Advanced Modeling & Simulations Seminar Series
NASA Ames Research Center, November 16th, 2021

The Nonlinear Harmonic Method

Rank-n interactions

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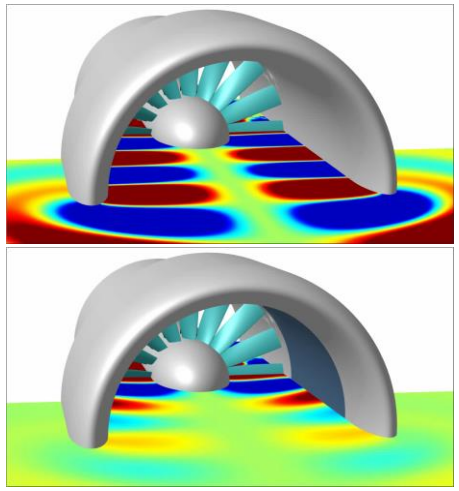
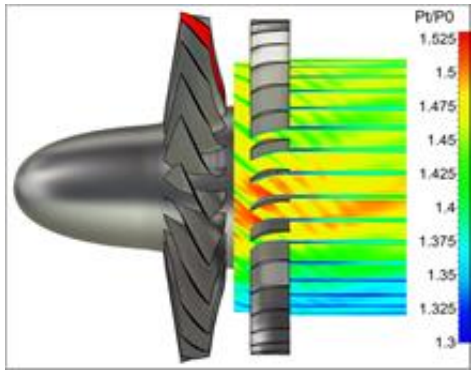
Nonlinear Harmonic Method

Applications

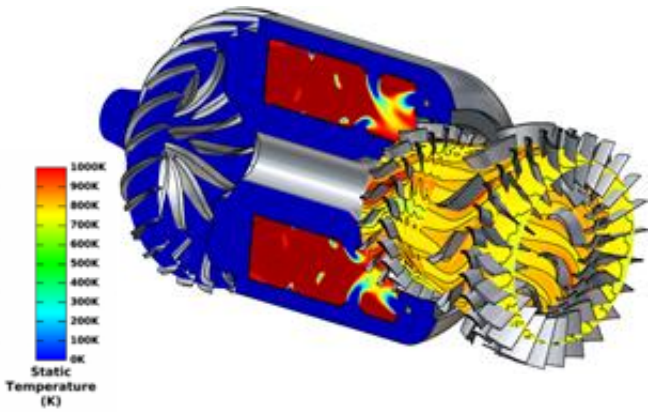
- The **Nonlinear Harmonic Method** represents an **efficient** and **accurate** unsteady flow solution method in the frequency domain[6-11].
- Originally linked to **blade passing phenomena**, the model can be applied to a range of turbomachinery problems: rotor/stator, clocking, impeller-volute interactions, conjugate heat transfer, tonal noise modeling, hot streaks propagation and inlet distortion.
- A recent development of the NLH method enabled a vast expansion of its applicability: the **rank-n**.



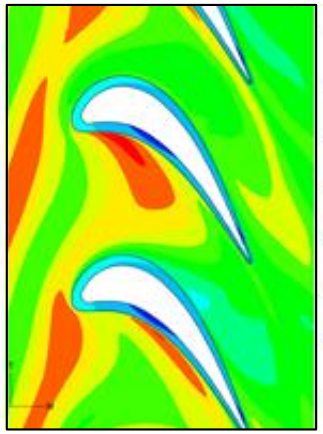
Tonal noise attenuation due to acoustic liners in a turbofan (real part of acoustic pressure at 1*BPF)[1]



NLH computation including a total pressure inlet distortion profile of an axial compressor configuration using 4 harmonics to model the distortion and 3 harmonics for BPF. The distortion effects are noticed in the downstream rows and Tyler-Sofrin modes can be identified[2]



Impeller and HPT coupled in a micro gas turbine engine[3,4]



Hot streak migration to the pressure side of a turbine rotor downstream of a combustion chamber[5]

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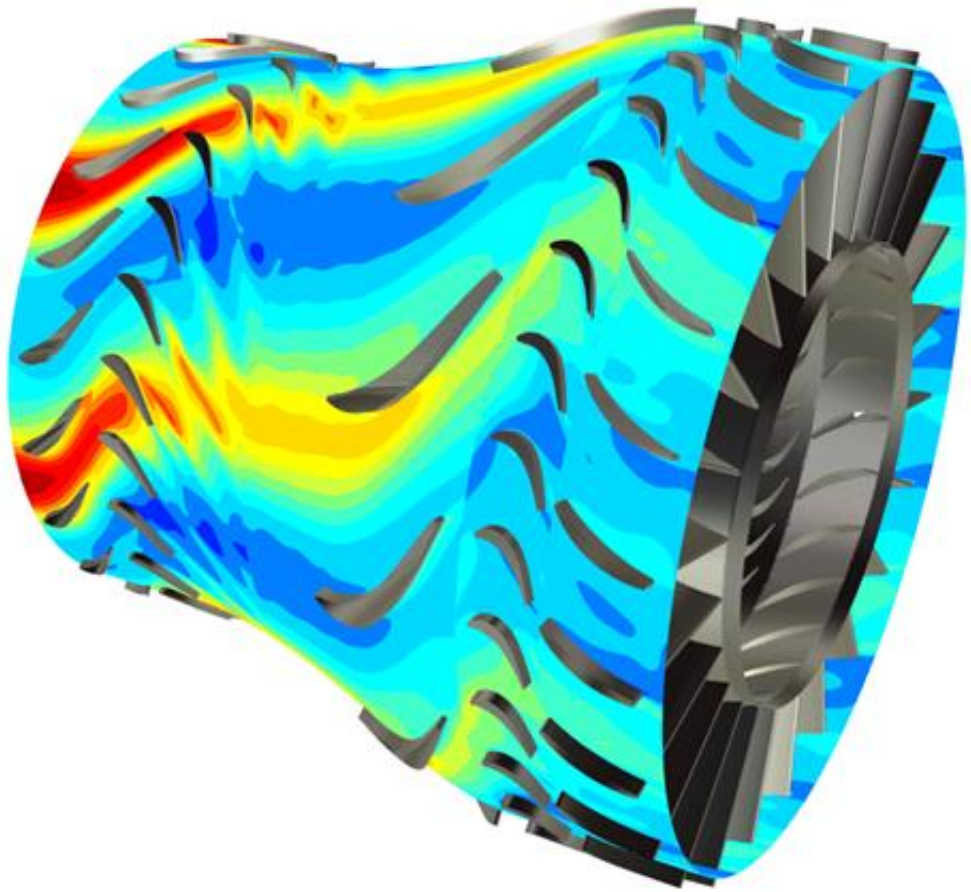
Engineering problem

Rank-n



NLH rank-n computation of a multi-stage axial turbine

Instantaneous field of static temperature reconstructed in time at 90% span. Blade passing and hot streaks flow distortion unsteady perturbations modeled with 3 Fourier harmonics

*How to **accurately** model **unsteady perturbations** linked to component interaction across a **limitless number of turbomachinery rows**?*

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Outline

Background

Unsteady flow in turbomachinery

Flow models

Rotor-stator modeling

01

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NLH

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Concept of rank

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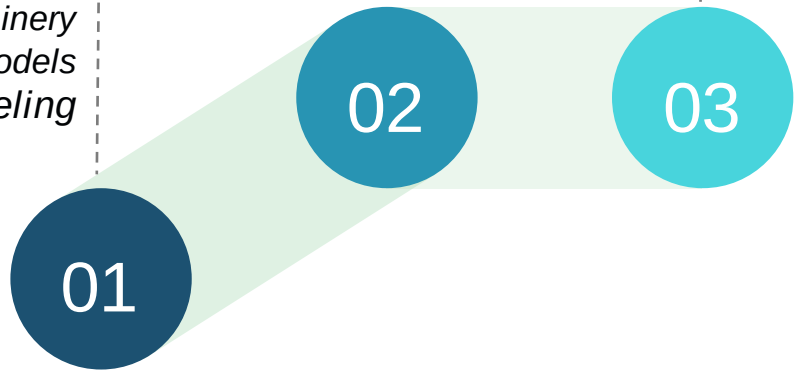

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01 02 03 04

Results analysis
Computational cost
Steady-state vs NLH
Rotor stator data
DFT
Time reconstruction

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Conclusions
Summary

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Unsteady flow in turbomachinery

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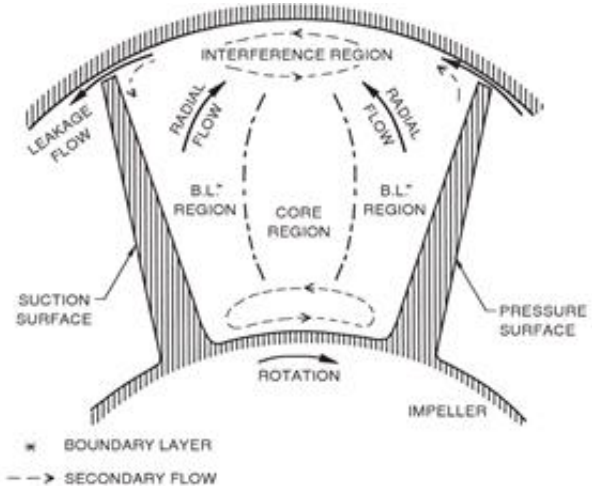
Unsteady flow in turbomachinery

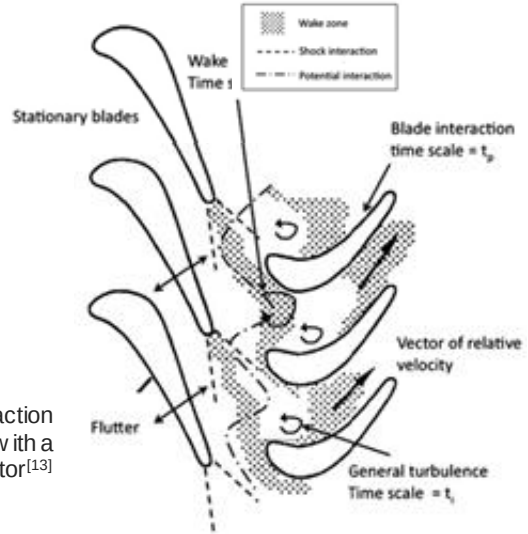
Physical phenomena

Turbomachinery flows are inherently unsteady and can be described as a combination of three-dimensional, viscous unsteady effects.

- **Multi-row interactions**: potential effects, wake propagation
- **Turbulence**: vortex shedding, secondary flows
- **Off-design effects**: rotating stall, surge
- **External distortion**: gust, wind with incidence
- **Blade vibration**: flutter, forced response

Cross section of a blade passage in an axial flow impeller showing the tip leakage flow, boundary layer radial flow, and other secondary flows[12]





Schematic of wake interaction from an upstream stator with a downstream rotor[13]

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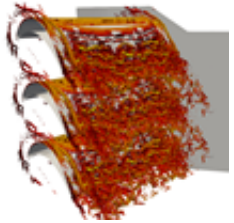
Unsteady flow in turbomachinery

Classification

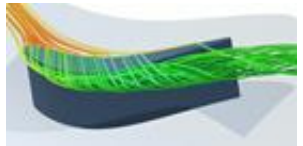
Unsteady phenomena in turbomachines

- **Non-periodic**
  - **Chaotic**
    - Turbulence vortices
    - Boundary layers
    - Jets
    - Waker
  - **Transient running**
    - Variable rotational speed
- **Periodic**
  - **Not correlated to ω**
    - Von Karman vortices
    - Surge
    - Flutter
    - Upstream distortion
  - **Correlated to ω**
    - **Unstable**
      - Rotating stall
    - **Stable**
      - Rotor-stator interactions
      - Potential effects
      - Wake effects
      - Secondary flows
      - Shocks

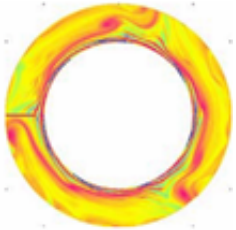
Laminar-turbulent transition on a T161 Cascade



The blade motion induces a displacement of this vortex leading to a pressure fluctuation in the inter-blade channel



Evolution of the vorticity in a vaneless diffuser of a radial flow pump in partial flow operating conditions



Normalized entropy contours, at midspan of a centrifugal compressor

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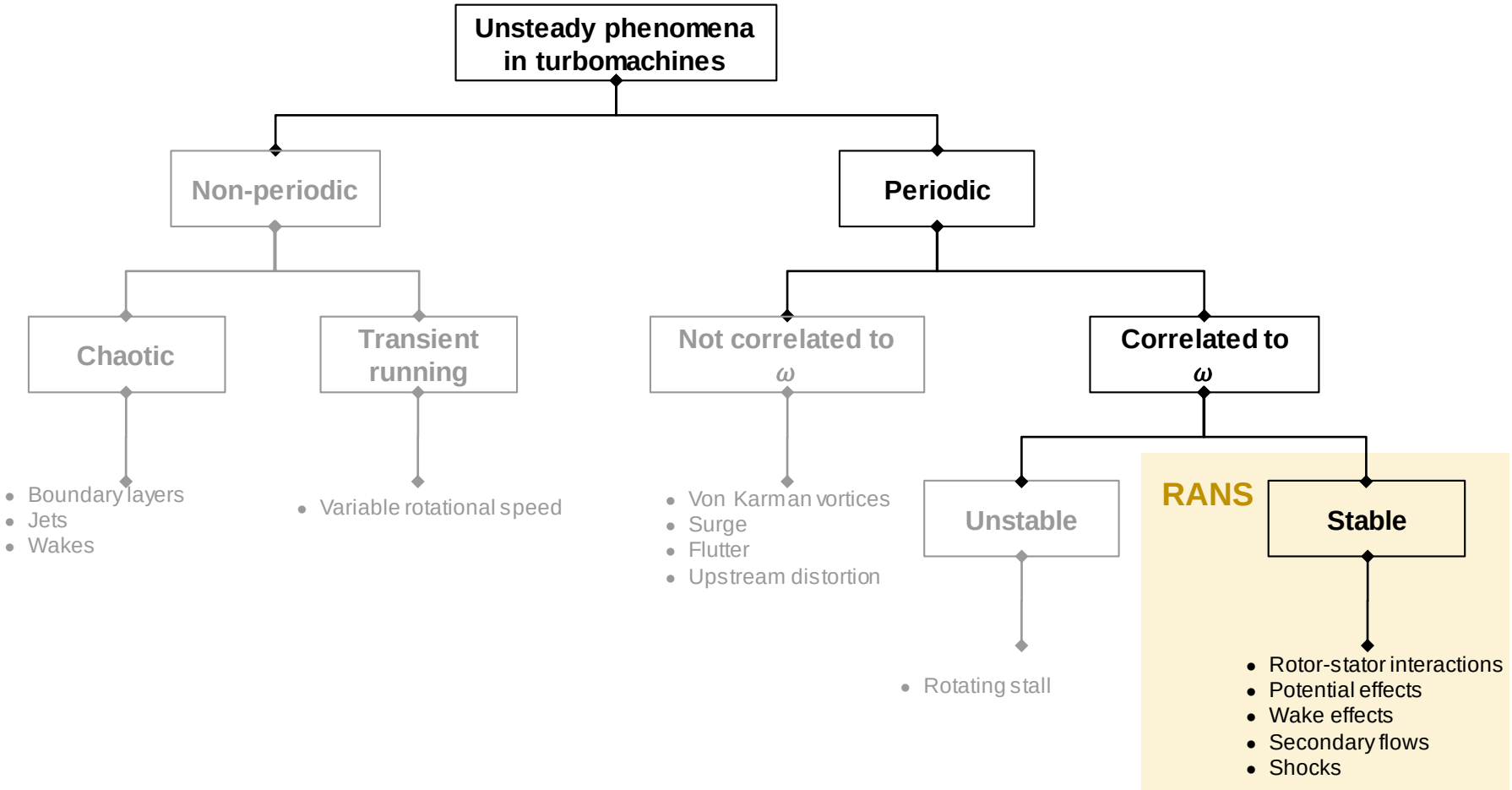
Unsteady flow in turbomachinery

Classification



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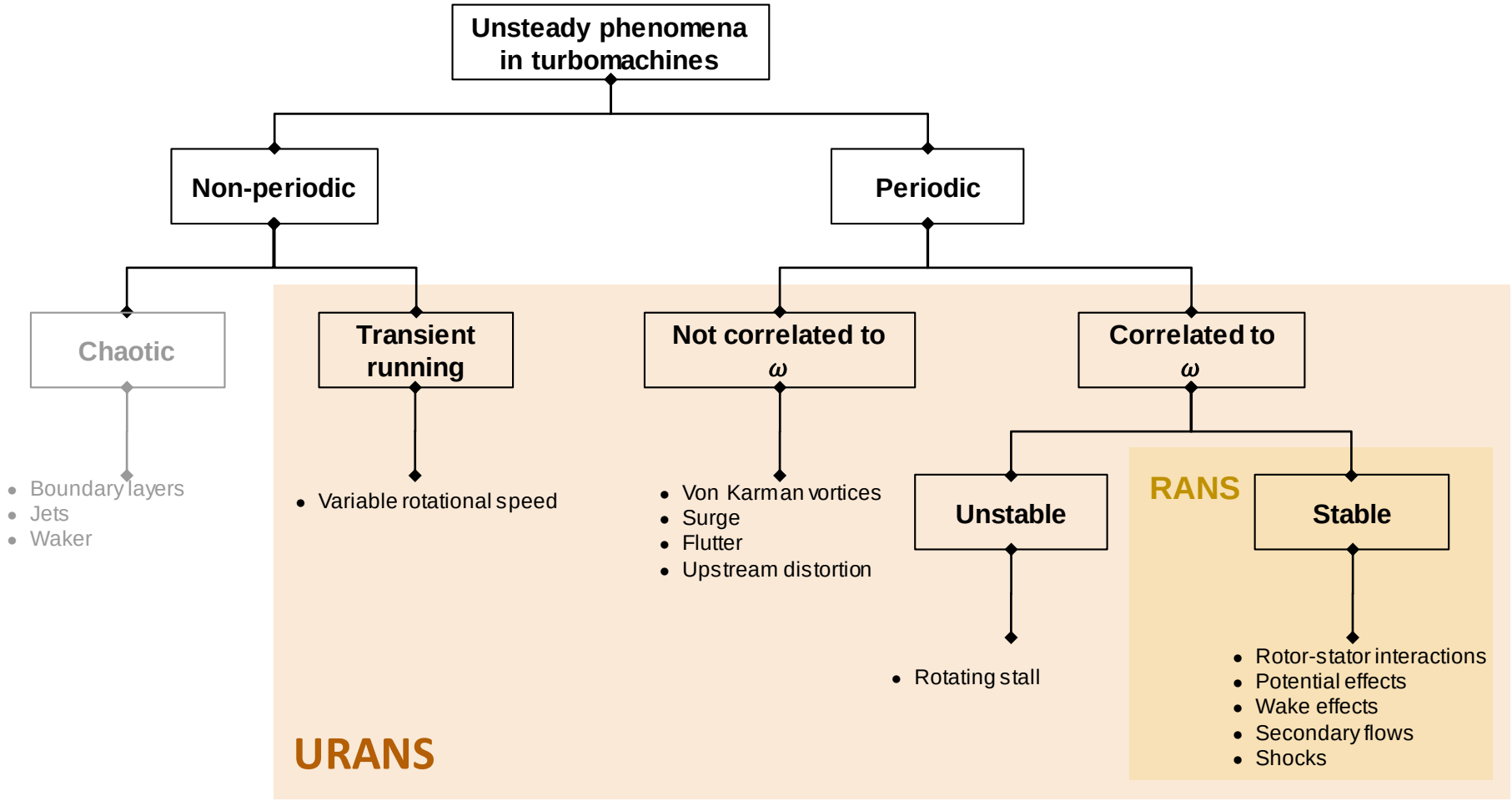

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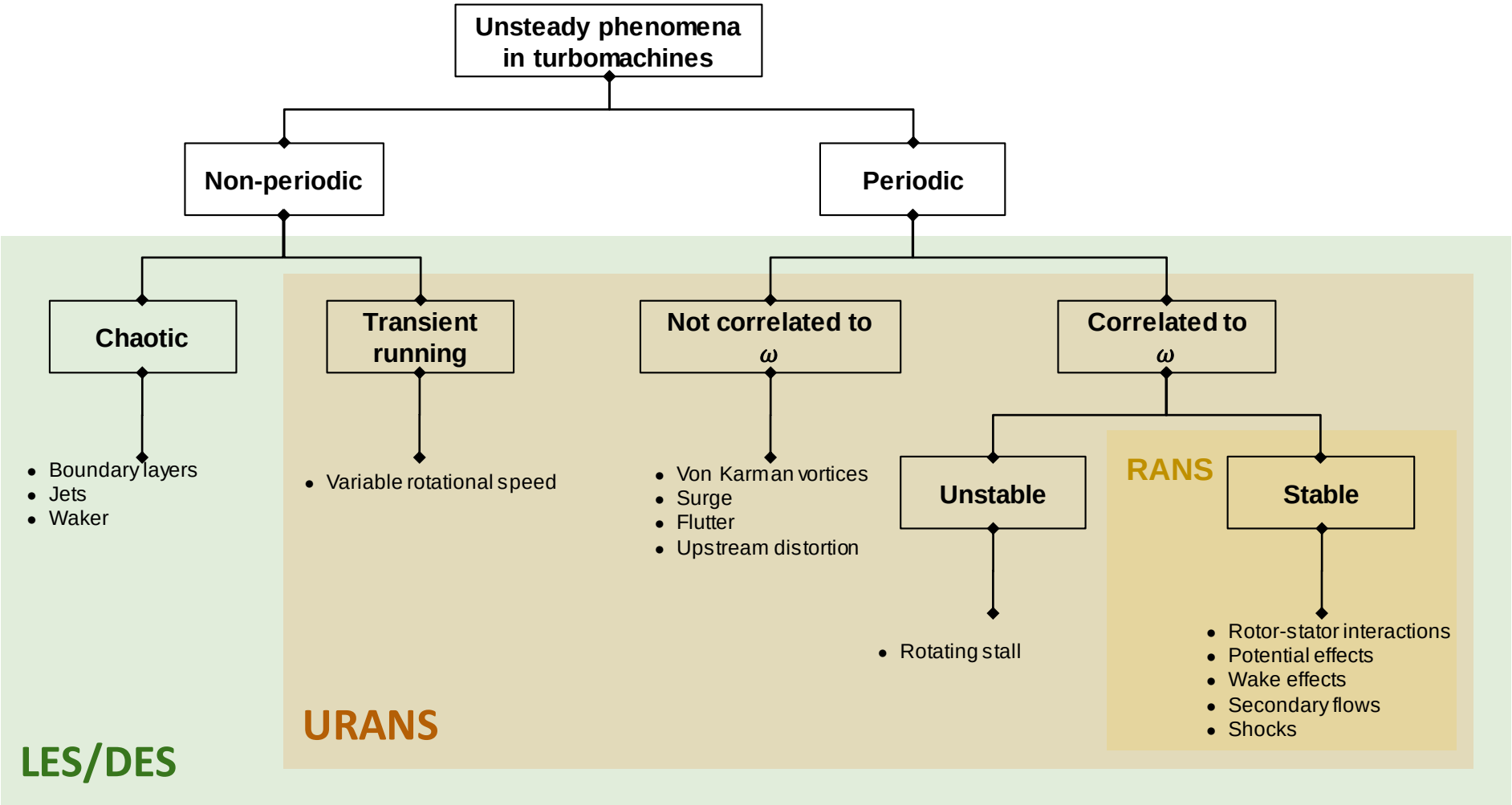

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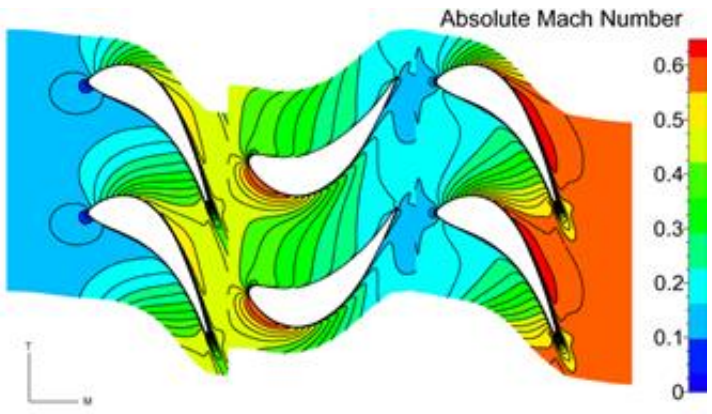
Rotor-stator modeling

Multi-row interactions



Mixing plane simulation of an axial turbine (shown with repetition)

The **mixing-plane interface** averages circumferentially the flow variables and mixes them out at the rotor-stator interface.

- Low computational cost
- Single blade passage
- Entropy jumps at the interface
- Not realistic, unless blade rows are widely spaced or for high rotation speed

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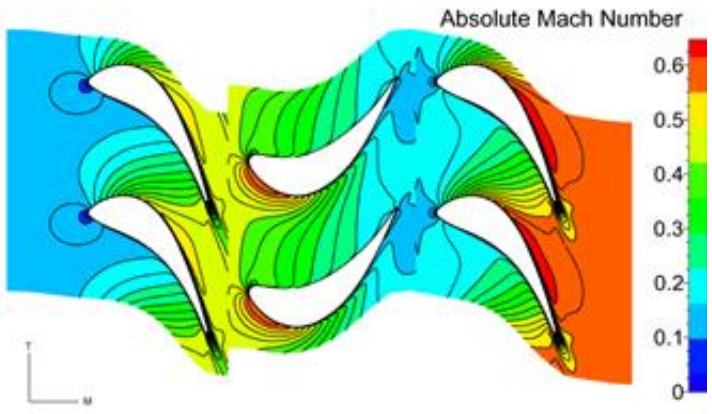
Rotor-stator modeling

Multi-row interactions



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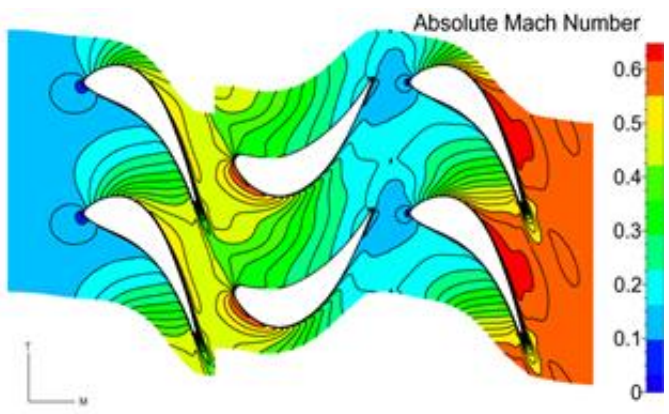

Frozen rotor simulation of an axial turbine (shown with repetition)

The **mixing-plane interface** averages circumferentially the flow variables and mixes them out at the rotor-stator interface.

- Low computational cost
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- Not realistic, unless blade rows are widely spaced or for high rotation speed

The **frozen rotor method** uses a moving reference frame equation but with no actual relative motion: the mesh remains fixed.

- Accurate flow reconstruction at the interface
- Common periodicity required
- Requires one computation for each rotor-stator position.
- Analogous to freezing the rotor motion and observing a snapshot of the flow field.
- Can be used for turbomachinery applications in which rotor-stator interactions are relatively weak

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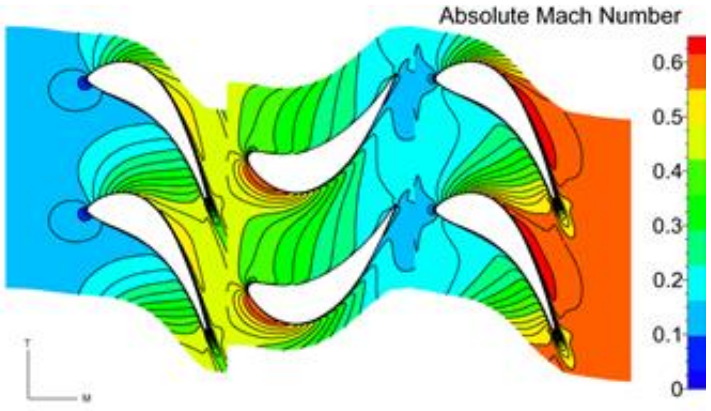
Rotor-stator modeling

Multi-row interactions



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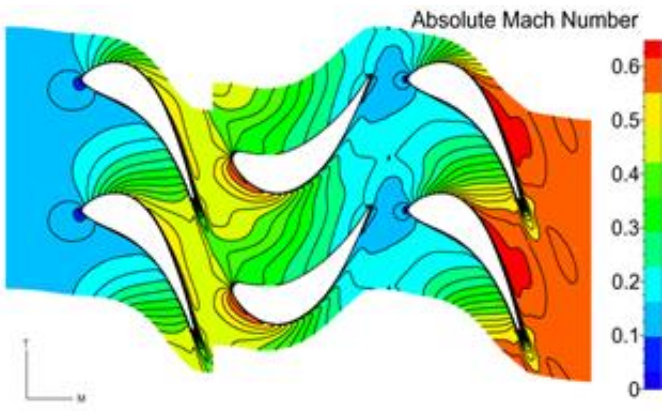
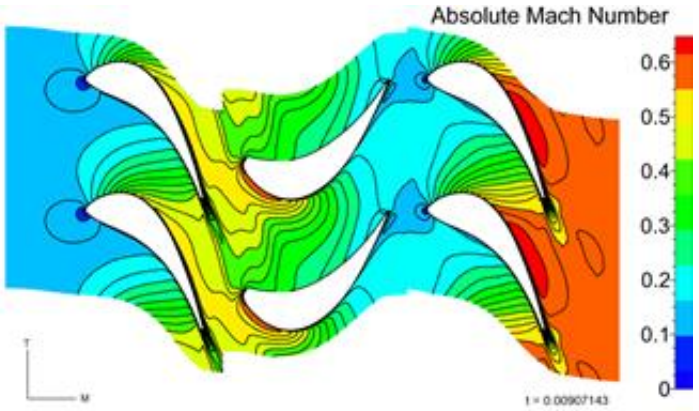


Frozen rotor simulation of an axial turbine (shown with repetition)



Sliding mesh simulation an axial turbine with common periodicities (shown with repetition)

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The **sliding-mesh method** connects domains moving in time.

- Time-accurate solution for rotor-stator interaction
- Most accurate RANS method for simulating flows in multiple moving reference frames
- High computation cost
- Common periodicity, domain scaling or full wheel required
- Realistic results but often computationally infeasible

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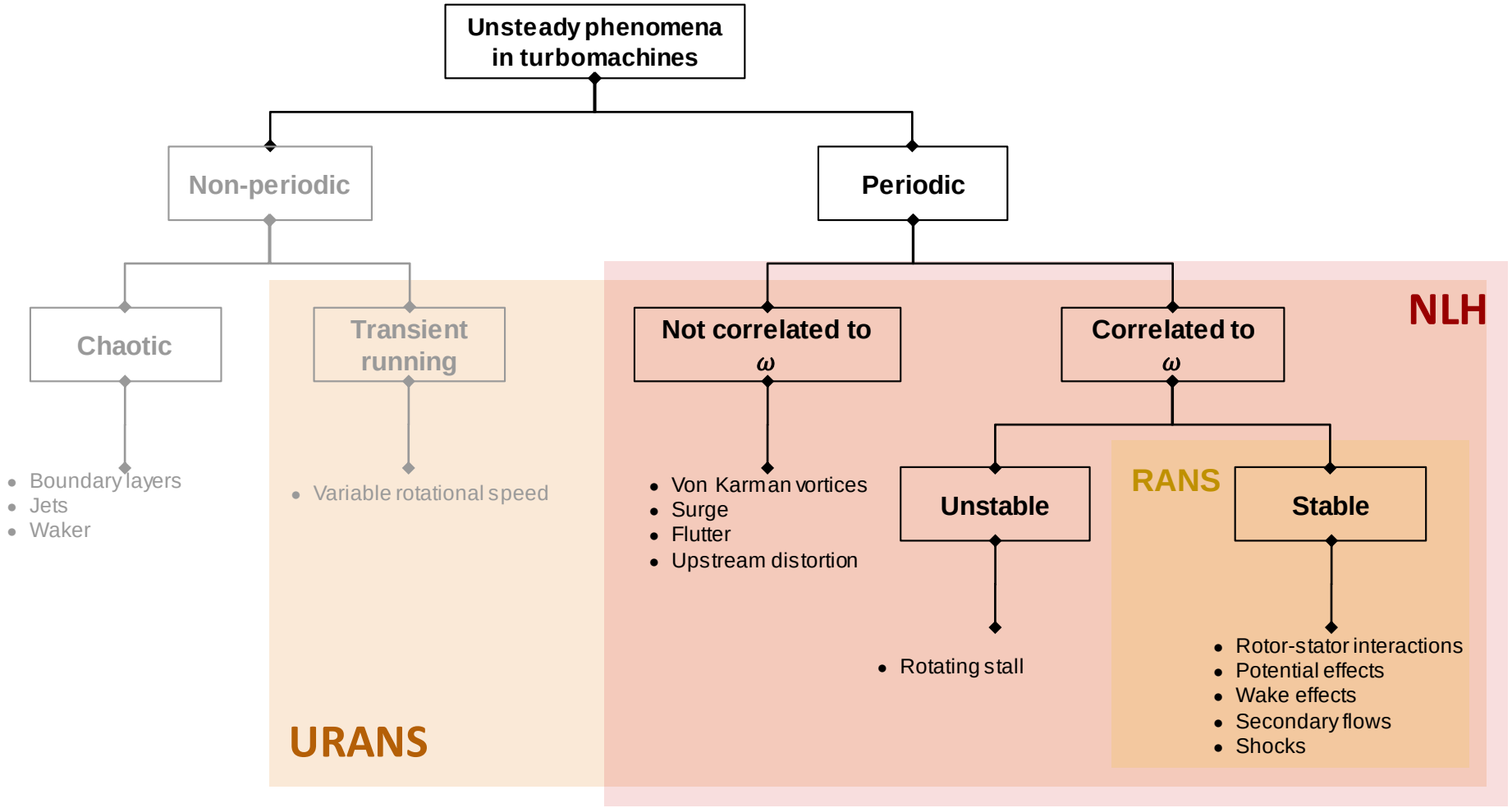
Unsteady flows in turbomachinery

Classification



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Theoretical background

Concept of rank

- ***Rank-1***: a periodic perturbation affecting a reference row coming from any row which is *one interface away* (adjacent row).
- ***Rank-2***: a periodic perturbation affecting a reference row coming from any row which is *two interfaces away* (adjacent-to-adjacent row).

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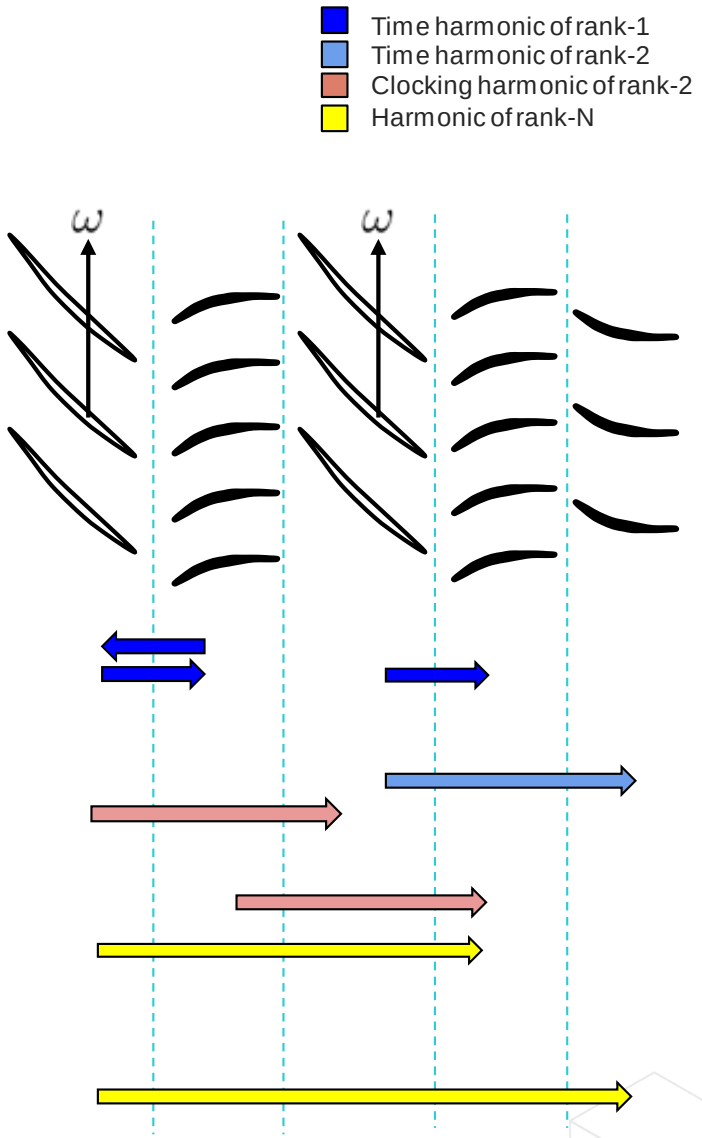
- ***Rank-n***: a periodic perturbation affecting a reference row coming from any row which is *n interfaces away.*

The **flexibility** allows the solution of the **selected perturbations.**



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Case study

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Case study

KJ66 MGT

Objective: assess the aerodynamic performance of an axial turbine downstream of a combustion chamber.

Test case: KJ66 micro gas turbine HPT and LPT[3,4].

Input:

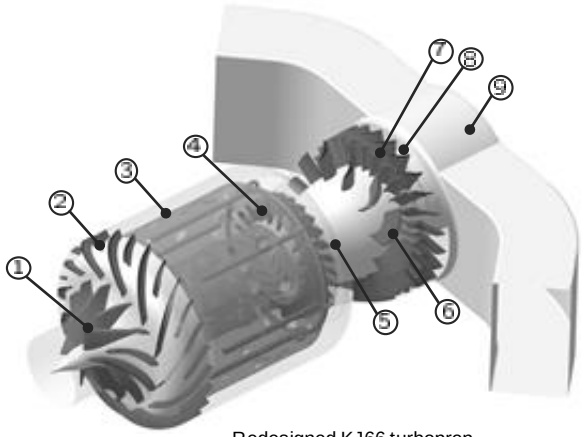
- geometry;
- operating conditions;
- combustor outlet absolute total temperature profile.

Challenges:

- capturing *unsteady phenomena*;
- capturing *hot streaks propagation*;
- computational *cost* vs *accuracy*.

Core diameter	108.00 mm
Length	270.50 mm
Compressor rotation speed	-80000 rpm
Combustor periodicity	6
HPT rotation speed	-80000 rpm
LPT rotation speed	-27500 rpm
Compressor total pressure ratio	1.61

1 – Impeller
2 – Vaned Diffuser
3 – Combustion chamber
4 – HPT nozzle
5 – HPT rotor
6 – LPT nozzle
7 – LPT rotor
8 – LPT deswirler
9 – Exhaust system



Redesigned KJ66 turboprop (propeller omitted)

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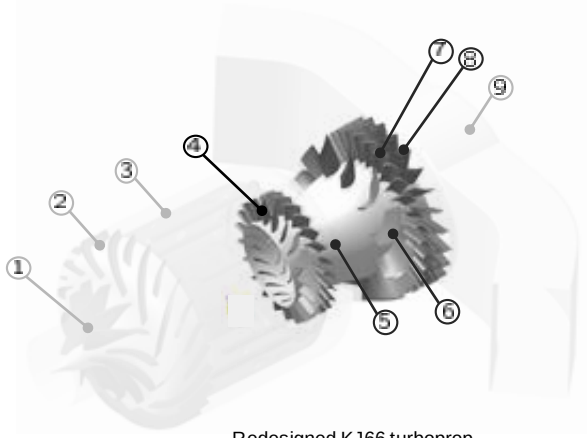

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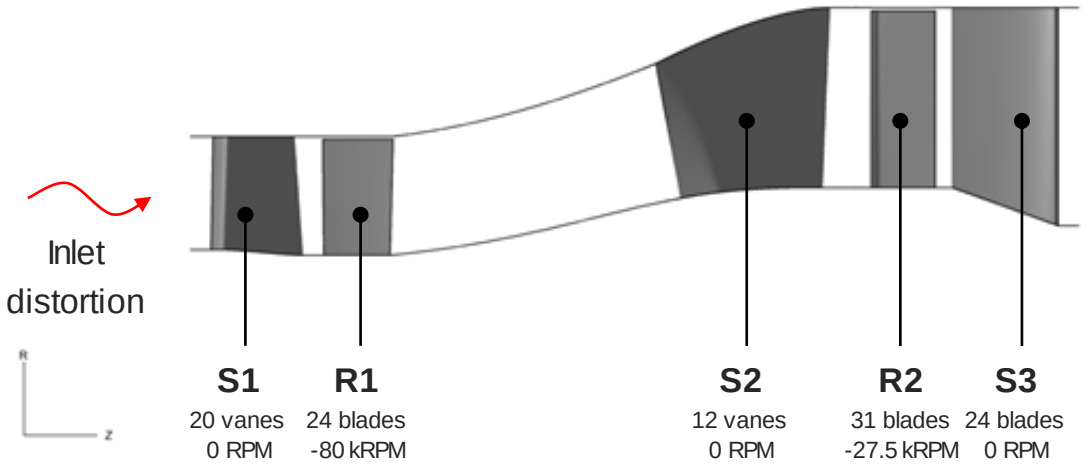

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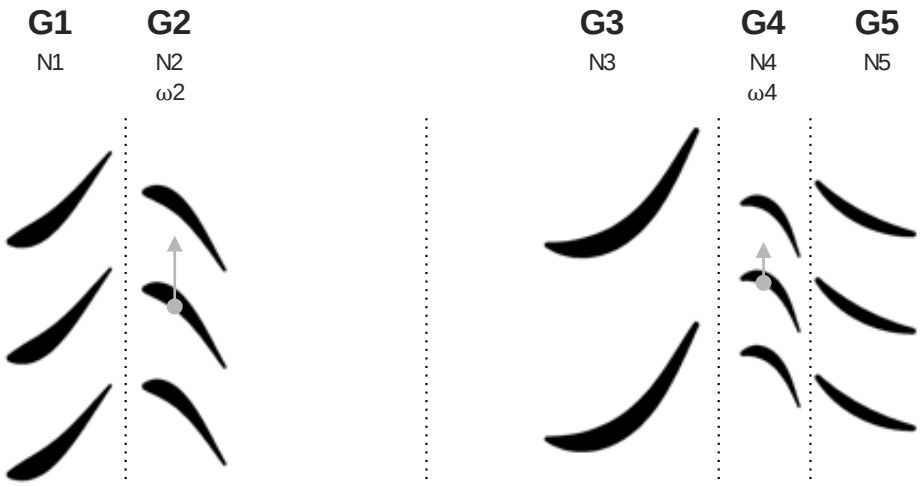
Problem definition

Unsteady phenomena



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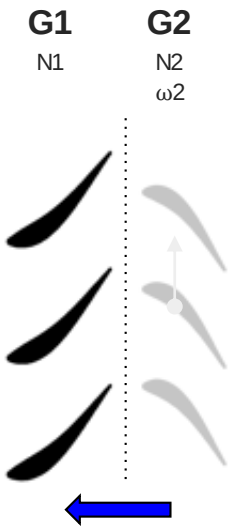

Unsteady perturbation linked to blade passing frequency from **G2** into **G1**

$$1G2, BPF = N_2\omega_2 = 24 \times 8377.58\ rad/s \approx 201062\ rad/s$$
$$2G2, 2BPF \approx 402124\ rad/s$$
$$3G2, 3BPF \approx 603186\ rad/s$$
$$4G2, 4BPF \approx 804248\ rad/s$$
$$\vdots$$

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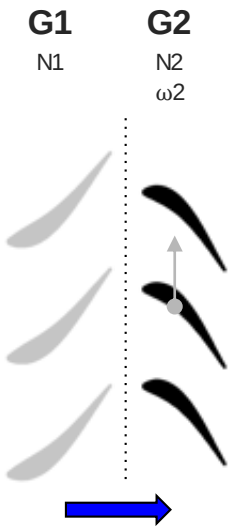
Problem definition

Unsteady phenomena



Unsteady perturbation linked to blade passing frequency from **G1** into **G2**

$$1G1, BPF = N_1\omega_2 = 20 \times 8377.58\ rad/s \approx 167551\ rad/s$$
$$2G1, 2BPF \approx 335103\ rad/s$$
$$3G1, 3BPF \approx 502655\ rad/s$$
$$4G1, 4BPF \approx 837758\ rad/s$$
$$\vdots$$

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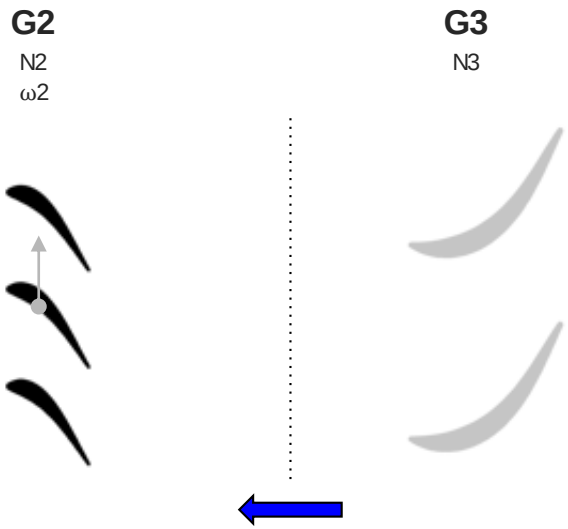
Problem definition

Unsteady phenomena



Unsteady perturbation linked to blade passing frequency from **G3** into **G2**

$$1G3, BPF = N_3\omega_2 = 12 \times 8377.58\ rad/s \approx 100531\ rad/s$$
$$2G3, 2BPF \approx 201062\ rad/s$$
$$3G3, 3BPF \approx 301593\ rad/s$$
$$4G3, 4BPF \approx 402124\ rad/s$$
$$\vdots$$

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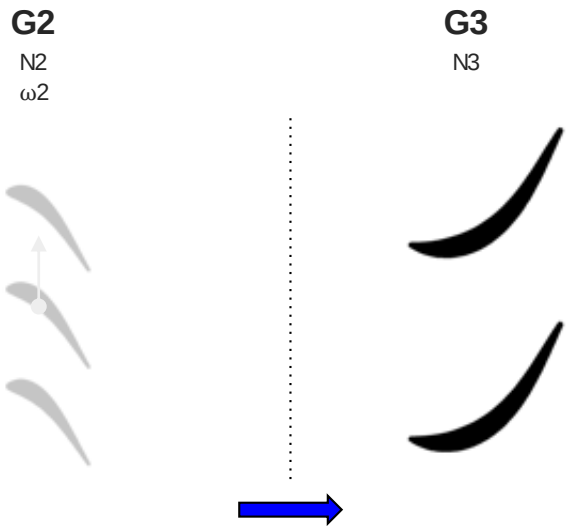
Problem definition

Unsteady phenomena



Unsteady perturbation linked to blade passing frequency from **G2** into **G3**

$$1G2, BPF = N_2\omega_2 = 24 \times 8377.58\ rad/s \approx 201062\ rad/s$$
$$2G2, 2BPF \approx 402124\ rad/s$$
$$3G2, 3BPF \approx 603186\ rad/s$$
$$4G2, 4BPF \approx 804248\ rad/s$$
$$\vdots$$

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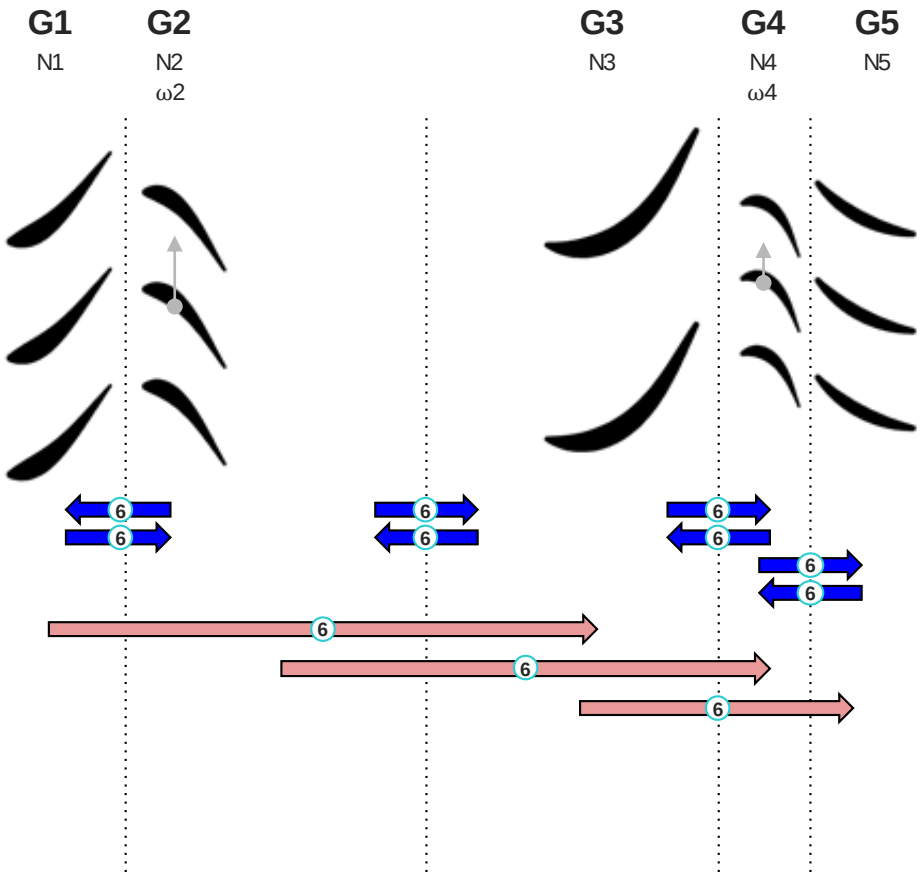
Problem definition

Unsteady phenomena



User choices:

- Selection of 6 harmonics per perturbation.
- Deactivation of rank-2 perturbations from downstream rows (e.g. from **G3** into **G1**).

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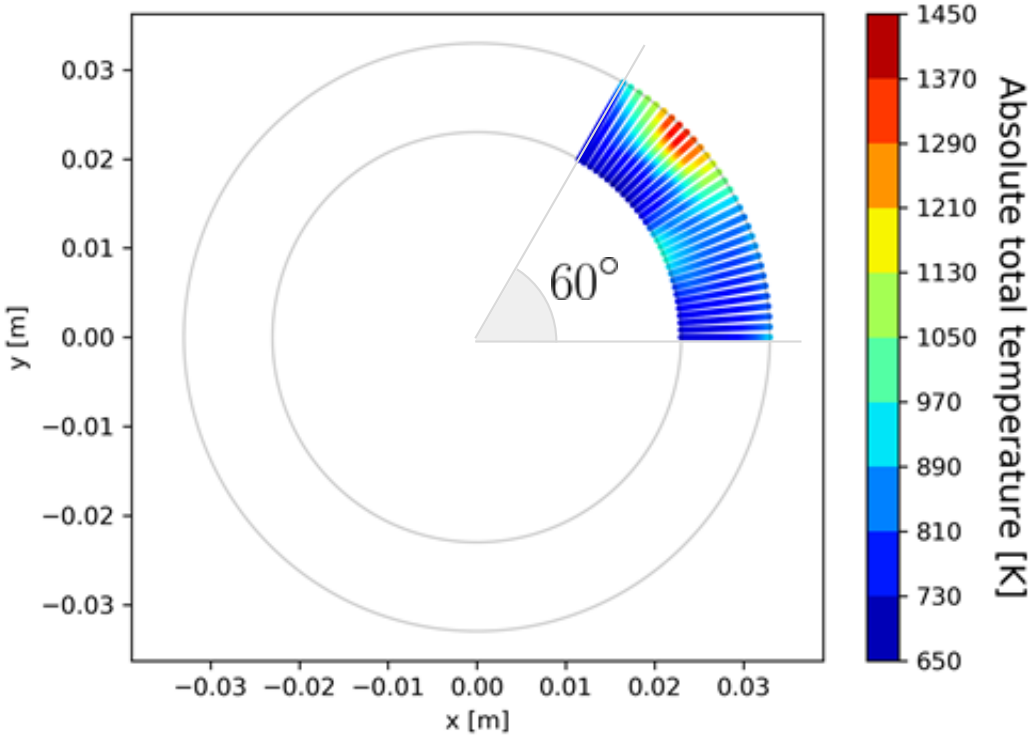
Problem definition

Inlet distortion



1488 structured data points
31 points in $\theta \times 48$ points in r
Spatial mode = 6
0 RPM

- FINE™/Turbo enables modeling inlet distortion conditions using the NLH method[7,8].
- 2D structured profile(s) in r-θ or θ-z can be imposed for static or total quantities boundary conditions.
- The profile(s) can have any periodicity and only a single passage is required per row.
- The profile(s) can be imposed with or without rotational speed.

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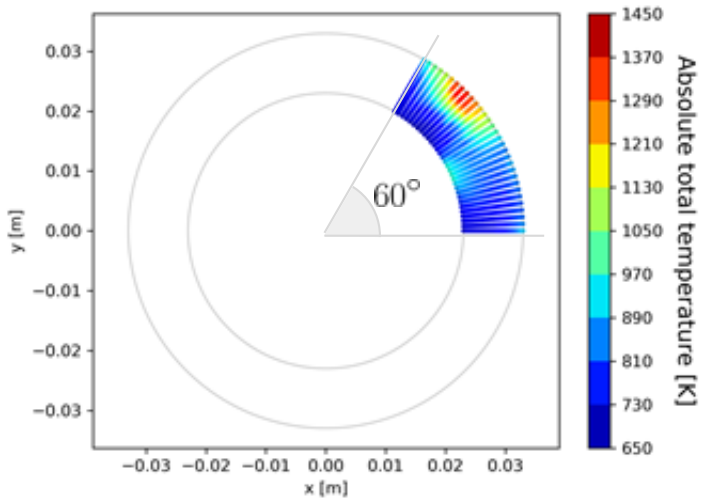
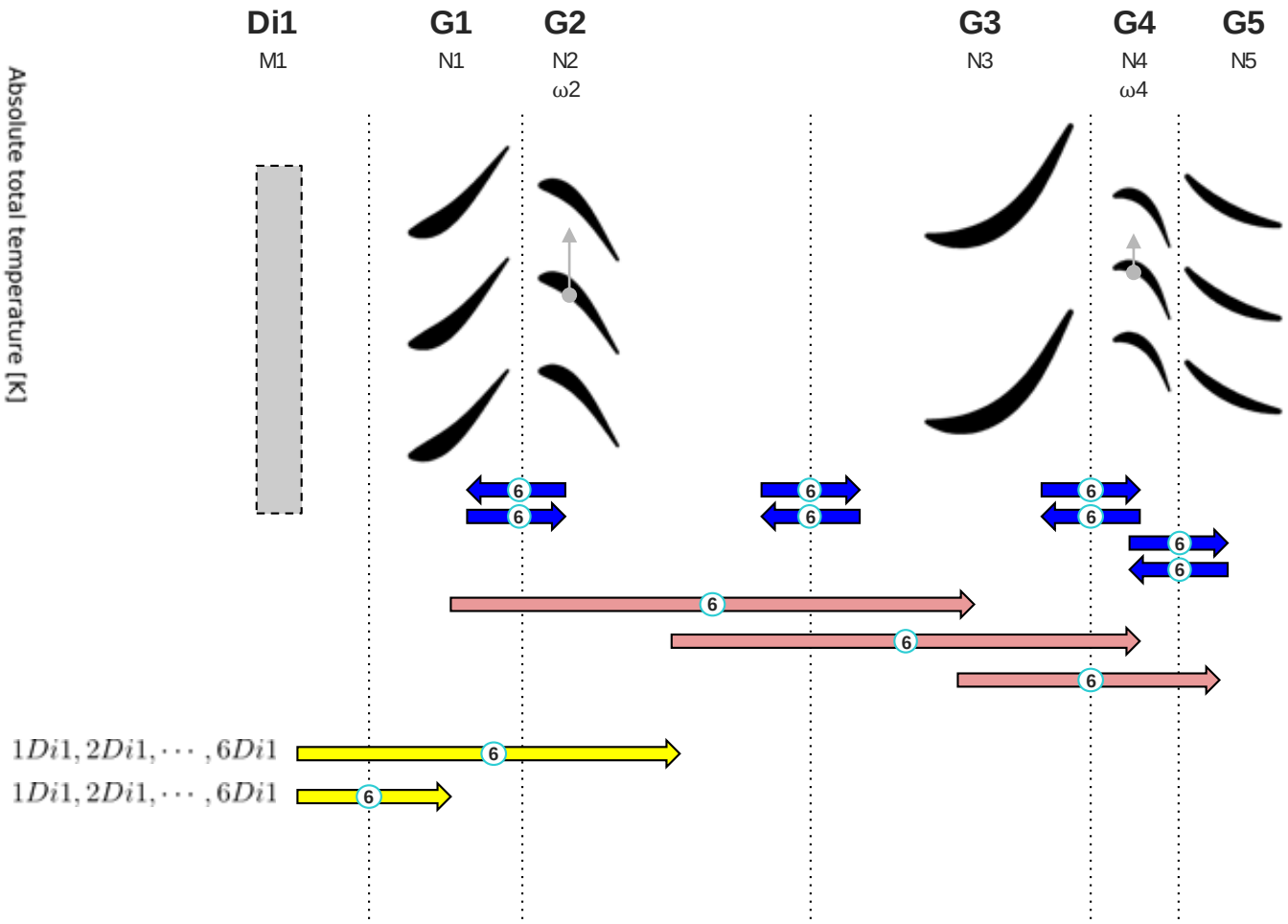
Problem definition

Rank-n



- The inlet distortion is modelled with 6 harmonics and by default is rank-2.



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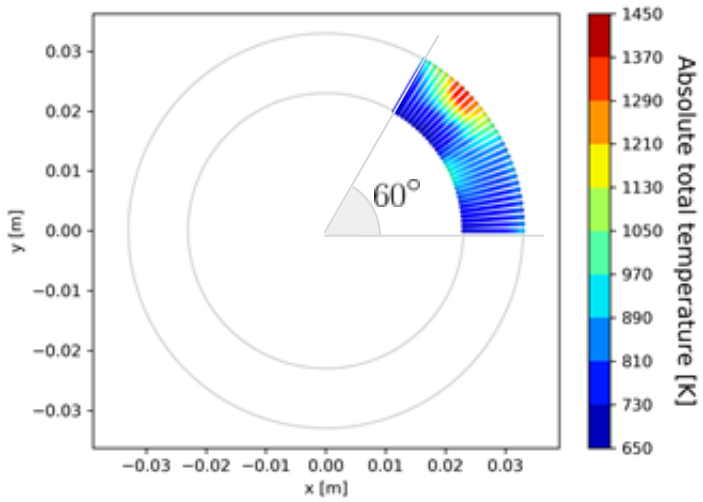
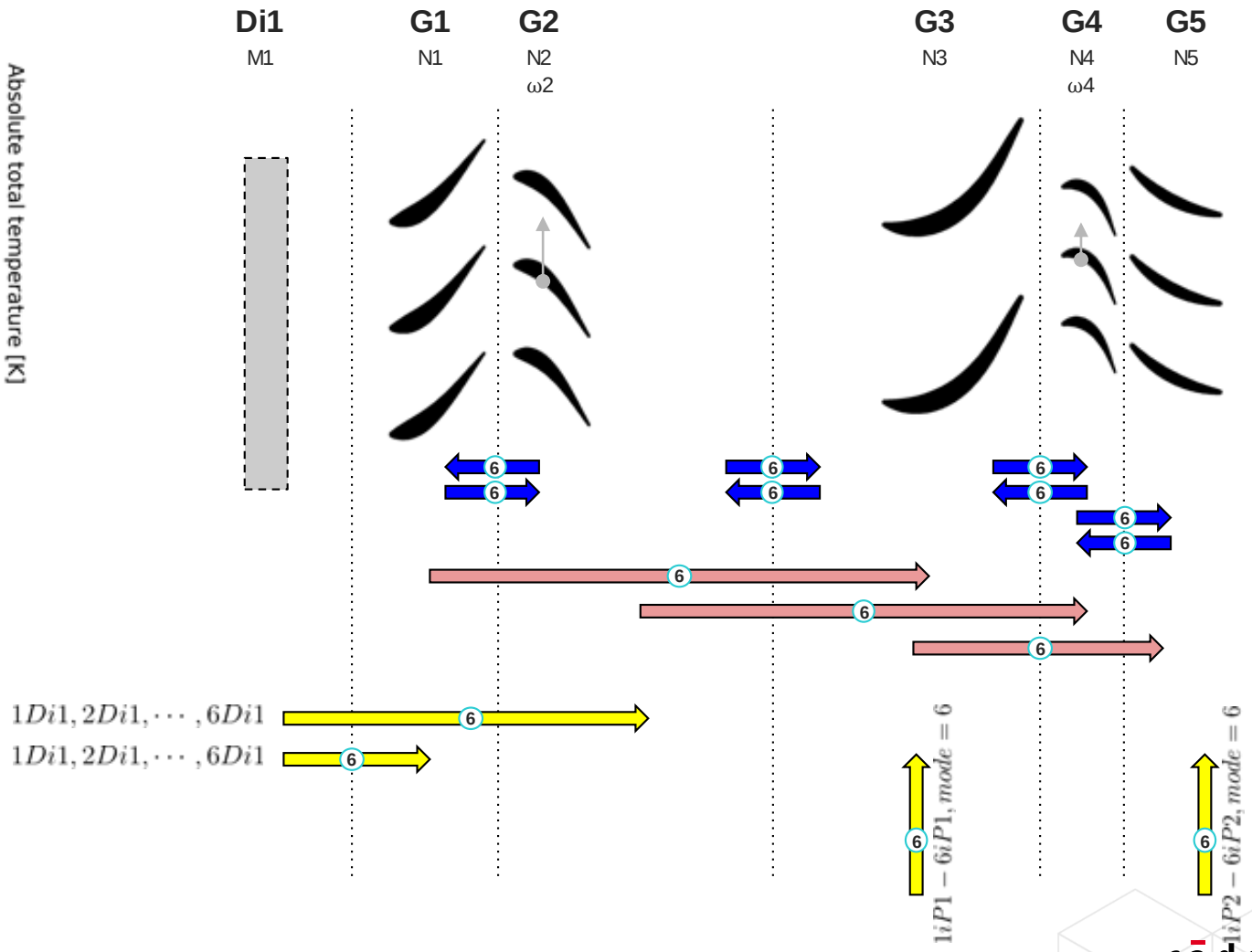
Problem definition

Rank-n



- The inlet distortion is modelled with 6 harmonics and by default is rank-2.
- Inner perturbation harmonics are injected into **G3** and **G5**. The user inputs are f = 0 Hz and mode = 6.



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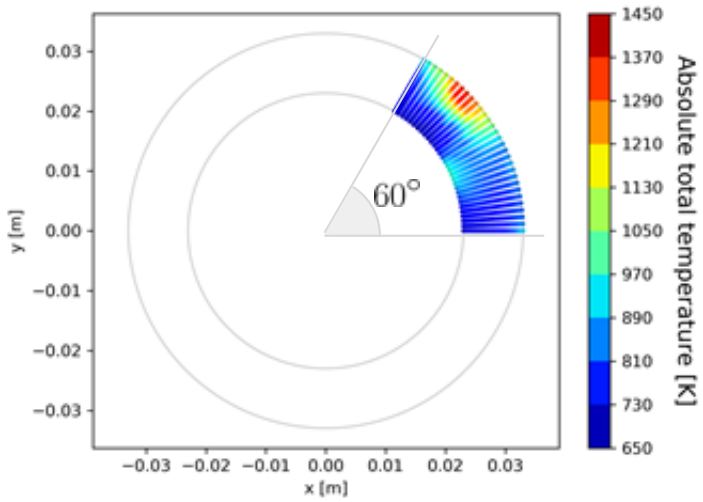
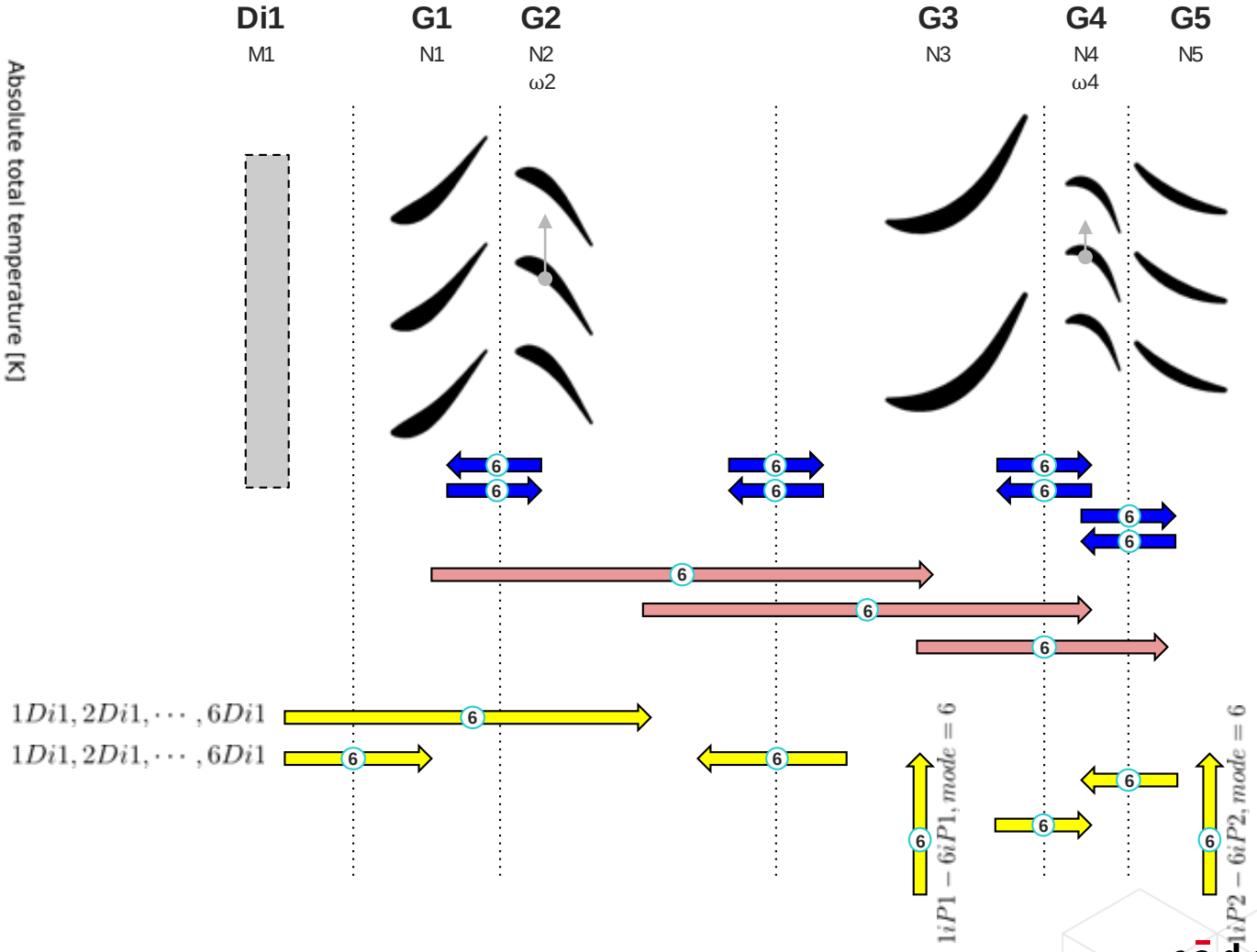
Problem definition

Rank-n



- The inlet distortion is modelled with 6 harmonics and by default is rank-2.
- Inner perturbation harmonics are injected into **G3** and **G5**. The user inputs are f = 0 Hz and mode = 6.
- The merging of harmonics (i.e. *aliases*) and their propagation is done automatically by the solver.



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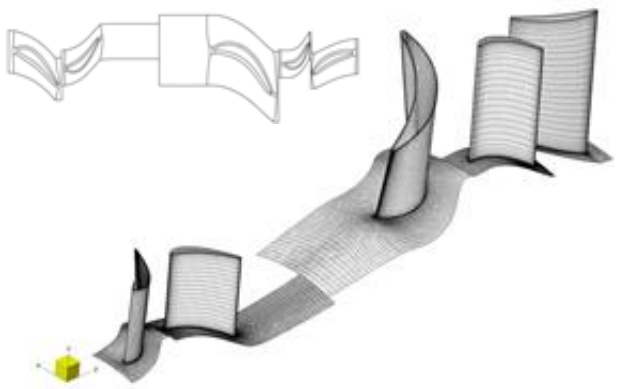
Computation setup

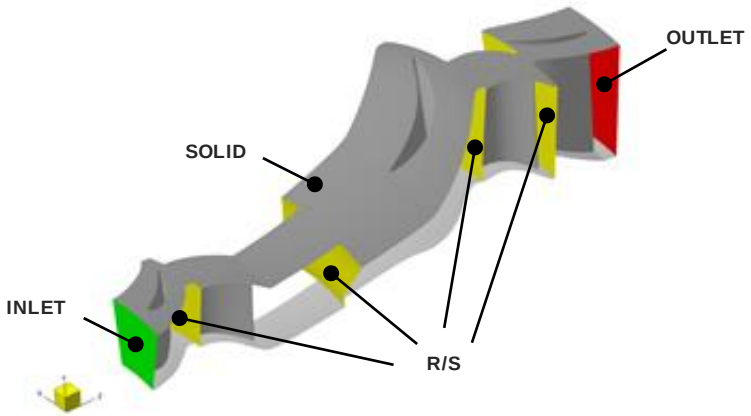
AutoGrid5™ + FINE™/Turbo

AutoGrid5™



Automatic Full Hexahedral Multi-block Structured Meshing of Multi-stage Turbomachinery
7.39M points

FINE™/Turbo



Turbomachinery dedicated CFD solver
CPU Booster™
CFL = 1000
Air (perfect gas)
Spalart-Allmaras
3 grid levels

NLH Rank-n harmonics

	Di ↓	G1 ↓	G2 ↓	G3 ↓	G4 ↓	G5 ↓
Di						
G1	6		6			
G2	6	6		6		
G3	6	6	6		6	
G4	6		6	6		6
G5	6			6	6	

NLH computation restarts from steady-state

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Results analysis

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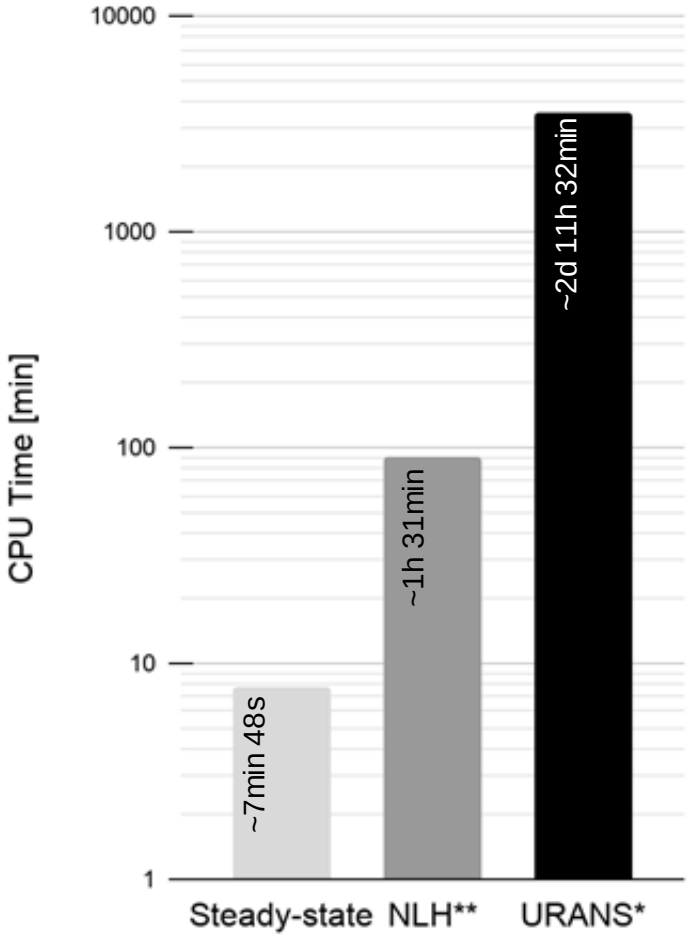
Computational cost

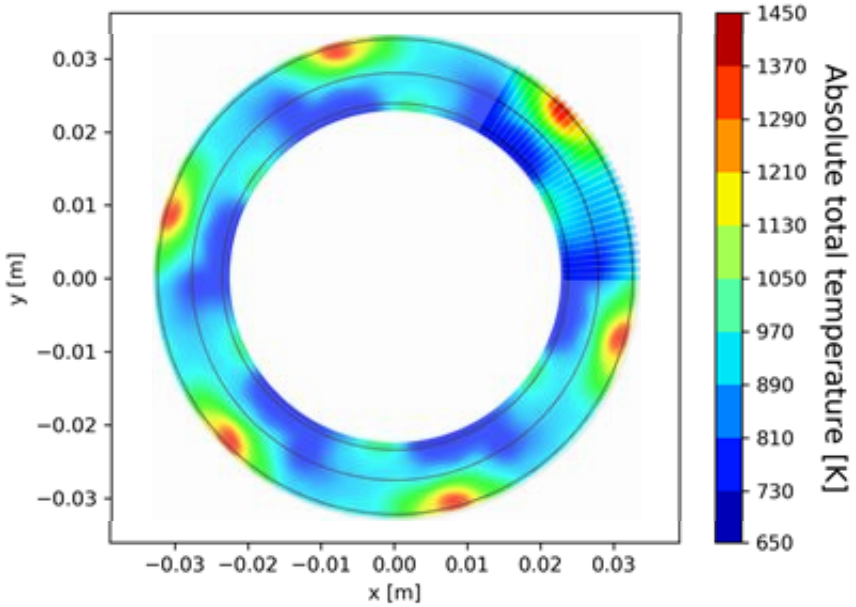
Are the results consistent?



768 CPUs Intel® E5-2697 V2 - 2,60 GHz
Intel® Ivy Bridge with InfiniBand
256 GB (5GB/core) RAM DDR3 1600MHz

*Estimation using domain scaling technique (5/6/3/8/6 passages), 80 angular positions, 20 periods, 50 inner iterations = 80k iterations

**NLH Rank-n with 6 harmonics per perturbation



Instantaneous absolute total temperature profile at the inlet from the NLH computation with overlaid inlet input data.

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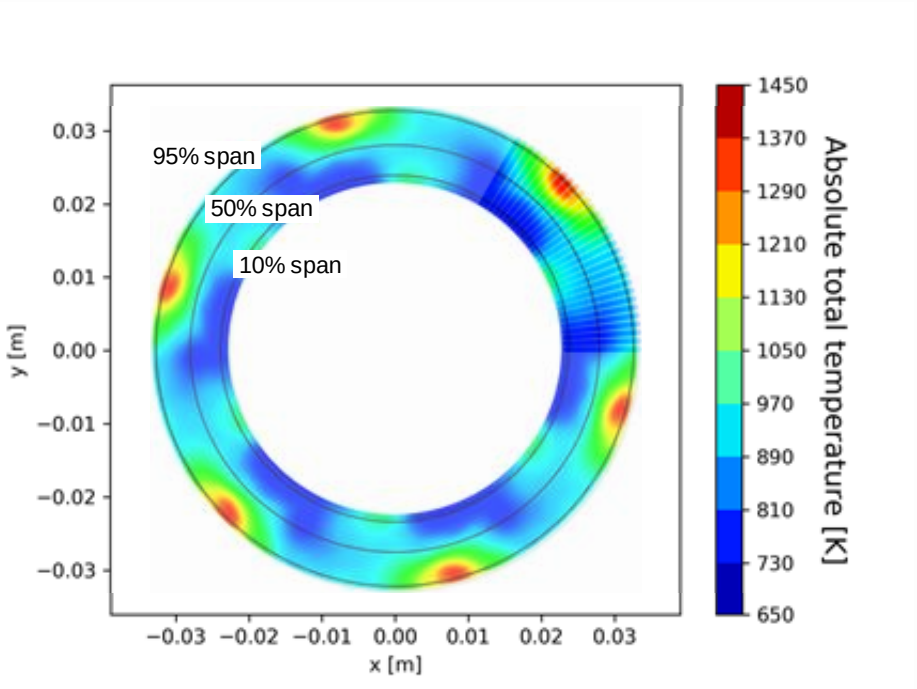
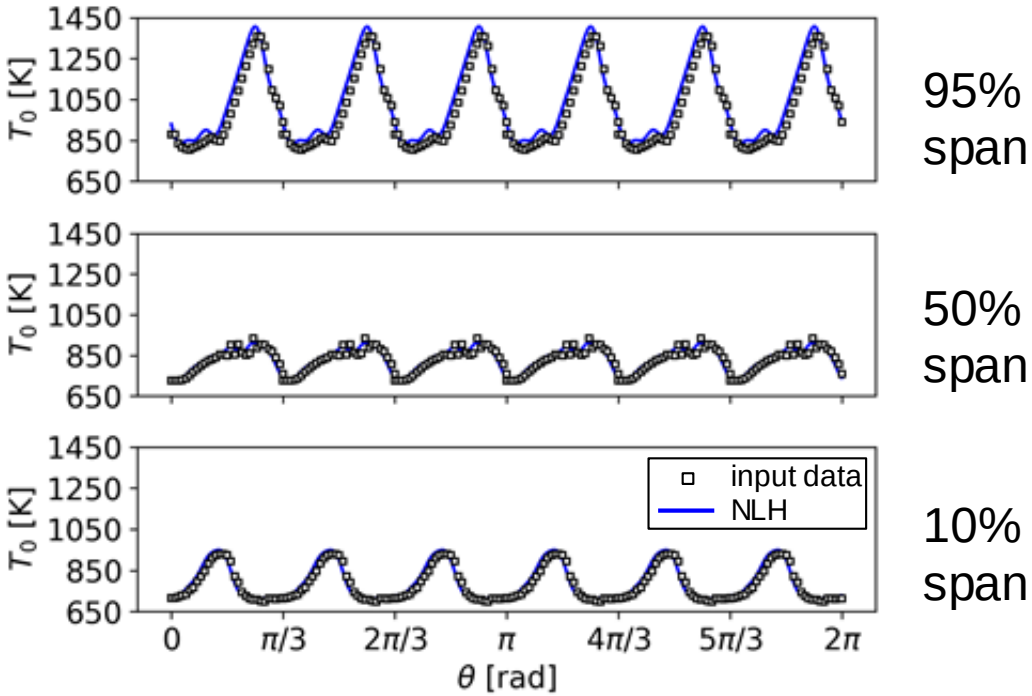
Capturing the inlet distortion

Span profiles



Instantaneous absolute total temperature profile at the inlet from the NLH computation with overlaid inlet input data.



Instantaneous absolute total temperature along the azimuthal direction. Inlet input data shown with repetition.

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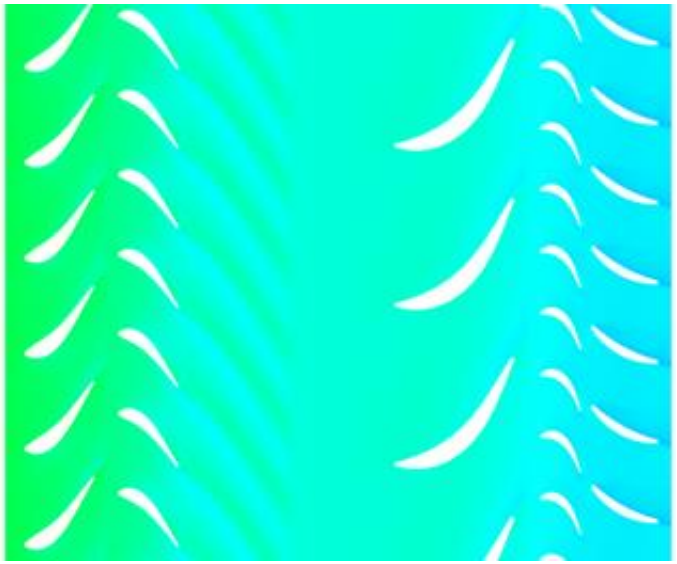
Steady-state vs NLH

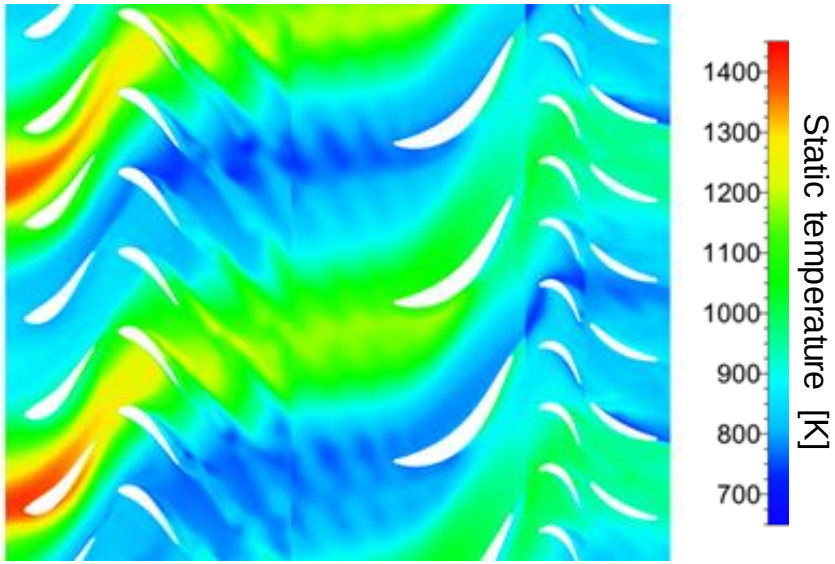
Hot streaks propagation, global performance

Steady-state



Steady-state color contour of static temperature at 95% span (shown with channel repetition).

NLH



NLH time-reconstructed color contour of static temperature at 95% span.

The absolute total temperature at the outlet **directly impacts** the turbine efficiency.

$$\eta = \frac{1 - T_{02}/T_{01}}{1 - (p_{02}/p_{01})^{\frac{\gamma-1}{\gamma}}}$$

Computation	Mass flow [kg/s]	Max temp @ LPT nozzle blade [K]	Total pressure ratio	Avg temp @ outlet [K]	Isentropic efficiency
Steady-state	0.14173	950.511	0.70699	789.57	0.85732
NLH	0.14114	1175.86	0.70858	796.36	0.84786

→ **-1.10% drop!**

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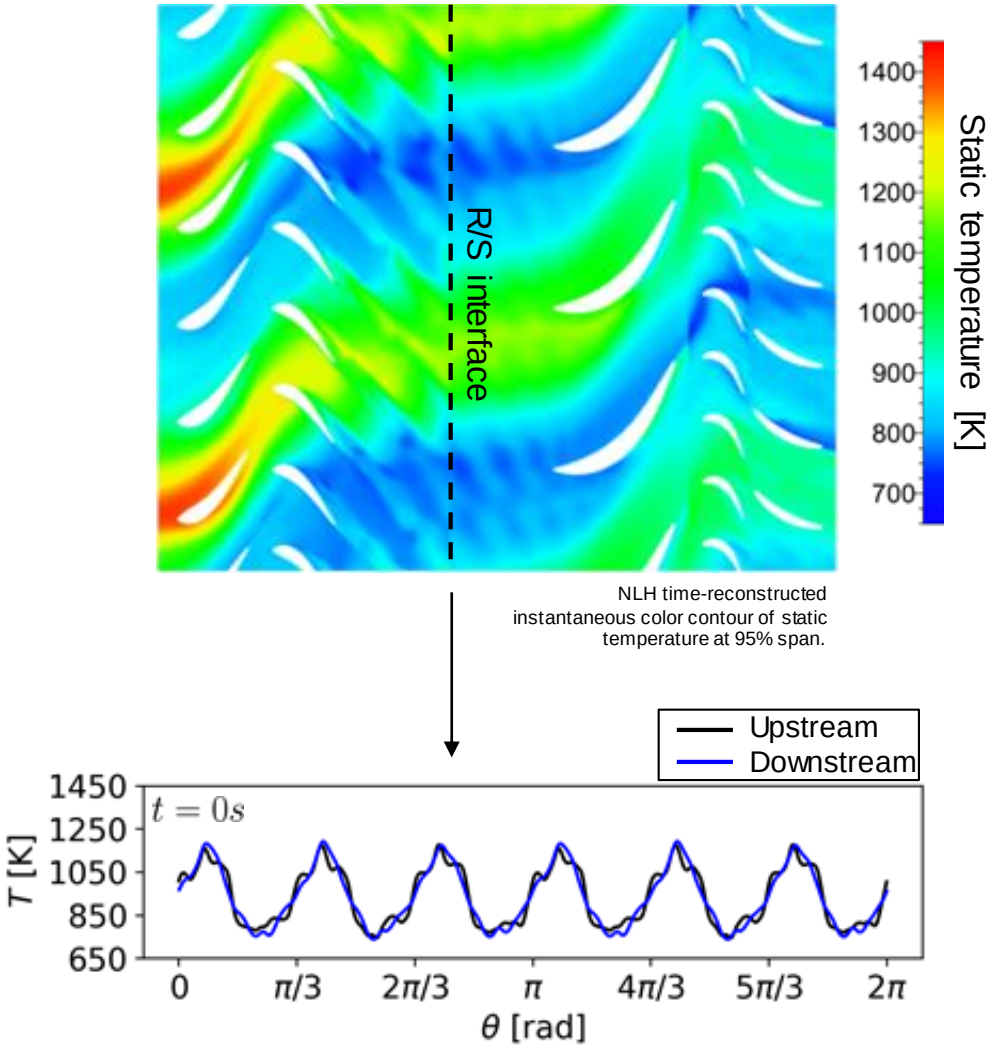
Rotor-stator data

Are the hot streaks really captured?



The profiles along the azimuth at the interface between the HPT rotor and the LPT nozzle are in good agreement.

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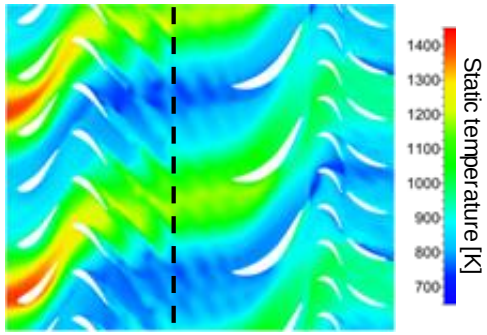
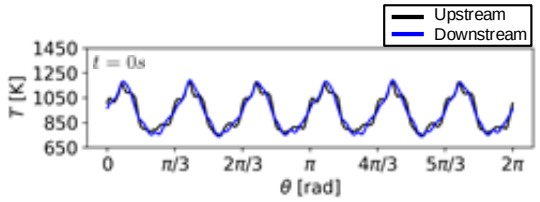
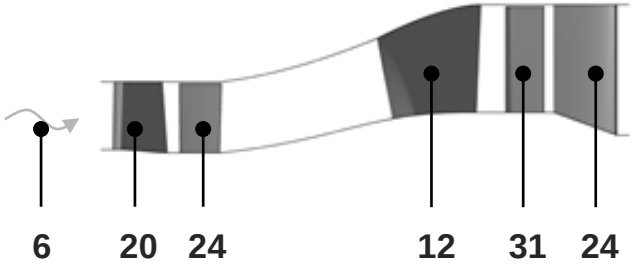
Spatial modes

Are the hot streaks really captured?

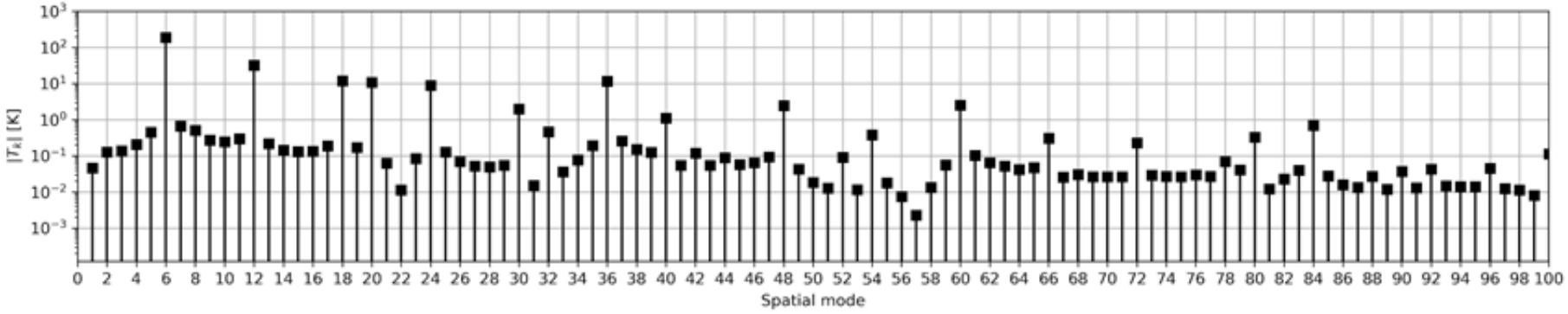




CFView™
DFT
Discrete Fourier Transformation



A ***Discrete Fourier Transformation*** of the profile along the azimuth can be help to identify the spatial modes linked to the instantaneous profile of static temperature



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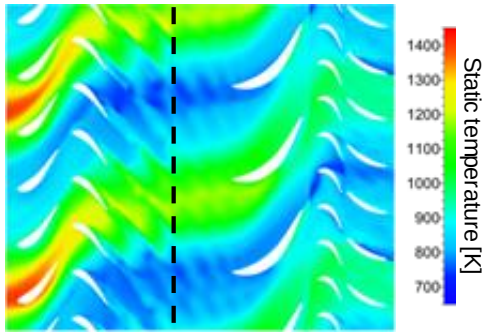
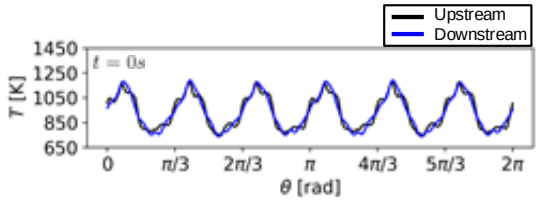
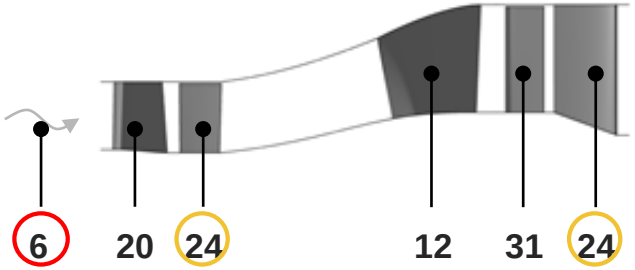
Spatial modes

Are the hot streaks really captured?

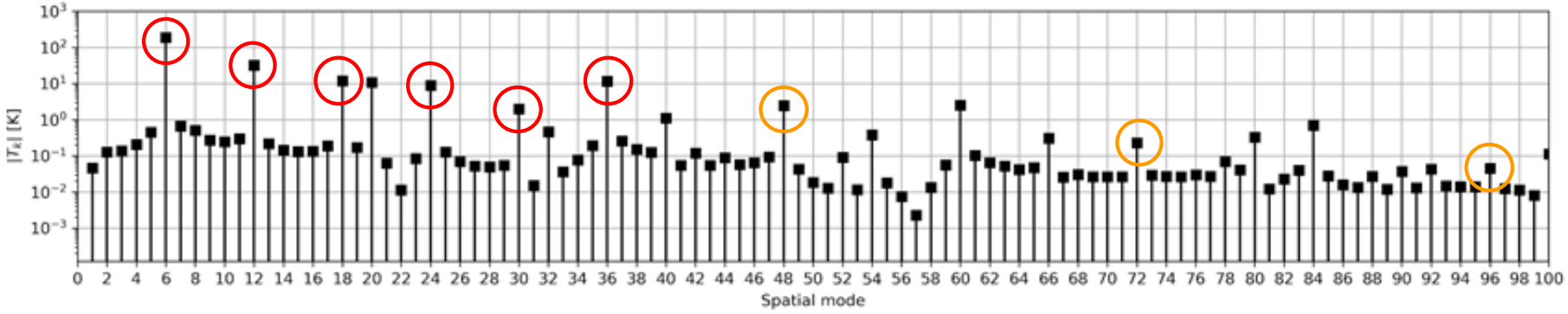




CFView™
DFT
Discrete Fourier Transformation



Multiples of **mode** = **6** are linked to the inlet distortion (as well as to the HPT rotor and LPT deswirler).



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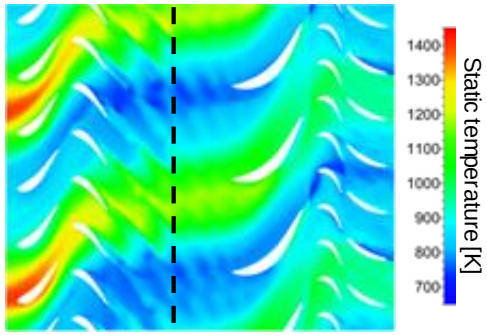
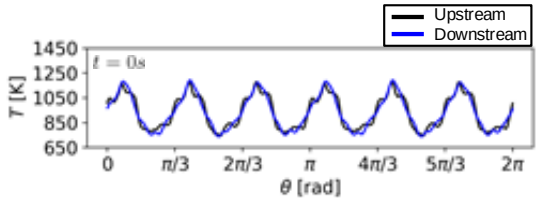
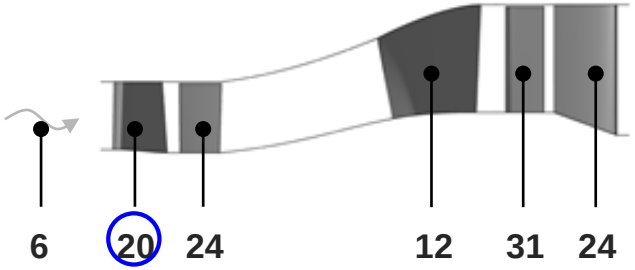
Spatial modes

Are the hot streaks really captured?

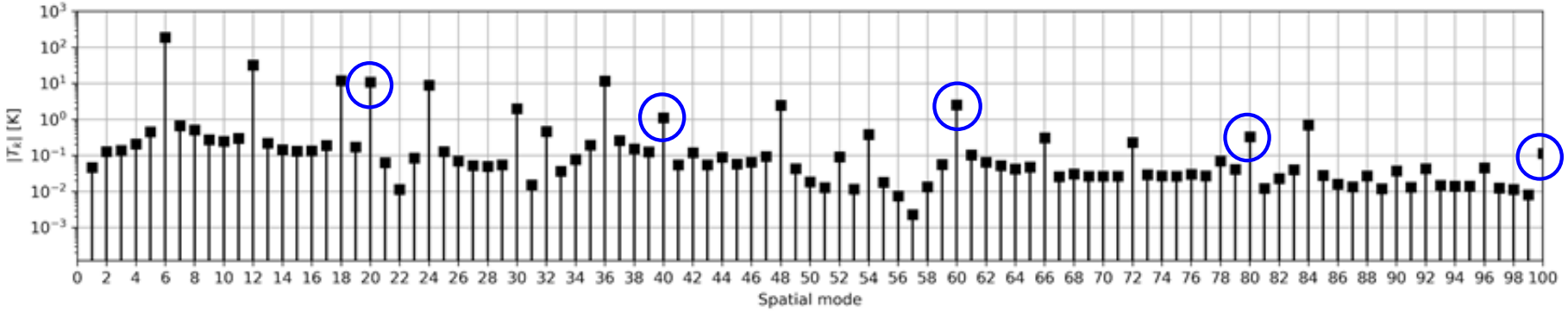




CFView™
DFT
Discrete Fourier Transformation



Multiples of **mode** = **20** are linked to the clocking position of the HPT nozzle.



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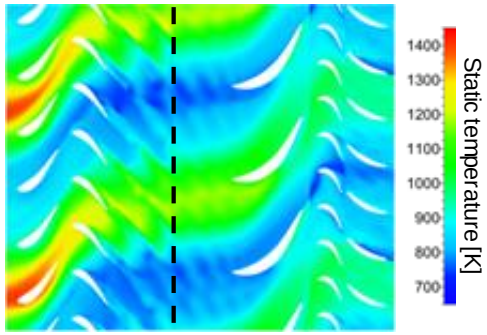
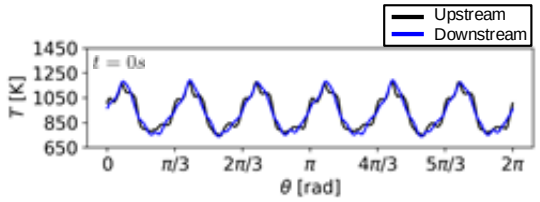
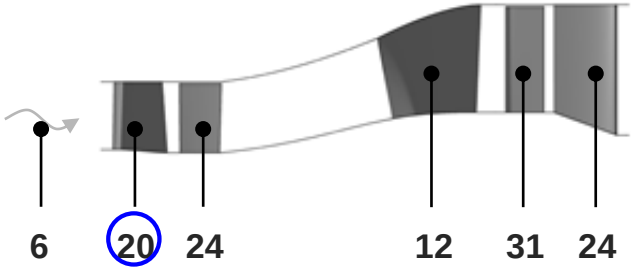
Spatial modes

Are the hot streaks really captured?

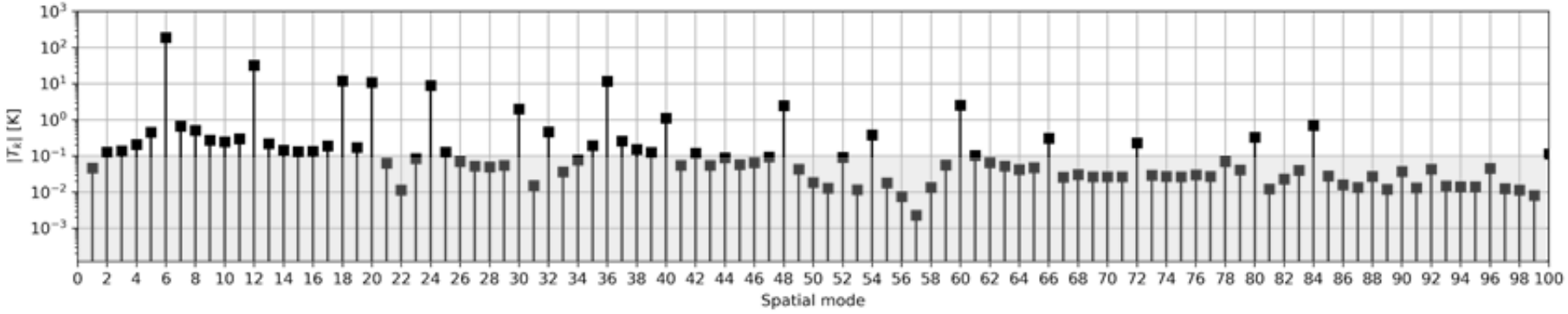




CFView™
DFT
Discrete Fourier Transformation



Less significant contributions can arise from the higher order of modes.



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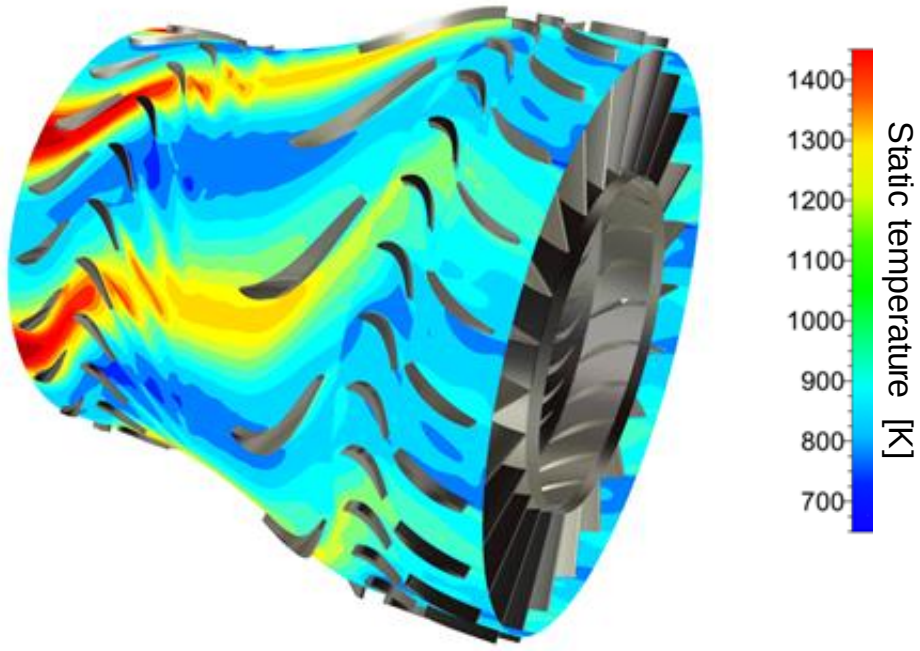
Time reconstruction

Focused analysis

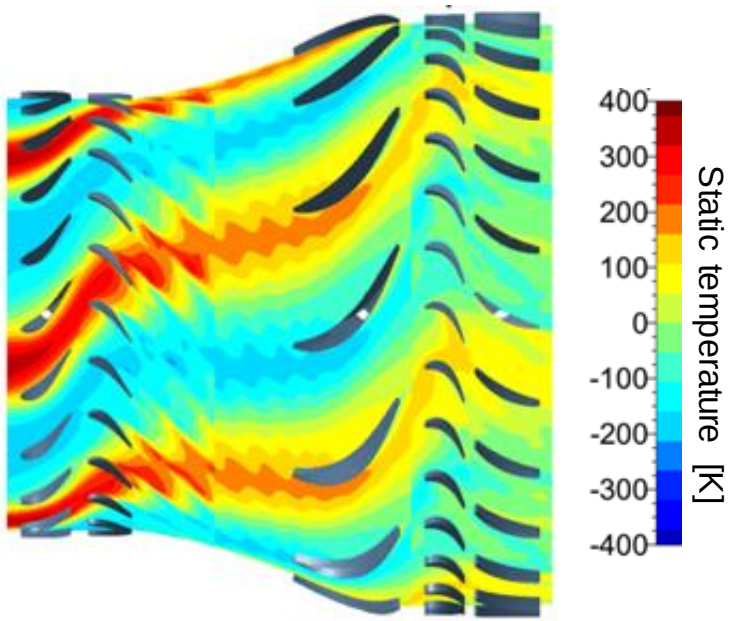
$$U(\vec{r},t) = \overline{U}(\vec{r},t) + \sum \sum_{k=1}^{N} \left[\tilde{U}_k(\vec{r}) e^{j\omega_k t} + \tilde{U}_{-k}(\vec{r}) e^{-j\omega_k t} \right]$$

Time-averaged + perturbations



NLH time-reconstruction of time-average and perturbations color contour of static temperature at 95% span

Perturbations only



NLH time-reconstruction of perturbations color contour of static temperature at 95% span

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Conclusion

The Nonlinear Harmonic Method

1. Review of main **unsteady flow effects in turbomachinery**
   a. Multi-rows interactions
   b. Turbulence
   c. Off design effects
   d. External distortion
   e. Blade vibration
2. Review of **multi-row interaction** modeling in **RANS-based turbomachinery CFD**:
   a. Mixing plane, frozen rotor, average passage;
   b. Sliding mesh, domain scaling, phase-lagged;
   c. Nonlinear Harmonic Method
3. NLH features:
   a. Based on the **existing steady flow solver**.
   b. Models periodic **unsteady perturbations** using **Fourier harmonics**.
   c. Computes the impact of unsteadiness on the time-averaged solution through the **deterministic stresses**.

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Conclusion

The Nonlinear Harmonic Method

4. NLH features:
   a. Only **one blade channel** is required per row.
   b. **100-1000 faster** than standard unsteady calculations.
   c. All unsteady flow can be **reconstructed in space and time**.
   d. **Reduces the loss of continuity** at the rotor/stator interface.
   e. **Clocking effects** can be analyzed with only one simulation.

5. NLH Rank-n:
   a. Application to an **axial turbine** subject to **inlet distortion**.
   b. **Flexibility** on harmonic selection.
   c. Considerable **reduction of time** compared to time-domain methods.
   d. Consistent **hot streak propagation**.
   e. Reconstruction of the flow in time.
   f. Impact of **unsteady perturbation modeling** on the machine performance.

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