
- Fixed Structural topology for wing box design



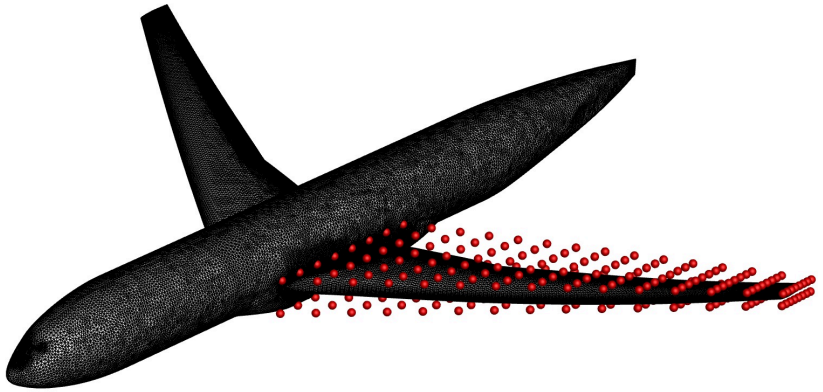
AERODYNAMIC DESIGN SPACE



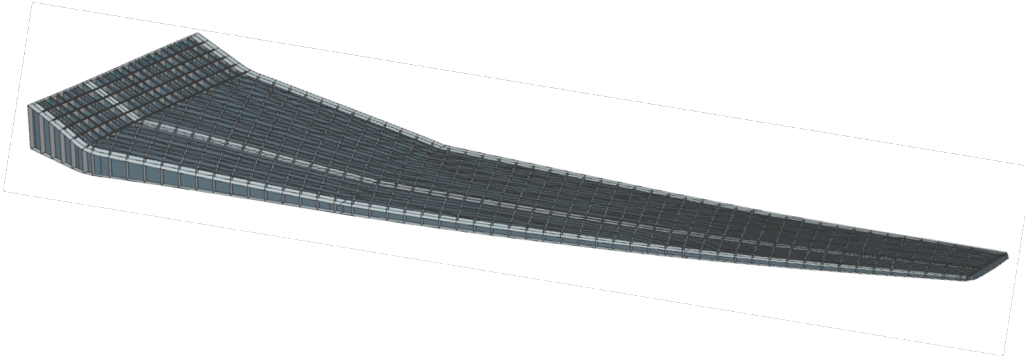
STRUCTURAL DESIGN SPACE

Conventional Design Approach

- **OBJECTIVE:** Minimize aerodynamic drag and structural weight
- Sequential or coupled approach
- Shape optimization for reduced drag
- Structural shape, sizing and thickness optimization
- Fixed Structural topology for wing box design



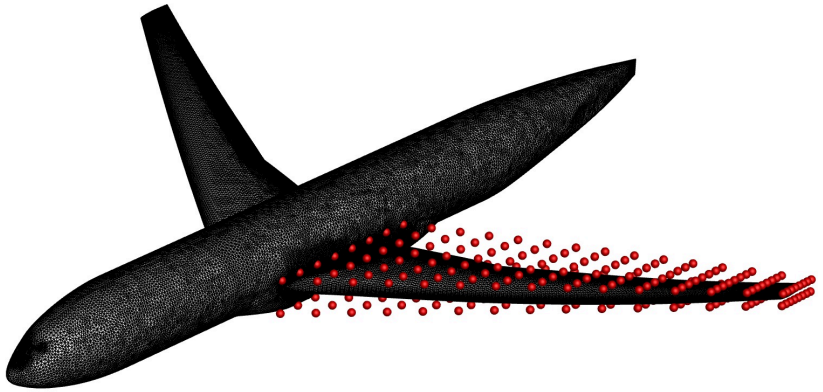
AERODYNAMIC DESIGN SPACE



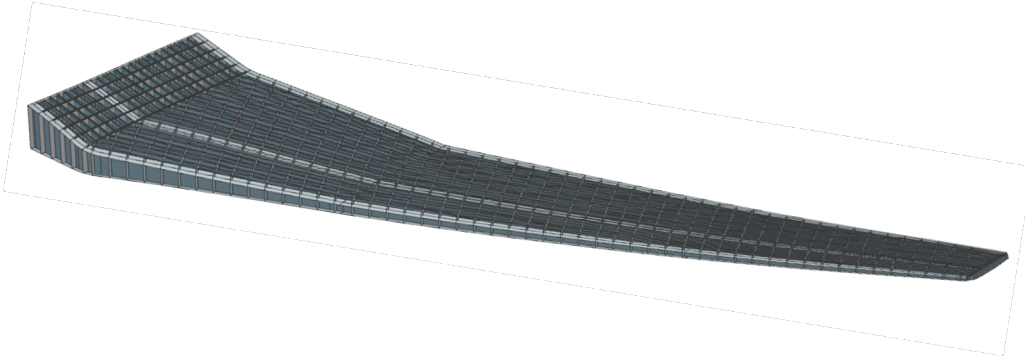
STRUCTURAL DESIGN SPACE

Conventional Design Approach

- **OBJECTIVE:** Minimize aerodynamic drag and structural weight
- Sequential or coupled approach
- Shape optimization for reduced drag
- Structural shape, sizing and thickness optimization for optimal elastic response
- Fixed Structural topology for wing box design



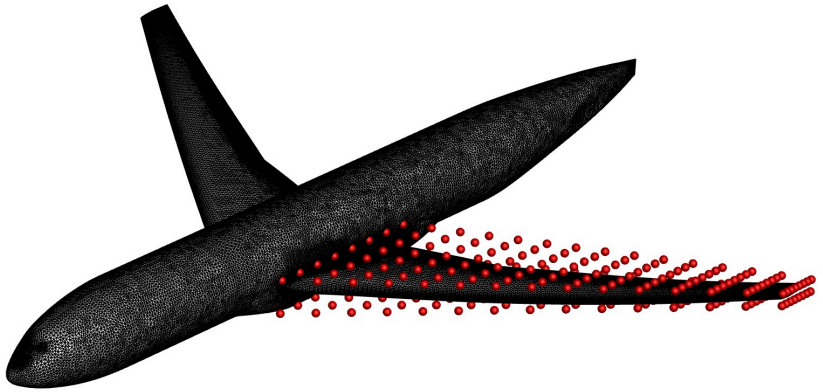
AERODYNAMIC DESIGN SPACE



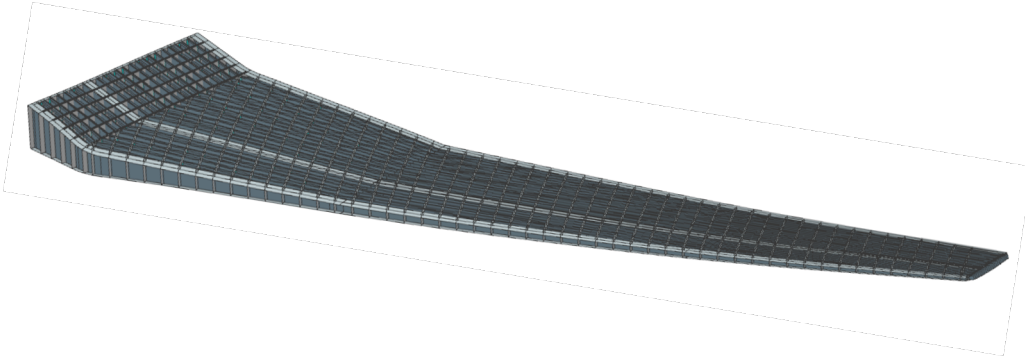
STRUCTURAL DESIGN SPACE

Conventional Design Approach

- **OBJECTIVE:** Minimize aerodynamic drag and structural weight
- Sequential or coupled approach
- Shape optimization for reduced drag
- Structural shape, sizing and thickness optimization for optimal elastic response
- Fixed Structural topology for wing box design



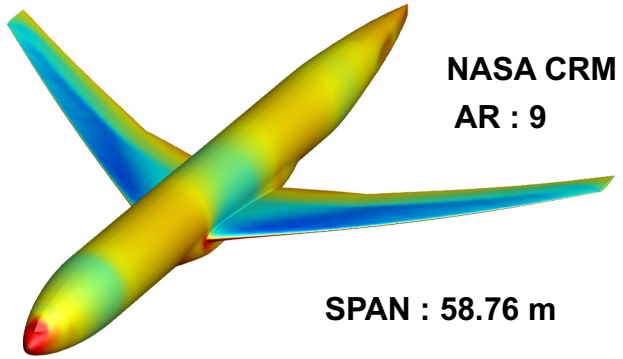
AERODYNAMIC DESIGN SPACE

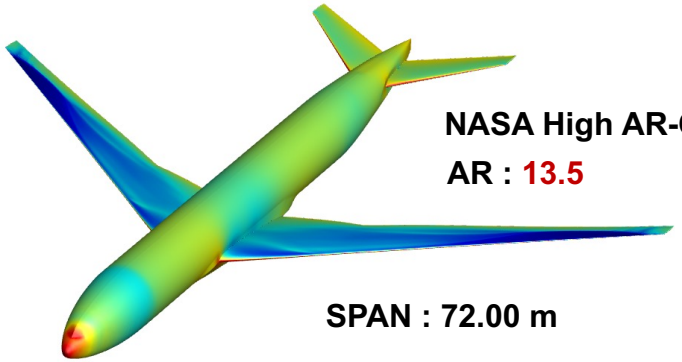


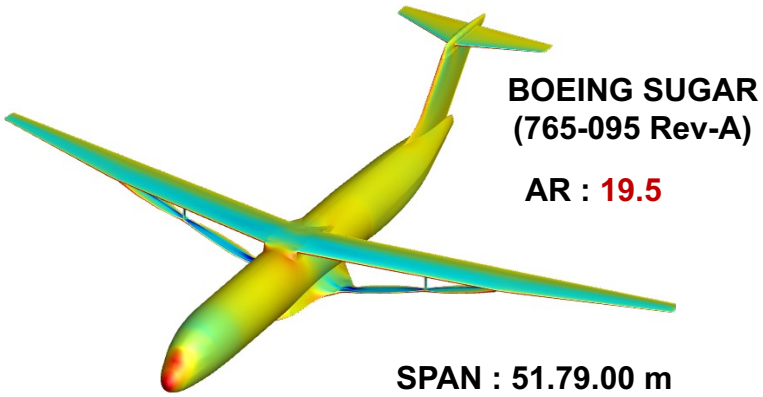
STRUCTURAL DESIGN SPACE

Need for optimal wing box design

- Gradual transition to high **Aspect Ratio** wings for greater *L/D*
- High Aspect Ratio **Truss-based** configurations
- Electric aircraft configurations with **constant structural** weight

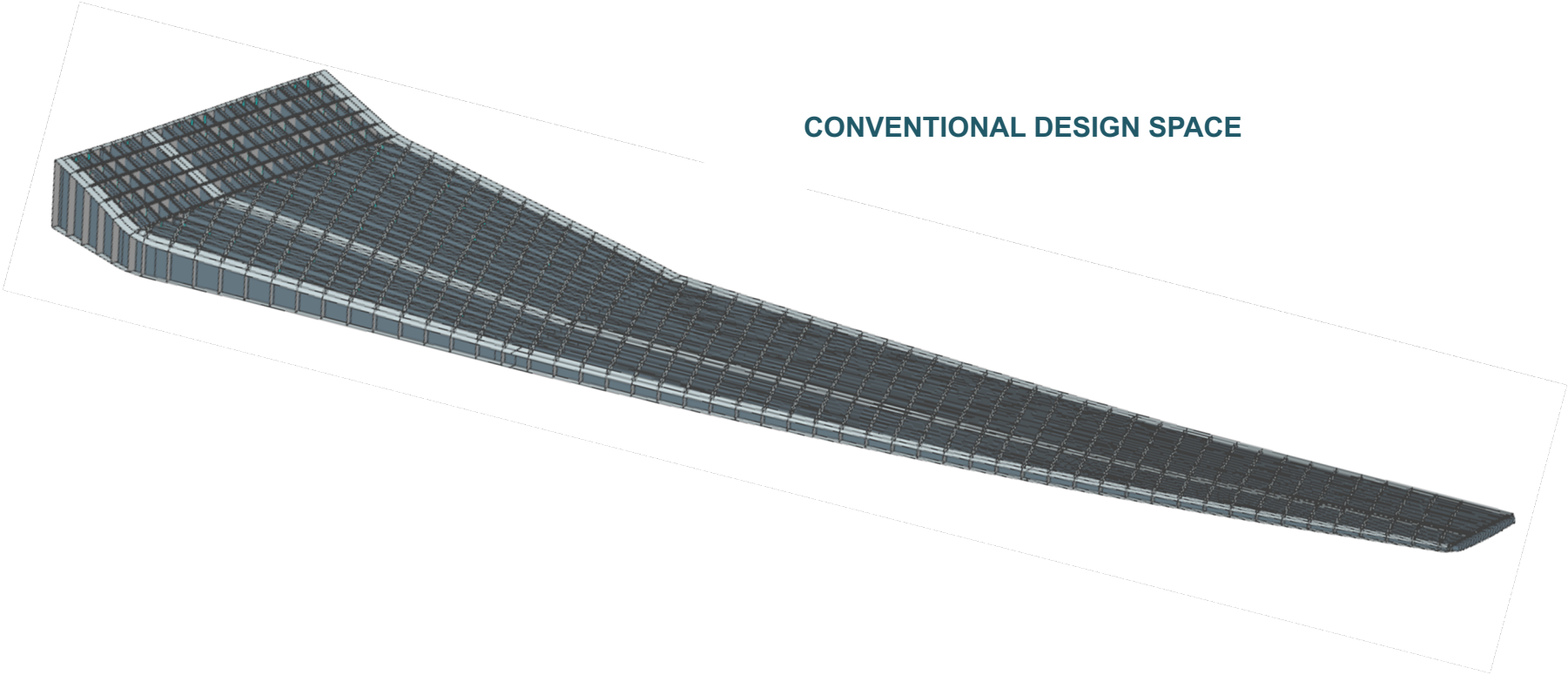




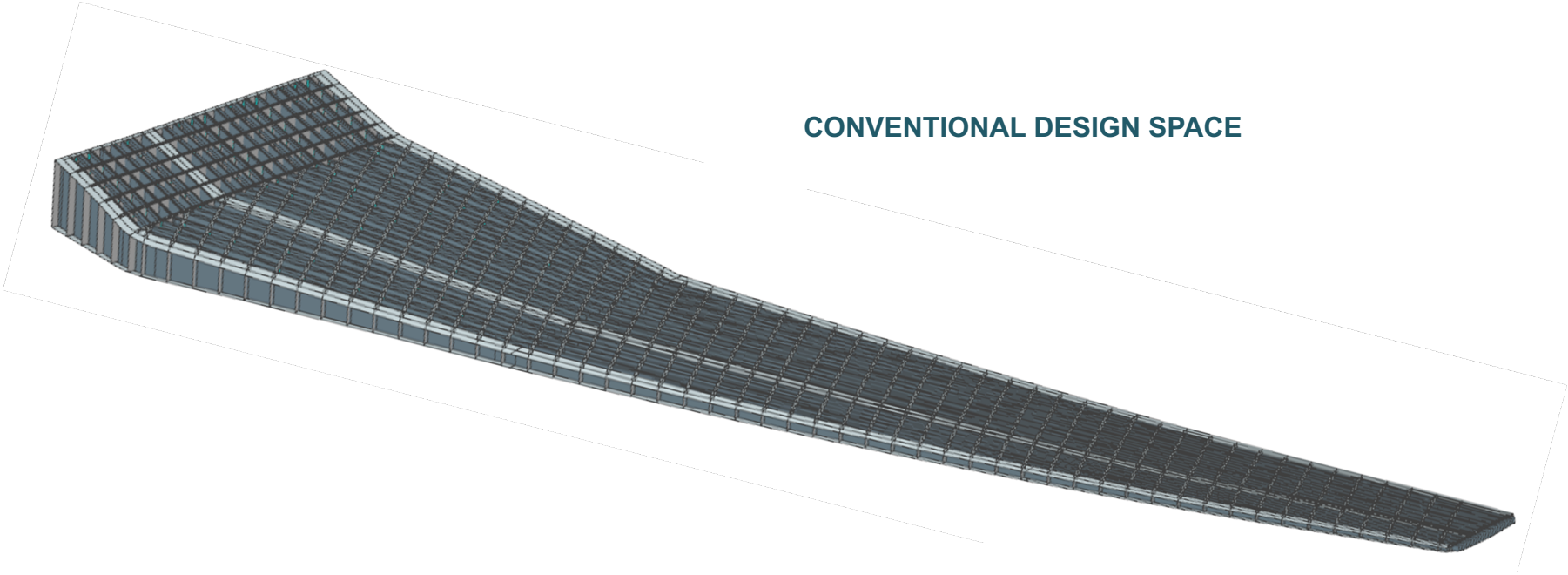


STRUCTURAL TOPOLOGY OPTIMIZATION

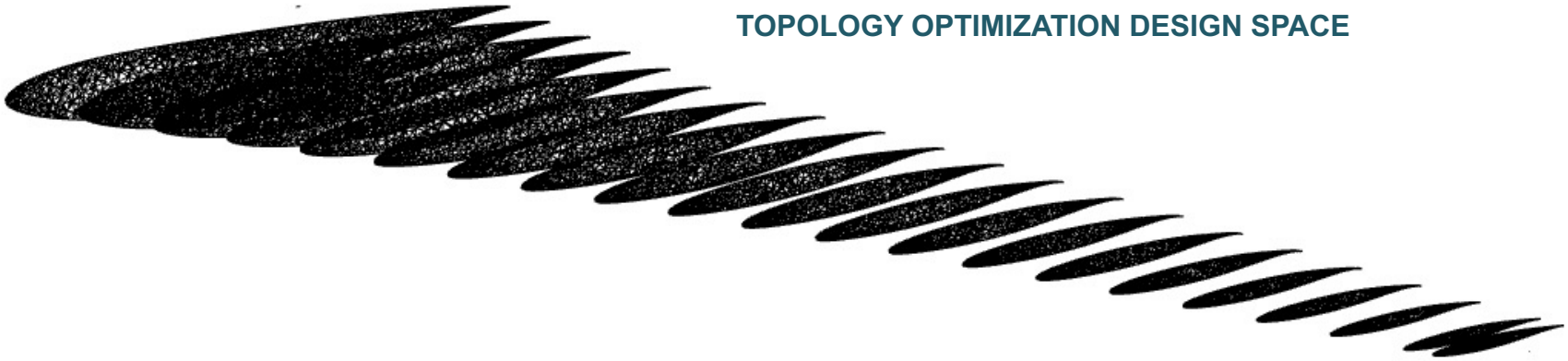
Structural Topology Optimization



Structural Topology Optimization

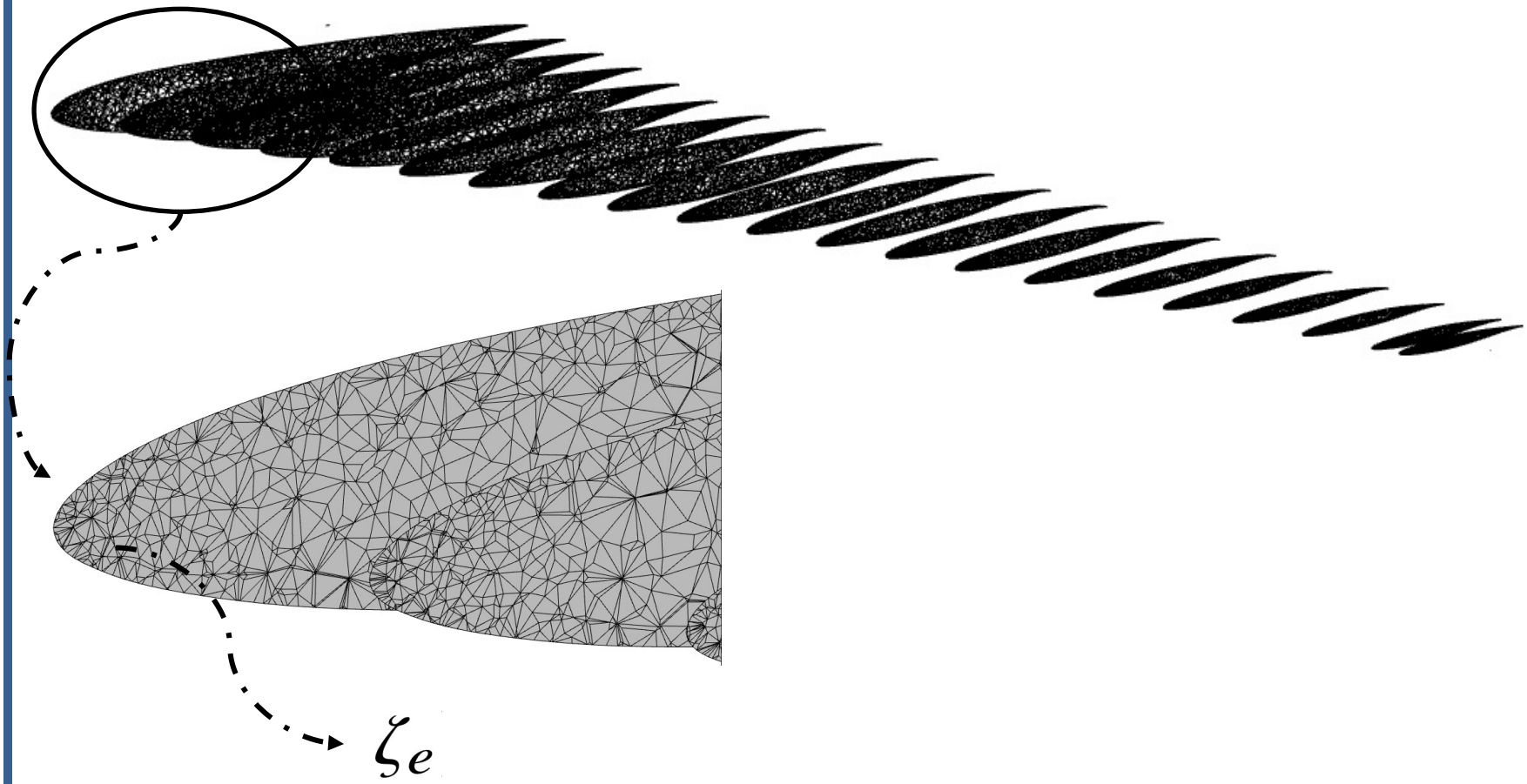


CONVENTIONAL DESIGN SPACE



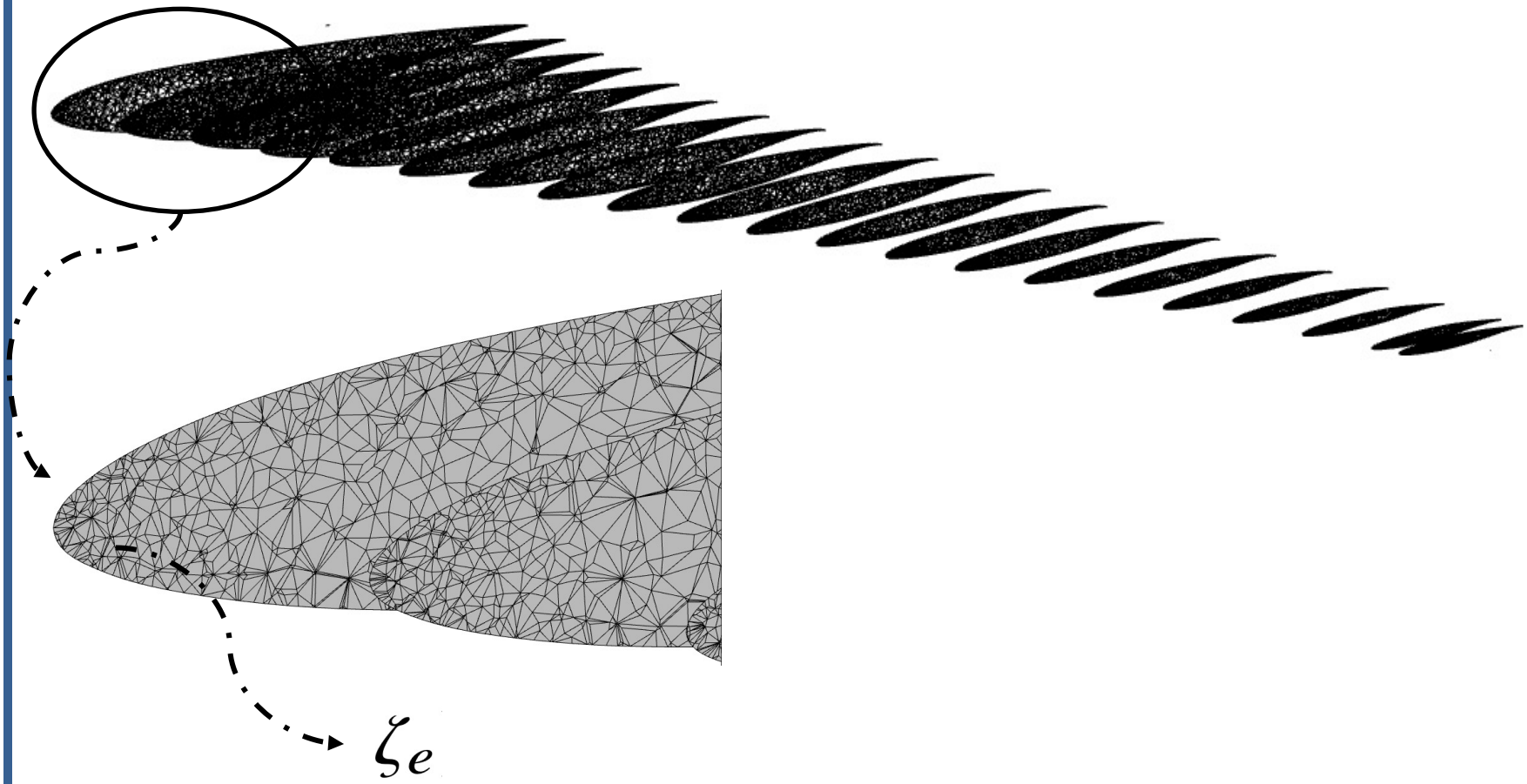
TOPOLOGY OPTIMIZATION DESIGN SPACE

Structural Topology Optimization



$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$

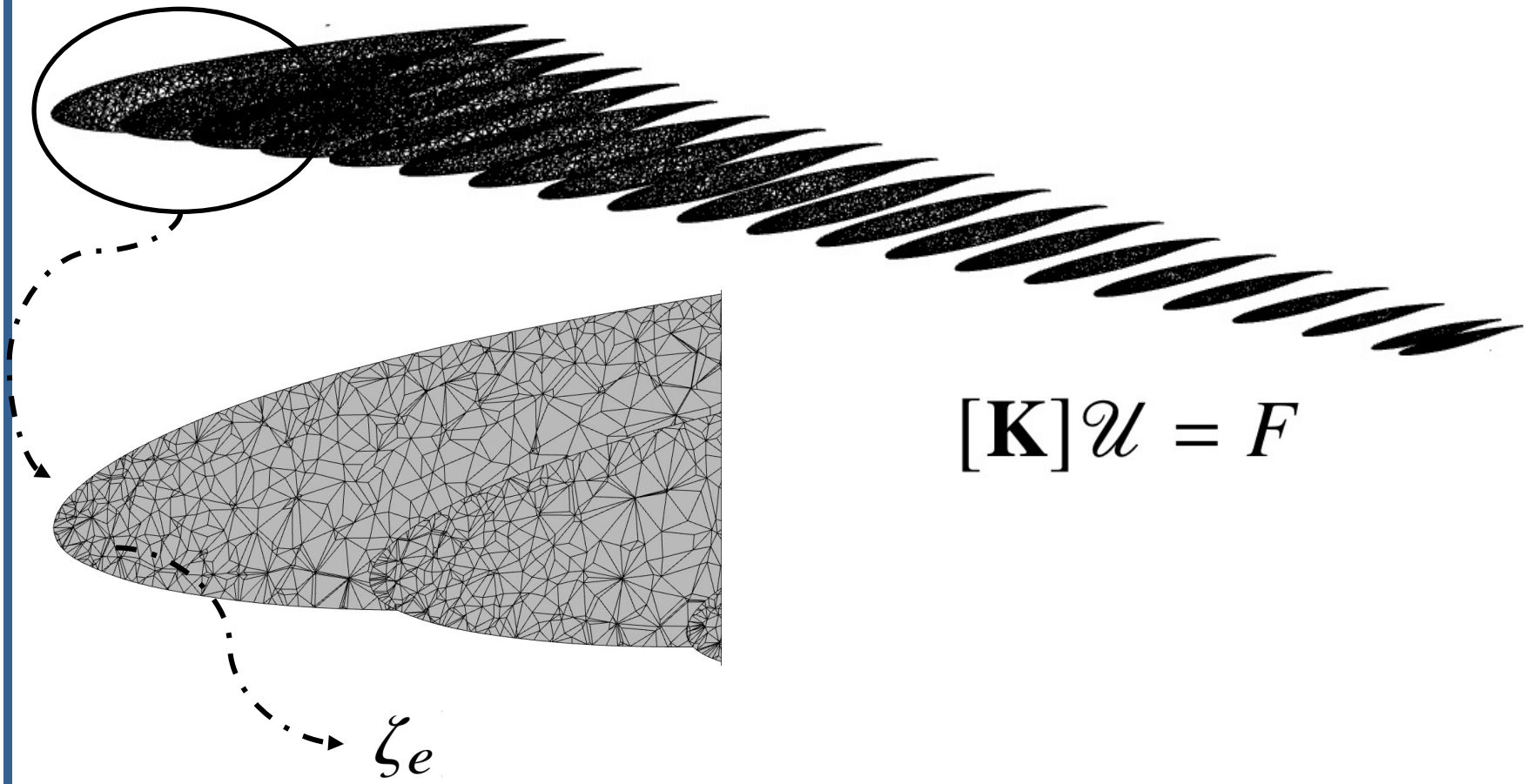
Structural Topology Optimization



$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \qquad E_e(\zeta_e) = \zeta_e^P E_0$$

LOCAL ELEMENT STIFFNESS

Structural Topology Optimization

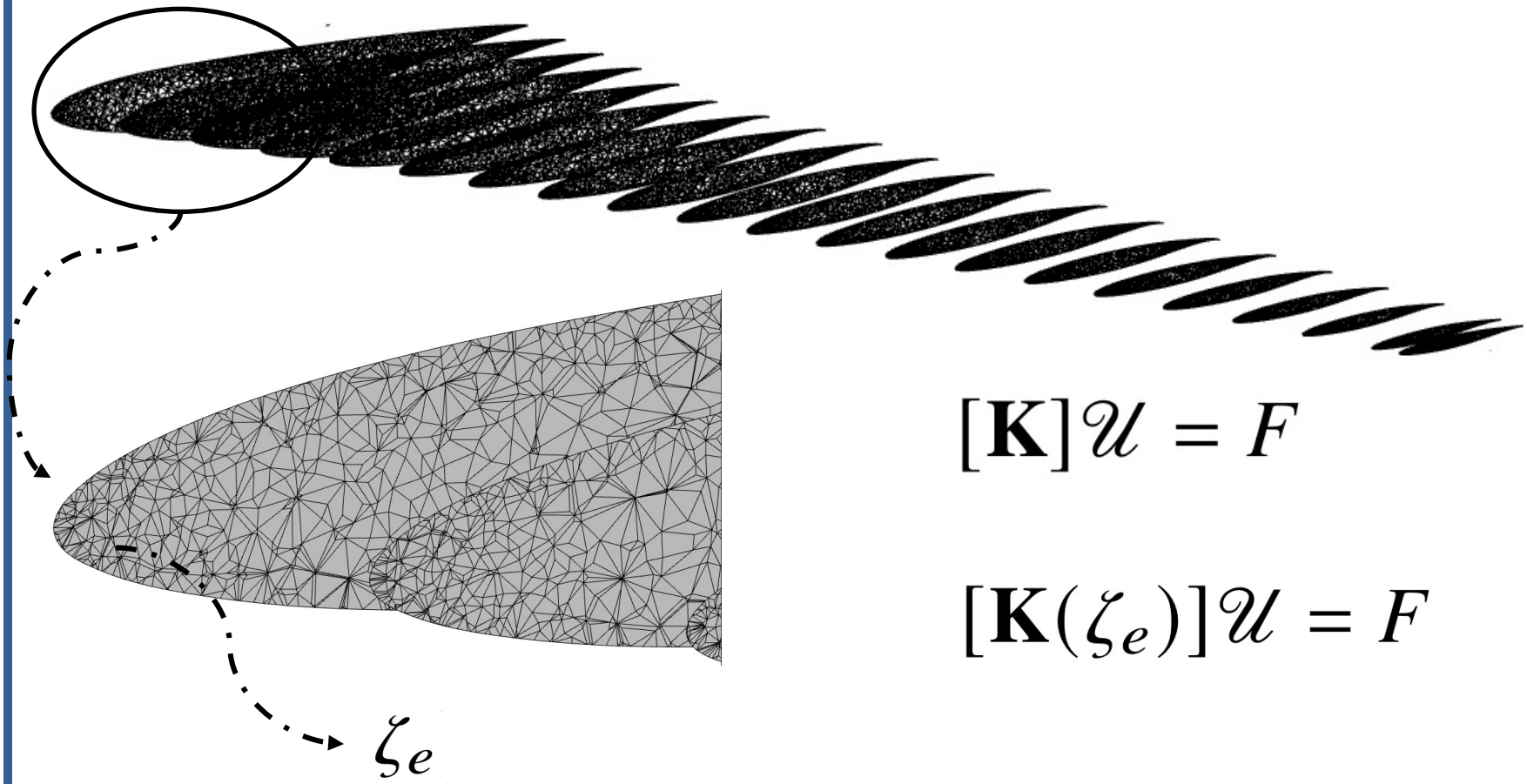


$$[\mathbf{K}]\mathcal{U} = F$$

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \qquad E_e(\zeta_e) = \zeta_e^P E_0$$

LOCAL ELEMENT STIFFNESS

Structural Topology Optimization



$$[\mathbf{K}]\mathcal{U} = F$$

$$[\mathbf{K}(\zeta_e)]\mathcal{U} = F$$

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \qquad E_e(\zeta_e) = \zeta_e^P E_0$$

LOCAL ELEMENT STIFFNESS

Typical problem formulation

$$\min_{\zeta} \quad C_{elastic}$$

MAXIMIZE STIFFNESS

$$\text{s.t.} \quad \sum_{i=1}^{n_e} \frac{\zeta_i V}{V_{max}} = v f$$

AVAILABLE MATERIAL

$$[\mathbf{K}(\zeta_\mathbf{e})]\{\mathcal{U}\} = F$$

EQUILLIBRIUM

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$

Typical problem formulation

$$\min_{\zeta} \quad C_{elastic}$$

MAXIMIZE STIFFNESS

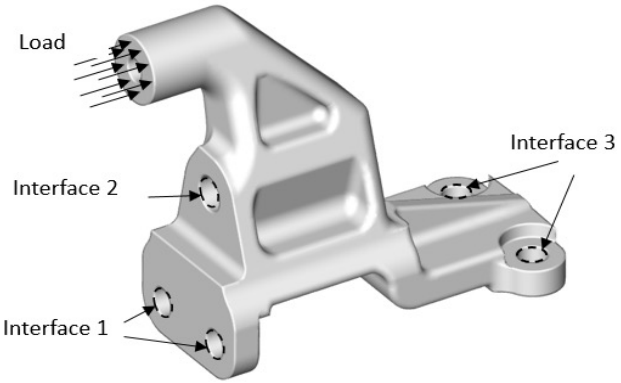
$$\text{s.t.} \quad \sum_{i=1}^{n_e} \frac{\zeta_i V}{V_{max}} = vf$$

AVAILABLE MATERIAL

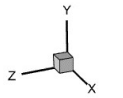
$$[\mathbf{K}(\zeta_e)]\{\mathcal{U}\} = F$$

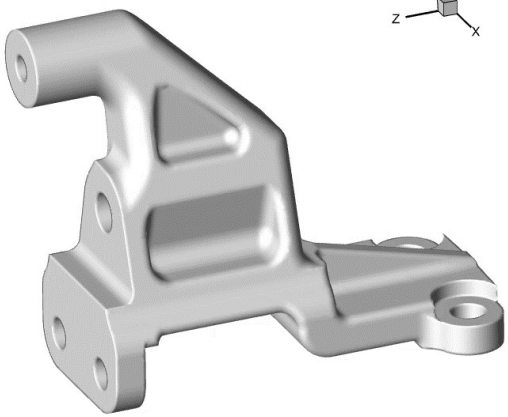
EQUILLIBRIUM

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$

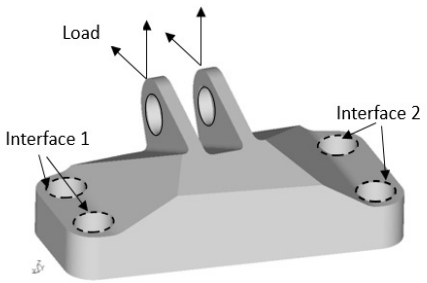


GENERATOR ADAPTER





GE JET ENGINE BRACKET



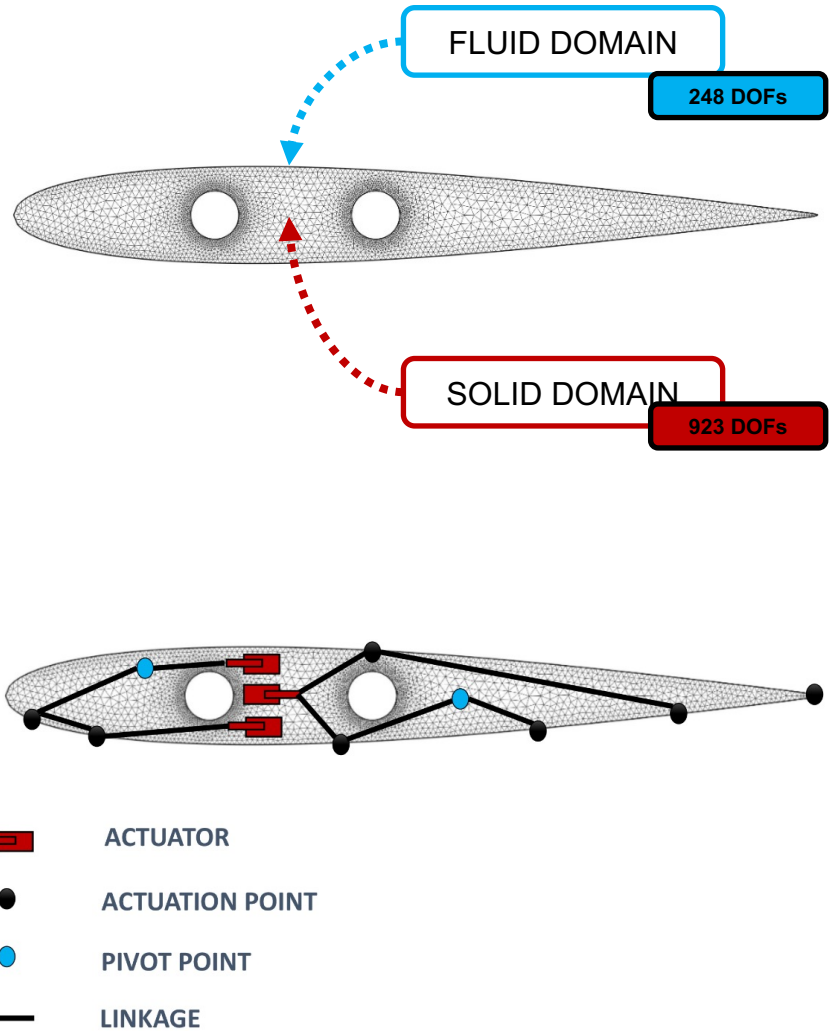


Compliant mechanism

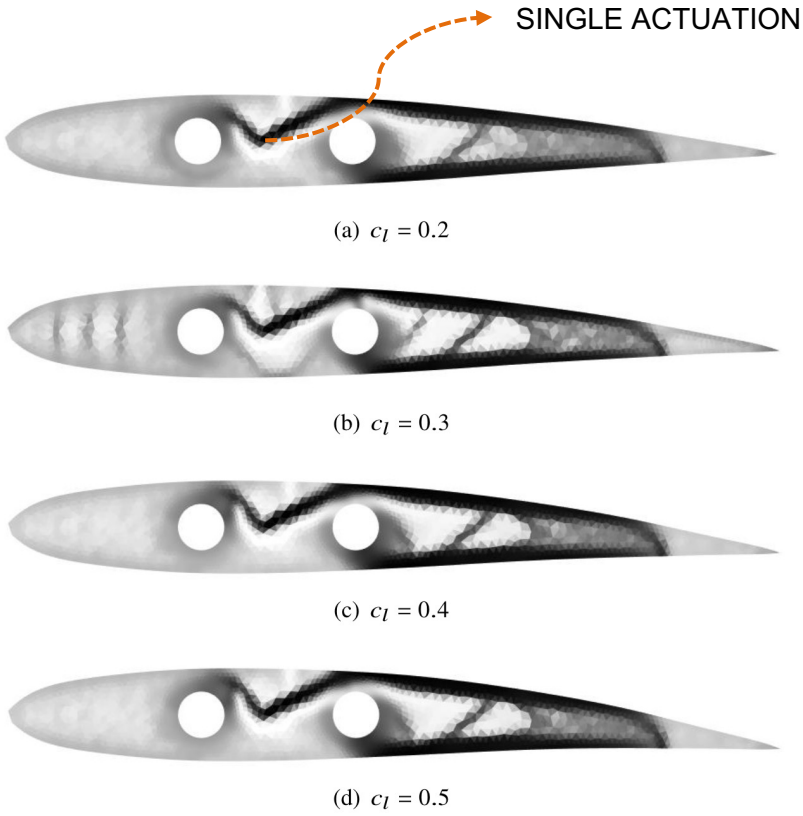
$$\begin{aligned}
\min_{\mathscr{X}} \quad & c_d \\
\text{s.t.} \quad & M_{pitch} - M_{max} \leq 0 \\
& c_l / c_l^* - 1 = 0 \\
& \sigma - \sigma_{yield} \leq 0 \\
& |U_{stroke}| - U_{max} \leq 0 \\
& m - m_{max} \leq 0 \\
& \omega - \omega_{max} \leq 0
\end{aligned}$$

$$\frac{dU_{out}}{dx} = -K^{-1} L \frac{\partial K}{\partial x}$$

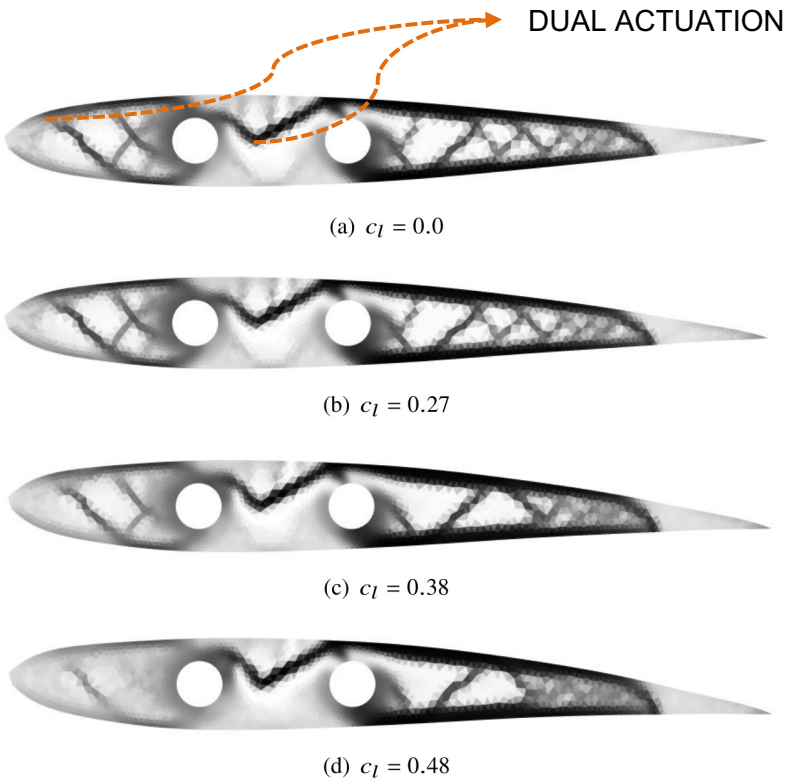
$$\frac{df_{comp}}{dx} = -u^T \frac{\partial K}{\partial x} u$$



Results: Compliant mechanism



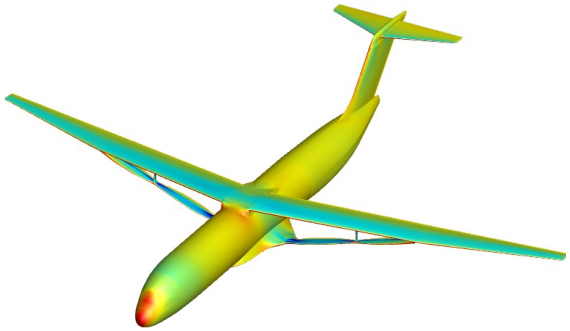
LOW-ORDER DENSITY FILTERING



HIGHER-ORDER DENSITY FILTERING

LARGE-SCALE OPTIMIZATION FRAMEWORK

Cornerstones



$$
\begin{aligned}
\min_{x,\alpha,\zeta} \quad & C_D \\
\text{s.t.} \quad & L = W \\
& C_{My} \leq -0.17 \\
& (C_L - C_L^*) \leq \epsilon \\
& t/c_{i=1..30} \geq 0.25 t/c_{i=1..30} \\
& V \geq= V_{initial} \\
& \sum_{i=1}^{n_e} \frac{\rho_i V}{V_{max}} = vf \\
& (C_{Elastic} - C^*_{Elastic}) \leq \epsilon \\
& [\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = \{F\} \\
& 0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \\
& -0.01 < x \leq 0.01
\end{aligned}
$$

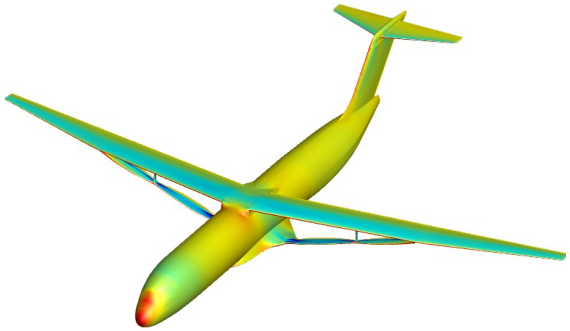
SOLID DYNAMICS

FLUID DYNAMICS

COUPLED ADJOINT SENSITIVITIES

NUMERICAL OPTIMIZATION

Cornerstones



$$
\begin{aligned}
\min_{x,\alpha,\zeta} \quad & C_D \\
\text{s.t.} \quad & L = W \\
& C_{My} \leq -0.17 \\
& (C_L - C_L^*) \leq \epsilon \\
& t/c_{i=1..30} \geq 0.25 t/c_{i=1..30} \\
& V \geq= V_{initial} \\
& \sum_{i=1}^{n_e} \frac{\rho_i V}{V_{max}} = vf \\
& (C_{Elastic} - C_{Elastic}^*) \leq \epsilon \\
& [\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = \{F\} \\
& 0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \\
& -0.01 < x \leq 0.01
\end{aligned}
$$

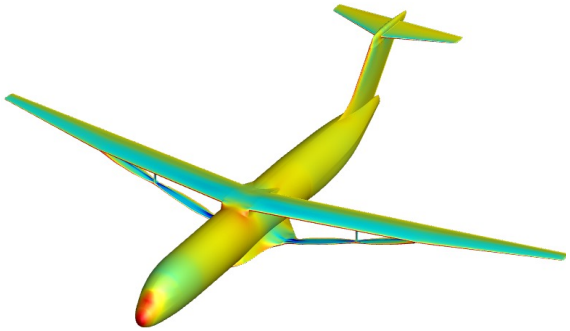
SOLID DYNAMICS

FLUID DYNAMICS

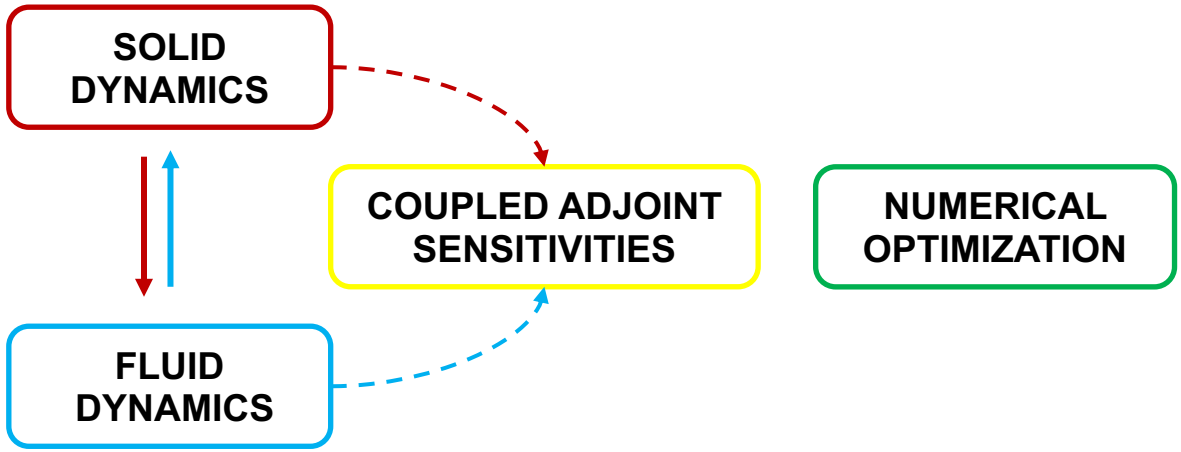
COUPLED ADJOINT SENSITIVITIES

NUMERICAL OPTIMIZATION

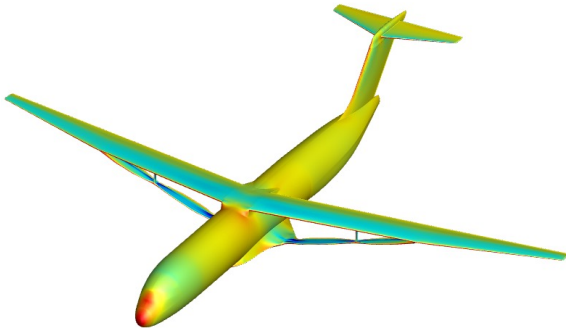
Cornerstones



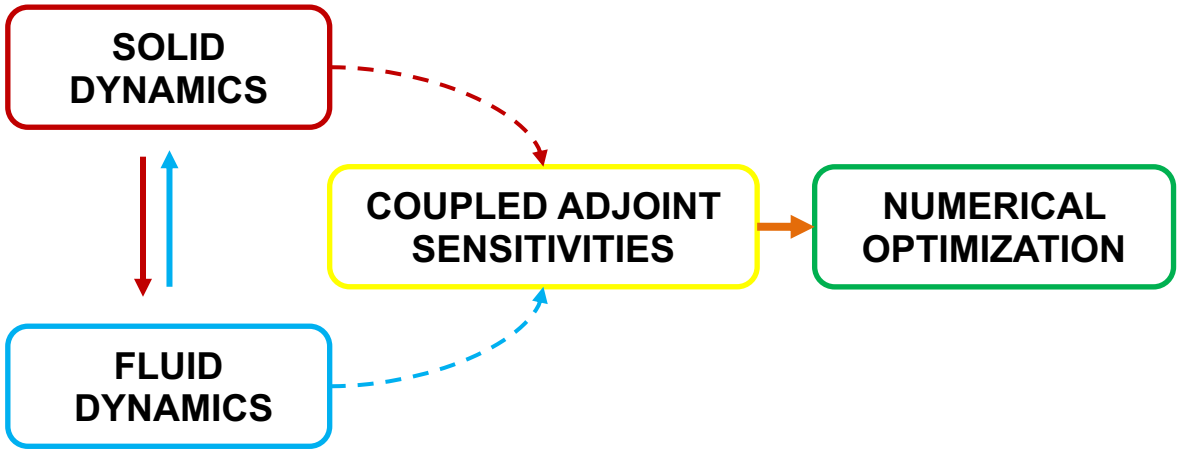
$$
\begin{aligned}
\min_{x,\alpha,\zeta} \quad & C_D \\
\text{s.t.} \quad & L = W \\
& C_{My} \leq -0.17 \\
& (C_L - C_L^*) \leq \epsilon \\
& t/c_{i=1..30} \geq 0.25 t/c_{i=1..30} \\
& V \geq= V_{initial} \\
& \sum_{i=1}^{n_e} \frac{\rho_i V}{V_{max}} = vf \\
& (C_{Elastic} - C^*_{Elastic}) \leq \epsilon \\
& [\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = \{F\} \\
& 0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \\
& -0.01 < x \leq 0.01
\end{aligned}
$$



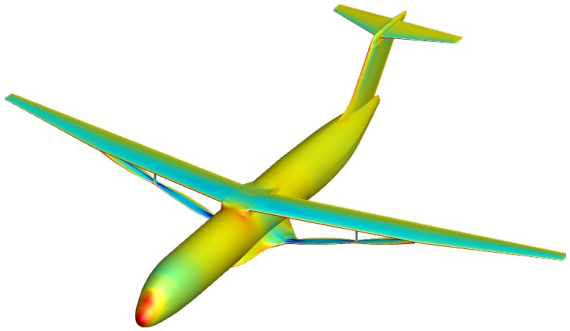
Cornerstones



$$
\begin{aligned}
\min_{x,\alpha,\zeta} \quad & C_D \\
\text{s.t.} \quad & L = W \\
& C_{My} \leq -0.17 \\
& (C_L - C_L^*) \leq \epsilon \\
& t/c_{i=1..30} \geq 0.25 t/c_{i=1..30} \\
& V \geq= V_{initial} \\
& \sum_{i=1}^{n_e} \frac{\rho_i V}{V_{max}} = vf \\
& (C_{Elastic} - C_{Elastic}^*) \leq \epsilon \\
& [\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = \{F\} \\
& 0 < \zeta_{e_{min}} \leq \zeta_e \leq 1 \\
& -0.01 < x \leq 0.01
\end{aligned}
$$

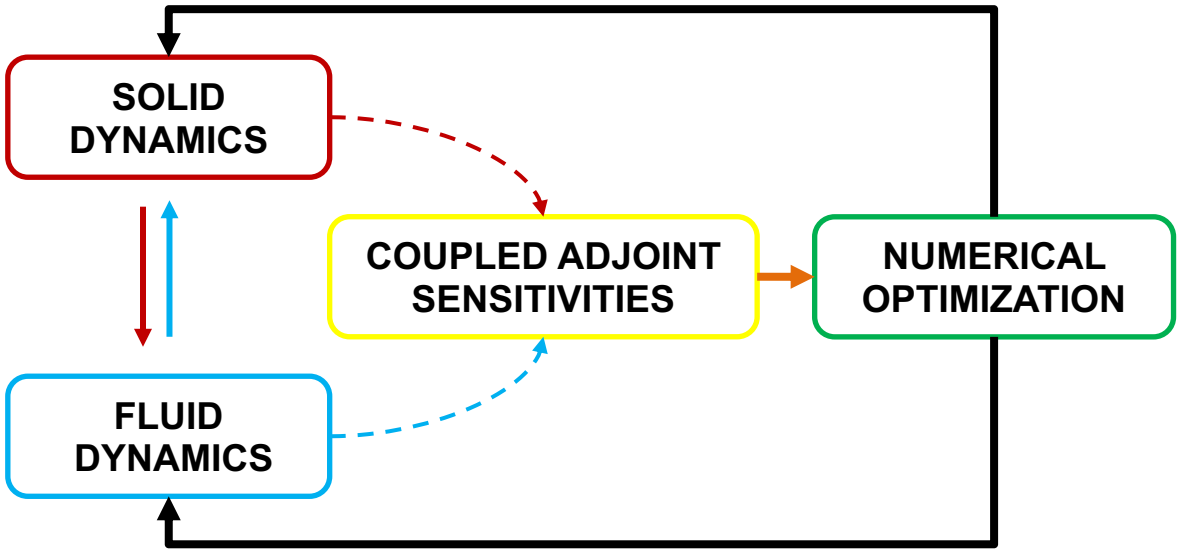


Cornerstones



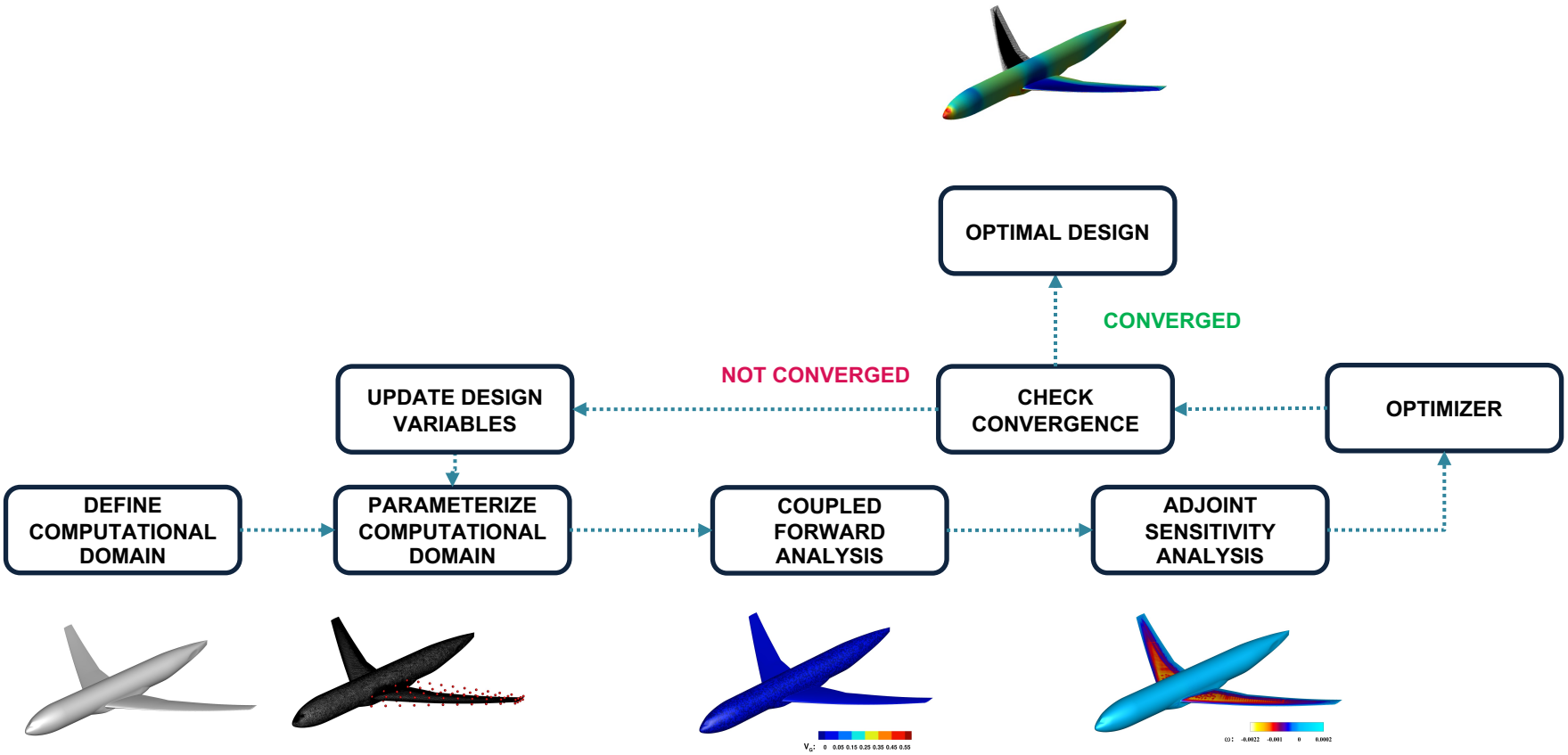
$$
\begin{aligned}
\min_{x,\alpha,\zeta} \quad & C_D \\
\text{s.t.} \quad & L = W \\
& C_{My} \le -0.17 \\
& (C_L - C_L^*) \le \epsilon \\
& t/c_{i=1..30} \ge 0.25 t/c_{i=1..30} \\
& V \ge= V_{initial} \\
& \sum_{i=1}^{n_e} \frac{\rho_i V}{V_{max}} = vf \\
& (C_{Elastic} - C^*_{Elastic}) \le \epsilon \\
& [\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = \{F\} \\
& 0 < \zeta_{e_{min}} \le \zeta_e \le 1 \\
& -0.01 < x \le 0.01
\end{aligned}
$$

DENSITY VARIABLE UPDATE

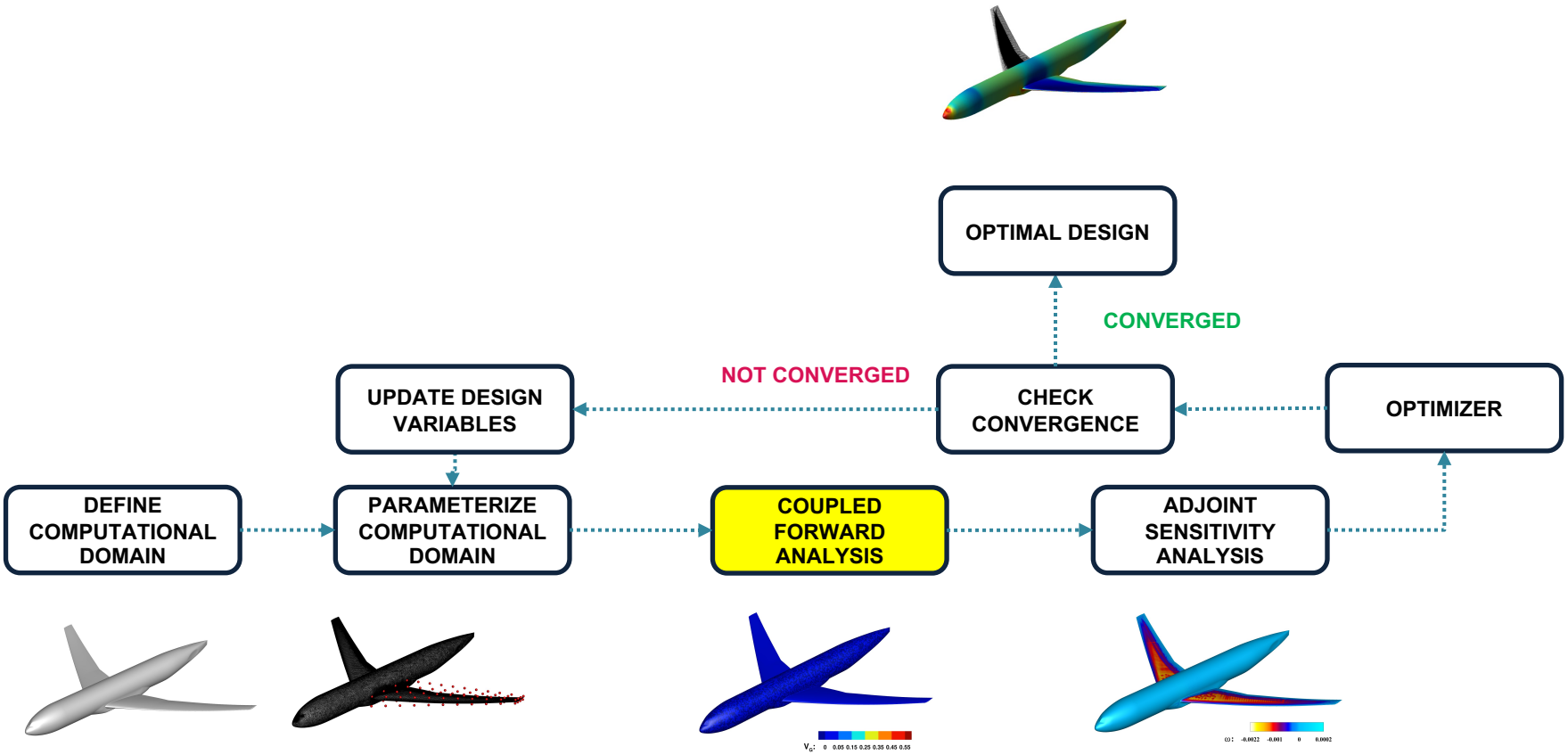


SHAPE VARIABLE UPDATE

Aircraft optimization workflow

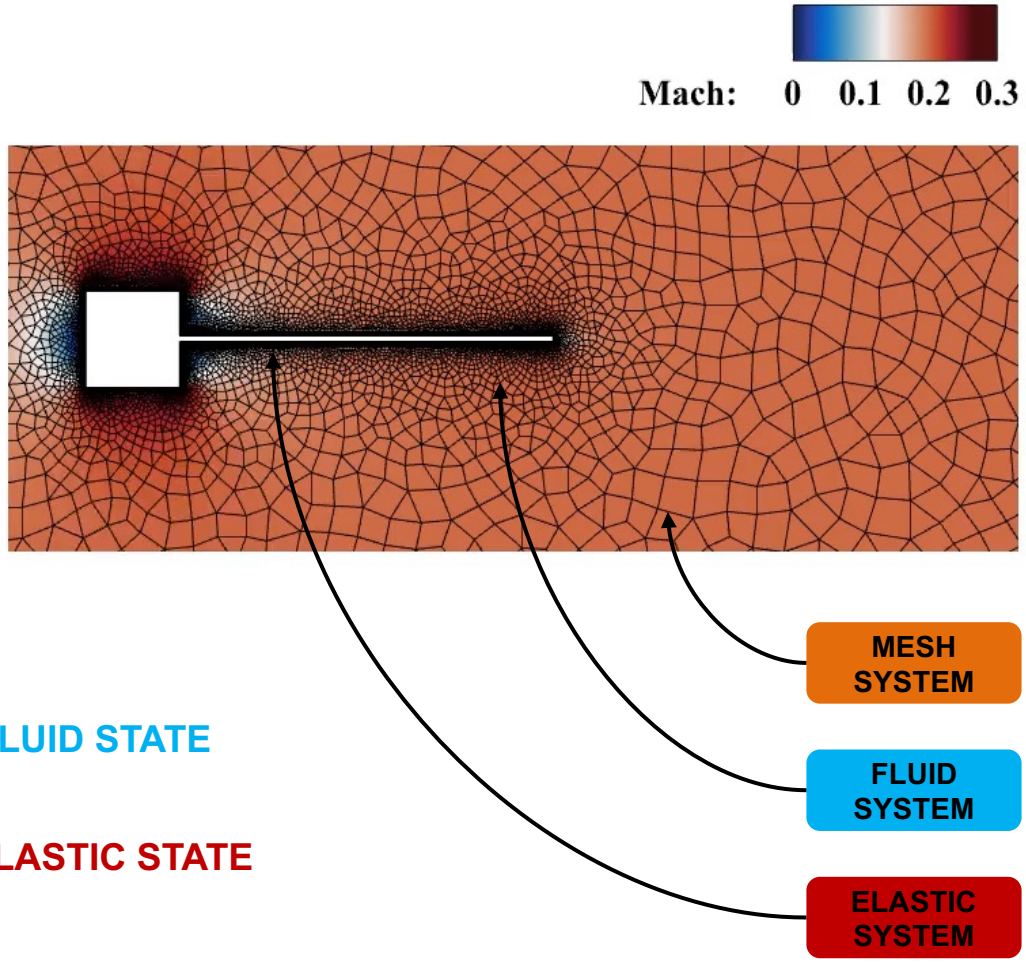


Aircraft optimization workflow



High-Fidelity FSI Framework

$$\mathcal{G} = \begin{Bmatrix} \mathcal{R}(\mathcal{F}, \mathcal{X}) \\ \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}) \\ \mathcal{M}(\mathcal{X}, \mathcal{U}) \end{Bmatrix} = 0$$



$\mathcal{R}$ FLUID RESIDUAL

$\mathcal{F}$ FLUID STATE

$\mathcal{S}$ ELASTIC RESIDUAL

$\mathcal{U}$ ELASTIC STATE

$\mathcal{M}$ MESH RESIDUAL

$\mathcal{X}$ MESH STATE

High-Fidelity FSI Framework

$$\mathcal{G} = \begin{Bmatrix} \mathcal{R}(\mathcal{F}, \mathcal{X}) \\ \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}) \\ \mathcal{M}(\mathcal{X}, \mathcal{U}) \end{Bmatrix} = 0$$

NOT AVAILABLE

$$\begin{bmatrix} \frac{\partial \mathcal{R}}{\partial \mathcal{F}} & \frac{\partial \mathcal{R}}{\partial \mathcal{U}} & \frac{\partial \mathcal{R}}{\partial \mathcal{X}} \\ \frac{\partial \mathcal{S}}{\partial \mathcal{F}} & \frac{\partial \mathcal{S}}{\partial \mathcal{U}} & \frac{\partial \mathcal{S}}{\partial \mathcal{X}} \\ \frac{\partial \mathcal{M}}{\partial \mathcal{F}} & \frac{\partial \mathcal{M}}{\partial \mathcal{U}} & \frac{\partial \mathcal{M}}{\partial \mathcal{X}} \end{bmatrix} \begin{bmatrix} \Delta\mathcal{F} \\ \Delta\mathcal{U} \\ \Delta\mathcal{M} \end{bmatrix} = \begin{bmatrix} \mathcal{R} \\ \mathcal{S} \\ \mathcal{M} \end{bmatrix}_0$$

AVAILABLE

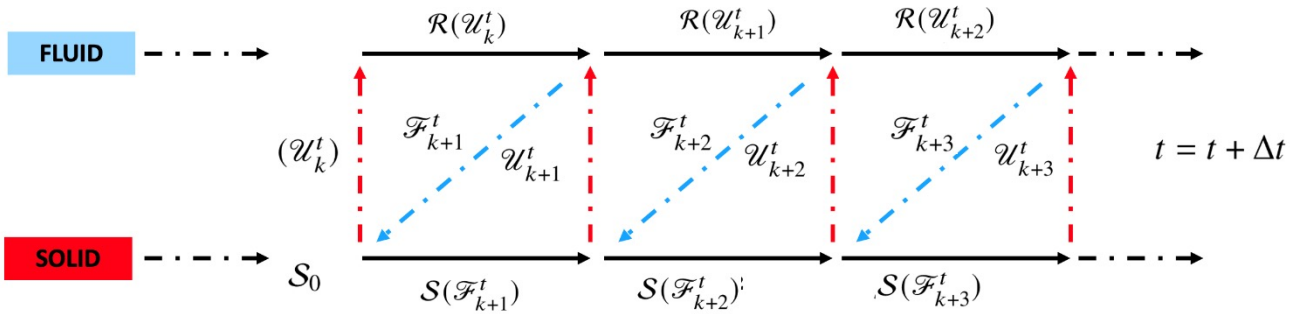
AVAILABLE

Partitioned Procedure

$$\begin{bmatrix} \frac{\partial \mathcal{R}}{\partial \mathcal{F}} & \frac{\partial \mathcal{R}}{\partial \mathcal{U}} & \frac{\partial \mathcal{R}}{\partial \mathcal{X}} \\ \frac{\partial \mathcal{S}}{\partial \mathcal{F}} & \frac{\partial \mathcal{S}}{\partial \mathcal{U}} & \frac{\partial \mathcal{S}}{\partial \mathcal{X}} \\ \frac{\partial \mathcal{M}}{\partial \mathcal{F}} & \frac{\partial \mathcal{M}}{\partial \mathcal{U}} & \frac{\partial \mathcal{M}}{\partial \mathcal{X}} \end{bmatrix} \begin{bmatrix} \Delta\mathcal{F} \\ \Delta\mathcal{U} \\ \Delta\mathcal{M} \end{bmatrix} = \begin{bmatrix} \mathcal{R} \\ \mathcal{S} \\ \mathcal{M} \end{bmatrix}_0 \dashrightarrow \begin{bmatrix} \frac{\partial \mathcal{R}}{\partial \mathcal{F}} & 0 & 0 \\ 0 & \frac{\partial \mathcal{S}}{\partial \mathcal{U}} & 0 \\ 0 & 0 & \frac{\partial \mathcal{M}}{\partial \mathcal{X}} \end{bmatrix} \begin{bmatrix} \Delta\mathcal{F} \\ \Delta\mathcal{U} \\ \Delta\mathcal{M} \end{bmatrix} = \begin{bmatrix} \mathcal{R} \\ \mathcal{S} \\ \mathcal{M} \end{bmatrix}_0$$

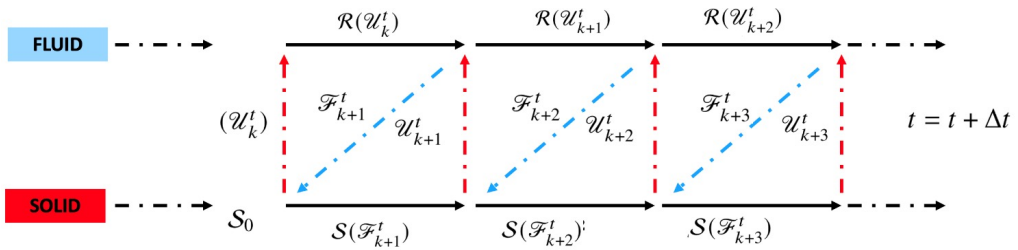
FULL JACOBIAN SYSTEM **REDUCED JACOBIAN SYSTEM**

$\mathcal{U}_0 = \mathcal{U}^t$;
$k = 0$;
while $t < T$ **do**
  while $\epsilon_{F_\Gamma} > tol_1$ & $\epsilon_{u_\Gamma} > tol_2$ **do**
    $\mathcal{R}(\mathcal{U}_k^t) = \mathcal{F}_{k+1}^t$;
    $\mathcal{S}(\mathcal{F}_{k+1}^t) = \mathcal{U}_{k+1}^{\tilde{t}}$;
    $\mathcal{U}_{k+1}^t = (1-\omega)\mathcal{U}_k^t + \omega\tilde{\mathcal{U}}_{k+1}^t$;
    $k = k + 1$;
  end
  $t = t + \Delta t$;
end



IMPLICIT STAGGERED SOLUTION PROCEDURE

Load and displacement transfer



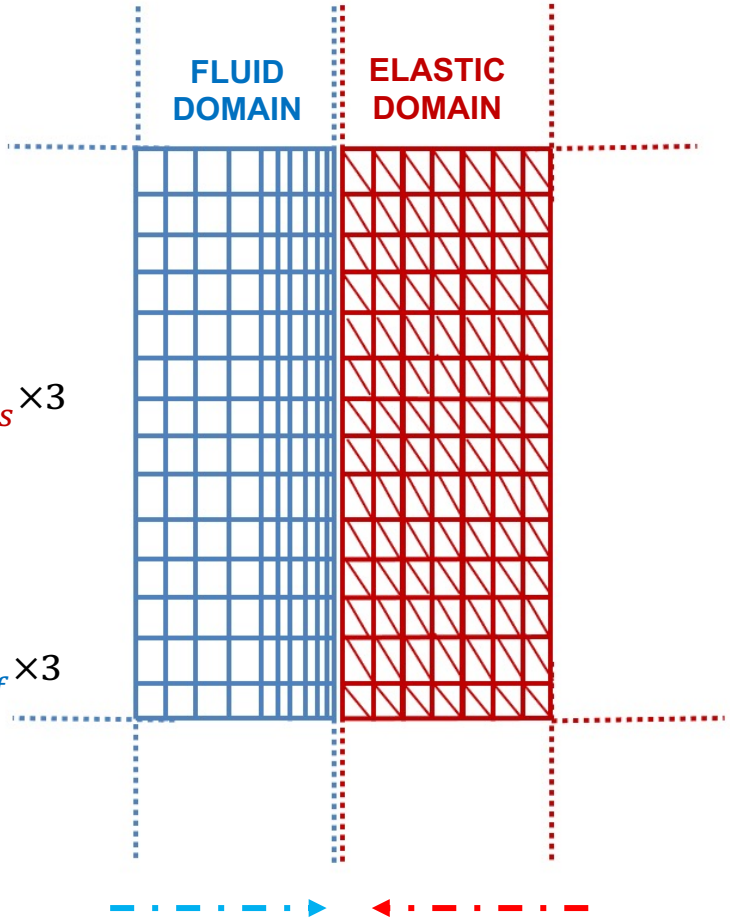
ELASTIC-> FLUID DISPLACEMENT TRANSFER

$$\{U_{\Omega_{\Gamma f}}\}_{n_{\Omega_{\Gamma f}}\times 3} = [H_{FS}]_{n_{\Omega_{\Gamma f}}\times n_{\Omega_{\Gamma s}}} \times \{U_{\Omega_{\Gamma s}}\}_{n_{\Omega_{\Gamma s}}\times 3}$$

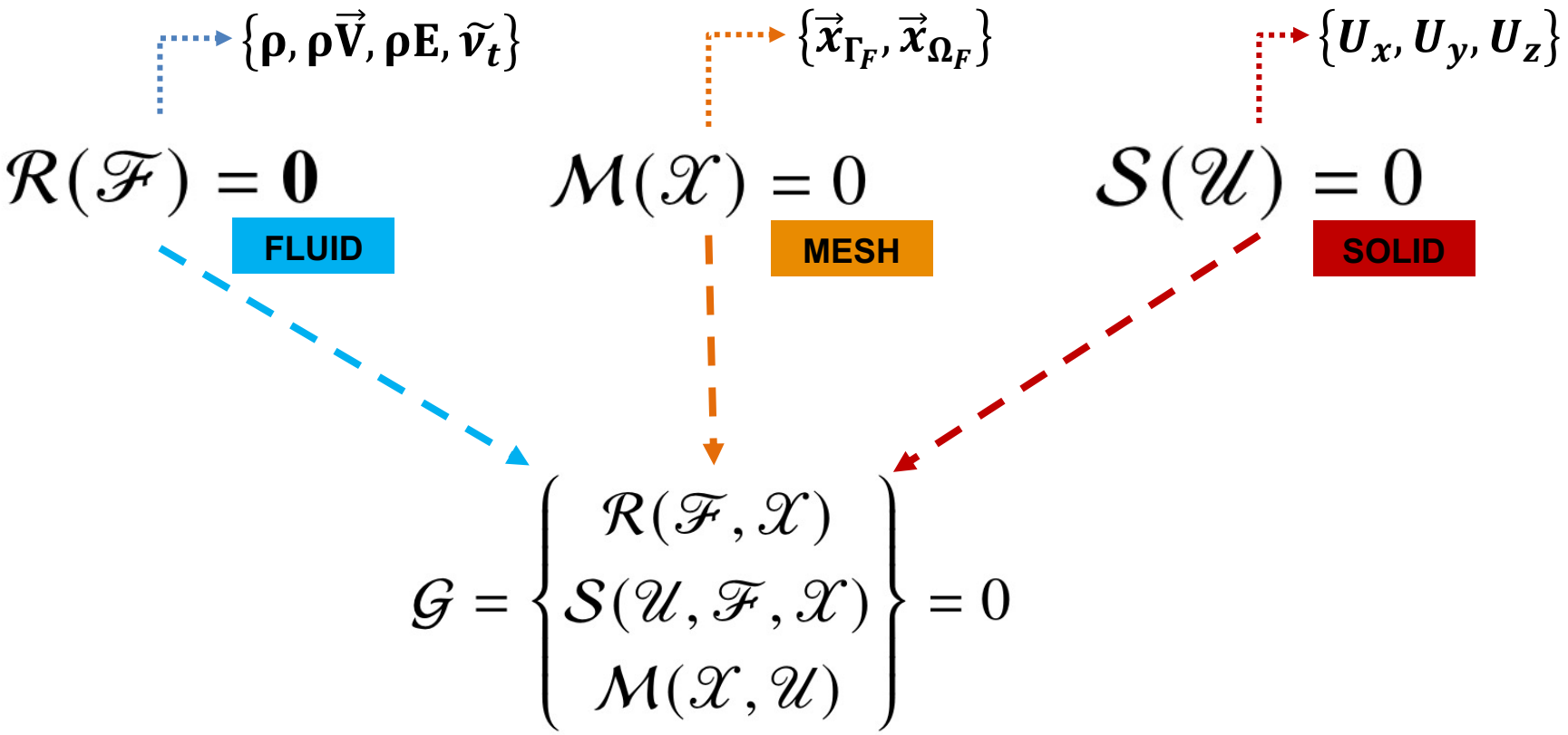
FLUID-> ELASTIC LOAD TRANSFER

$$\{P_{\Omega_{\Gamma s}}\}_{n_{\Omega_{\Gamma s}}\times 3} = [H_{SF}]_{n_{\Omega_{\Gamma S}}\times n_{\Omega_{\Gamma F}}} \times \{P_{\Omega_{\Gamma f}}\}_{n_{\Omega_{\Gamma f}}\times 3}$$

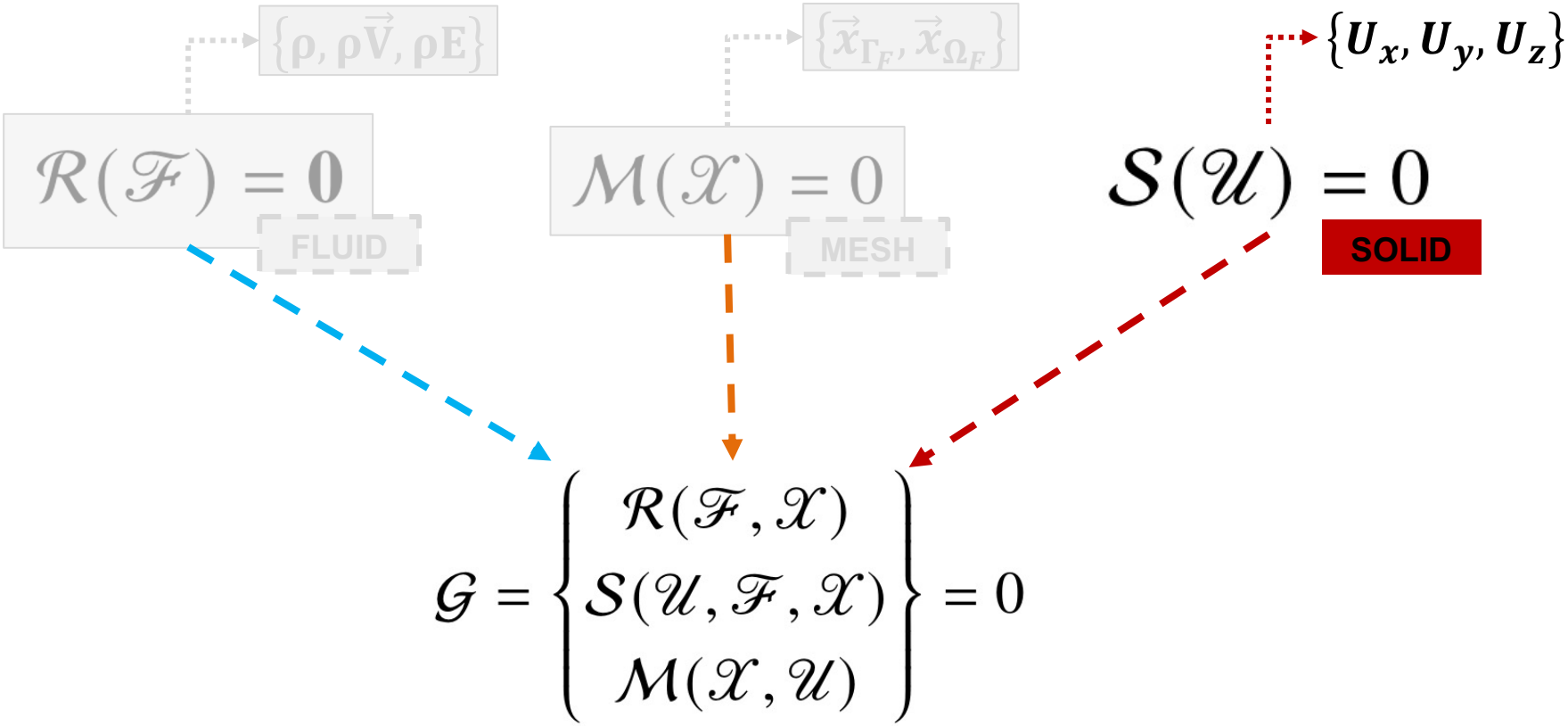
- **NEAREST NEIGHBOUR**
- **RADIAL BASIS FUNCTIONS**



High-Fidelity FSI Framework



High-Fidelity FSI Framework



Elastic System

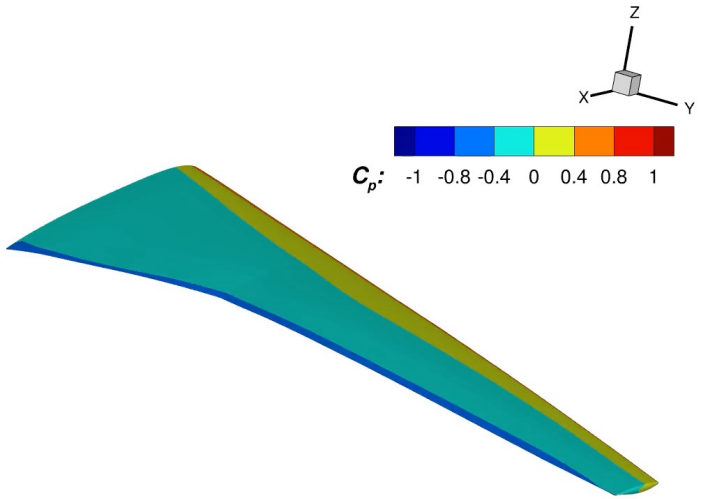
$$\mathcal{S}(\mathscr{U}) = \mathbf{M}\ddot{\mathscr{U}} + \mathbf{K}\mathscr{U} - F(\mathscr{F})$$

Elastic System

$$\mathcal{S}(\mathscr{U}) = \mathbf{M}\ddot{\mathscr{U}} + \mathbf{K}\mathscr{U} - F(\mathscr{F})$$

$$\mathcal{S}(\mathscr{U}) = \mathbf{M}\ddot{\mathscr{U}}_{n+1} + (1+\alpha)\mathbf{K}\mathscr{U}_{n+1} - \alpha\mathbf{K}\mathscr{U}_n - F(\mathscr{F})_\alpha$$

$$(1+\alpha)t_{n+1} - \alpha t_n = t_{n+1} + \alpha\Delta t$$



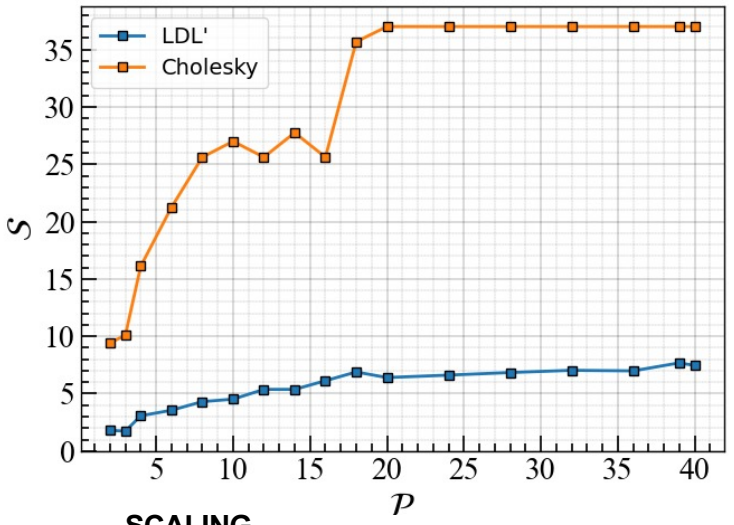
DYNAMIC RESPONSE FOR NASA CRM WING WITH LOW DENSITY WINGBOX

Elastic System

$$\mathcal{S}(\mathcal{U}) = \mathbf{M}\ddot{\mathcal{U}} + \mathbf{K}\mathcal{U} - F(\mathcal{F})$$

0

$$\mathcal{S}(\mathcal{U}) = \mathbf{K}\mathcal{U} - F(\mathcal{F})$$



SCALING CHARACTERITICS

Algorithm 4: Steady aero-elastic coupling algorithm

Result: Tightly coupled steady aeroelastic state

$k = 0;$

while $\epsilon_{F_\Gamma} > tol_1$ & $\epsilon_{u_\Gamma} > tol_2$ & $k < k_{max}$ **do**

  $\mathcal{R}(\mathcal{U}_k) = \mathcal{F}_{k+1};$

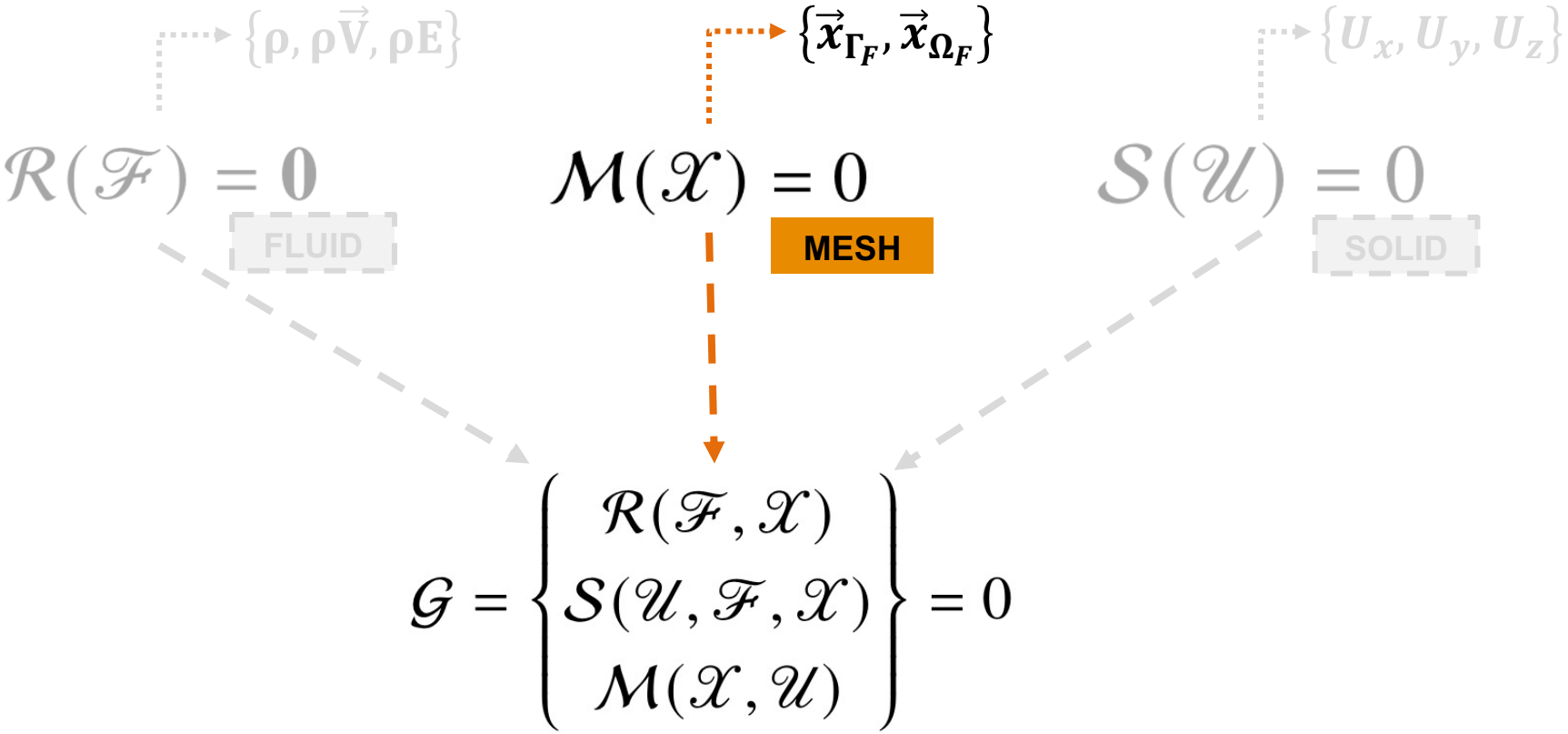
  $\mathcal{S}(\mathcal{F}_{k+1}) = \tilde{\mathcal{U}_{k+1}};$

  $\mathcal{U}_{k+1} = (1-\omega)\mathcal{U}_k + \omega\tilde{\mathcal{U}}_{k+1};$

  $k = k + 1;$

end

High-Fidelity FSI Framework



Mesh Displacement

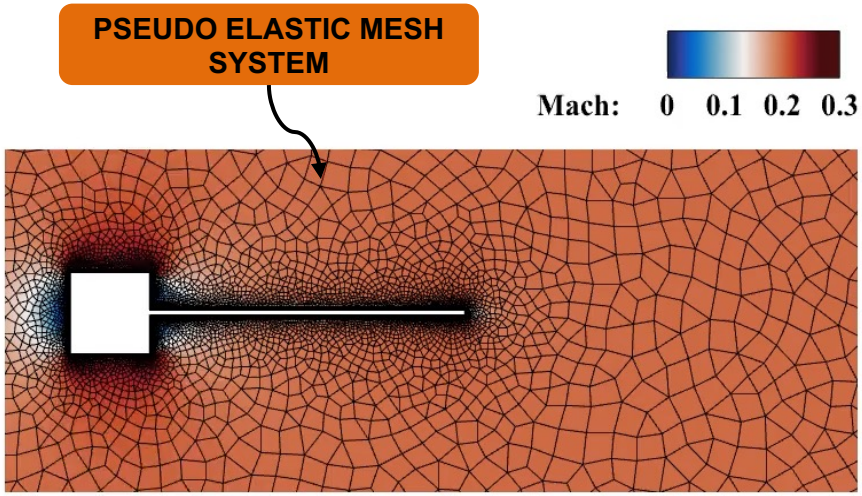
$$\mathcal{M}(\mathscr{X}) = \begin{bmatrix} \overline{\mathbf{K}}_{\Omega_F\Omega_F} & \overline{\mathbf{K}}_{\Omega_F\Omega_{F_\Gamma}} \\ \overline{\mathbf{K}}^T_{\Omega_F\Omega_F} & \overline{\mathbf{K}}_{\Omega_{F_\Gamma}\Omega_{F_\Gamma}} \end{bmatrix} \begin{bmatrix} \mathscr{X}_{\Omega_F} \\ \mathscr{X}_{\Omega_{F_\Gamma}} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left\{U_{\Omega_{\Gamma f}}\right\}_{n_{\Omega_{\Gamma f}}\times 3} = [H_{FS}]_{n_{\Omega_{\Gamma f}}\times n_{\Omega_{\Gamma s}}} \times \left\{U_{\Omega_{\Gamma s}}\right\}_{n_{\Omega_{\Gamma s}}\times 3}$$

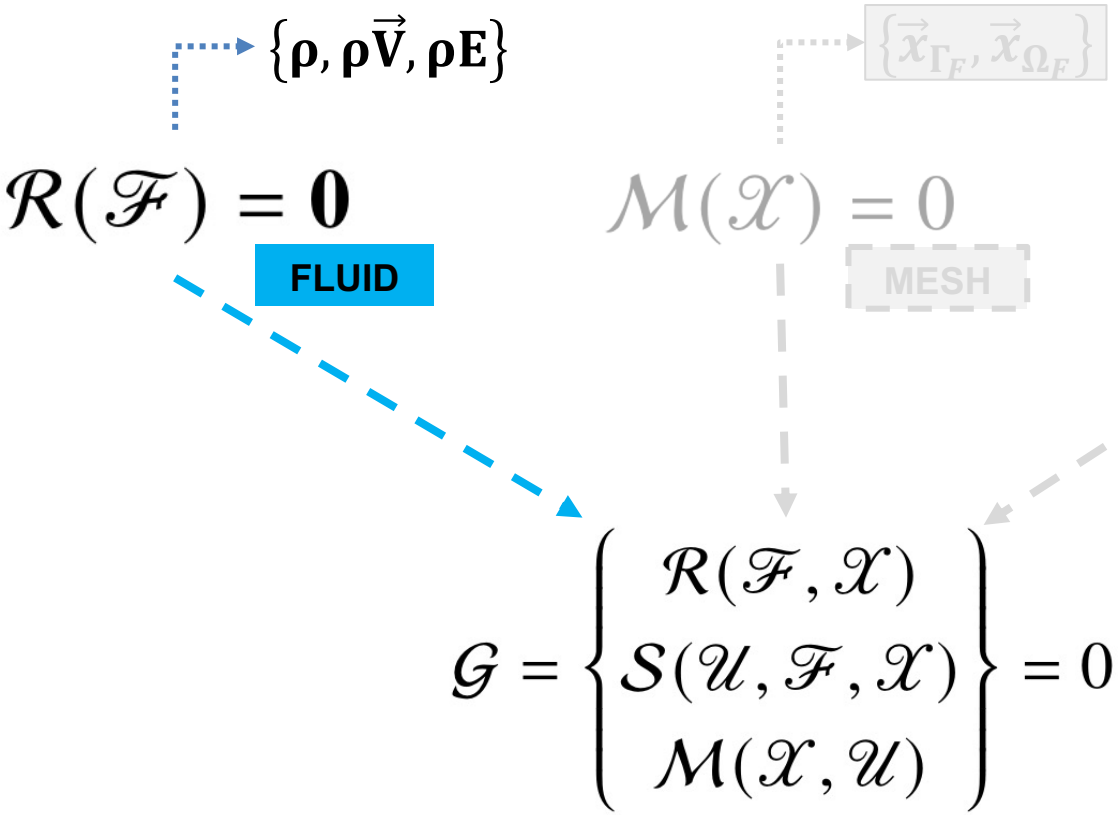
$$\mathscr{X}_{\Omega_{F_\Gamma}} = \mathbf{H_{FS}}\mathscr{U}_{\Omega_{S_\Gamma}}$$

$$\overline{\mathbf{K}}_{\Omega_F\Omega_F}\mathscr{X}_{\Omega_F} = -\overline{\mathbf{K}}_{\Omega_F\Omega_{F_\Gamma}}\mathscr{X}_{\Omega_{F_\Gamma}}$$

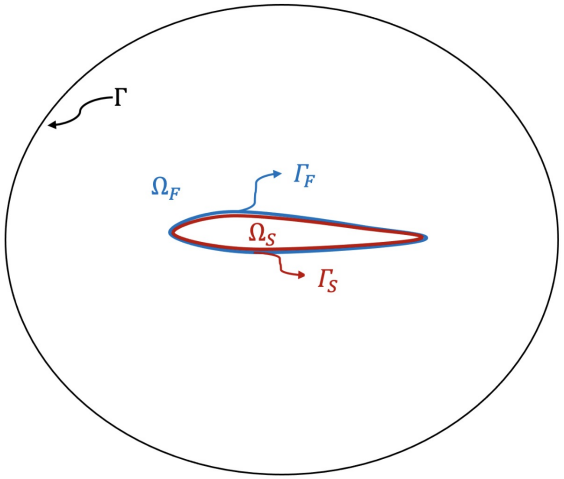
JABOBI UPDATE



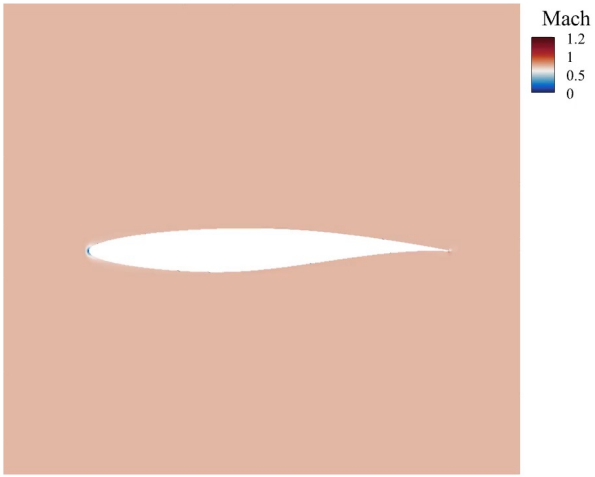
High-Fidelity FSI Framework



Three-dimensional Navier Stokes Solution



COUPLED DOMAIN



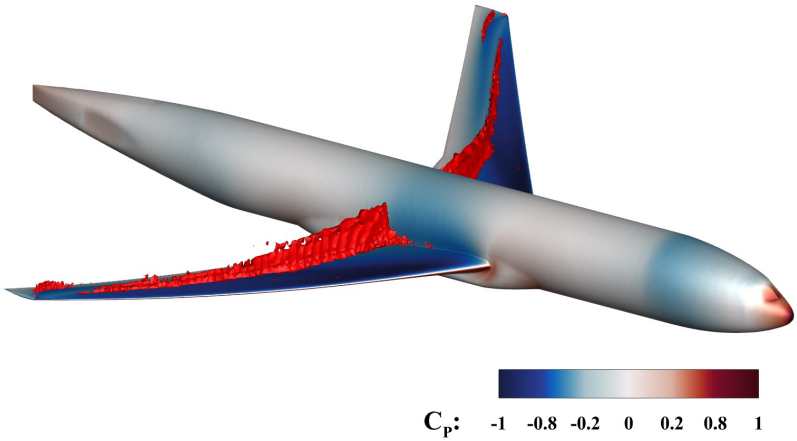
SHOCK FORMATION OVER RAE2822
AT Ma 0.8

$$\mathcal{R}(\mathscr{F}) = \frac{\partial \mathscr{F}}{\partial t} + \nabla \cdot F^{\vec{C}}_{ALE} - \nabla \cdot F^{\vec{V}} - Q = 0$$

$$\vec{V} = V_{\vec{\Gamma}_F}$$

$$\frac{\partial T}{\partial n} = 0$$

$$(W_+) = W_\infty$$



SHOCK FORMATION OVER NASA CRM
AT Ma 0.8

Navier Stokes ALE Formulation

$$\mathcal{R}(\mathscr{F}) = \frac{\partial \mathscr{F}}{\partial t} + \nabla \cdot F^{\vec{C}}_{ALE} - \nabla \cdot F^{\vec{V}} - Q = 0$$

LUMPED RESIDUAL

$$R_i(\mathscr{F}) = \sum_{j=N(i)} (\tilde{F^C_{ij}} - \tilde{F^V_{ij}})\Delta L_{ij} - |\Omega_F|_i Q_i$$

$$F^{\vec{C}}_{ALE} = \left\{ \begin{array}{c} \rho_f(\vec{V} - \vec{V_{\Omega_F}}) \\ \rho_f \vec{V} \otimes (\vec{V} - \vec{V_{\Omega_F}}) + \mathbf{I}p \\ \rho_f(\vec{V} - \vec{V_{\Omega_F}})E + p\vec{V} \end{array} \right\}$$

CONVECTIVE FLUXES

$$F^{\vec{v}2} = \begin{bmatrix} \cdot \\ \cdot \\ c_p \Delta T \end{bmatrix}$$

$$F^{\vec{v}1} = \begin{bmatrix} \cdot \\ \tau \\ \tau \cdot \vec{V} \end{bmatrix}$$

VISCOUS FLUXES

$$Q = \left\{ \begin{array}{c} q_\rho \\ \vec{q}_{\rho\vec{v}} \\ q_{\rho E} \end{array} \right\}$$

SOURCE TERMS

Time Integration : Steady Aeroelastic

$$\frac{d}{dt}\int_{\Omega_{F_i}} \mathscr{F}\, d\Omega_F + R_i(\mathscr{F}) = 0$$

$$|\Omega_F|\frac{\Delta \mathscr{F}_i}{\Delta \xi_i} = -\mathcal{R}(\mathscr{F}^{n+1})$$

EULER IMPLICIT TIME INTEGRATION

$$\Delta\xi_i = \mathcal{C} \times min\left(\frac{|\Omega_i|}{\lambda_i^C}, \frac{|\Omega_i|}{\lambda_i^V}\right)$$

LOCAL TIME-STEP

Algorithm 4: Steady aero-elastic coupling algorithm

Result: Tightly coupled steady aeroelastic state

$k = 0$;

while $\epsilon_{F_\Gamma} > tol_1$ & $\epsilon_{u_\Gamma} > tol_2$ & $k < k_{max}$ **do**

  $\mathcal{R}(\mathcal{U}_k) = \mathcal{F}_{k+1}$;

  $\mathcal{S}(\mathcal{F}_{k+1}) = \tilde{\mathcal{U}_{k+1}}$;

  $\mathcal{U}_{k+1} = (1-\omega)\mathcal{U}_k + \omega\tilde{\mathcal{U}}_{k+1}$;

  $k = k + 1$;

end

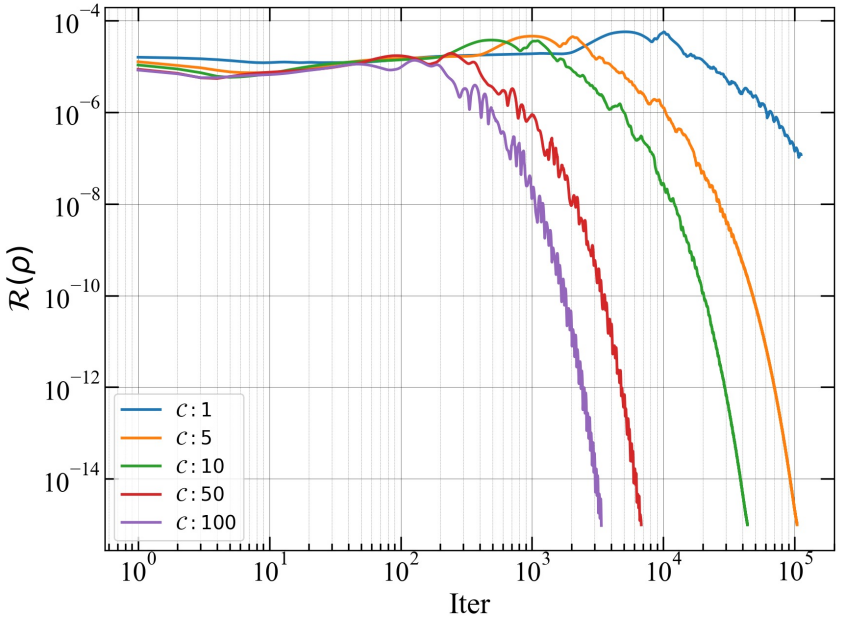
Time Integration : Steady Aeroelastic

$$\left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right) \cdot \Delta\mathscr{F}_i^n = \mathcal{R}_i(\mathscr{F}^n)$$

LINEAR SYSTEM WITH APPROXIMATE JACOBIAN

$$\mathscr{F} = \begin{Bmatrix} \rho \\ \rho(\vec{V_{rel}}) \\ \rho E \\ \tilde{\nu}_t \end{Bmatrix}$$

MEAN-FLOW STATE



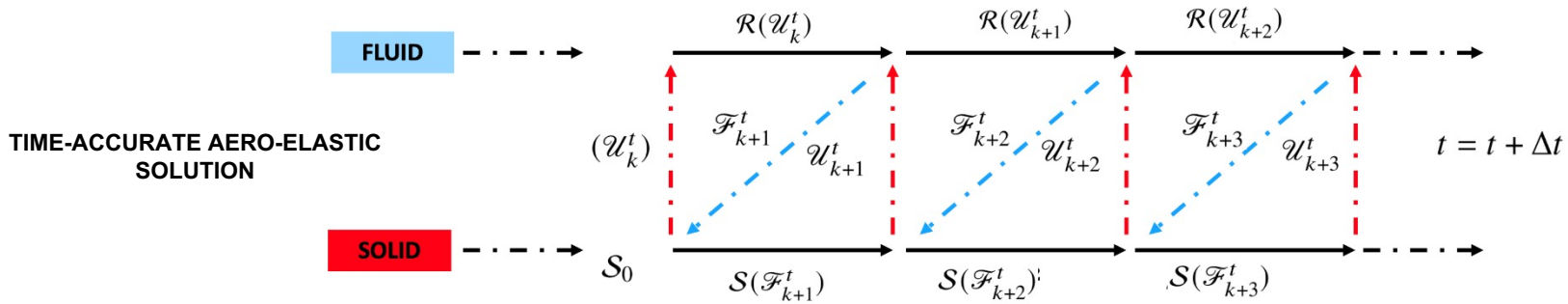
RESIDUAL CONVERGENCE CHARACTERISTICS

Time Integration : Unsteady Aeroelastic

$$\frac{d}{dt}\int_{\Omega_{F_i}} \mathscr{F}\, d\Omega_F + R_i(\mathscr{F}) = 0$$

$$R_i(\mathscr{F}^*) = \underbrace{\frac{3}{\Delta t}}_{\text{COUPLING TIME-STEP}}\mathscr{F}_i + \underbrace{\frac{1}{|\Omega|^{n+1}_{F_i}}}_{\text{FILE I/O}}\left(R_i(\mathscr{F}) - \frac{2}{\Delta t}|\Omega_F|^n_i \mathscr{F}^n_i + \frac{1}{2\Delta t}\underbrace{|\Omega_F|^{n-1}_i \mathscr{F}^{n-1}_i}_{\text{FILE I/O}}\right)$$

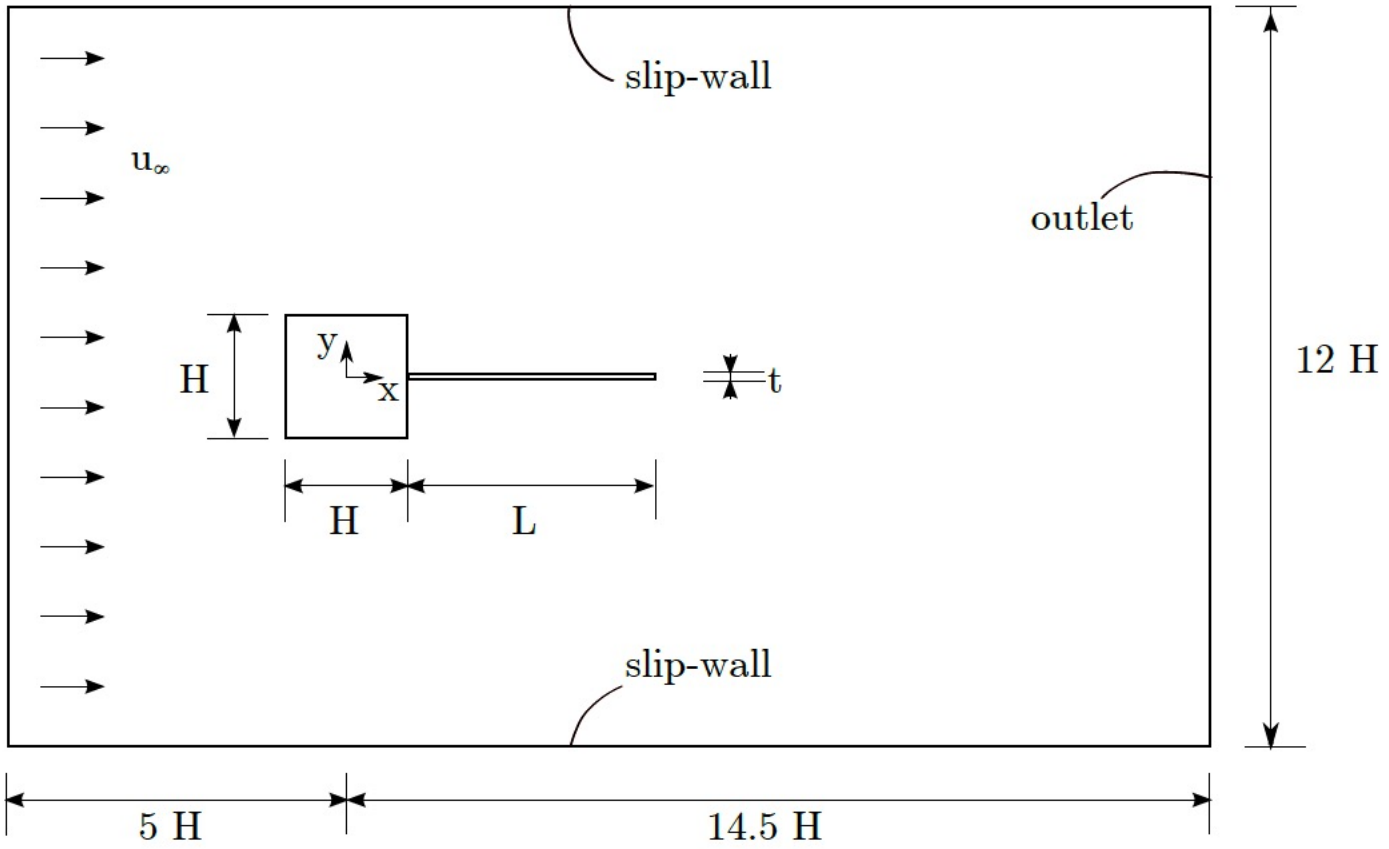
$$\frac{d\mathscr{F}}{d\xi} + R^*_i(\mathscr{F}) = 0$$



Results

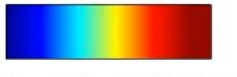
- Unsteady Aeroelasticity
- Steady Aeroelasticity

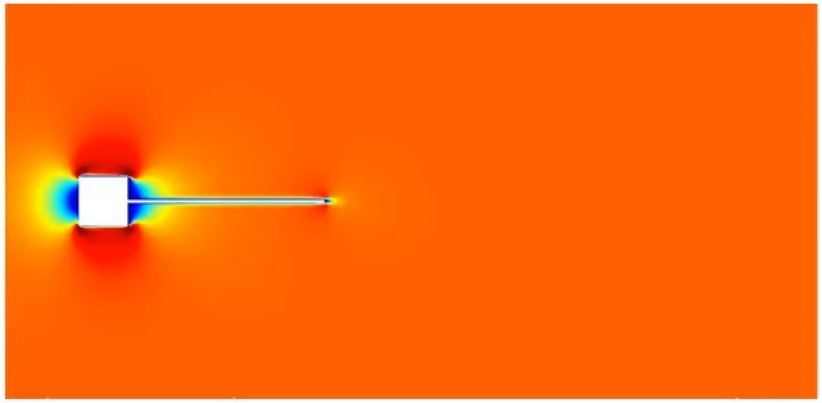
Results: Unsteady Aero-elasticity



Ramm, E., and W. Wall. "Fluid-structure interaction based upon a stabilized (ALE) finite element method." In *4th World Congress on Computational Mechanics: New Trends and Applications, CIMNE, Barcelona*, pp. 1-20. 1998.

Rigid Flap Flow-field





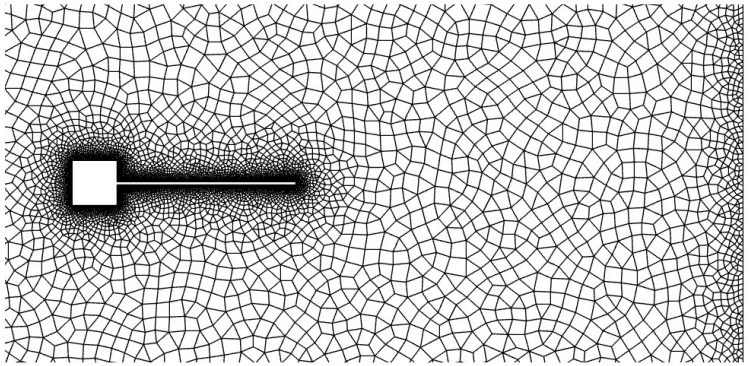
ELASTIC PROPERTIES		
E	2.5×10^5	Pa
ν	0.35	
ρ_s	100	kg/m^3

GEOMETRIC PROPERTIES		
H	0.01	m
L	0.04	m
t	0.0006	m

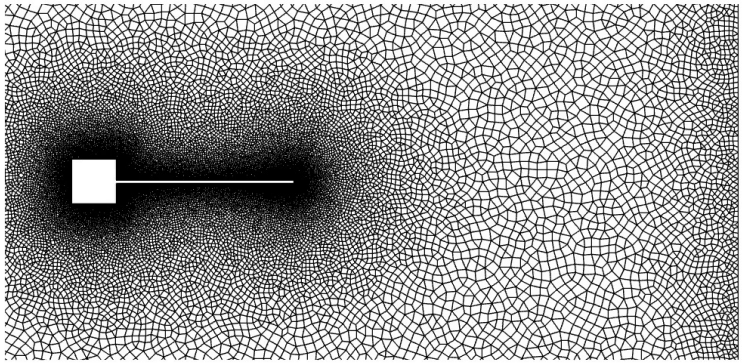
FLOW PROPERTIES		
$\vec{V}_\infty$	0.513	m/s
μ_f	1.82×10^{-5}	kg/m-s
ρ_f	1.18	kg/m^3
Ma_∞	0.2	
Re_t	332	

Fluid Structure Interaction

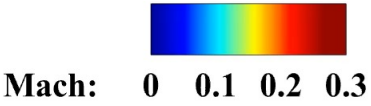
MESH DYNAMICS

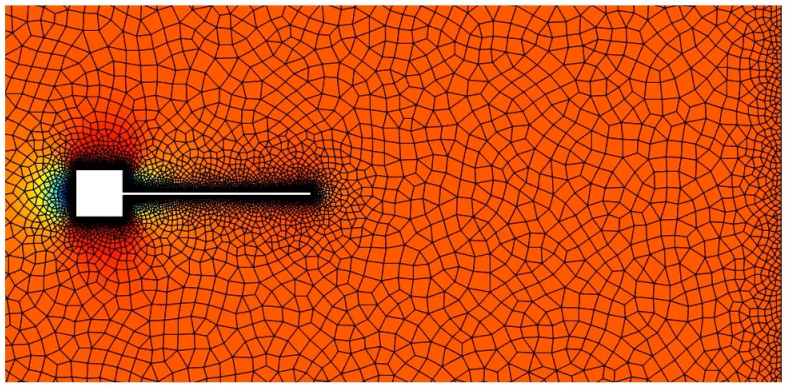


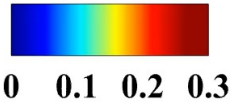
MEDIUM DISCRETIZATION

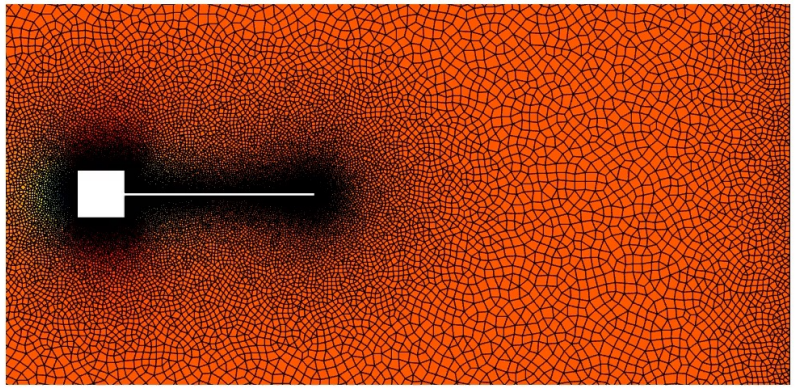


FINE DISCRETIZATION



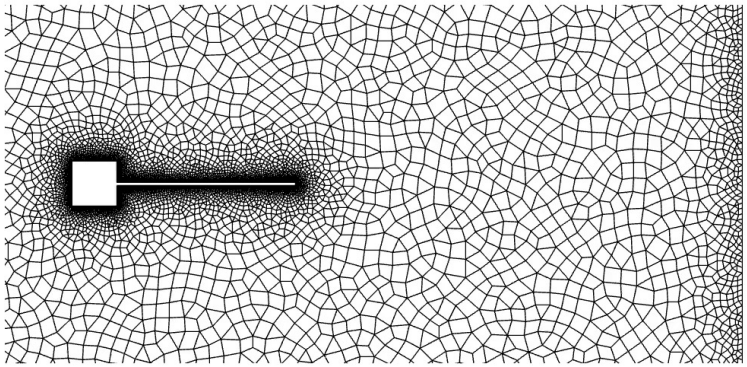




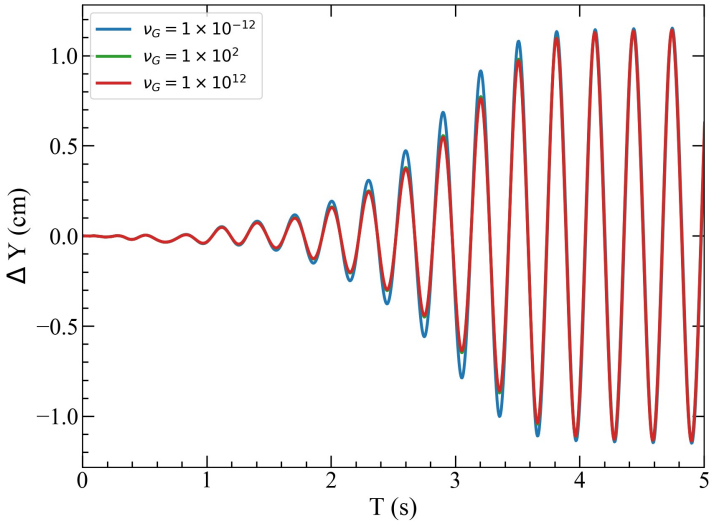


AEROELASTIC RESPONSE

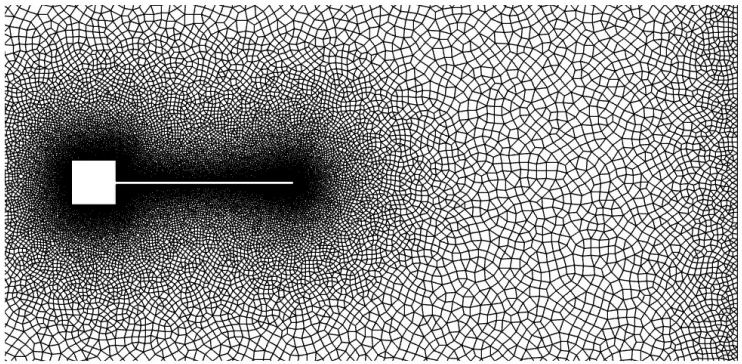
Influence of Fluid Mesh Elasticity



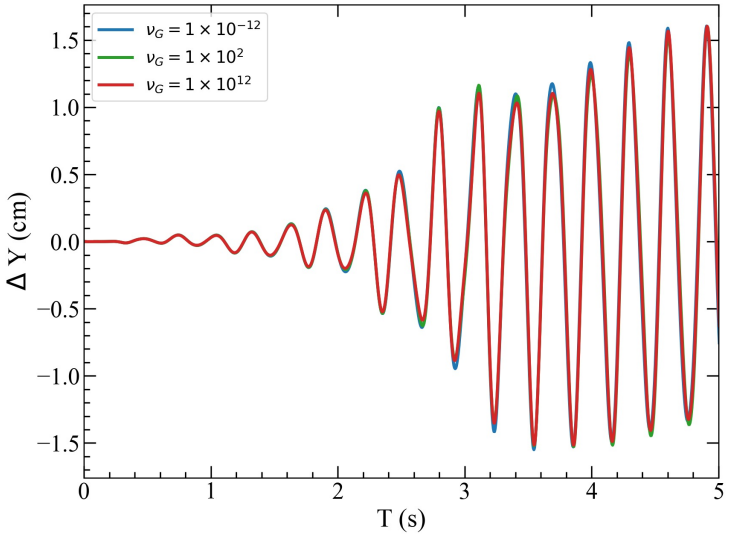
MEDIUM DISCRETIZATION



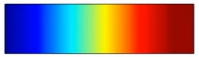
FLAP TIP DISPLACEMENT



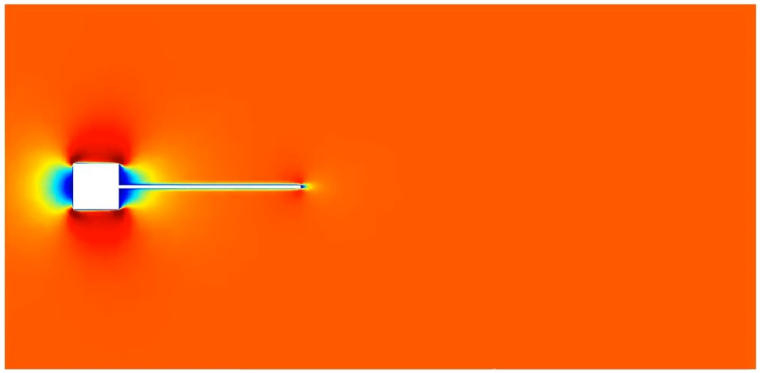
FINE DISCRETIZATION



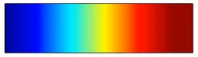
Influence of Elastic Solver



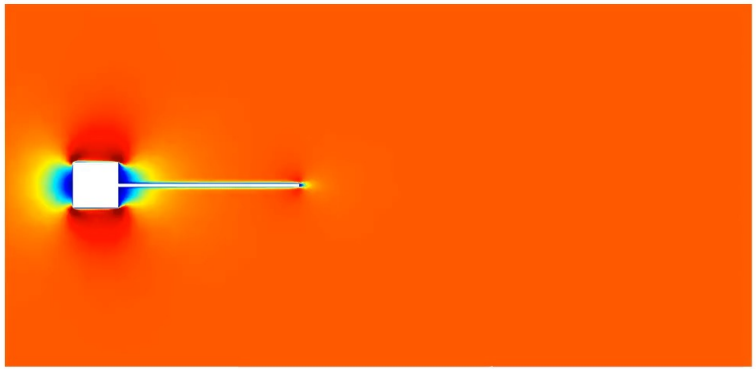
Mach: 0 0.1 0.2 0.3



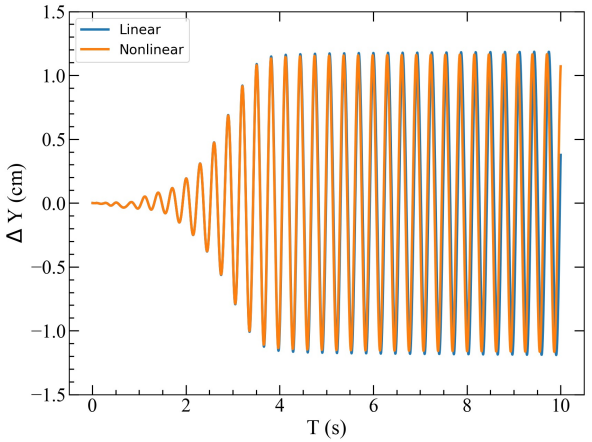
LINEAR ELASTIC



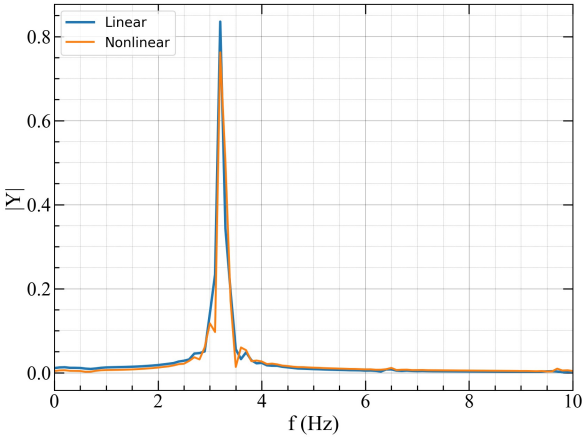
Mach: 0 0.1 0.2 0.3



NON-LINEAR ELASTIC



TIME RESPONSE



FREQUENCY RESPONSE

Validation

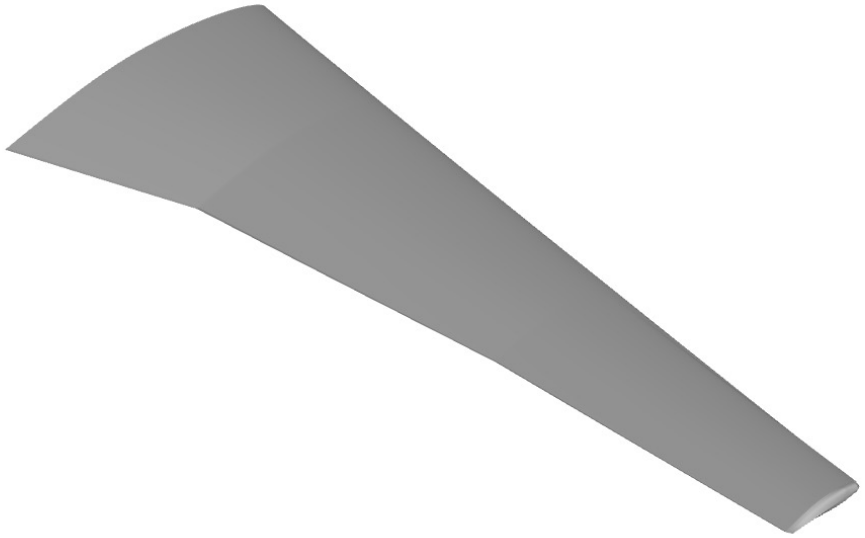
	$f_{avg}(Hz)$	$\|Y\|_{max}(cm)$	St. Number
Wall and Ramm	3.08	1.31	0.06
Kassiotis *et al.*	3.17	1.00	0.062
Habchi *et al.*	3.25	1.02	0.063
Matthies and Steindorf	2.99	1.34	0.058
Wood *et al.*	2.96-3.125	1.1-1.2	0.054-0.061
Sanchez *et al.*	3.05-3.15	1.02-1.25	0.059-0.061
Current	3.18	1.17	0.061

Results

- Unsteady Aeroelasticity
- Steady Aeroelasticity

High Reynolds Number Aerostructural Dynamics Wing (HIRENASD)

- **OBJECTIVE:** Validate static aero-elastic response
- **MOTIVATION:**
  - Well-defined static aero-elastic problem with stable modes
  - Numerous documented results from studies at NASA Langley (AePW)
  - Flow conditions and wing-box structure similar to modern aircraft flight and design conditions



FLOW PROPERTIES		
$\vec{V}_\infty$	256.5	m/s
μ_f	1.15×10^{-5}	kg/m-s
Ma_∞	0.8	
$Re_{C_{ref}}$	7×10^6	

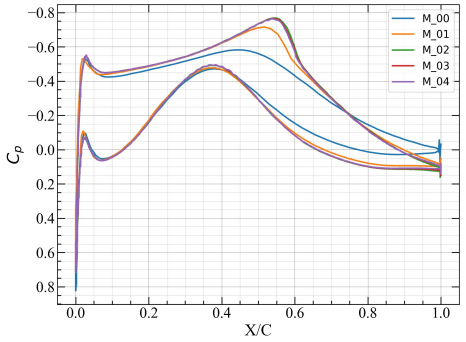
GEOMETRIC PROPERTIES		
C_{Ref}	0.3445	m
A_{Ref}	0.3926	m^2
Semi-Span	1.28571	m
$\Lambda_{c/4}$	34	deg
$Airfoil_R$	BAC 3-11/RES/30/21	
$Airfoil_T$	BAC 3-11/RES/30/21	

ELASTIC PROPERTIES		
E	1.81×10^{11}	Pa
ν	0.264	
ρ_s	7920	kg/m^3

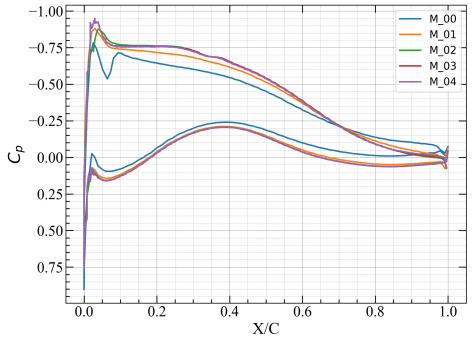
Test conditions and computational domain

TEST CONDITIONS		
Q/E:	2.20×10^{-7}	$\alpha : 0^0 : 4^0$
Q/E:	3.40×10^{-7}	$\alpha : 0^0 : 4^0$
Q/E:	4.80×10^{-7}	$\alpha : 0^0 : 4^0$
Q/E:	7.00×10^{-7}	$\alpha : 0^0 : 4^0$

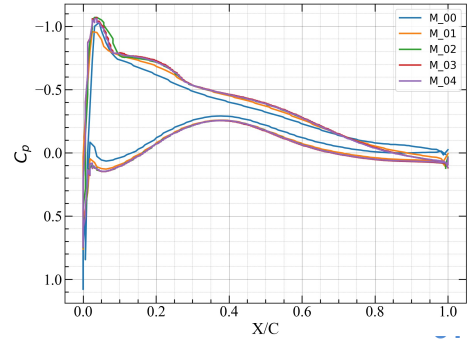
	Resolution	C_L	C_D	C_{M_y}	$\Delta s(m)$
M00:	4.5×10^6	0.2688	0.0201	-0.3992	1×10^{-3}
M01:	7.0×10^6	0.33648	0.0156	-0.4949	1×10^{-4}
M02:	18×10^6	0.3569	0.0146	-0.5267	5×10^{-5}
M03:	22×10^6	0.3571	0.0146	-0.5252	1×10^{-5}
M04:	30×10^6	0.3572	0.0146	-0.5231	1×10^{-6}



η: 14.5 %

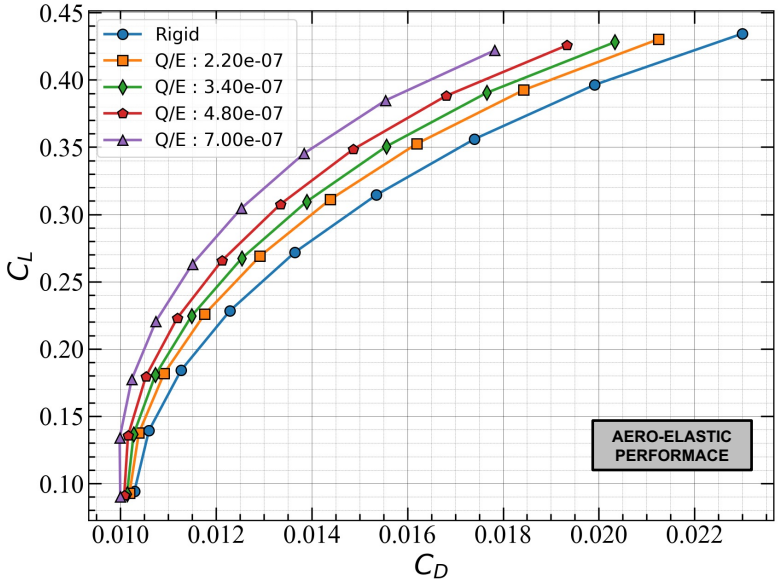


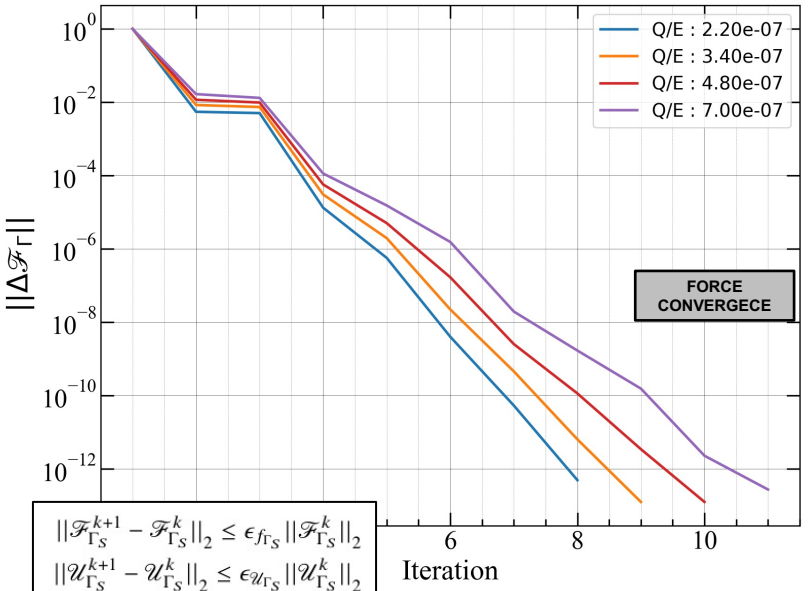
η: 63.3 %



η: 95.3 %

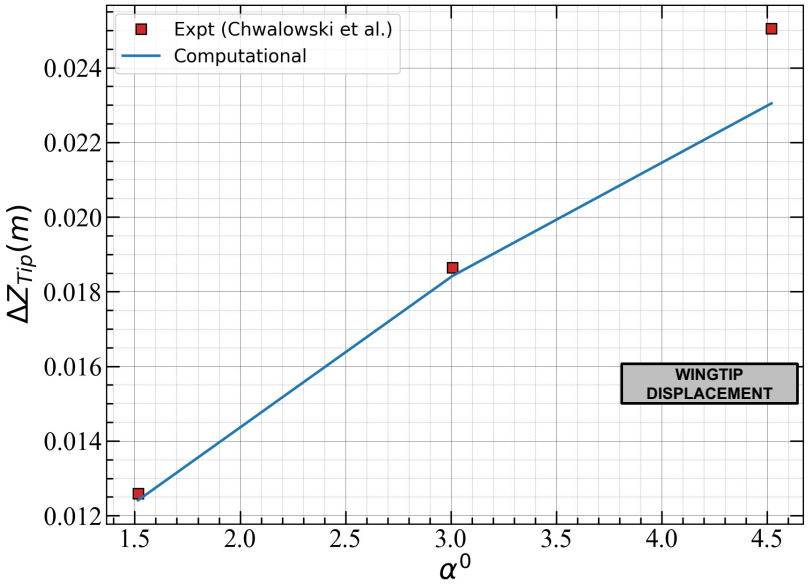
Results

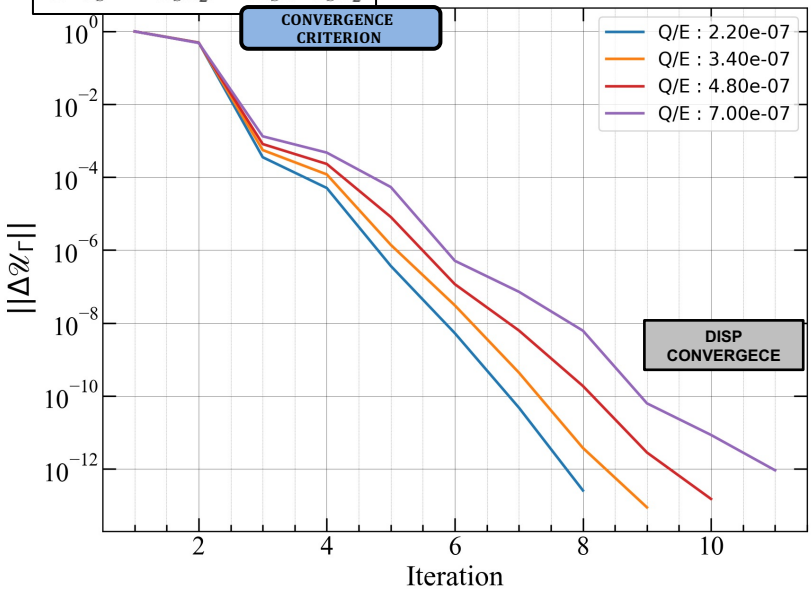




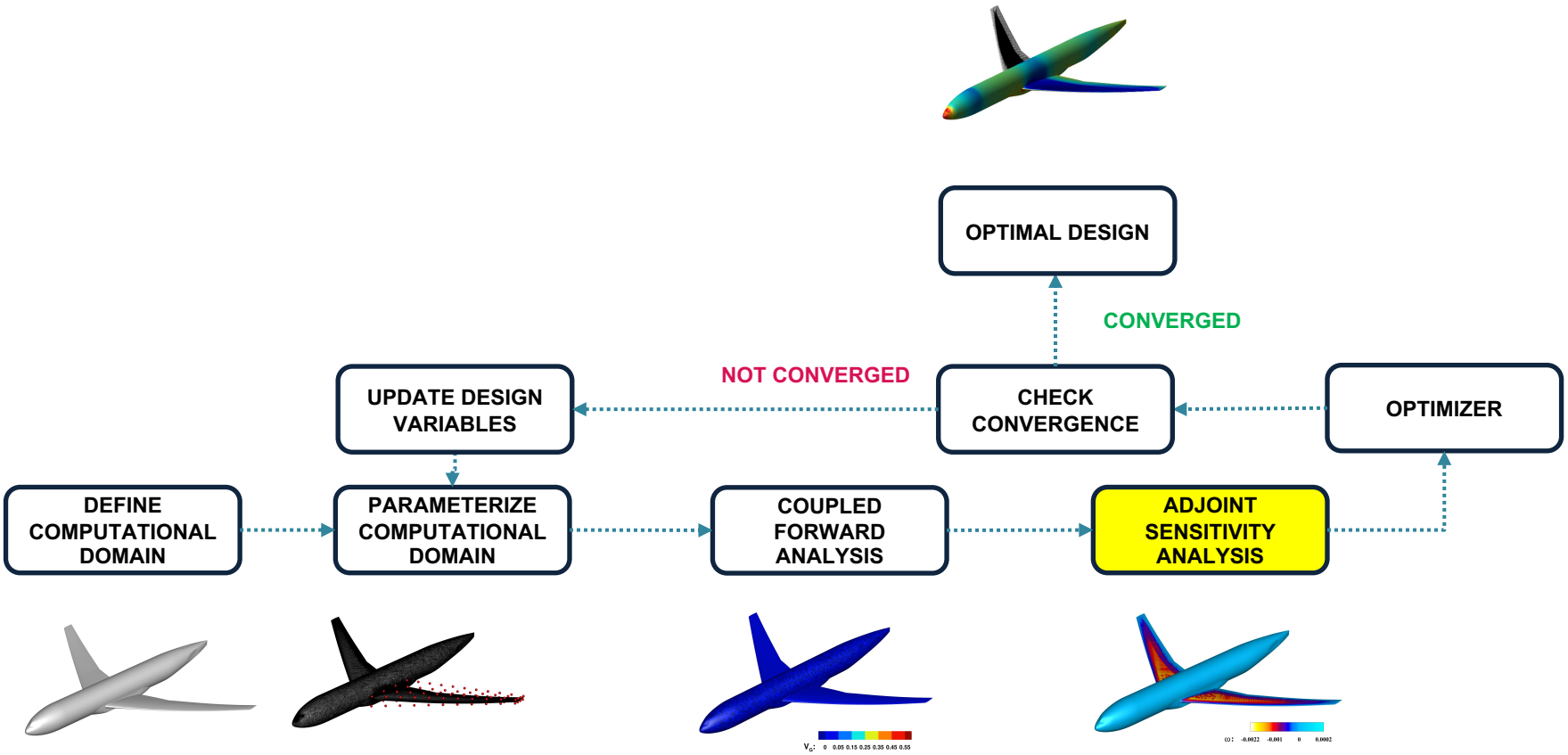
$$||\mathcal{F}_{\Gamma_S}^{k+1} - \mathcal{F}_{\Gamma_S}^{k}||_2 \leq \epsilon_{f_{\Gamma_S}} ||\mathcal{F}_{\Gamma_S}^{k}||_2$$
$$||\mathcal{U}_{\Gamma_S}^{k+1} - \mathcal{U}_{\Gamma_S}^{k}||_2 \leq \epsilon_{\mathcal{U}_{\Gamma_S}} ||\mathcal{U}_{\Gamma_S}^{k}||_2$$

CONVERGENCE CRITERION





Optimization Workflow



ADJOINT SENSITIVITY ANALYSIS

Coupled Adjoint Sensitivities

$$\mathcal{G} = \begin{Bmatrix} \mathcal{R}(\mathcal{F}, \mathcal{X}) \\ \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}) \\ \mathcal{M}(\mathcal{X}, \mathcal{U}) \end{Bmatrix} = 0$$

Coupled Adjoint Sensitivities

$$\mathcal{G} = \begin{Bmatrix} \mathcal{R}(\mathcal{F}, \mathcal{X}) \\ \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}) \\ \mathcal{M}(\mathcal{X}, \mathcal{U}) \end{Bmatrix} = 0$$

$$\begin{aligned} \min_{\alpha} \quad & J(\mathcal{U}, \mathcal{F}, \mathcal{X}, \alpha) \\ \text{s.t.} \quad & \mathcal{R}(\mathcal{F}, \mathcal{X}, \alpha) - \mathcal{F} = 0 \\ & \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}, \alpha) = 0 \\ & \mathcal{M}(\mathcal{U}, \alpha) - \mathcal{X} = 0 \end{aligned}$$

Coupled Adjoint Sensitivities

$$\mathcal{G} = \begin{Bmatrix} \mathcal{R}(\mathcal{F}, \mathcal{X}) \\ \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}) \\ \mathcal{M}(\mathcal{X}, \mathcal{U}) \end{Bmatrix} = 0$$

$$\begin{aligned} \min_{\alpha} \quad & J(\mathcal{U}, \mathcal{F}, \mathcal{X}, \alpha) \\ \text{s.t.} \quad & \mathcal{R}(\mathcal{F}, \mathcal{X}, \alpha) - \mathcal{F} = 0 \\ & \mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}, \alpha) = 0 \\ & \mathcal{M}(\mathcal{U}, \alpha) - \mathcal{X} = 0 \end{aligned}$$

$$\begin{aligned} \mathcal{L}(\alpha, \mathcal{U}, \mathcal{F}, \mathcal{X}) = J(\mathcal{U}, \mathcal{F}, \mathcal{X}, \alpha) + \lambda_F^T [\mathcal{R}(\mathcal{F}, \mathcal{X}, \alpha) - \mathcal{F} = 0] \\ + \lambda_S^T [\mathcal{S}(\mathcal{U}, \mathcal{F}, \mathcal{X}, \alpha)] + \lambda_M^T [\mathcal{M}(\mathcal{U}, \alpha) - \mathcal{X} = 0] \end{aligned}$$

Coupled Adjoint Sensitivities

$$\mathscr{L}(\alpha, \mathscr{U}, \mathscr{F}, \mathscr{X}) = J(\mathscr{U}, \mathscr{F}, \mathscr{X}, \alpha) + \lambda_F^T [\mathcal{R}(\mathscr{F}, \mathscr{X}, \alpha) - \mathscr{F} = 0]$$
$$+ \lambda_S^T [\mathcal{S}(\mathscr{U}, \mathscr{F}, \mathscr{X}, \alpha)] + \lambda_M^T [\mathcal{M}(\mathscr{U}, \alpha) - \mathscr{X} = 0]$$

Coupled Adjoint Sensitivities

$$\mathscr{L}(\alpha,\mathcal{U},\mathcal{F},\mathcal{X}) = J(\mathcal{U},\mathcal{F},\mathcal{X},\alpha) + \lambda_F^T[\mathcal{R}(\mathcal{F},\mathcal{X},\alpha) - \mathcal{F} = 0] + \lambda_S^T[\mathcal{S}(\mathcal{U},\mathcal{F},\mathcal{X},\alpha)] + \lambda_M^T[\mathcal{M}(\mathcal{U},\alpha) - \mathcal{X} = 0]$$

$$\begin{aligned}\frac{d\mathscr{L}}{d\alpha} = {} & \frac{\partial J}{\partial \mathcal{U}}\frac{d\mathcal{U}}{d\alpha} + \frac{\partial J}{\partial \mathcal{F}}\frac{d\mathcal{F}}{d\alpha} + \frac{\partial J}{\partial \mathcal{X}}\frac{d\mathcal{X}}{d\alpha} + \frac{\partial J}{\partial \alpha} \\ & + \lambda_F^T\left[\frac{\partial \mathcal{R}}{\partial \mathcal{F}}\frac{d\mathcal{F}}{d\alpha} + \frac{\partial \mathcal{R}}{\partial \mathcal{X}}\frac{d\mathcal{X}}{d\alpha} + \frac{\partial \mathcal{R}}{\partial \alpha} - \frac{d\mathcal{F}}{d\alpha}\right] \\ & + \lambda_S^T\left[\frac{\partial \mathcal{S}}{\partial \mathcal{U}}\frac{d\mathcal{U}}{d\alpha} + \frac{\partial \mathcal{S}}{\partial \mathcal{F}}\frac{d\mathcal{F}}{d\alpha} + \frac{\partial \mathcal{S}}{\partial \mathcal{X}}\frac{d\mathcal{X}}{d\alpha} + \frac{\partial \mathcal{S}}{\partial \alpha}\right] \\ & + \lambda_M^T\left[\frac{\partial \mathcal{M}}{\partial \mathcal{U}}\frac{d\mathcal{U}}{d\alpha} + \frac{\partial \mathcal{M}}{\partial \alpha} - \frac{d\mathcal{X}}{d\alpha}\right]\end{aligned}$$

$\frac{d*}{d\alpha}$ **Implicit terms**

$$\frac{dL}{d\alpha} = \cancel{\frac{d\mathcal{U}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{U}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{U}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{U}}\right]$$
$$+ \cancel{\frac{d\mathcal{F}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{F}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{F}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{F}} - \lambda_F^T\right]$$
$$+ \cancel{\frac{d\mathcal{X}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{X}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{X}} - \lambda_M^T\right]$$
$$+ \frac{\partial J}{\partial \alpha} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \alpha} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \alpha} + \lambda_S^T \frac{\partial S}{\partial \alpha}$$

$$\frac{dL}{d\alpha} = \cancel{\frac{d\mathcal{U}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{U}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{U}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{U}}\right]$$

$$+ \cancel{\frac{d\mathcal{F}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{F}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{F}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{F}} - \lambda_F^T\right]$$

$$+ \cancel{\frac{d\mathcal{X}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{X}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{X}} - \lambda_M^T\right]$$

$$+ \frac{\partial J}{\partial \alpha} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \alpha} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \alpha} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \alpha}$$

$$\frac{\partial J}{\partial \mathcal{U}} = -\lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{U}} - \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{U}}$$ } STRUCTURAL ADJOINT

$$\lambda_F^T = \frac{\partial J}{\partial \mathcal{F}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{F}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{F}}$$ } FLUID ADJOINT

$$\lambda_M^T = \frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{X}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{X}}$$ } MESH ADJOINT

$$\frac{dL}{d\alpha} = \cancel{\frac{d\mathcal{U}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{U}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{U}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{U}}\right]$$

$$+ \cancel{\frac{d\mathcal{F}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{F}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{F}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{F}} - \lambda_F^T\right]$$

$$+ \cancel{\frac{d\mathcal{X}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{X}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{X}} - \lambda_M^T\right]$$

$$+ \frac{\partial J}{\partial \alpha} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \alpha} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \alpha} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \alpha}$$

$$\frac{\partial J}{\partial \mathcal{U}} = -\lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{U}} - \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{U}}$$ } STRUCTURAL ADJOINT

$$\lambda_F^T = \frac{\partial J}{\partial \mathcal{F}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{F}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{F}}$$ } FLUID ADJOINT

$$\lambda_M^T = \frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{X}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{X}}$$ } MESH ADJOINT

Fluid adjoint evaluation

$$\left(\frac{|\Omega_{F_i}|}{\Delta \xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j} \right) \cdot \Delta \mathscr{F}_i^n = \mathcal{R}_i(\mathscr{F}^n)$$

APPROXIMATE JACOBIAN

$$\mathscr{F}_i^{(n+1)} = \mathscr{F}_i^n - \left(\frac{|\Omega_{F_i}|}{\Delta \xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j} \right)^{-1} \mathcal{R}_i(\mathscr{F}^n)$$

Fluid adjoint evaluation

$$\left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right) \cdot \Delta\mathscr{F}_i^n = \mathcal{R}_i(\mathscr{F}^n)$$

$$\mathscr{F}_i^{(n+1)} = \mathscr{F}_i^n - \left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right)^{-1} \mathcal{R}_i(\mathscr{F}^n)$$

$$\mathscr{F}^{(n+1)} = \underbrace{\mathscr{F}^n - \mathscr{B}(\mathscr{F}^n)\mathcal{R}(\mathscr{F}^n)}_{\mathscr{A}(\mathscr{F})}$$

Fluid adjoint evaluation

$$\left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right) \cdot \Delta\mathscr{F}_i^n = \mathcal{R}_i(\mathscr{F}^n)$$

$$\mathscr{F}_i^{(n+1)} = \mathscr{F}_i^n - \left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right)^{-1} \mathcal{R}_i(\mathscr{F}^n)$$

$$\mathscr{F}^{(n+1)} = \underbrace{\mathscr{F}^n - \mathscr{B}(\mathscr{F}^n)\mathcal{R}(\mathscr{F}^n)}_{\mathscr{A}(\mathscr{F})}$$

$$\lambda_F = \frac{\partial J}{\partial \mathscr{F}}^T + \lambda_F \frac{\partial \mathscr{A}}{\partial \mathscr{F}}^T + \lambda_S \frac{\partial \mathcal{S}}{\partial \mathscr{F}}^T$$

Fluid adjoint evaluation

$$\left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right) \cdot \Delta\mathscr{F}_i^n = \mathcal{R}_i(\mathscr{F}^n)$$

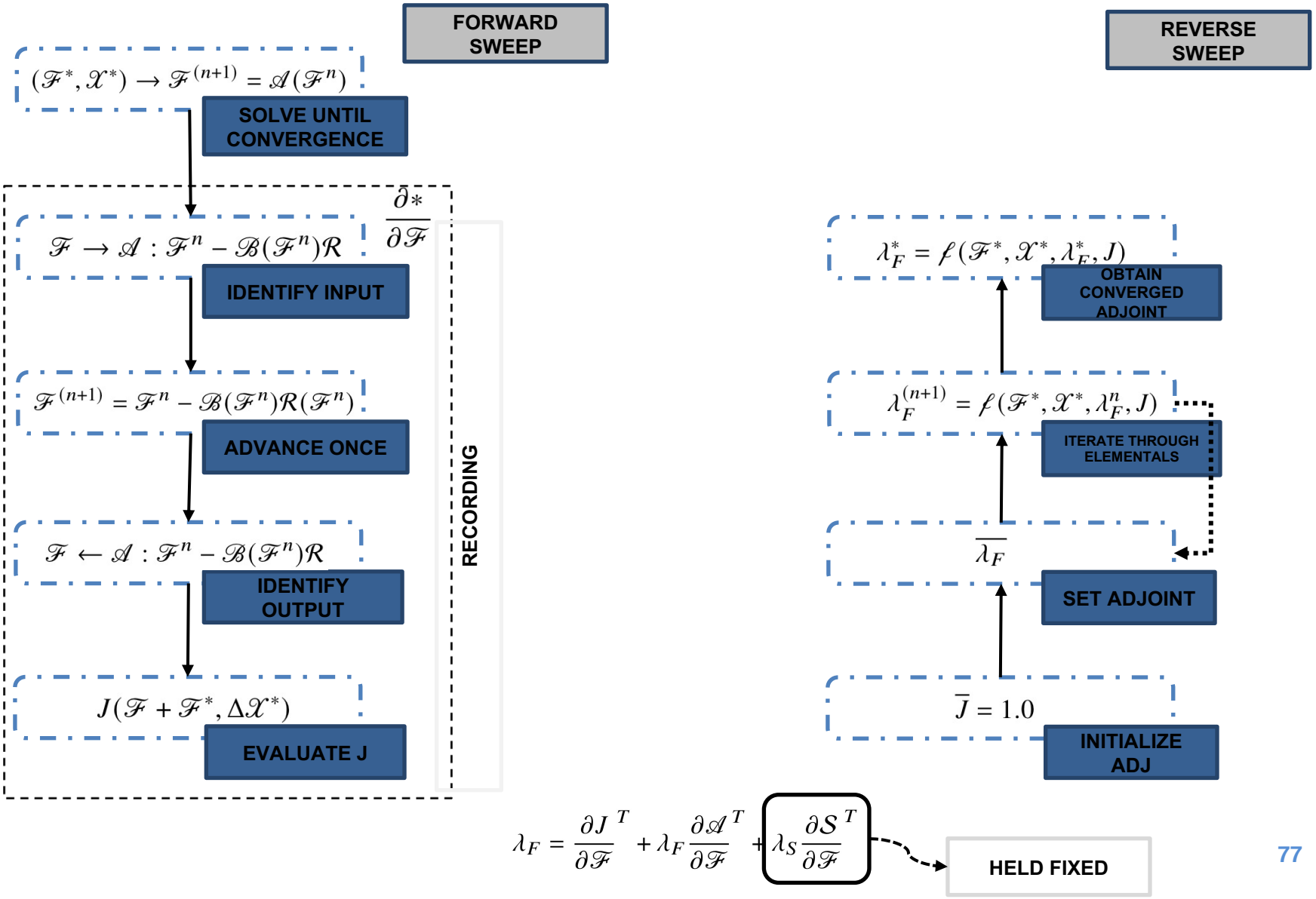
$$\mathscr{F}_i^{(n+1)} = \mathscr{F}_i^n - \left(\frac{|\Omega_{F_i}|}{\Delta\xi_i^n} + \frac{\partial \mathcal{R}_i(\mathscr{F}^n)}{\partial \mathscr{F}_j}\right)^{-1} \mathcal{R}_i(\mathscr{F}^n)$$

$$\mathscr{F}^{(n+1)} = \underbrace{\mathscr{F}^n - \mathscr{B}(\mathscr{F}^n)\mathcal{R}(\mathscr{F}^n)}_{\mathscr{A}(\mathscr{F})}$$

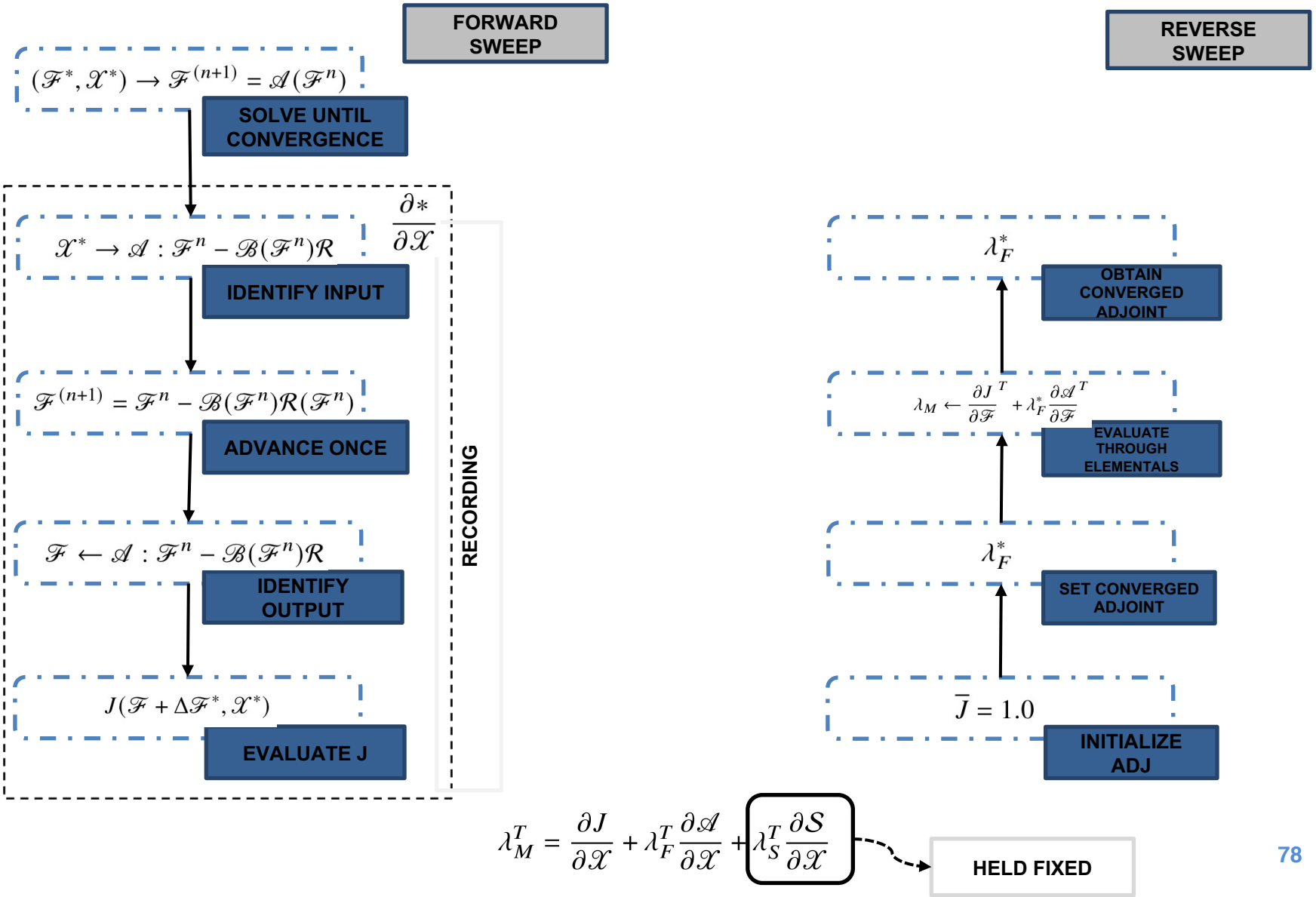
$$\lambda_F = \frac{\partial J}{\partial \mathscr{F}}^T + \lambda_F \frac{\partial \mathscr{A}}{\partial \mathscr{F}}^T + \lambda_S \frac{\partial \mathcal{S}}{\partial \mathscr{F}}^T$$

$$\lambda_F^{(n+1)} = \ell(\mathscr{F}^*, \mathscr{X}^*, \lambda_F^n, J)$$

Flow Adjoint Implicit Evaluation



Mesh Adjoint Evaluation



$$\lambda_M^T = \frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{A}}{\partial \mathcal{X}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{X}}$$

HELD FIXED

Global sensitivity

$$\frac{dL}{d\alpha} = \cancel{\frac{d\mathcal{U}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{U}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{U}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{U}}\right]$$

$$+ \cancel{\frac{d\mathcal{F}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{F}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{F}} + \lambda_S^T \frac{\partial \mathcal{S}}{\partial \mathcal{F}} - \lambda_F^T\right]$$

$$+ \cancel{\frac{d\mathcal{X}}{d\alpha}}\left[\frac{\partial J}{\partial \mathcal{X}} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \mathcal{X}} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \mathcal{X}} - \lambda_M^T\right]$$

$$+ \frac{\partial J}{\partial \alpha} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \alpha} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \alpha} + \lambda_S^T \frac{\partial S}{\partial \alpha}$$

$$\boxed{\frac{dL}{d\alpha} = \frac{\partial J}{\partial \alpha} + \lambda_F^T \frac{\partial \mathcal{R}}{\partial \alpha} + \lambda_M^T \frac{\partial \mathcal{M}}{\partial \alpha} + \lambda_S^T \frac{\partial S}{\partial \alpha}}$$

GLOBAL SENSITIVITY

ELASTIC-> FLUID DISPLACEMENT TRANSFER

$$\{U_{\Omega_{\Gamma f}}\}_{n_{\Omega_{\Gamma f}}\times 3} = [H_{FS}]_{n_{\Omega_{\Gamma f}}\times n_{\Omega_{\Gamma s}}} \times \{U_{\Omega_{\Gamma s}}\}_{n_{\Omega_{\Gamma s}}\times 3}$$

FLUID-> ELASTIC LOAD TRANSFER

$$\{P_{\Omega_{\Gamma s}}\}_{n_{\Omega_{\Gamma s}}\times 3} = [H_{SF}]_{n_{\Omega_{\Gamma S}}\times n_{\Omega_{\Gamma F}}} \times \{P_{\Omega_{\Gamma f}}\}_{n_{\Omega_{\Gamma f}}\times 3}$$

TRANSFORMATION MATRICES

Results : Coupled Topology Optimization

- NASA BSCW (Small Aspect Ratio)
- NASA CRM (High Aspect Ratio)

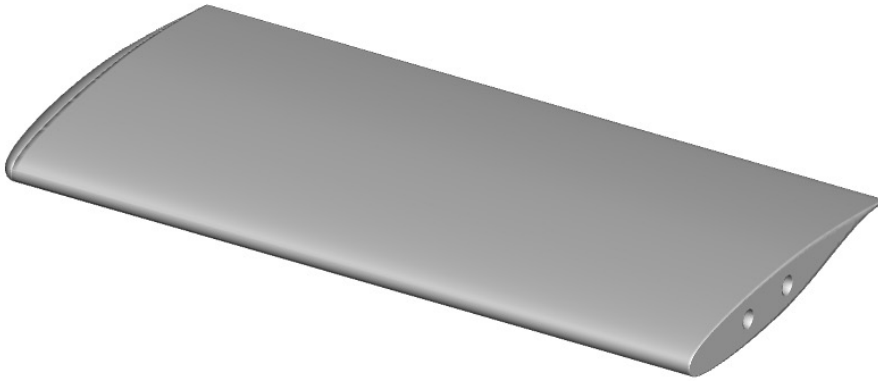
NASA Benchmark Supercritical Wing

$$\min_{\zeta} \quad C_{elastic}$$

$$\text{s.t.} \quad L = \rho_S \times (v_f \times V) \times g$$

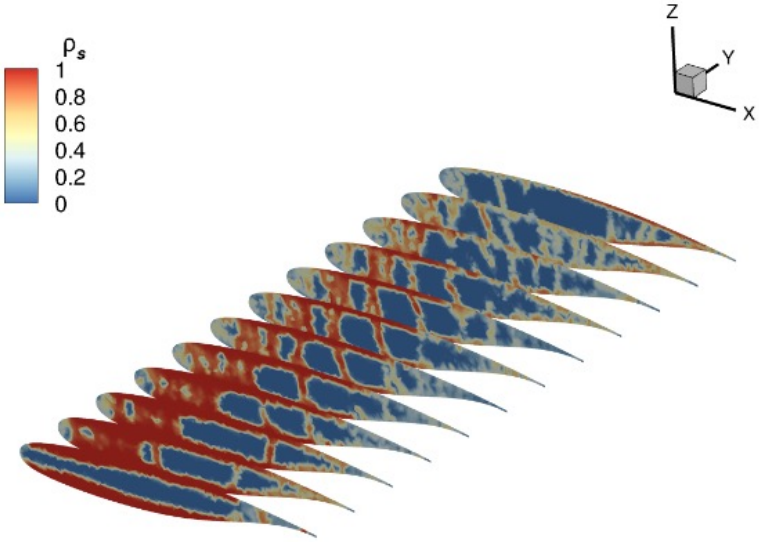
$$[\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = (\zeta_e) + F_g(\zeta_e)$$

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$

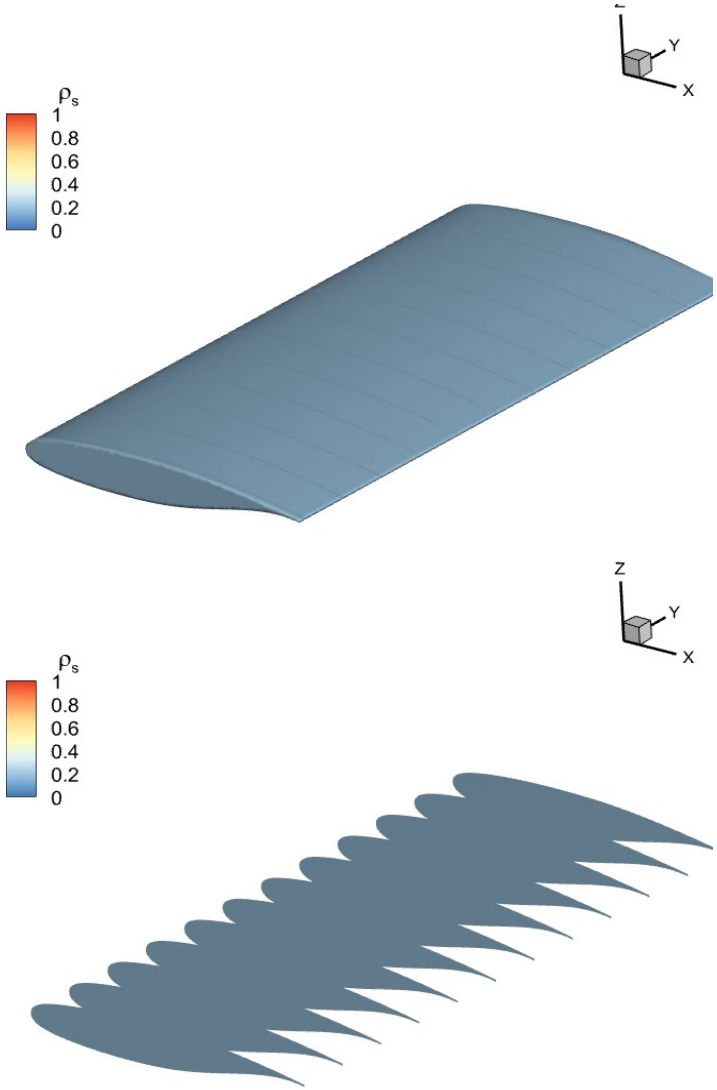


Parameter	Value
Ma_∞	0.85
Re_c	4.491×10^6
$\rho_f\,(kg/m^3)$	0.957
p_{total} (Pa)	36260.0028
$\rho_s(kg/m^3)$	2700
$E(GPa)$	69
ν	0.35
Medium	R-134a

Optimal material distribution

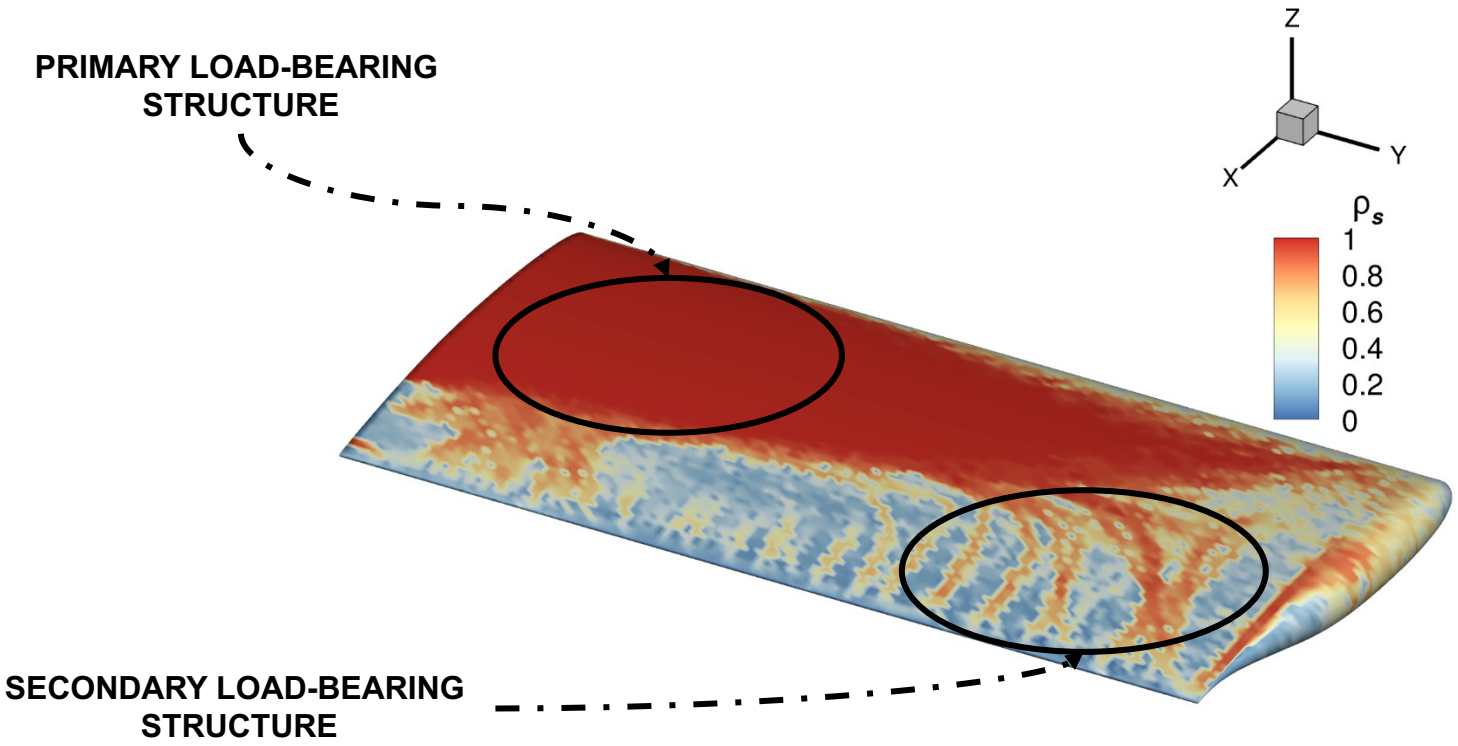


v_f= 0.2



OPTIMAL MATERIAL DISTRIZBUTION

Surface material contour



83

Results : Coupled Topology Optimization

- NASA BSCW (Small Aspect Ratio)
- NASA CRM (High Aspect Ratio)

NASA CRM Wing in Jig State

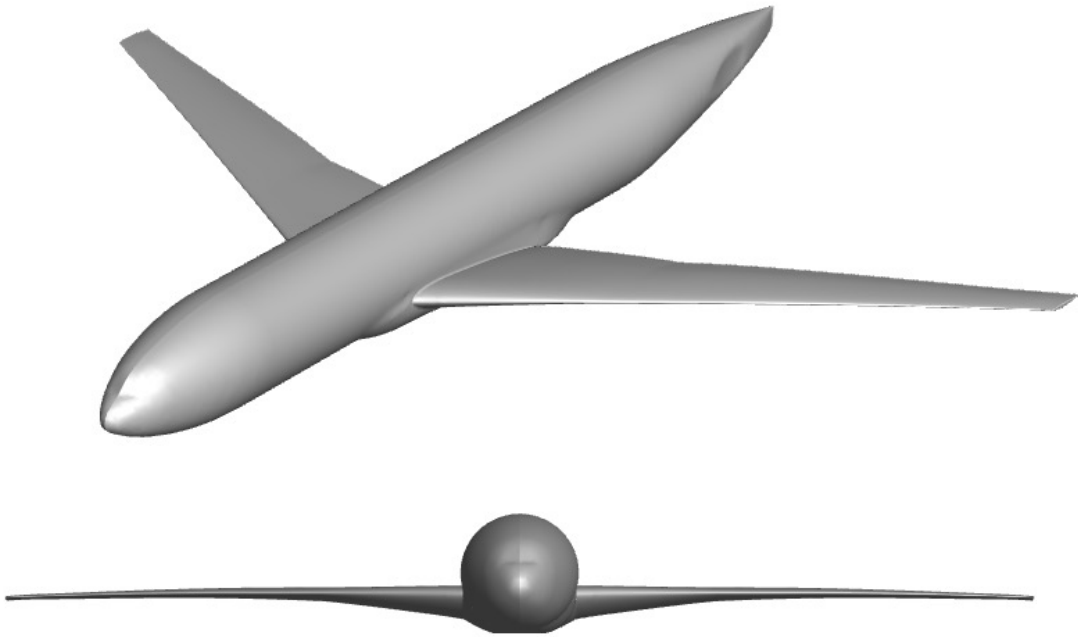
$$\min_{\zeta,\alpha} \quad C_{elastic}$$

$$\text{s.t.} \quad L = \rho_S \times (v_f \times V) \times g$$

$$C_L = 0.5$$

$$[\mathbf{K}(\zeta_\mathbf{e})]\{\mathbf{U}\} = F(\zeta_e) + F_g(\zeta_e)$$

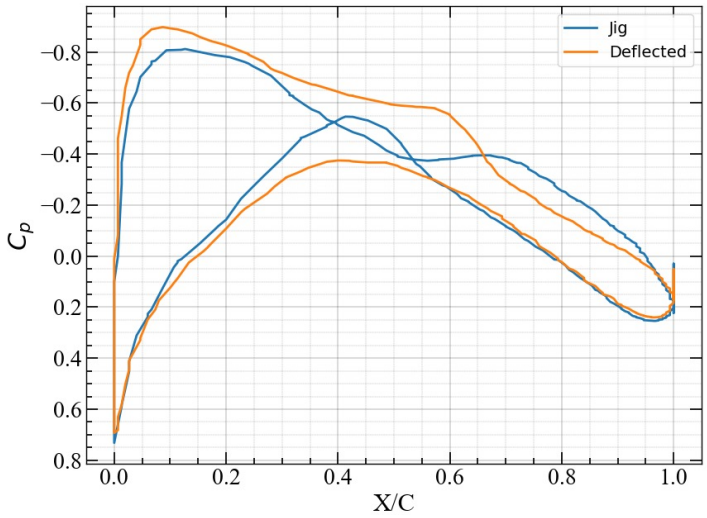
$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$



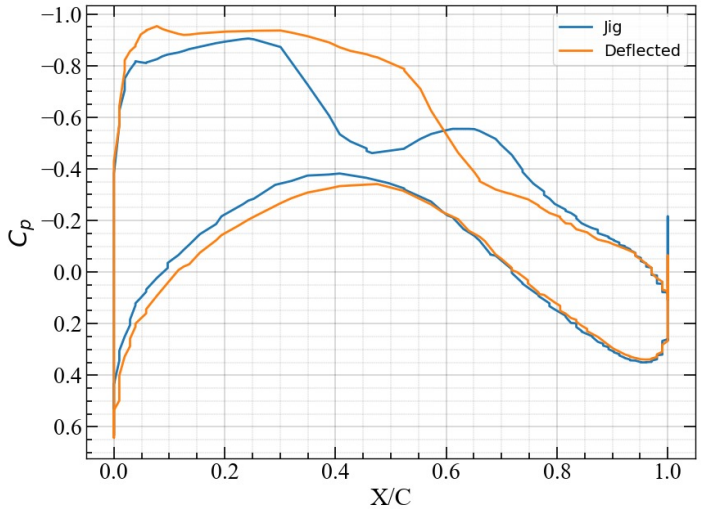
GEOMETRIC PROPERTIES		
C_{Ref}	1	m
A_{Ref}	3.4038	m^2
Semi-Span	1.2857	m
$\Lambda_{c/4}$	35	deg

ELASTIC PROPERTIES		
E	6.9×10^{10}	Pa
ν	0.33	
ρ_s	2700	kg/m^3

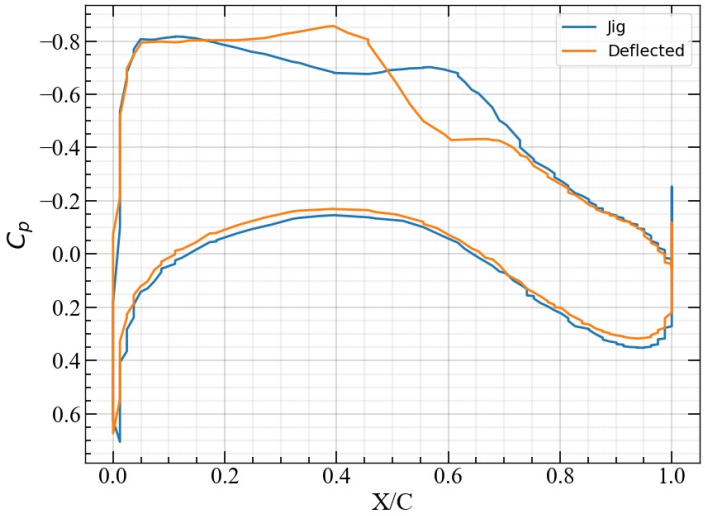
Local pressure distributions



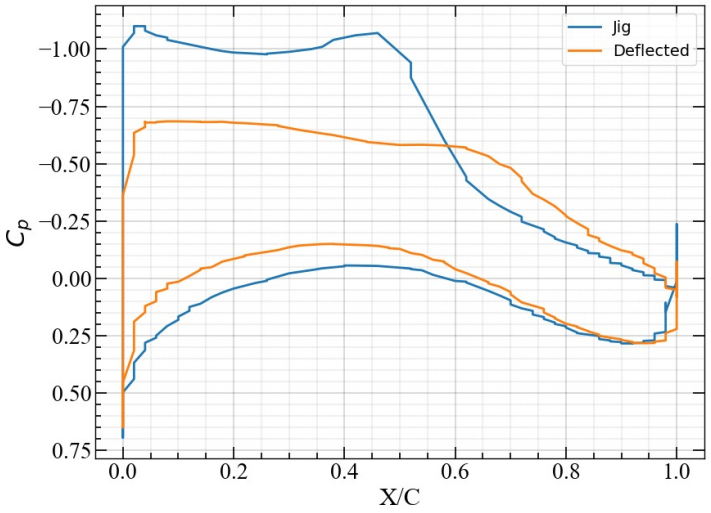
$\eta = 18\%$



$\eta = 37\%$



$\eta = 59\%$

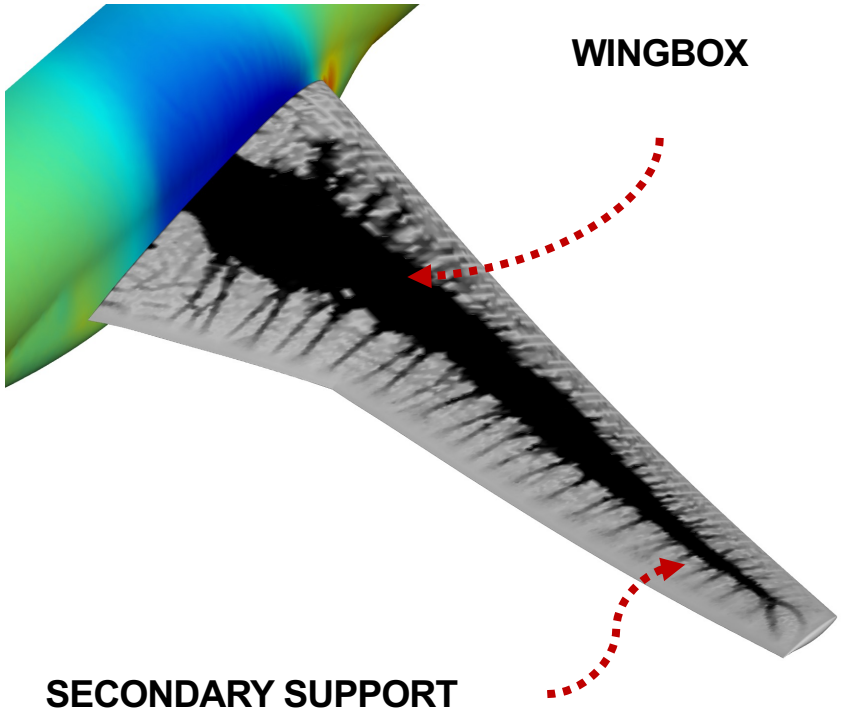


$\eta = 89\%$

Wing-box optimal material distribution



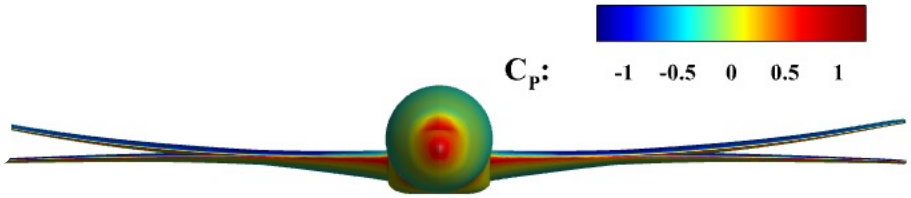
UNDERLYING WING BOX



SECONDARY SUPPORT STRUCTURES

$\alpha_{initial}$: 0.758^o

α_{final}: 2.253^o



Structural Weight/Wing: 11756 Kgs

Problem Formulation

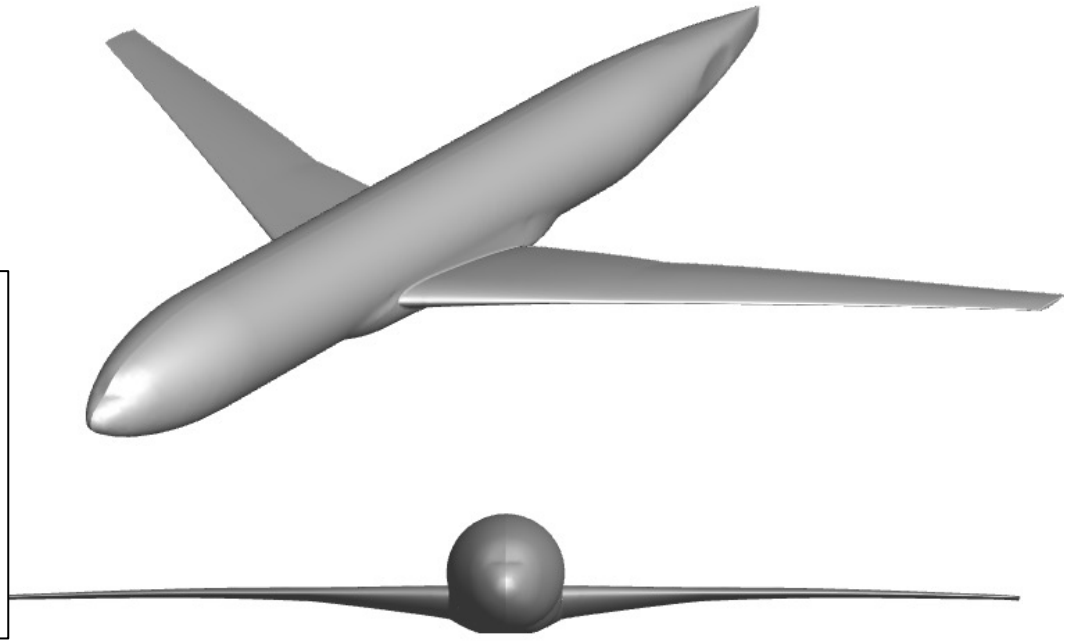
$$\min_{\zeta,\alpha} \quad C_{elastic}$$

$$\text{s.t.} \quad L = \rho_S \times (v_f \times V) \times g$$

$$C_L = 0.5$$

$$[\mathbf{K}(\zeta_e)]\{\mathbf{U}\} = F(\zeta_e) + F_g(\zeta_e)$$

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$



GEOMETRIC PROPERTIES		
C_{Ref}	1	m
A_{Ref}	3.4038	m^2
Semi-Span	1.2857	m
$\Lambda_{c/4}$	35	deg

ELASTIC PROPERTIES		
E	6.9×10^{10}	Pa
ν	0.33	
ρ_s	2700	kg/m^3

Coupled Aero-Elastic Topology Optimization

$$\min_{\zeta,\alpha,x} \quad C_{elastic}$$

s.t.

$$L = \rho_S \times (v_f \times V) \times g$$

$$C_L = 0.5$$

$$C_{My} \leq -0.17$$

AERODYNAMIC CONSTRAINTS

$$t/c_{i=1..30} \geq 0.25 t/c_{i=1..30}$$

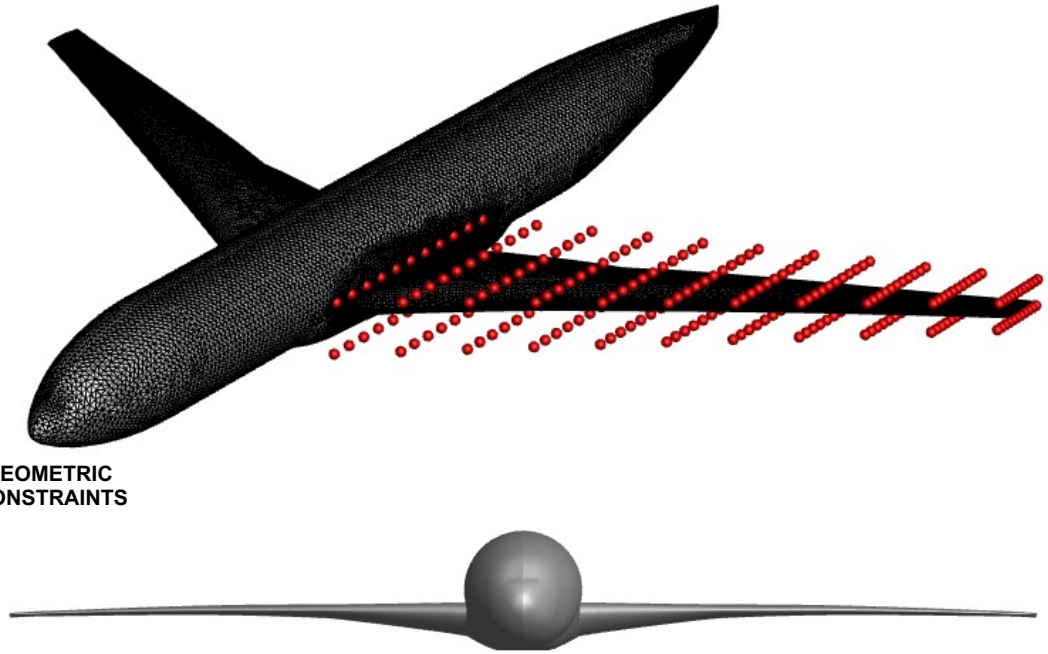
$$V \geq= V_{initial}$$

GEOMETRIC CONSTRAINTS

$$[\mathbf{K}(\zeta_e)]\{\mathbf{U}\} = (\zeta_e) + F_g(\zeta_e)$$

$$0 < \zeta_{e_{min}} \leq \zeta_e \leq 1$$

$$-0.01 < x \leq 0.01$$

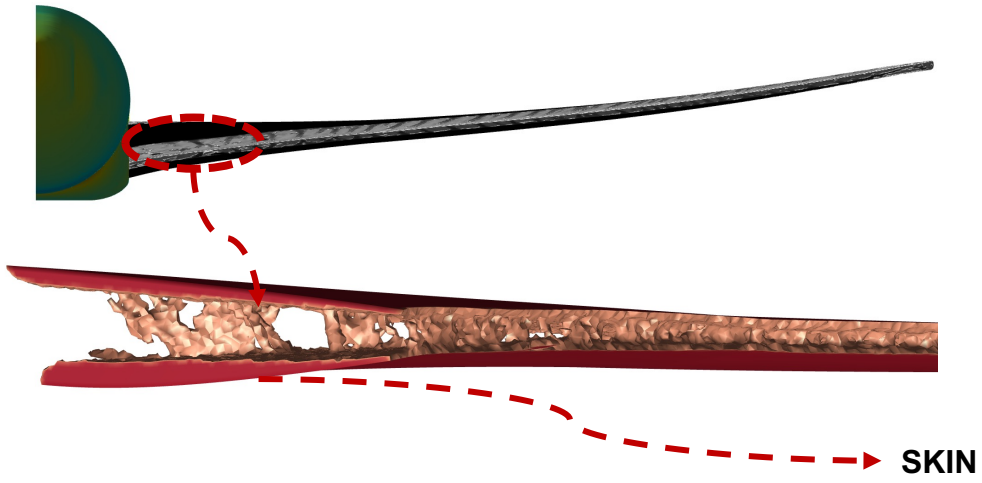


GEOMETRIC PROPERTIES		
C_{Ref}	1	m
A_{Ref}	3.4038	m^2
Semi-Span	1.2857	m
$\Lambda_{c/4}$	35	deg

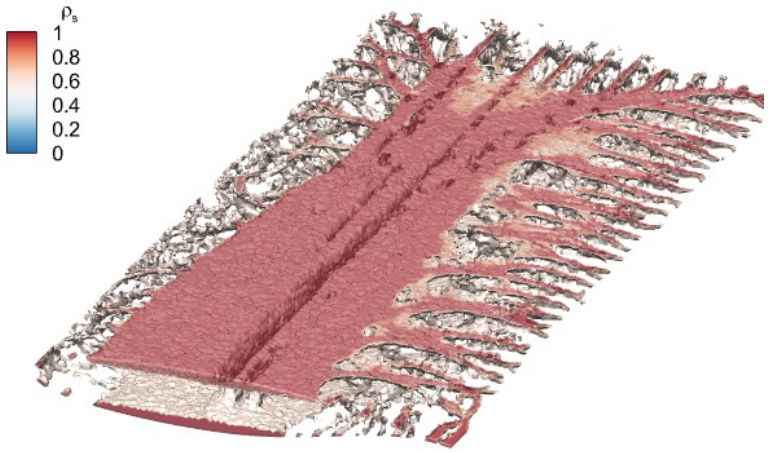
ELASTIC PROPERTIES		
E	6.9×10^{10}	Pa
ν	0.33	
ρ_s	2700	kg/m^3

Challenges in Topology Optimization

- Intermediate material densities
- Difficult to manufacture
- Current approach cannot scale to aircraft configurations



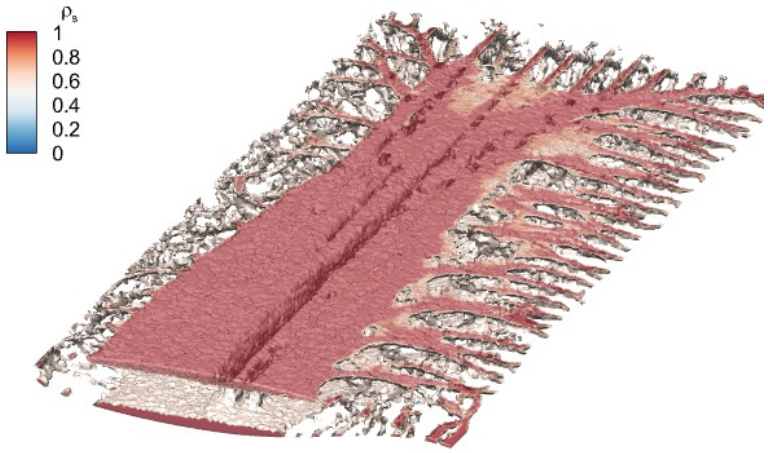
Reduced design space





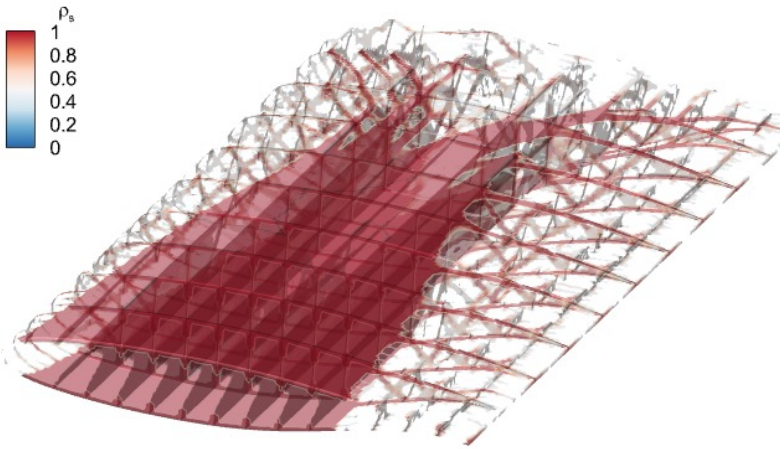
TET-BASED DISCRETIZATION

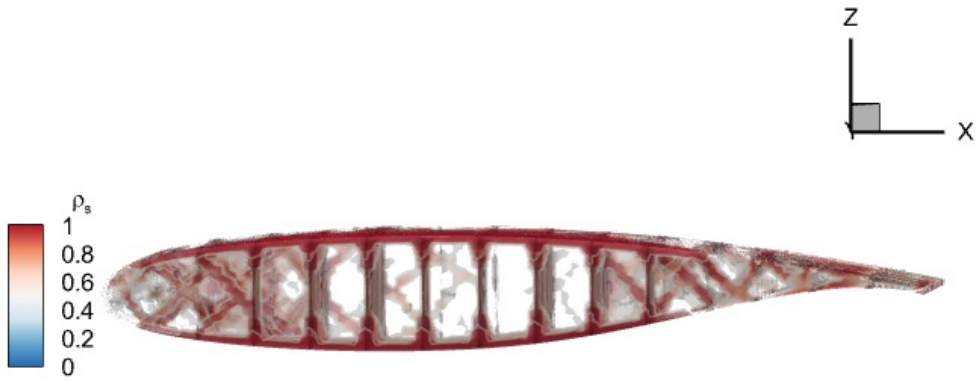
Reduced design space





TET-BASED DISCRETIZATION

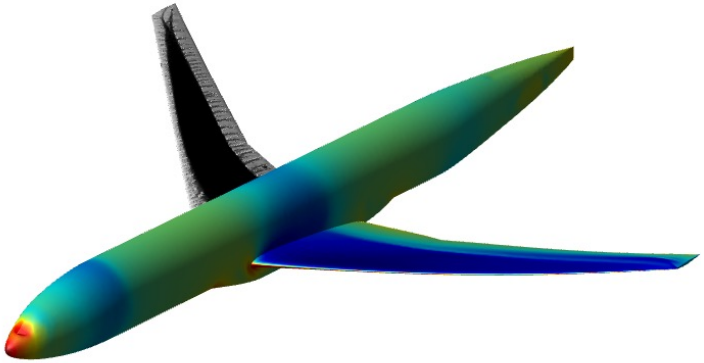


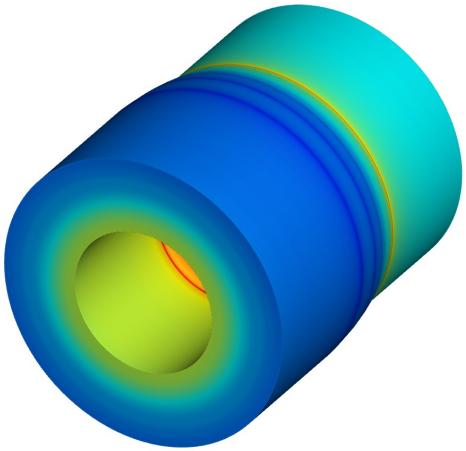


SHELL-BASED DISCRETIZATION

Future work

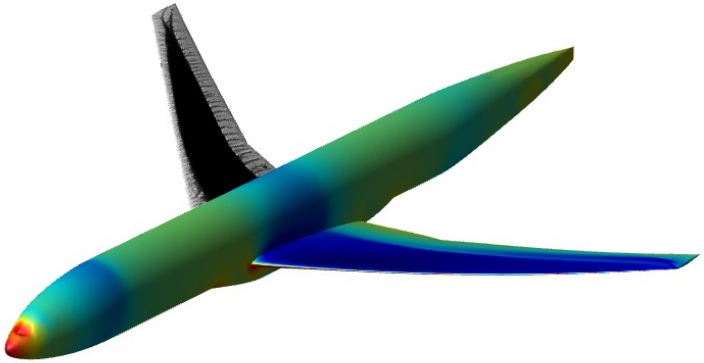
- Improving scalability of the aero-elastic system
- Integrated aero-structural-propulsion optimization
- Robust shape optimization framework with minimal user input
- Manufacturable load-bearing structures

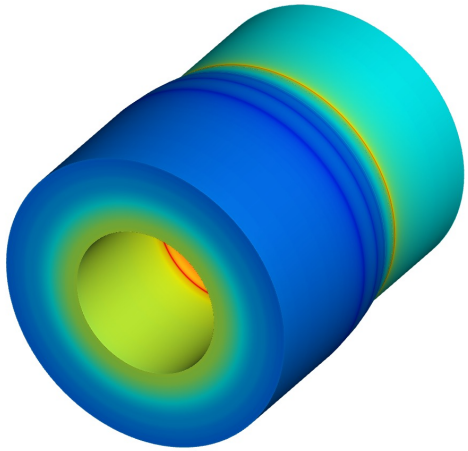




Future work

- Improving scalability of the aero-elastic system
- Integrated aero-structural-propulsion optimization
- **Robust shape optimization framework with minimal user input**
- Manufacturable load-bearing structures





Aerodynamic design exploration using SU2

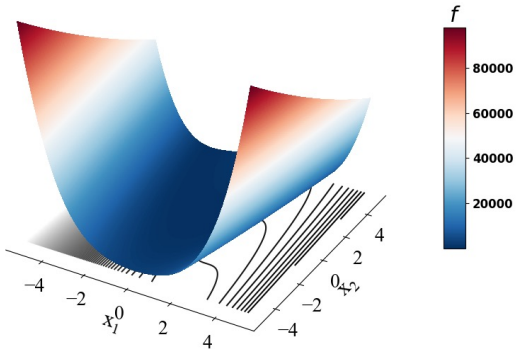


- **OBJECTIVE**: Understand the influence of :
  1. Choice of gradient-based optimization algorithm
  2. Design variable pre-conditioning
  3. Design space dimension

  On aerodynamic shape optimization for two and three-dimensional configurations

- **MOTIVATION:**
  1. Establish best practices for large-scale shape optimization
  2. Identify elements of the workflow the affect robustness

Influence of Optimization Algorithms

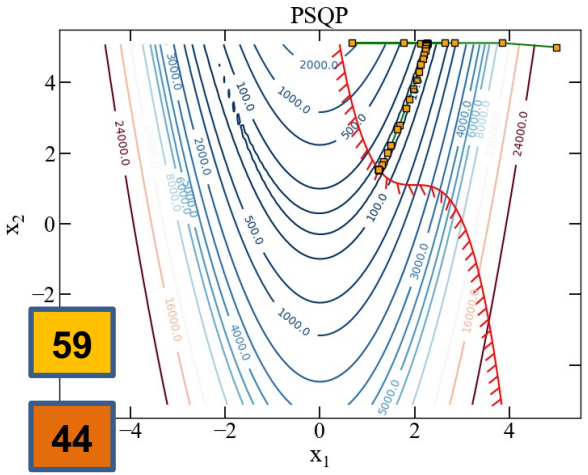


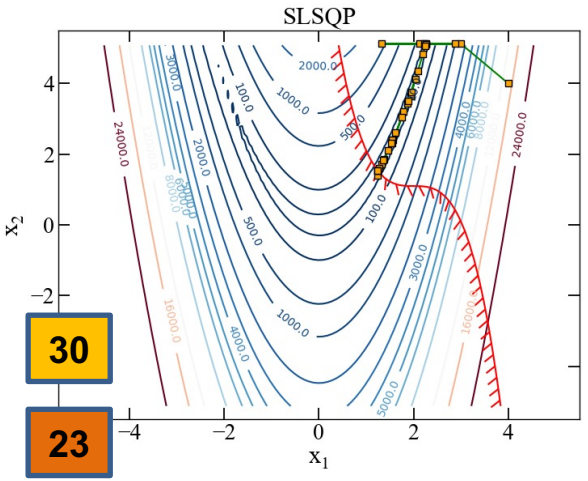
ROSENBROCK FUNCTION

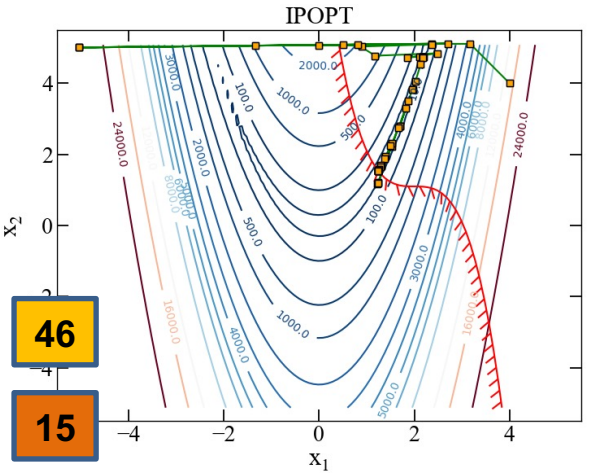
$$\min_{x \in \mathcal{R}^n} \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2)$$

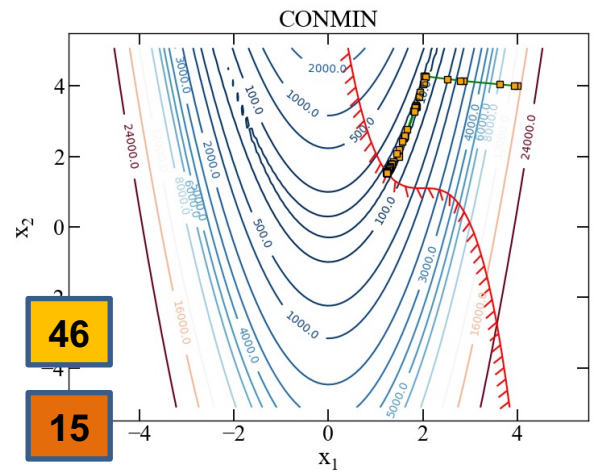
$$\text{s.t.} \sum_{i=1}^{n-1} (1.1 - (x_i - 2)^3 - x_{i+1}) \geq 0$$

$$-5.14 \leq x_i \leq 5.14$$









GRAD EVAL

FUNC EVAL

ADODG Case 4.1

$$\min_{x,\alpha} \quad C_D$$

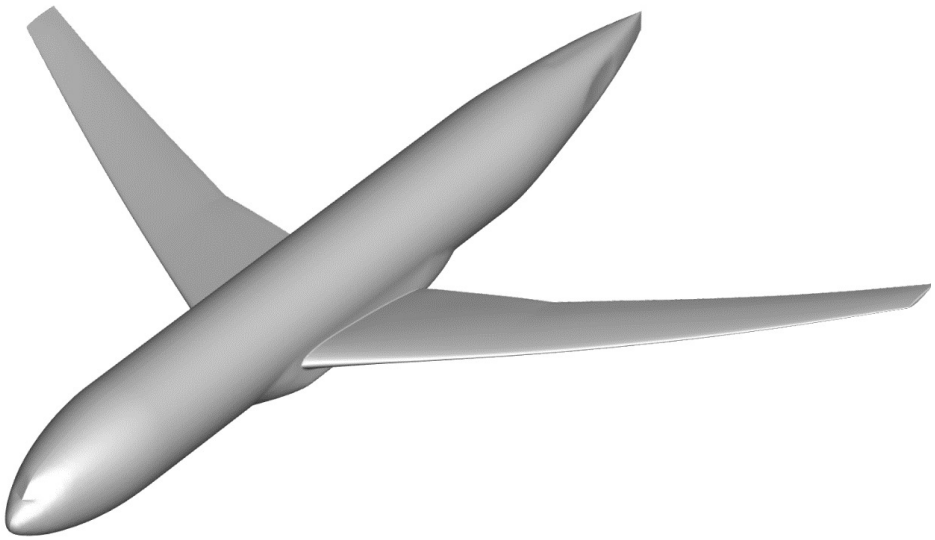
$$\text{s.t.} \quad C_L = 0.5$$

$$C_{My} \leq -0.17$$

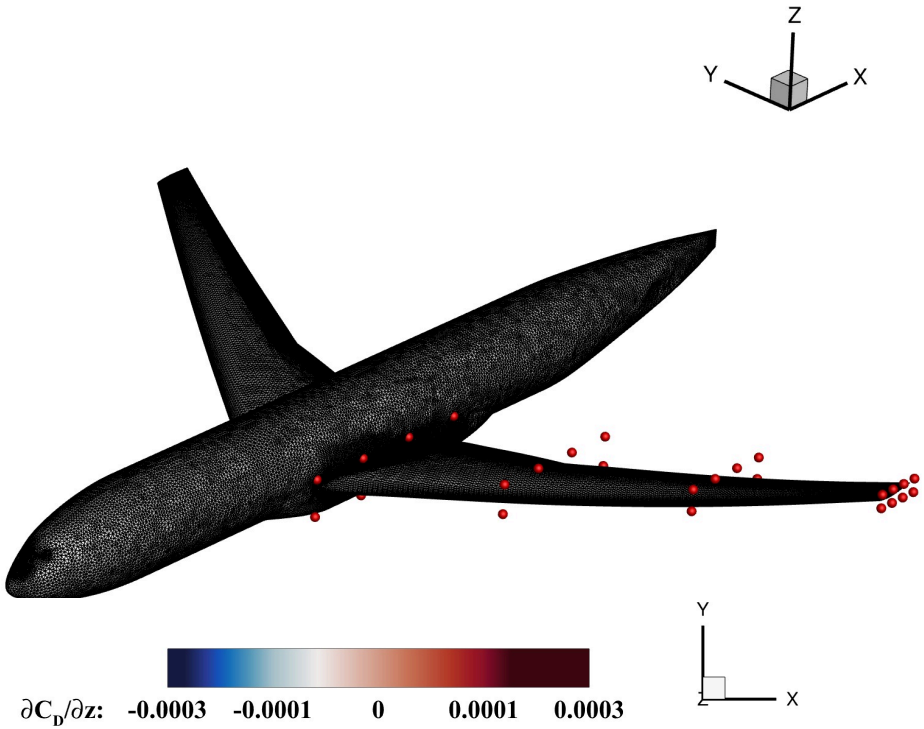
$$t/c_{i=1..30} \geq 0.25 t/c_{i=1..30}$$

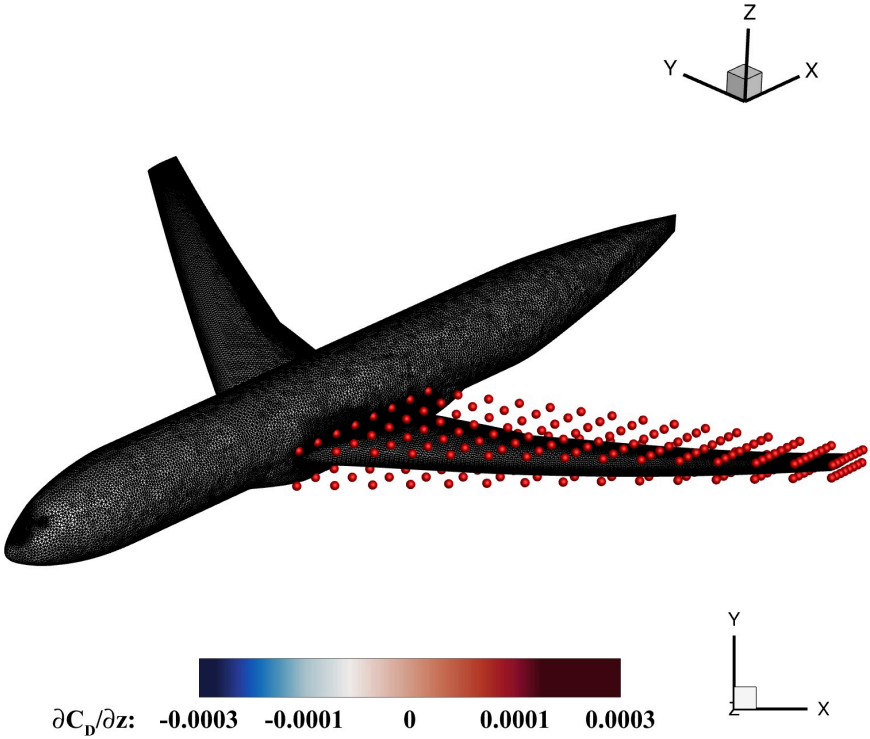
$$V \geq= V_{initial}$$

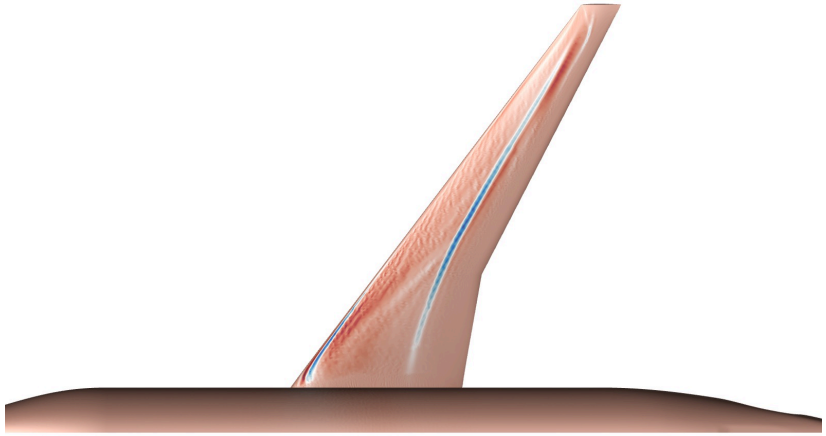
$$lb < x \leq ub$$

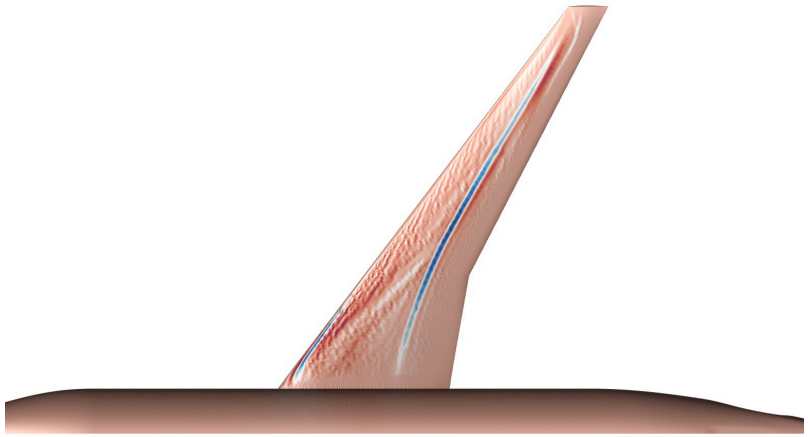


Influence of design space dimension

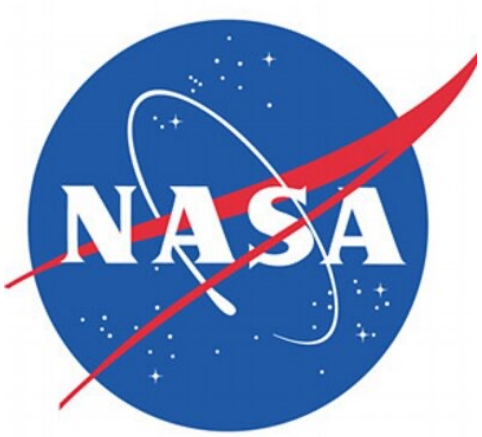








Acknowledgements



This work was supported by NASA under award number 80NSSC19M0125 as part of the Center for High-Efficiency Electrical Technologies for Aircraft (CHEETA)

Questions?