LES of Wall-bounded Turbulent Flows

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Outline

- Introduction
- SGS and Wall Model
  - *Virtual wall boundary conditions*
- Codes used
  - *Code description*
  - *Previous examples of DNS*
- Results of large-eddy simulations
  - *LES of flow over flat plate (WM)*
    - *Power-law vs. log law*
    - *Flow with separation*
  - *LES of flow past a periodic hill (WM)*
  - *LES of flow past a cylinder (WR)*
  - *LES of flow past airfoils (WM)*
- Conclusion

Turbulent flows: all about Reynolds number







Reynolds number

$$Re = \frac{U_\infty L}{\nu}$$

10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10}

DNS ← | → ***Modeling***

- ***RANS***
- ***LES-RANS/DES***
- ***Wall-resolved LES***
- ***Wall-modeled LES***

- ***DNS: $Re^{9/4}$***
- ***TBL DNS: $Re^{37/14}$***
- ***TBL WR LES: $Re^{13/7}$***

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Wall Bounded Turbulence

- Applications Driver: Computing wall-bounded turbulent flows is extremely challenging. Most engineering applications of turbulent flows are of the wall-bounded variety
  - *Holy grail: computing flow over a sphere to reproduce the drag crisis (Re~ 300,000)*
  - *DNS: $Re^{9/4}$*
  - *TBL DNS: $Re^{37/14}$*
  - *TBL WR LES: $Re^{13/7}$*

- Research: Development of wall models and high-fidelity simulation codes for wall-bounded turbulent flows and applying these to flows over bluff bodies and airfoils (e.g. wind turbines components) (KAUST + Caltech collaboration)

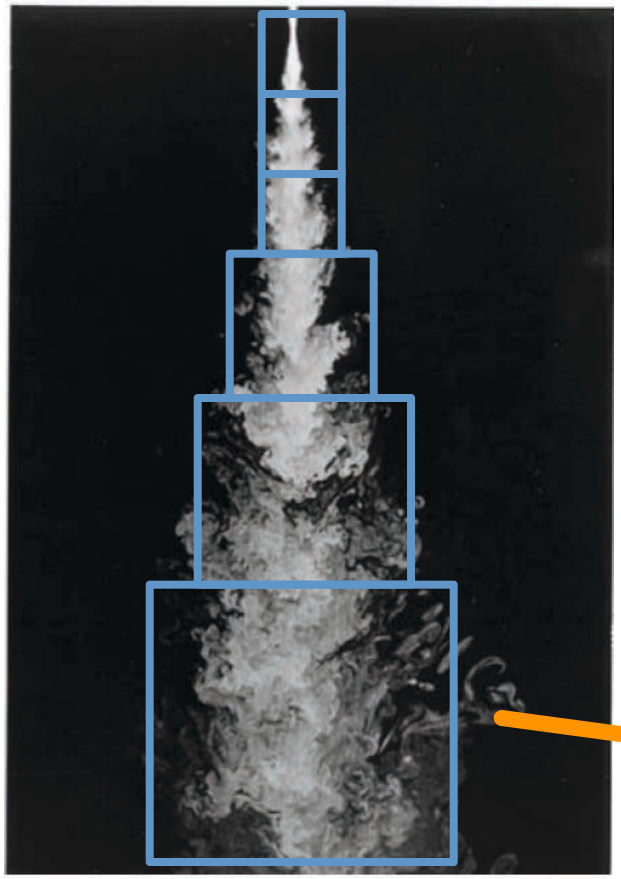
- Key physics issues: separation, adverse pressure gradient, curvature, roughness

Direct Numerical Simulation (DNS)
Large-Eddy Simulation (LES)

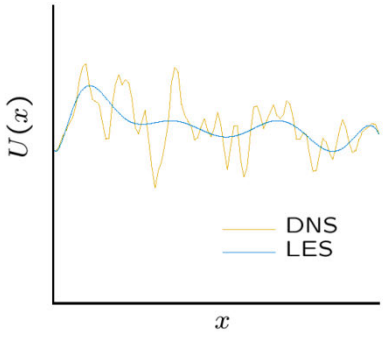


Re = 10k (Dimotakis et al. 1983)

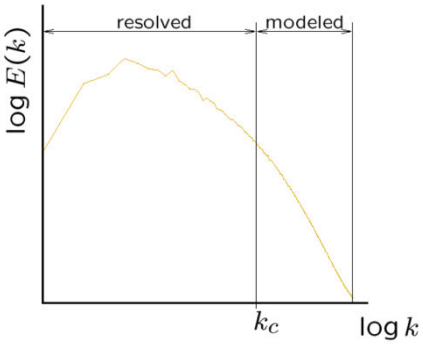
Direct Numerical Simulation (DNS)
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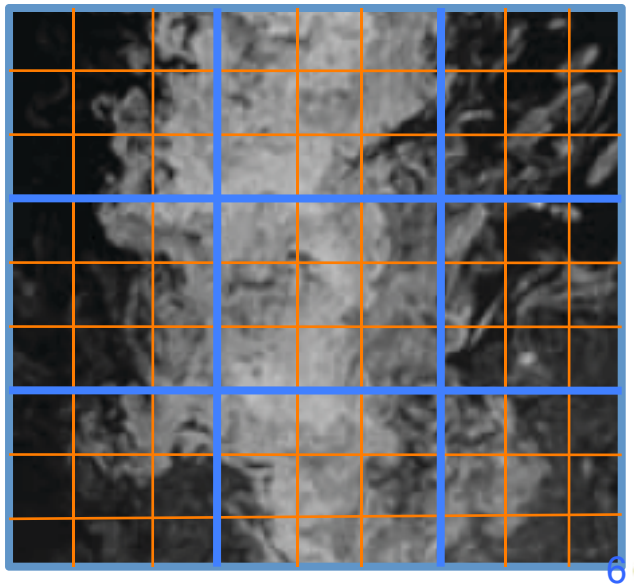
Re = 10k (Dimotakis et al. 1983)



physical space: fine-scale fluctuations not resolved, their influence is modeled.



spectral space: resolved range, $k < k_c$ (cutoff wavenumber k_c), subgrid range $k > k_c$.



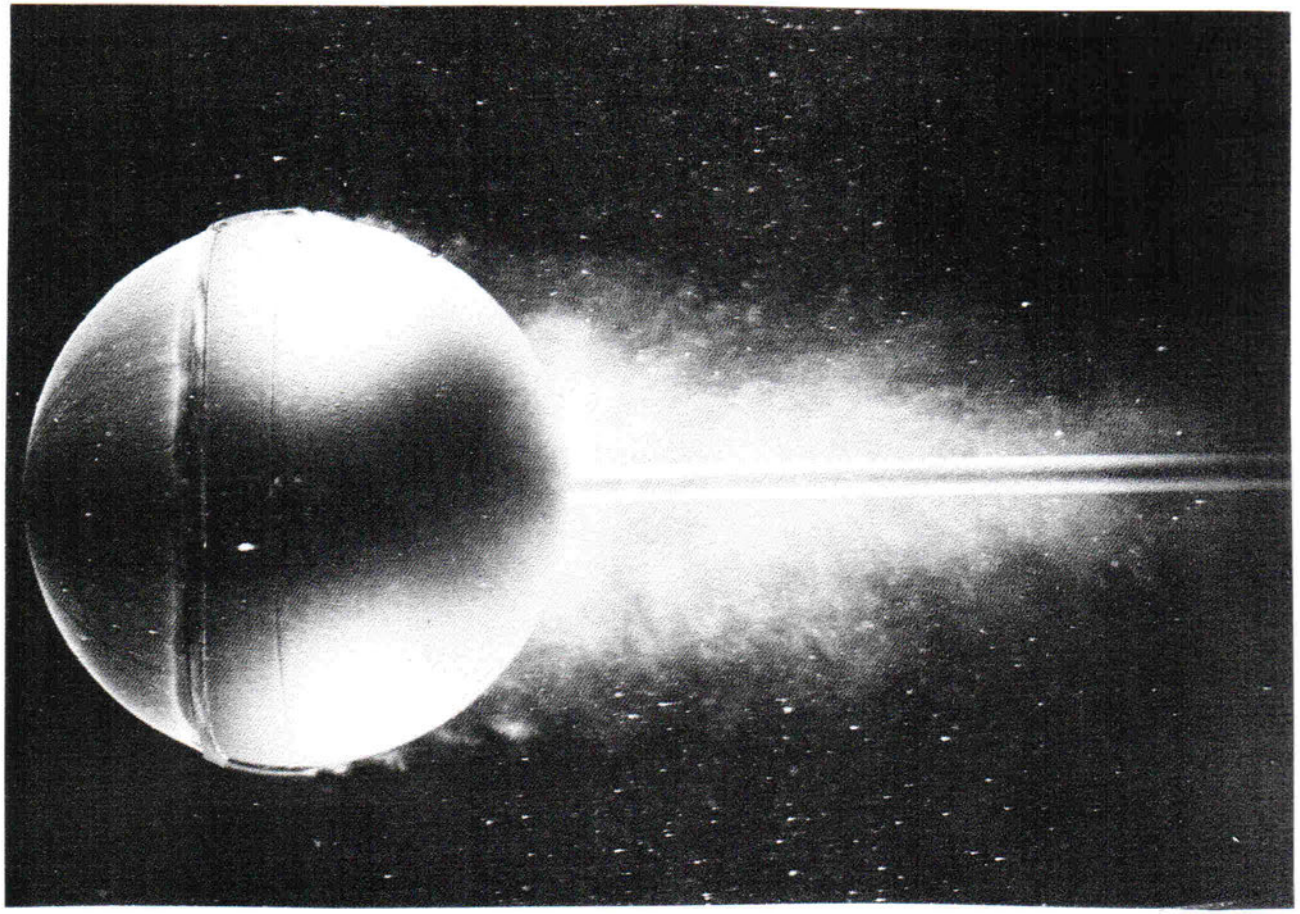
DNS

LES

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Flow past sphere (with trip wire)

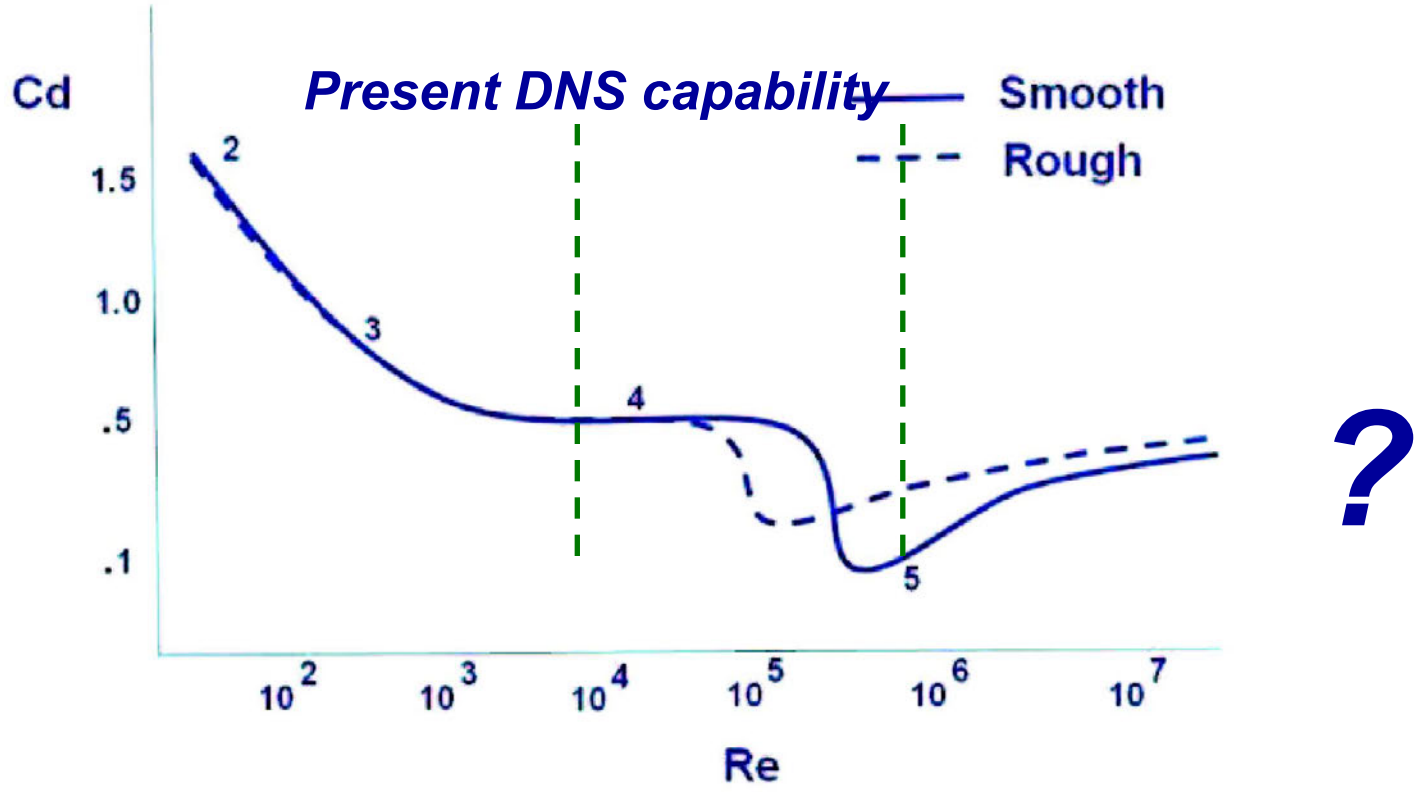


Re = 30k (Werle 1980)

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Drag coefficient of sphere

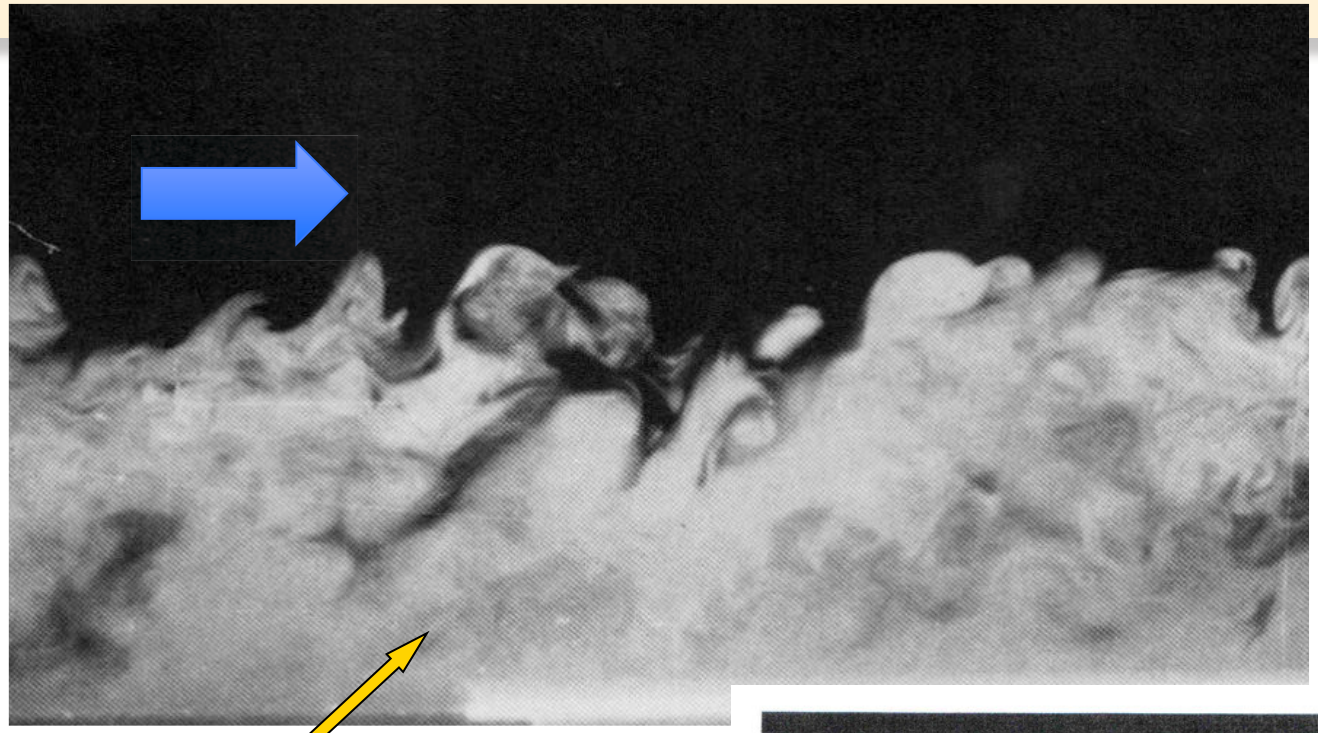


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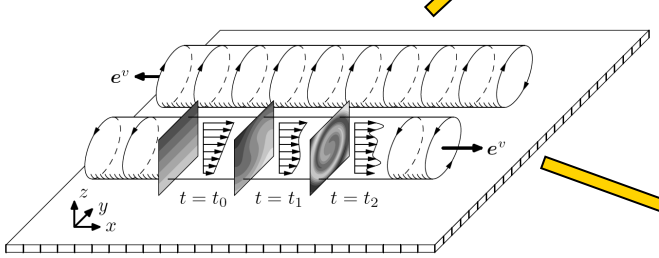
NASA

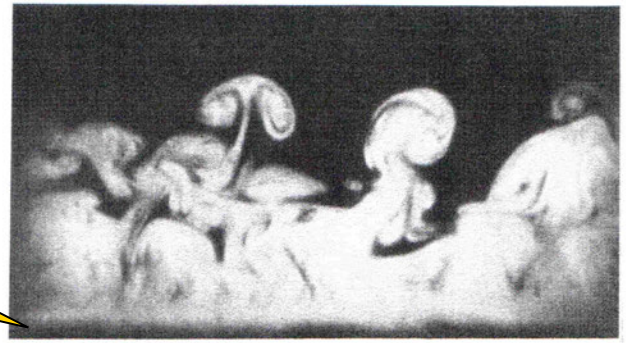
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LES for wall-bounded flows



(Falco 1977)





Head & Bandyopandhyay (1981)

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Large/extreme Reynolds numbers: $Re_\tau = u_\tau \delta / \nu$

- DNS:
  - *Hoyas & Jimenez (2012)*
  - *Lee & Moser et al (2015)*

$$Re_\tau \approx 2 - 3 \times 10^3 \qquad \log(Re_\tau) = 8.2$$

- Laboratory boundary layers:
  - *Oweis et al (2010)*

$$Re_\tau \approx 6.2 \times 10^4 \qquad \log(Re_\tau) = 11.0$$

- 747 wing:

$$Re_\tau \approx 2 \times 10^5 \qquad \log(Re_\tau) = 12.0$$

- Pipe flow: super pipe:
  - *Smits and Zagarola (1998)*

$$Re_\tau \approx 5 \times 10^5 \qquad \log(Re_\tau) = 13.2$$

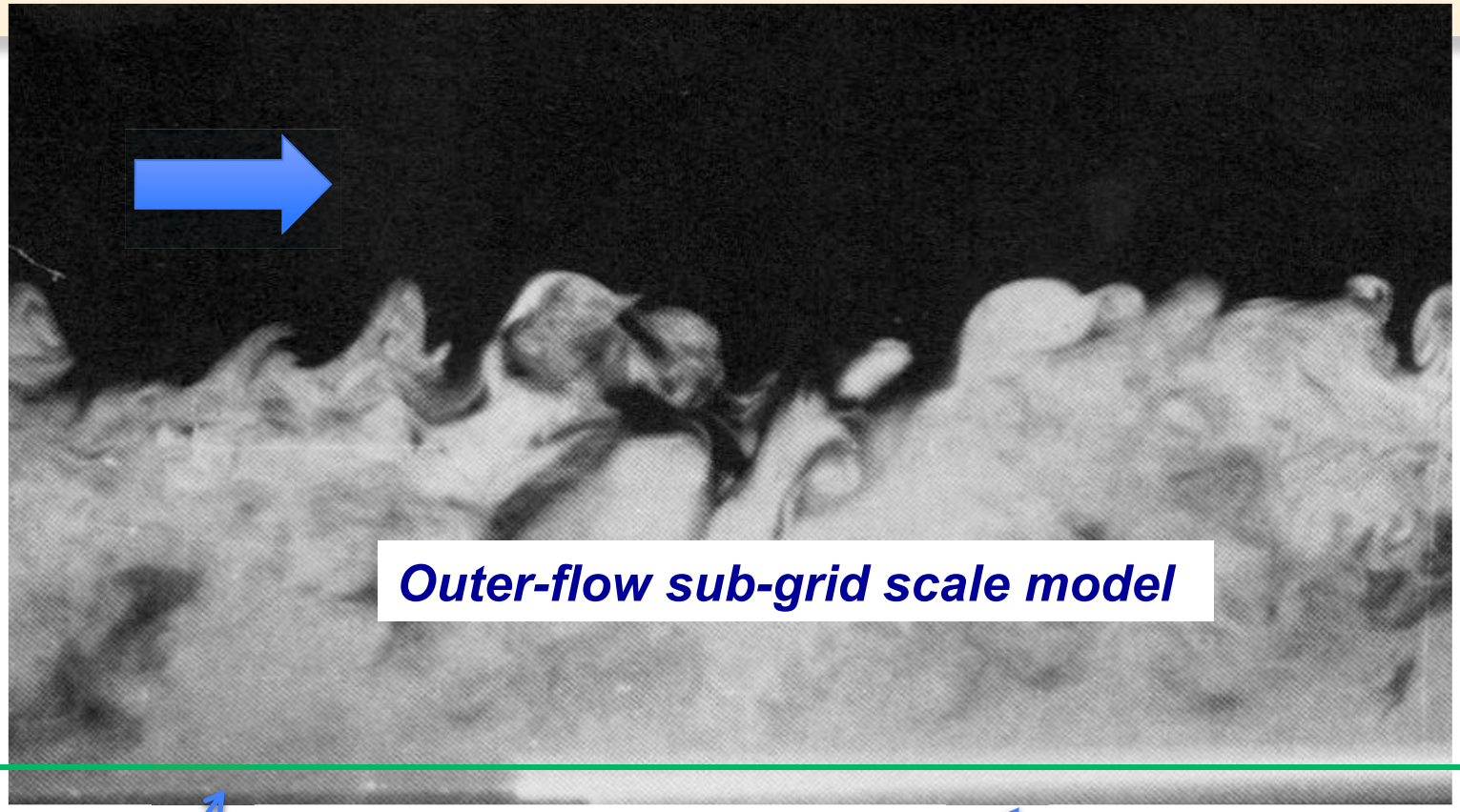
- Salt Lake TEST data:
  - *Metzger & Klewicki (2002),*
  - *Metzger et al (2007)*

$$Re_\tau \approx 10^6 \qquad \log(Re_\tau) = 13.8$$

- LES: ZPGTBL:
  - *Inoue & Pullin (2011)*

$$Re_\tau \approx 2 \times 10^{11} \qquad \log(Re_\tau) = 26.0$$

LES for wall-bounded flows



(Falco 1977)

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Filtered Navier-Stokes Equations

- Apply filtering operation to incompressible Navier-Stokes equations

$$\frac{\partial \widetilde{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \widetilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widetilde{u_i\, u_j}\right) = -\frac{\partial \widetilde{p}}{\partial x_i} + \nu \frac{\partial^2 \widetilde{u}_i}{\partial x_i^2}$$

$$\frac{\partial \widetilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widetilde{u}_i \widetilde{u}_j\right) = -\frac{\partial \widetilde{p}}{\partial x_i} - \frac{\partial}{\partial x_i}\underbrace{\left(\widetilde{u_i\, u_j} - \widetilde{u}_i\, \widetilde{u}_j\right)}_{T_{ij}} + \nu \frac{\partial^2 \widetilde{u}_i}{\partial x_i^2}$$

$$\widetilde{u_i\, u_j}(\boldsymbol{x}, t) \equiv \int_{-\infty}^{\infty} \mathcal{G}(\boldsymbol{x}' - \boldsymbol{x})\, u_i(\boldsymbol{x}')\, u_j(\boldsymbol{x}')\, d\boldsymbol{x}' \neq \widetilde{u}_i\, \widetilde{u}_j$$

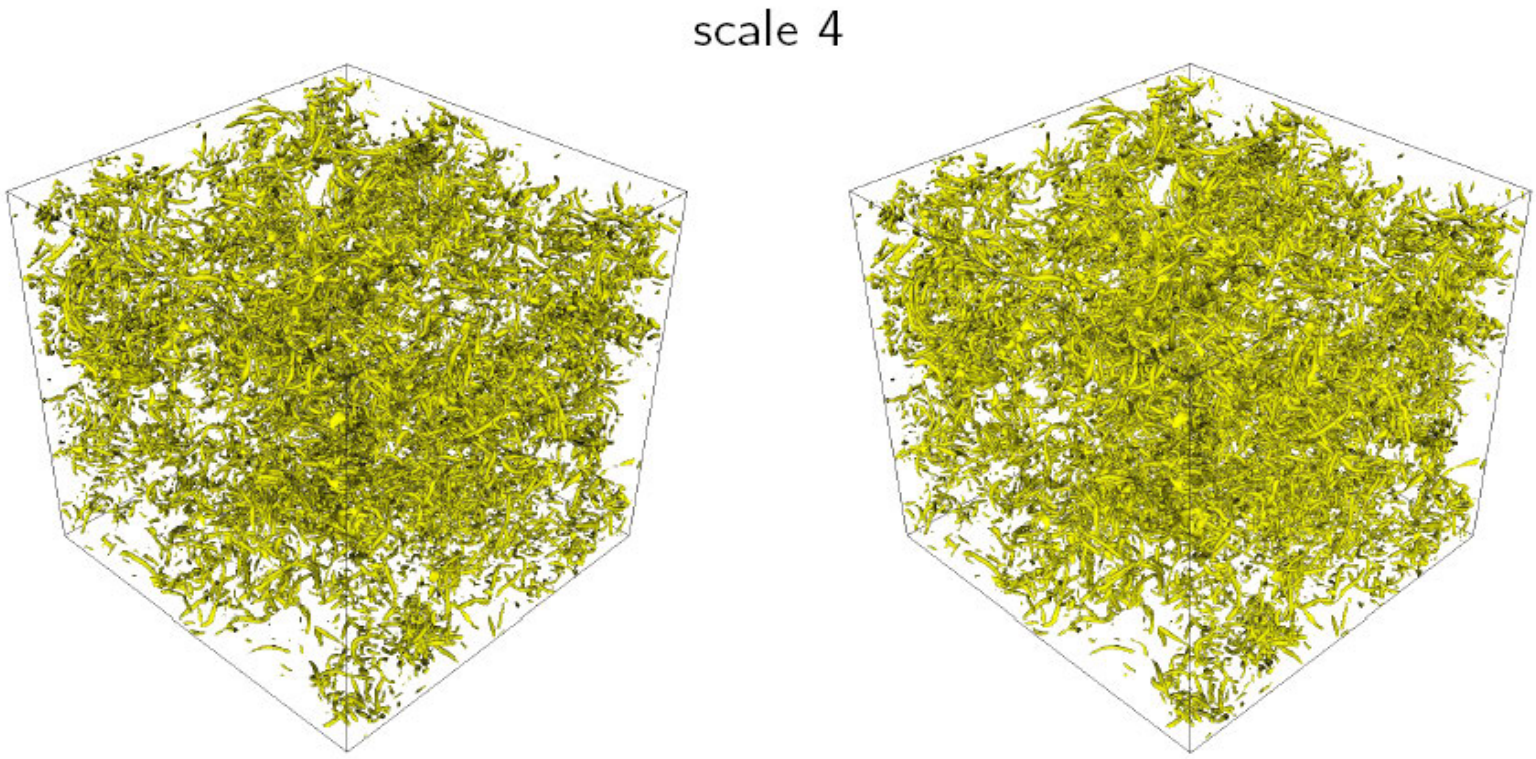
- T_{ij} ``unresolved stresses'' must be modeled: this is the ``closure problem''
- This equation set is NOT closed
- Filtering process on NS equations is strictly formal: no particular filter is actually needed

Discovery through simulation

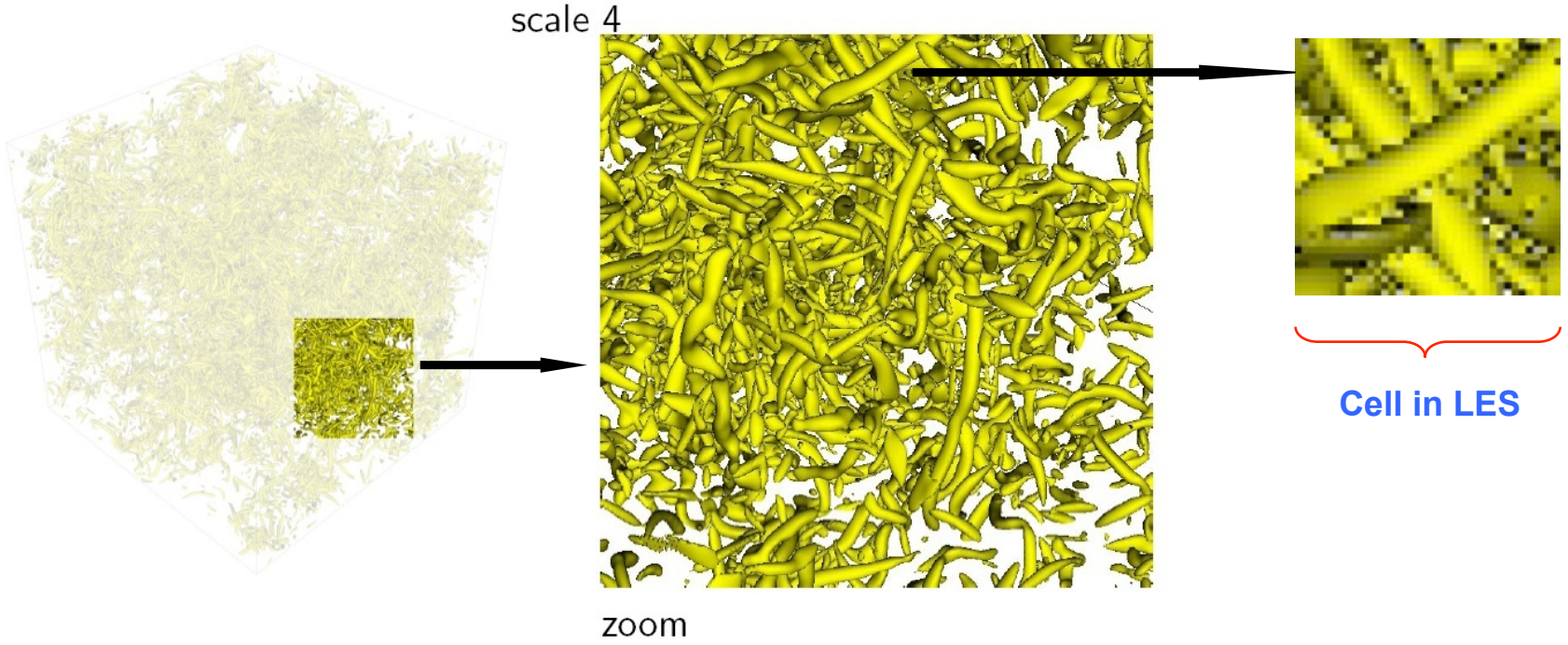
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Outer-flow sub-grid scale model

Multi-scale decomposition: isosurfaces of enstrophy (Bermejo-Moreno & Pullin 2009)



Explicit SGS model; stretched-vortex model



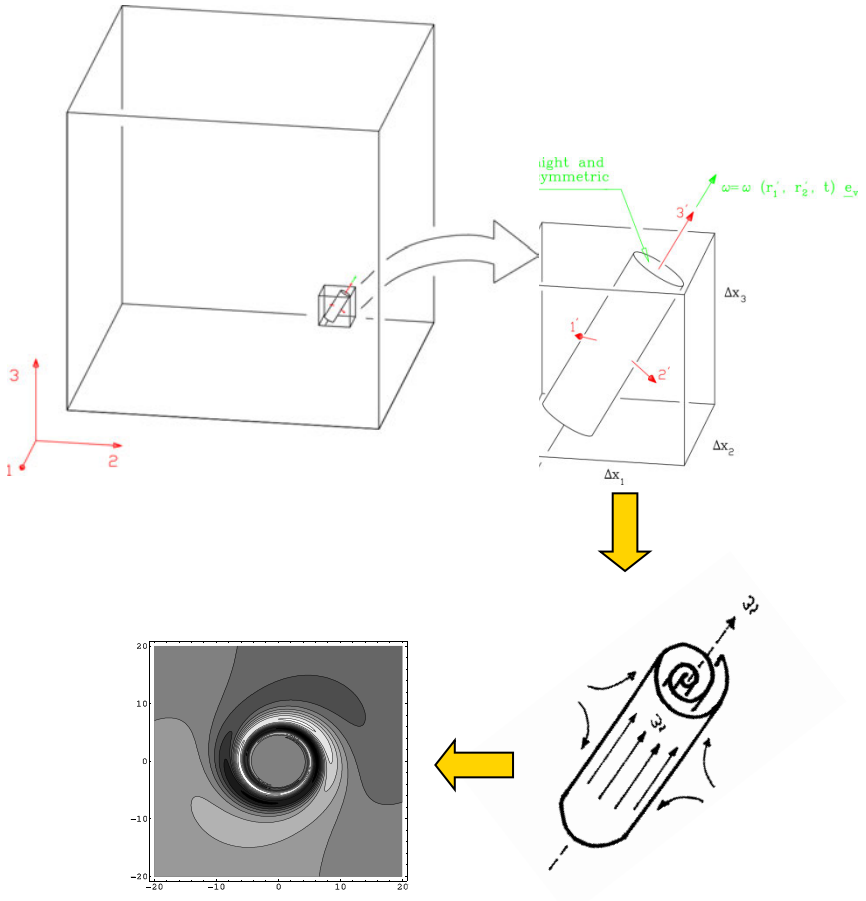
- **Small scales of turbulence Intense vorticity in form of tubes and sheets**

[Worms: Ashurst, Jimenez et al 1993]

- **Can this be used as a basis of a stucture-based sub-grid scale model?**

Explicit SGS model: stretched-vortex model

- Structure-based approach
- Subgrid motion represented by nearly axisymmetric vortex tube within each cell
- Local solution of NS equations for stretched-spiral vortex
  - *Lundgren (1982), Pullin & Lundgren (2001)*
- Subgrid stress:



$$T_{ij} = (\delta_{ij} - e_i^v e_j^v) K,$$

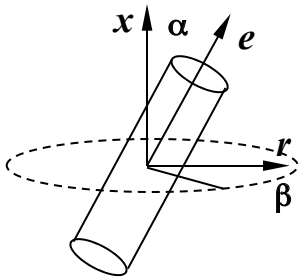
$$K = \int_{k_c}^{\infty} E(k)dk = \frac{1}{2}\mathcal{K}_0' \Gamma\left[-1/3, \kappa_c^2\right], \mathcal{K}_0' = \mathcal{K}_0 \epsilon^{2/3} \lambda_v^{2/3}$$

Model parameters

- Subgrid energy spectrum (Lundgren, 1982)

$$E(k) = \mathcal{K}_0 \epsilon^{2/3} k^{-5/3} \exp[-2k^2\nu/(3|\tilde{a}|)]$$

$$\tilde{a} = \tilde{S}_{ij} e_i^v e_j^v, \qquad \tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$



- Parameters obtained from resolved-scale, second order velocity structure-functions (Lesieur et al)

$$\mathcal{K}_0 \epsilon^{2/3} = \frac{\overline{\mathcal{F}_2}(\triangle)}{\triangle^{2/3} \mathrm{A}}, \qquad \mathrm{A} = 4\int_0^\pi s^{-5/3}(1 - s^{-1}\sin s)ds \approx 1.90695$$

$$\overline{\mathcal{F}_2}(\triangle) = \frac{1}{6}\sum_{j=1}^{3}\left(\delta\tilde{u}_1^{+\,2} + \delta\tilde{u}_2^{+\,2} + \delta\tilde{u}_3^{+\,2} + \delta\tilde{u}_1^{-\,2} + \delta\tilde{u}_2^{-\,2} + \delta\tilde{u}_3^{-\,2}\right)_j,$$

- Align SGS vortex axis with principal extensional eigenvector of $\tilde{S}_{ij}$

Requirements for wall-modeled LES

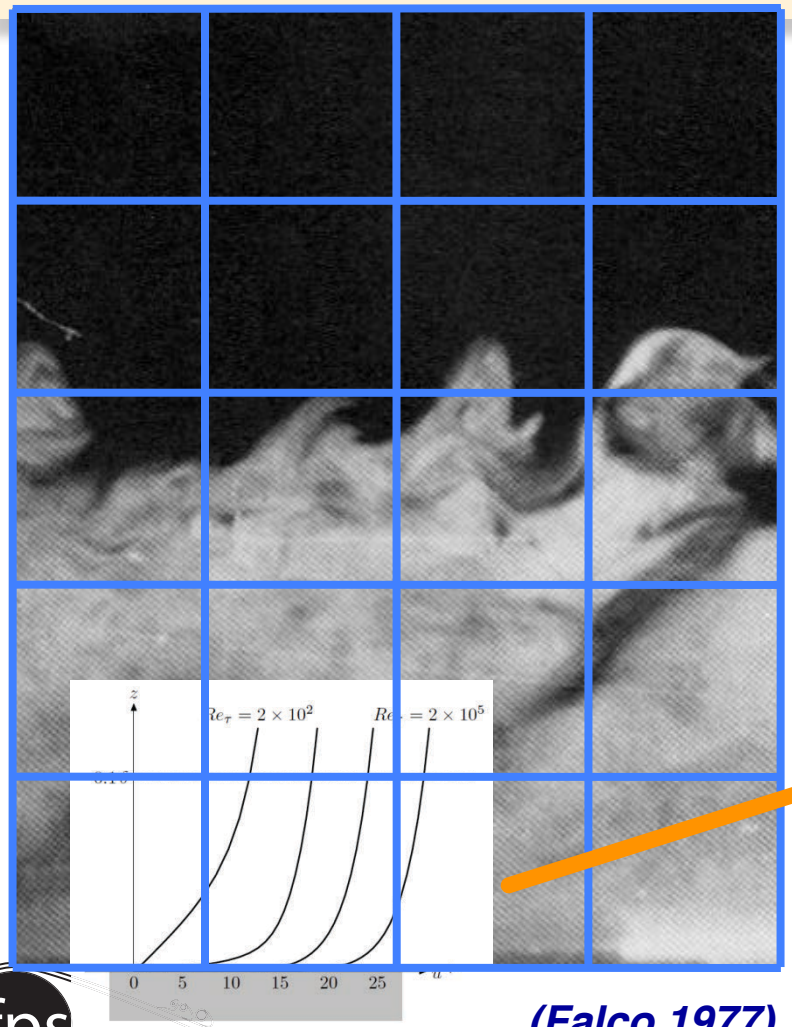
- Capture prior laminar boundary-layer
- Adverse/favorable pressure gradients
- Boundary-layer separation/reattachment
- Calculation of surface force vector : p, C_f, u_τ
- Robust: streamlined and bluff-bodied flows
- Complex boundary conditions
- ''Reasonable'' number of adjustable parameters



M. Gharib: Re = ?

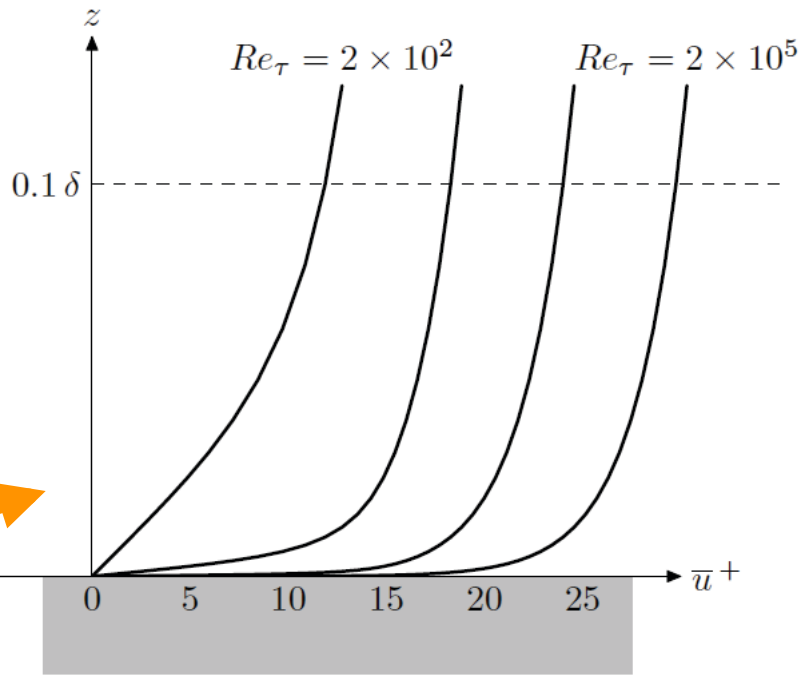
- <u>Strong LES</u>
  - Unsteady, under-resolved but the code does not crash
  - Capable of very large Reynolds number Re_τ=O(10^6);
  - Resolve Reynolds number effects; e.g. C_f(Re), drag catastrophe (sphere flow)
  - Capture laminar-turbulent transition (receptivity?)
  - Treatment of resolved/under-resolved-scale wall roughness

Mean profile near-singular near walls



(Falco 1977)

$$Re_\tau \equiv \frac{u_\tau \, \delta}{\nu}, \qquad u_\tau^2 \equiv \nu \left(\frac{\partial u_1}{\partial x_3} \right)_{wall} \equiv \nu \, \eta_0$$



Wall resolved LES: $N_x \, N_y \, N_z = O(Re^{1.8})$

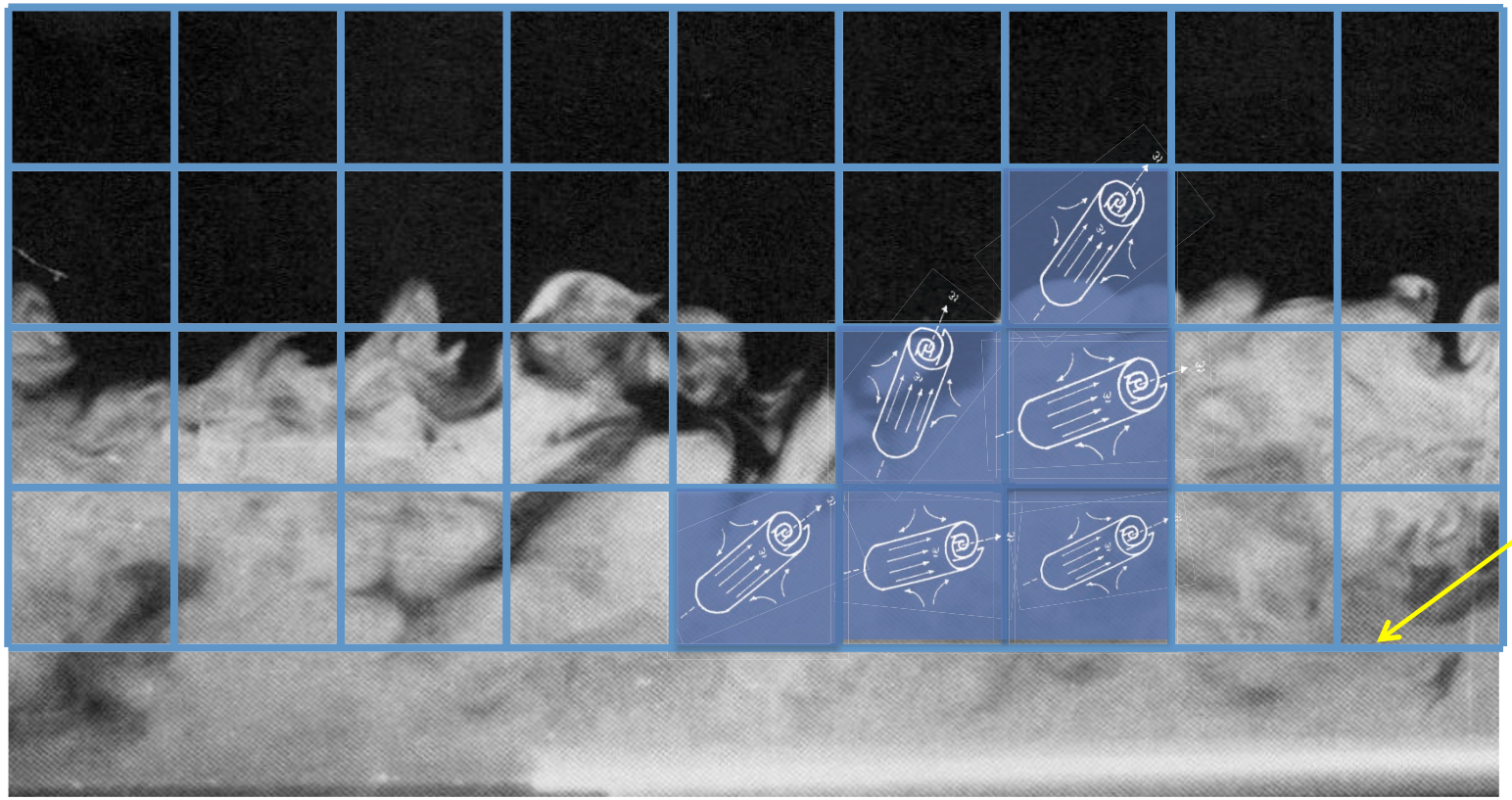
(Chapman 1979)

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LES + wall model for high Re flow

Outer flow: LES with stretched-vortex SGS model



Regular LES

Virtual wall

$h_0 \sim 0.01\delta$

Discovery through simulation

LES + wall model for high Re flow

Wall-model based on idea of attached eddies (Townsend)

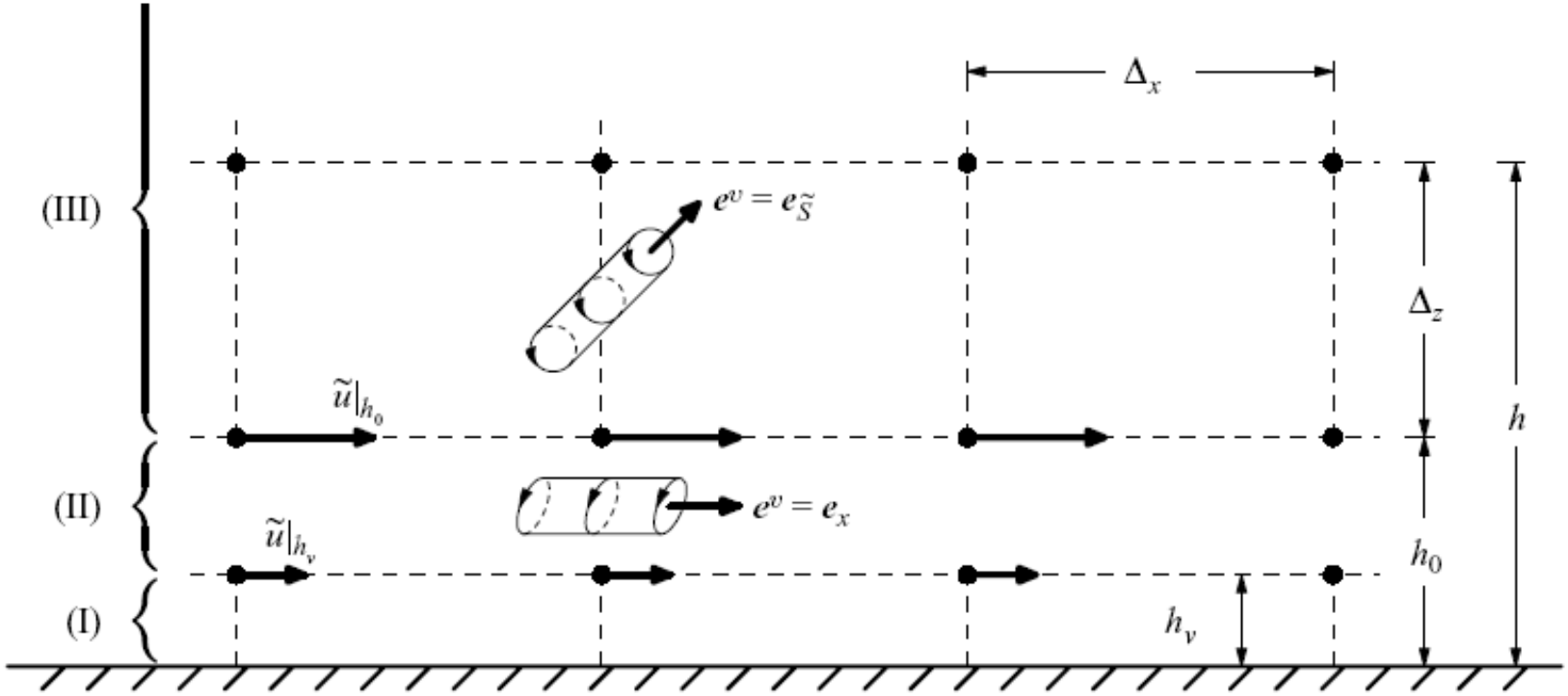


FIGURE 1. Schematic showing the near-wall set-up: h_0 locates the lifted virtual wall, where boundary conditions are applied; h locates the input plane to the wall shear stress equation, (3.10); h_v locates the outer edge of the viscous sublayer; $\boldsymbol{e}^v$ is the alignment of SGS vortices in their respective regions.

Wall Model – Essential Idea

$$u, v, w, p \longrightarrow \tilde{u}, \tilde{v}, \tilde{w}, \tilde{p} \longrightarrow \langle u \rangle, \langle v \rangle, \langle w \rangle, \langle p \rangle$$

A general wall-parallel filter

wall-normal integration filter

- Inner scaling combined with wall normal integration filter

$$\frac{\tilde{q}}{u_\tau} = F(z^+), \quad z^+ \equiv \frac{z}{l^+} = \frac{z u_\tau}{\nu}. \quad \Longrightarrow \quad \frac{\partial \langle q \rangle}{\partial t} = \frac{\tilde{q}|_h}{2\eta_0} \frac{\partial \eta_0}{\partial t}.$$

- In the original model, applying the wall parallel filter

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial \widetilde{uu}}{\partial x} + \frac{\partial \widetilde{uv}}{\partial y} + \frac{\partial \widetilde{uw}}{\partial z} = -\frac{\partial \tilde{p}}{\partial x} + \nu \frac{\partial^2 \tilde{u}}{\partial z^2},$$

- Main points to note
  - *Mean flow aligned with streamwise direction*
  - *Classical inner scaling*
  - *Near wall integration approach*

Wall Model

- ODE for wall shear stress (or u_τ) at every wall point
  - Wall-normal integration of streamwise momentum equatior $u_\tau^2 \equiv \nu\eta_0$
  - Top-hat filter normal to the wall, 0 <z < h : h = Δ z > h_0
  - Local inner-scaling reduction for unsteady term $\eta_0 \equiv \left.\frac{\partial \widetilde{u}}{\partial z}\right|_0$

$$\frac{\partial \eta_0}{\partial t} = \frac{2\eta_0}{\widetilde{u}|_h}\left[-\frac{\partial \widetilde{u}\widetilde{u}|_h}{\partial x} - \frac{\partial \widetilde{u}\widetilde{v}|_h}{\partial y} - \frac{\partial \widetilde{u}\widetilde{v}|_h}{\partial y} - \left.\frac{\partial \widetilde{p}}{\partial x}\right|_h + \frac{\nu}{h}\left(\left.\frac{\partial \widetilde{u}}{\partial z}\right|_h - \eta_0\right)\right]$$

- Attached-eddy ansatz in overlap region (Townsend, 1976)
  - Hierarchy of streamwise ``attached'' SGS vortices whose size scales with distance from wall
  - Extended stretched-vortex SGS model with attached-eddy assumption
  - SGS model gives log relationship for slip-velocity at lifted wall position z = h_0
  - ``Karman constant'' calculated dynamically

$$\widetilde{u}|_{h_0} = u_\tau\left(\frac{1}{\mathcal{K}_1}\log\left(\frac{h_0 u_\tau/\nu}{h_\nu^+}\right) + h_\nu^+\right) \qquad \mathcal{K}_1 = \frac{\gamma_{\mathrm{II}} K^{1/2}}{2\left(-T_{xz}|_{e_{\widetilde{S}}}\right)^{1/2}}$$

Chung & Pullin , JFM (2009)

Wall Model Development & Applications in LES

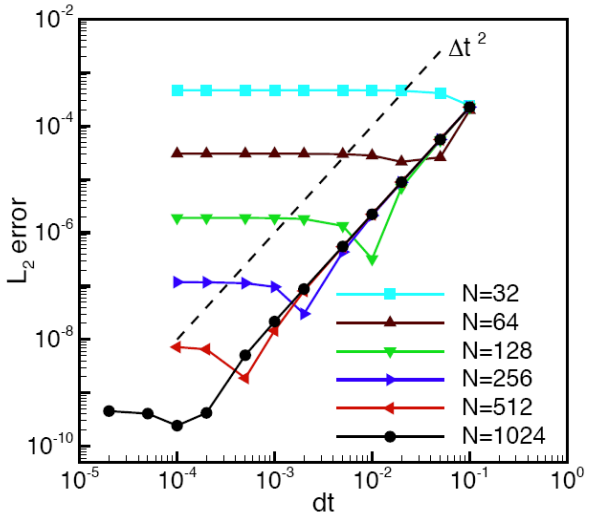
- LES of flat plate TBL: power law vs. log law
  - *Cheng & Samtaney, Phys. Fluids 2014*
- LES of separation/reattachment of flat plate TBL
  - *Cheng, Pullin, Samtaney: JFM 2015*
- LES of flow over a periodic hill array
  - *Cheng, Pullin, Samtaney*
- LES of flow past a cylinder
  - *Cheng, Pullin, Zhang, Samtaney*
- LES of flow past an airfoil
  - *Gao, Zhang, Samtaney*

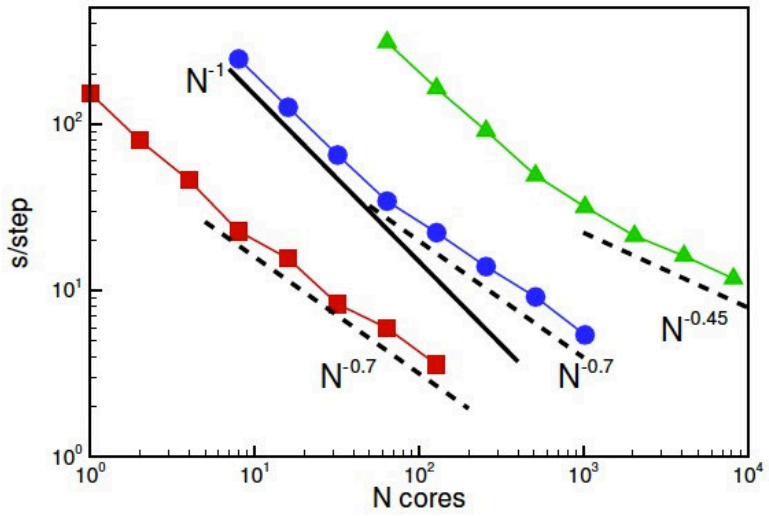
CODES

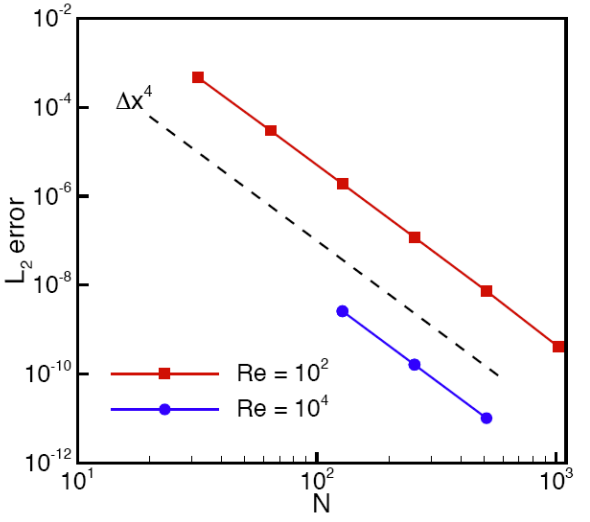
- Two In-house KAUST codes
  - DNS/LES Capability

TBL Code

- Fractional step method –second order
  - *Perot JCP 1993*
- 3rd Order RK time integration
  - *Spalart et al. JCP 1991*
- 4th Order staggered mesh finite difference
  - *Perot JCP 1993*
- Skew-symmetric convective form
- Multigrid solver for pressure Poisson equation



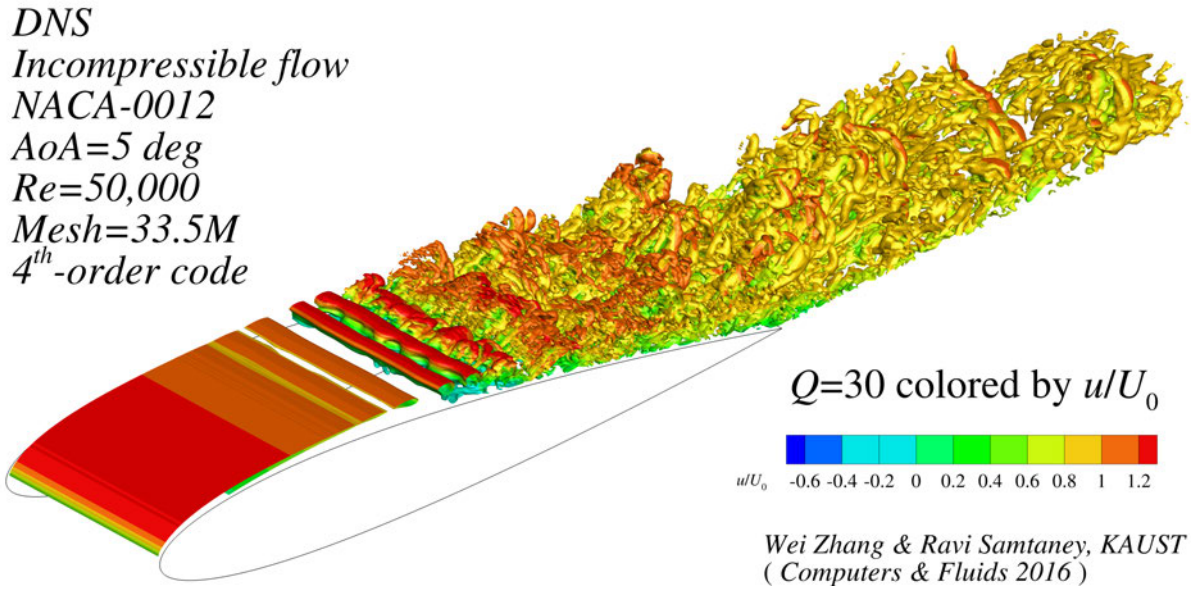




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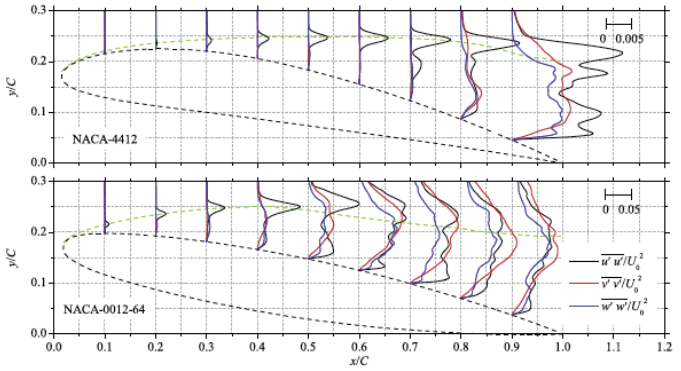
Curvilinear Coordinates Code

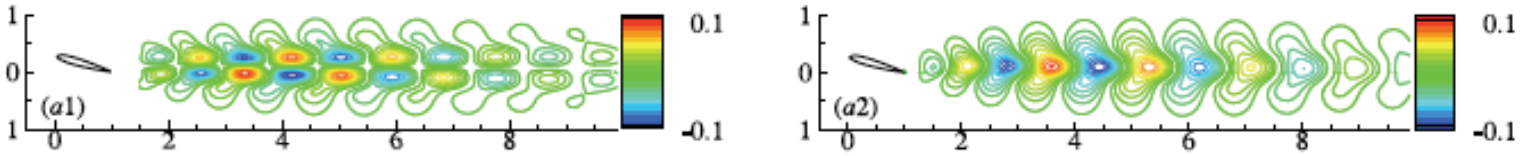
- Fractional step method in curvilinear coordinates
  - *Zang et al. JCP 1994*
- Energy conservative 4th order spatial discretization
  - *Spalart et al. JCP 1991*
- Point and line-relaxed Gauss-Seidel smoothers in geometric multigrid for pressure Poisson equation
- MPI persistent non-blocking communication

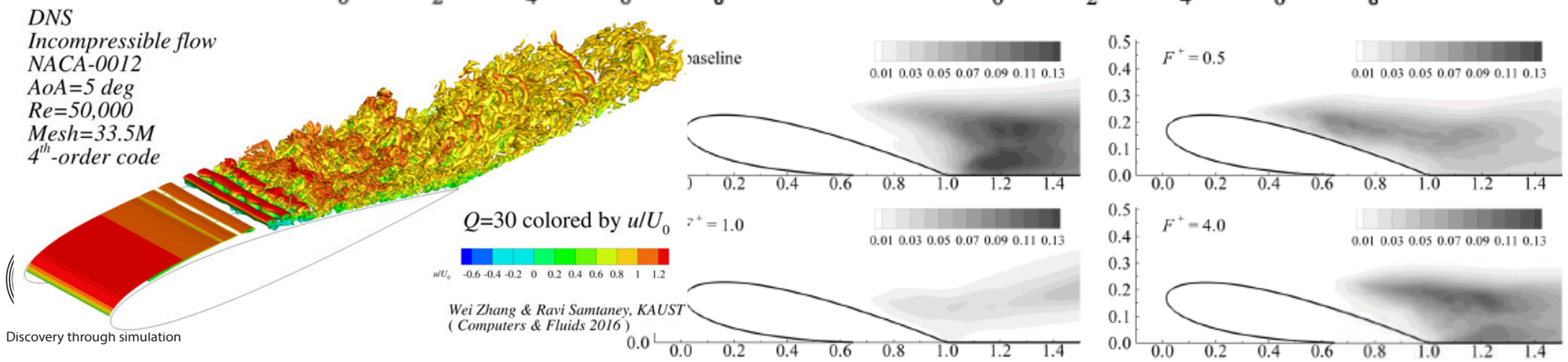


Curvilinear Coordinates Code: Examples

- Code used extensively for DNS, active control and BiGlobal stability analysis
- Geometric effects on flow past an airfoil
  - *Zhang et al. Comput. Fluids 2015*
- Effect of spanwise domain size in flow past an airfoil
- Active control of flow past an airfoil
  - *Zhang & Samtaney, Phys. Fluids 2015*
- BiGlobal linear stability analysis on flow past an airfoil
  - *Zhang & Samtaney, Phys. Fluids 2016*







Application 1

Log Law or Power Law?

Inner Wall Scaling Laws

- Question: Does the velocity profile in a wall-bounded turbulent flow obey a log-law or a power-law scaling?
  - *Controversy ongoing for over two decades*

$$U^+ = \frac{1}{k}\log z^+ + B$$

- Log law
  - *The law of the wall, Prandtl (1932)*
  - *The defect law, von Karman (1930)*
  - *Matching procedure, Millikan (1938)*

- Power Law
  - *Barenblatt, Chorin, Prostokishin (BCP Law)*
    - *JFM 1993, PNAS 1997, PNAS 2000, JFM 2000*

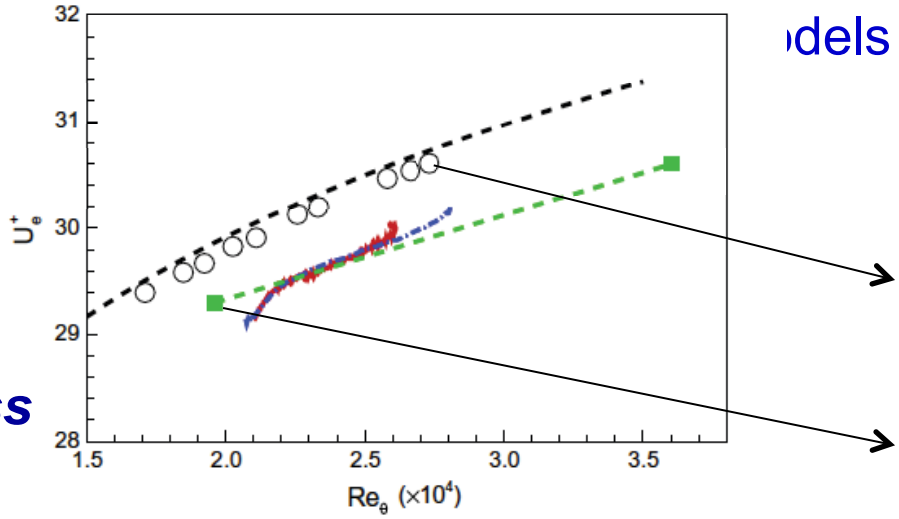
- George & Castillo (GC Law)
  - *Applied Mech Rev 1997, AIAA J. 2006*
  - *Power law for TBL*
  - *Log law for channel and pipe*

$$U^+ = C(z^+ + a)^\gamma$$

Log Law vs. Power Law

- Employed the Cartesian TBL code
- Systematically ...dels
  - *Each intrinsi...*
- Large scale ...

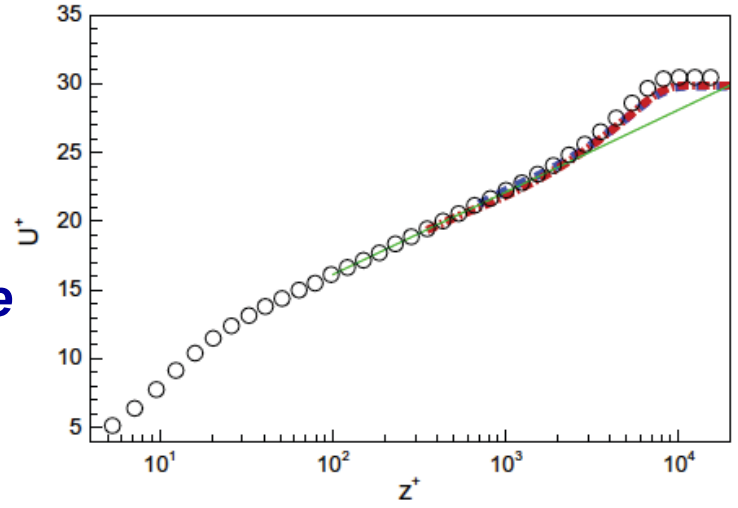
Wall shear stress



Osterlund, 1999

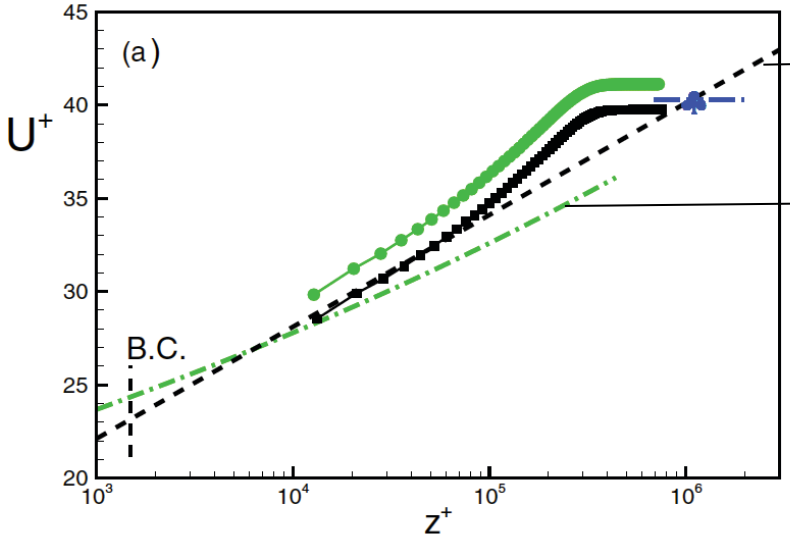
Mathis et al. 2011

Mean vel. profile



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Discovery through simulation

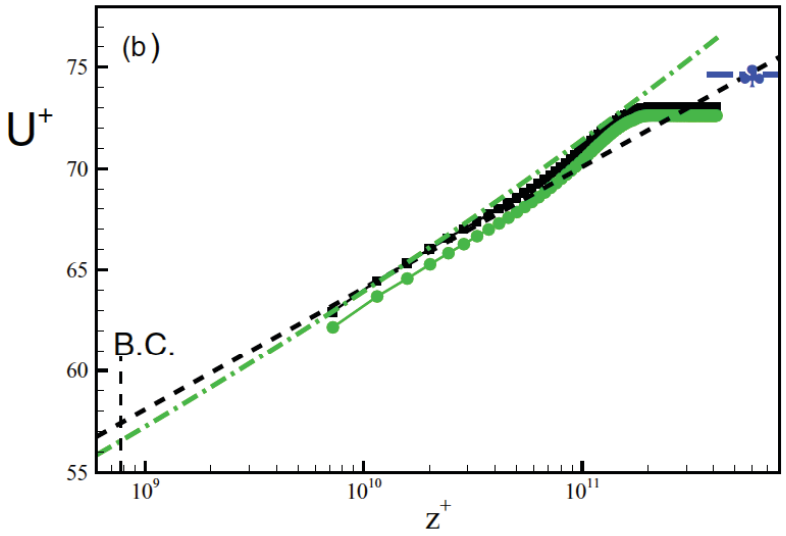
Log Law or Power Law?



Log-law fit

Power-law fit

$Re_\theta \sim 10^6$



$Re_\theta \sim 10^{11}$

Log Law or Power Law?

Relative difference between computed and theoretical velocity gradient

	Re_θ	Log-law	GC-law
LES-L	1.4×10^4	18%	20%
	1.2×10^5	14%	26%
	1.0×10^6	11%	24%
	9.2×10^6	9.6%	20%
	8.1×10^7	8.1%	15%
	7.8×10^8	7.0%	6.6%
	7.2×10^9	5.3%	5.4%
	6.9×10^{10}	4.6%	13%
	6.6×10^{11}	2.8%	20%
LES-P	1.3×10^4	12%	34%
	1.1×10^5	10%	70%
	1.0×10^6	6.8%	50%
	9.0×10^6	4.7%	23%
	8.2×10^7	3.2%	9.1%
	7.6×10^8	3.7%	1.7%
	7.3×10^9	2.5%	3.8%
	6.8×10^{10}	2.3%	13%
	6.7×10^{11}	0.63%	21%

Monotonic decrease

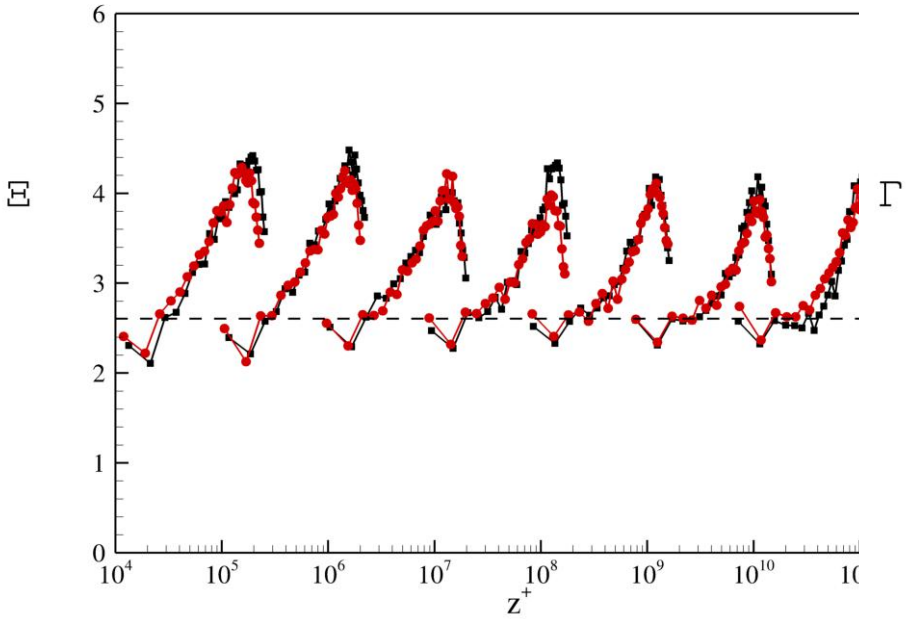
Scaled velocity gradient in overlap region

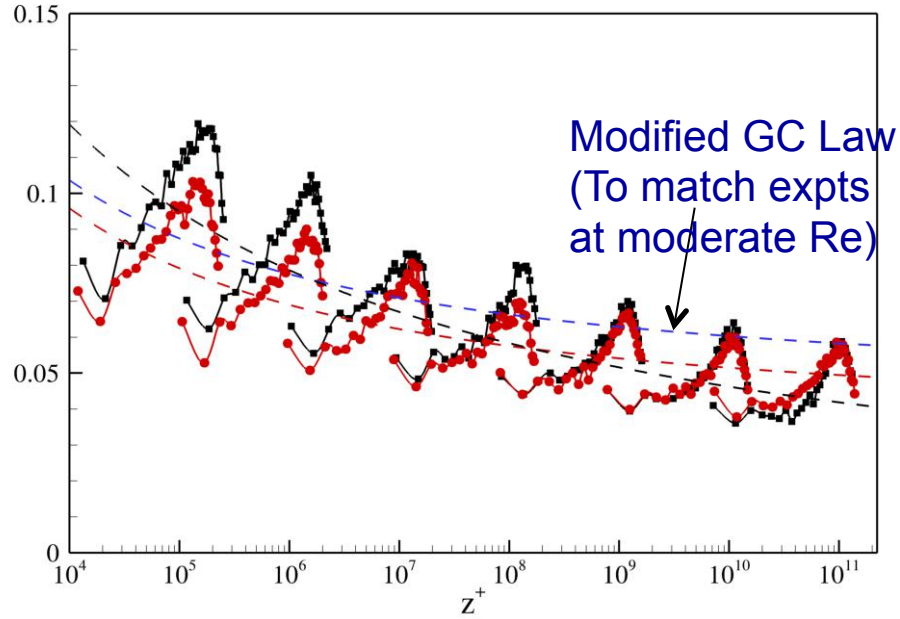
- Log law

$$\Xi = z^+ \frac{dU^+}{dz^+} = \frac{1}{\kappa}$$

- *Power law*

$$\Gamma = \frac{(z^+ + a^+)}{U^+} \frac{dU^+}{dz^+} = \gamma$$





- *For Ξ , We can still predict Karman constant with power law models.*
- *For Γ , power law is reasonable in theory.*
  Exponent γ needs modification.

Conclusion I

- Virtual wall model can be extended under the assumption of power law scaling
  - *Insignificant difference in integrated quantities (e.g. BL thickness)*
  - *Large difference for velocity profiles*

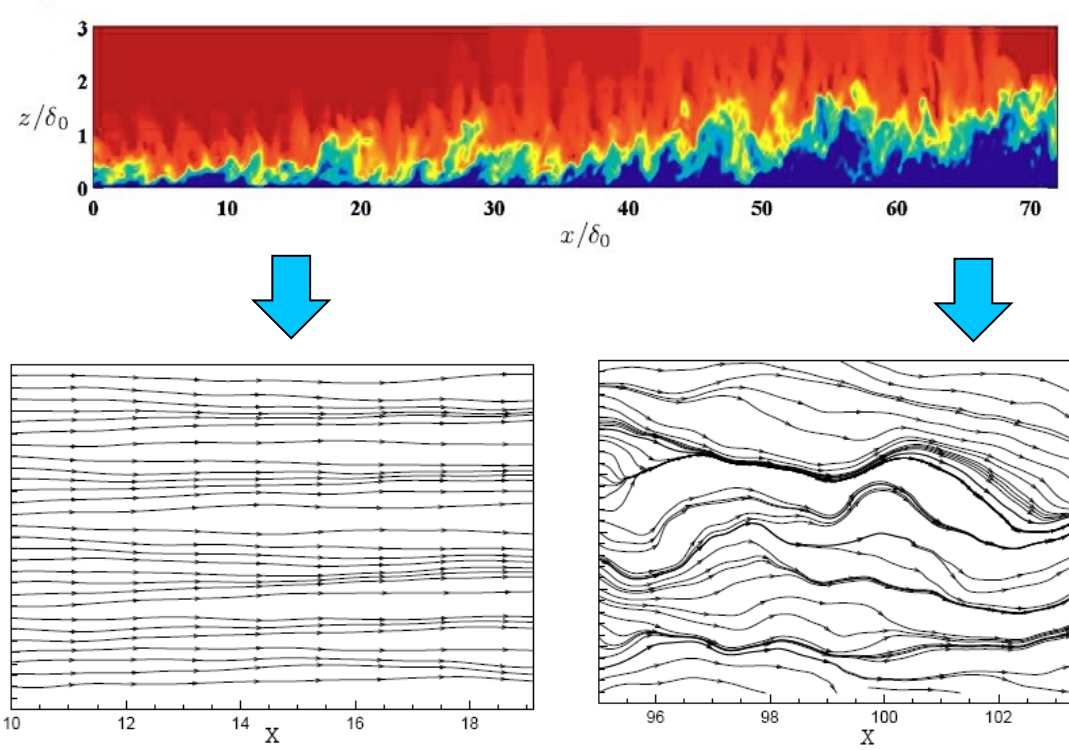
- LES perspective on GC power law vs. log law
  - *All LES results agree reasonably well with log law*
  - *Cannot recover power law results with present GC power law model*

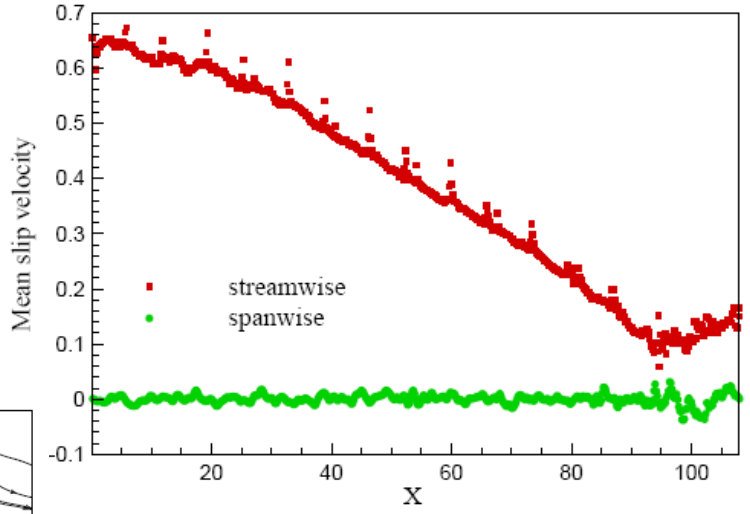
- Surprising result
  - *Under all power law assumptions, we still predict a reasonable Karman constant*
    - *Proof of log law?*

Application 2

FPTBL separation

Wall model: 2D version



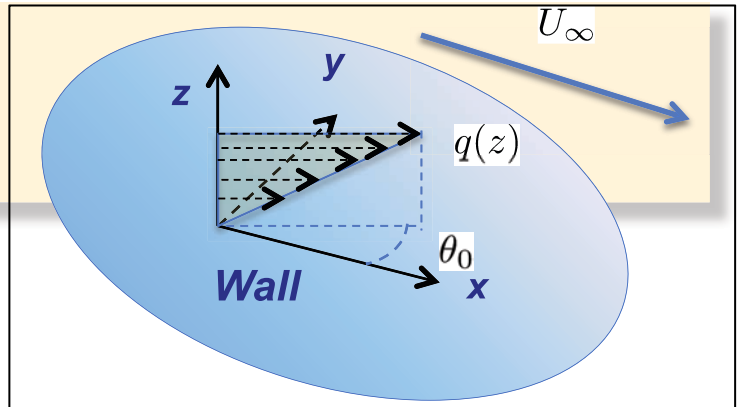


1D is not sufficient. We need 2D wall model

Discovery through simulation

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Wall model in 2D form



- ***ODEs:*** $\eta_0 = \left.\frac{\partial \tilde{q}}{\partial z}\right|_w$, θ_0 $u_\tau = \sqrt{\nu\,\eta_0}$

$$\frac{\tilde{q}|_h^2}{\eta_0}\frac{d\eta_0}{dt} - \nu\frac{\tilde{q}|_h}{h}\eta_0 = -\tilde{u}|_h\tilde{F}_x - \tilde{v}|_h\tilde{F}_y + \nu\frac{\tilde{q}|_h}{h}\left.\frac{\partial\tilde{q}}{\partial z}\right|_h$$

$$\frac{\partial\theta_0}{\partial t} = -\frac{2\sin\theta_0}{\tilde{q}|_h}\left(\tilde{F}_x + \frac{\nu}{h}\left(\left.\frac{\partial\tilde{u}}{\partial z}\right|_h\right)\right) + \frac{2\cos\theta_0}{\tilde{q}|_h}\left(\tilde{F}_y + \frac{\nu}{h}\left(\left.\frac{\partial\tilde{v}}{\partial z}\right|_h\right)\right)$$

$$\tilde{F}_x = -\frac{\partial\widetilde{uu}|_h}{\partial x} - \frac{\partial\widetilde{uv}|_h}{\partial y} - \frac{1}{h}\widetilde{uw}|_h - \left.\frac{\partial\tilde{p}}{\partial x}\right|_h, \quad \tilde{F}_y = -\frac{\partial\widetilde{uv}|_h}{\partial x} - \frac{\partial\widetilde{vv}|_h}{\partial y} - \frac{1}{h}\widetilde{vw}|_h - \left.\frac{\partial\tilde{p}}{\partial y}\right|_h.$$

$$u = q\cos\theta, \quad v = q\sin\theta$$

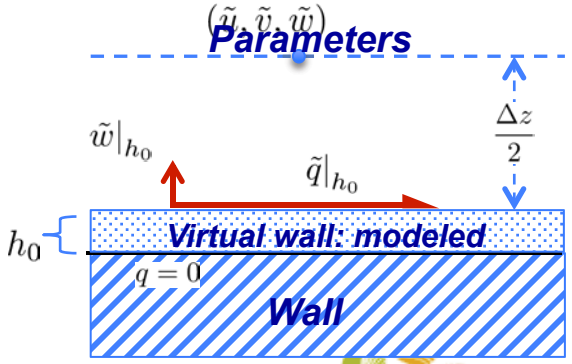
$$\eta_0 = \left.\frac{\partial q}{\partial z}\right|_w, \quad u_\tau^2 = \nu\,\eta_0$$

$$\tau_{w,x} = \rho\,\nu\,\eta_0\cos\theta_0, \quad \tau_{w,y} = \rho\,\eta_0\sin\theta_0$$

- ***Slip at virtual wall :*** $h_0^+ = \frac{h_0\,u_\tau}{\nu}$, $h_\nu^+ = 11$

$$\tilde{q}|_{h_0} = \begin{cases} \begin{cases} u_\tau\left(\frac{1}{\kappa_1}\log\left(\frac{h_0^+}{h_\nu^+}\right) + h_\nu^+\right), & h_0^+ > h_\nu^+, \\ u_\tau h_0^+, & h_0^+ < h_\nu^+, \end{cases} & \tau_{w,x} > 0, \\ u_\tau\,h_0^+, & \tau_{w,x} \le 0. \end{cases}$$

Note: q, η_0 and u_τ are always positive.



$$\tilde{w}|_{h_0} = -\frac{\tilde{q}|_h\,h}{2\eta_0}\eta_{0,n}, \qquad \eta_{0,n} = \frac{\partial\eta_0}{\partial x}\cos\theta + \frac{\partial\eta_0}{\partial y}\sin\theta$$

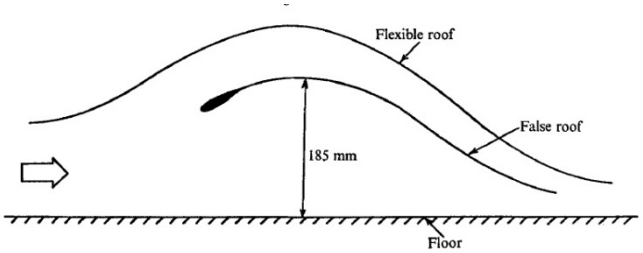
(continuity)

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Discovery through simulation
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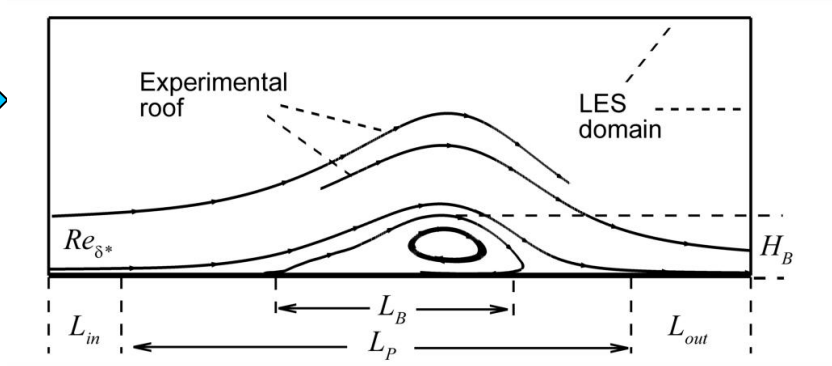
Turbulent separation/reattachment bubble

- DNS:
  - *Na & Moin (1998), Skote & Henningson (2002)*
- Experiments
  - *Perry & Fairlie (1975), Patrick (1987), …, Lögdberg (2006)*

Experiment



LES

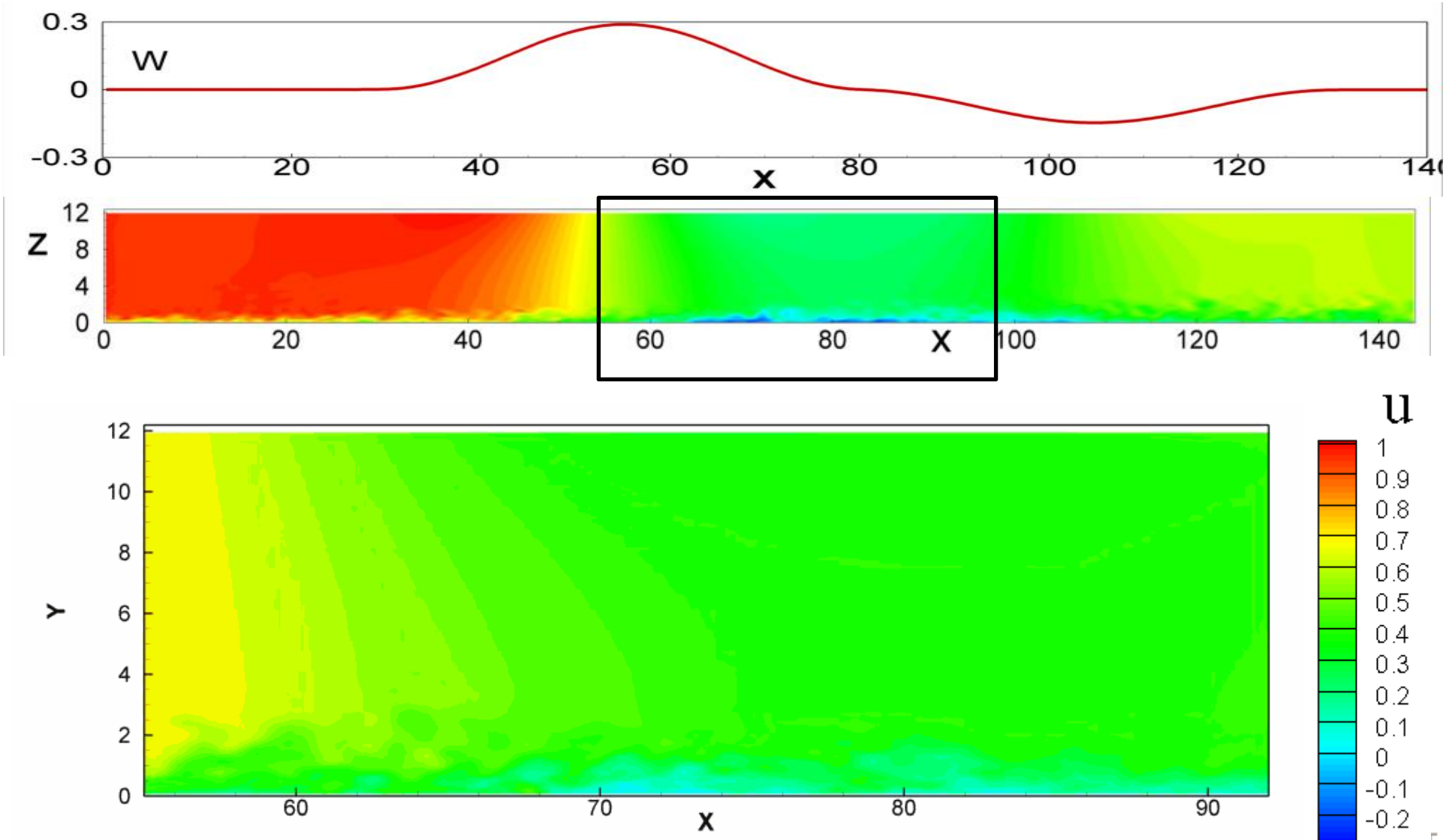


- ***Presently:***
  - ***Use parallelepiped (rectangular domain)***
  - ***Replace upper wall by a stream-wise distribution of wall-normal velocity***
  - ***Match wall pressure distribution to experiments***

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Discovery through simulation

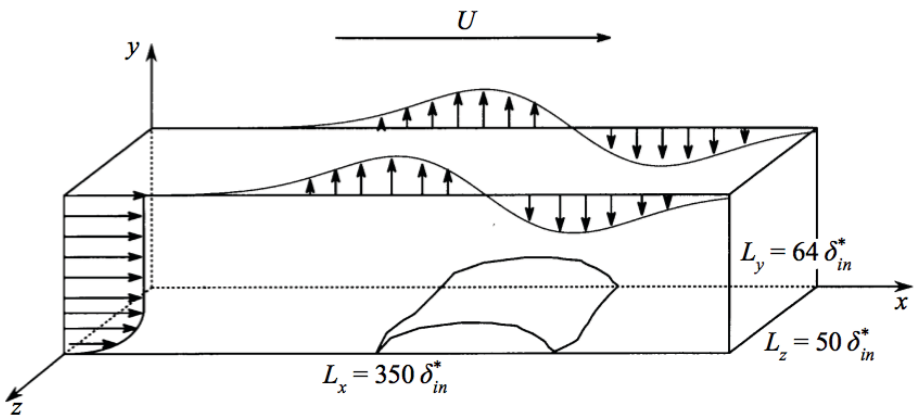
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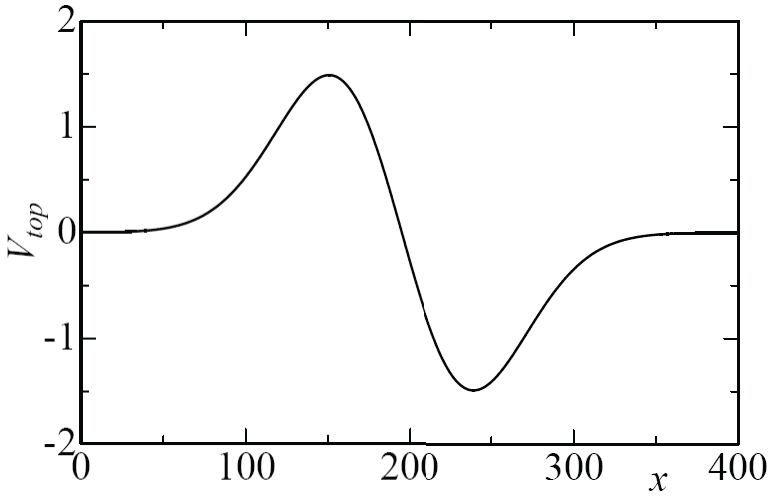
Case 0: Instantaneous Streamwise Velocity

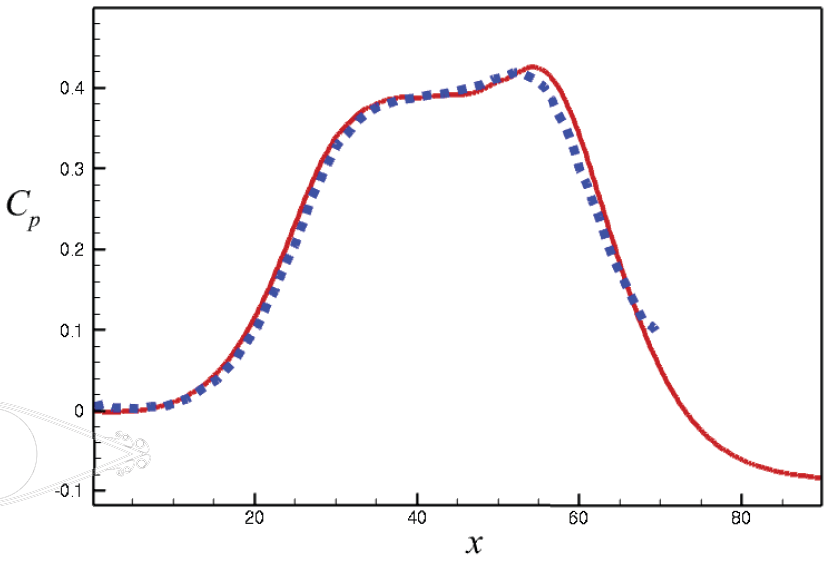


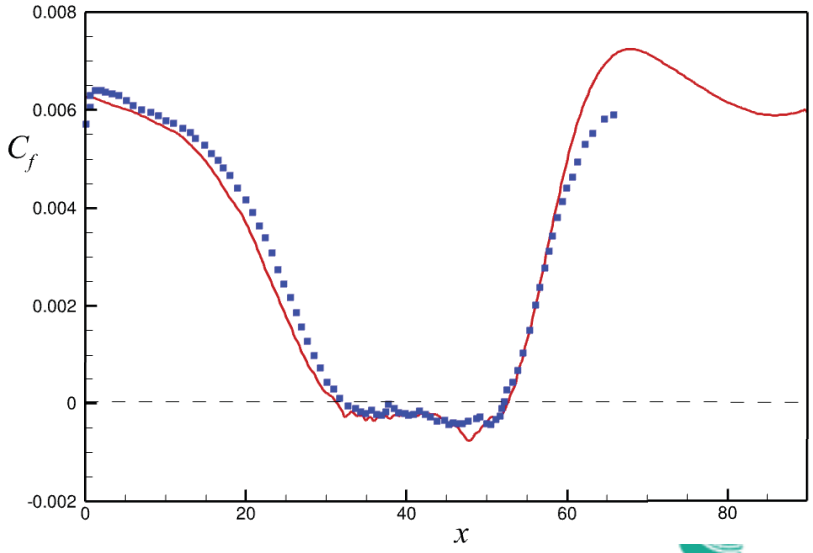
Case 0: Comparison with DNS by Na & Moin

Re_θ=300





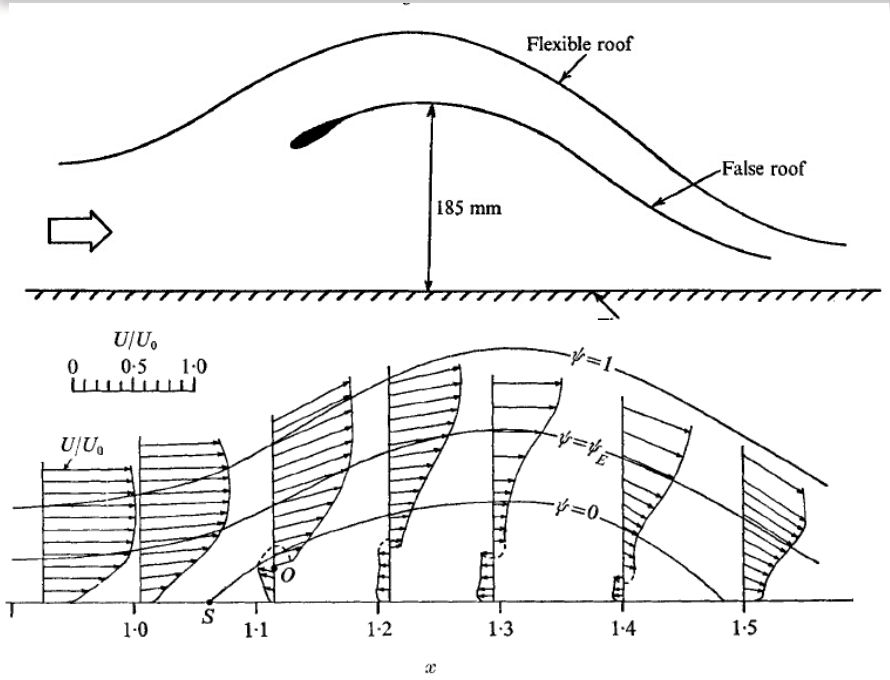


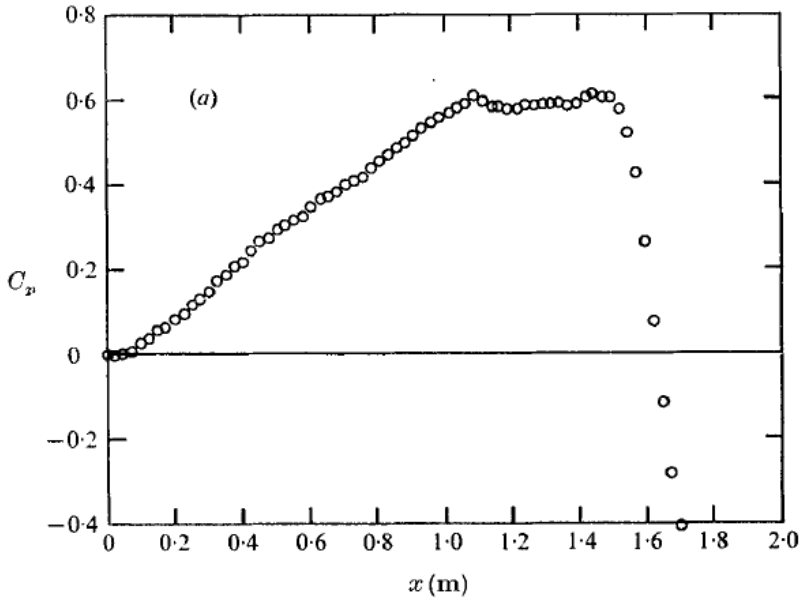


Discove

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Case 1 : Perry & Fairlie (1975) $Re_\theta=2000$



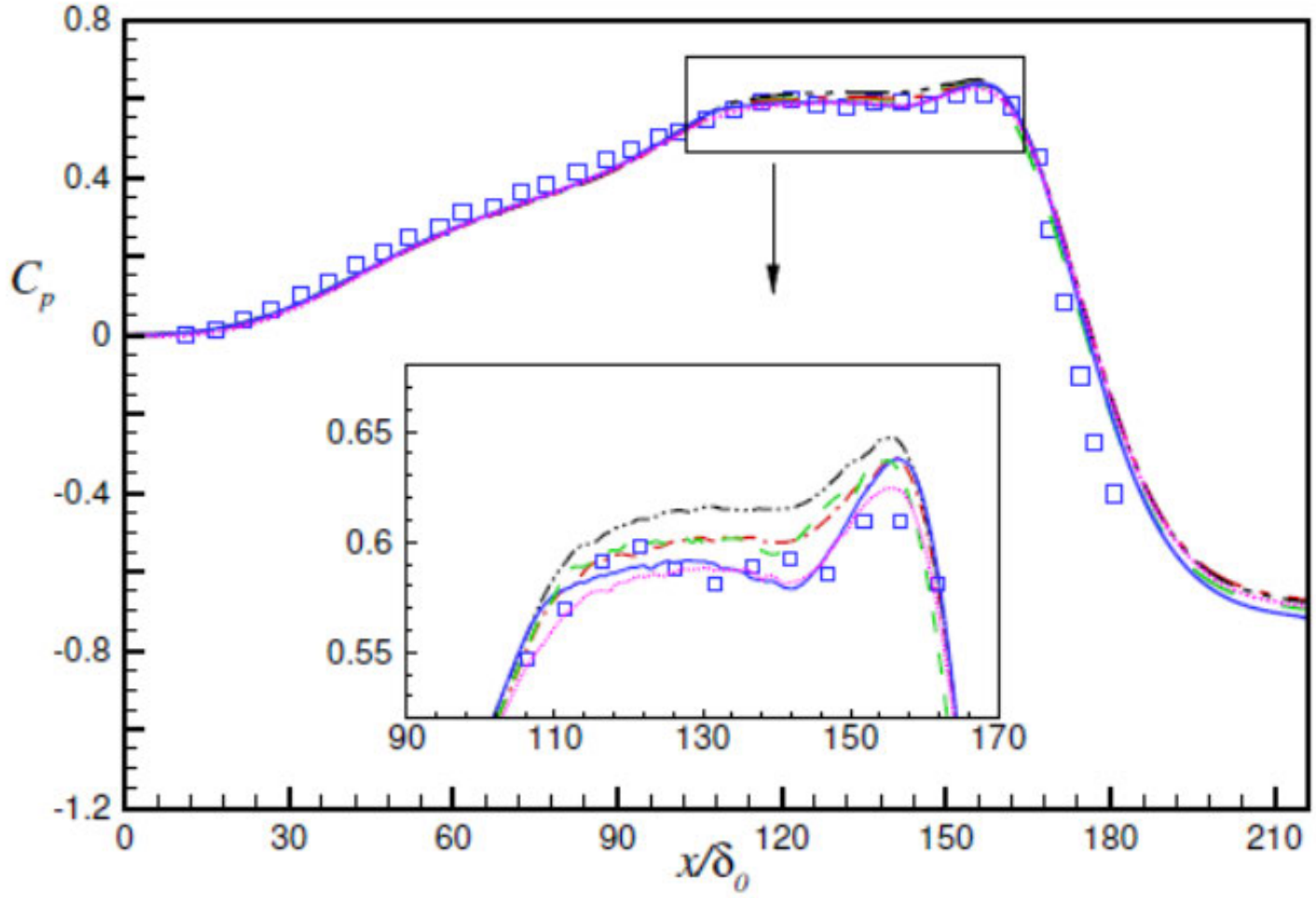


x (m)	C_f'†	π_c	H	δ^* (mm)	Δ (mm)	ξ
Case 1. Reynolds number = $1{\cdot}25 \times 10^6$ m^{-1}						
0·150	0·0034	0·55	1·44	2·30	—	—
0·300	0·0029	0·95	1·47	3·47	18·8	0·53
0·450	0·0025	1·25	1·51	4·67	23·1	0·58
0·600	0·0021	1·70	1·57	6·55	29·8	0·63
0·750	0·0017	2·40	1·65	9·01	38·0	0·68
0·900	0·0011	4·15	1·85	13·43	48·0	0·80

Figures from Perry & Fairlie, 1975, JFM

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Discovery through simulation
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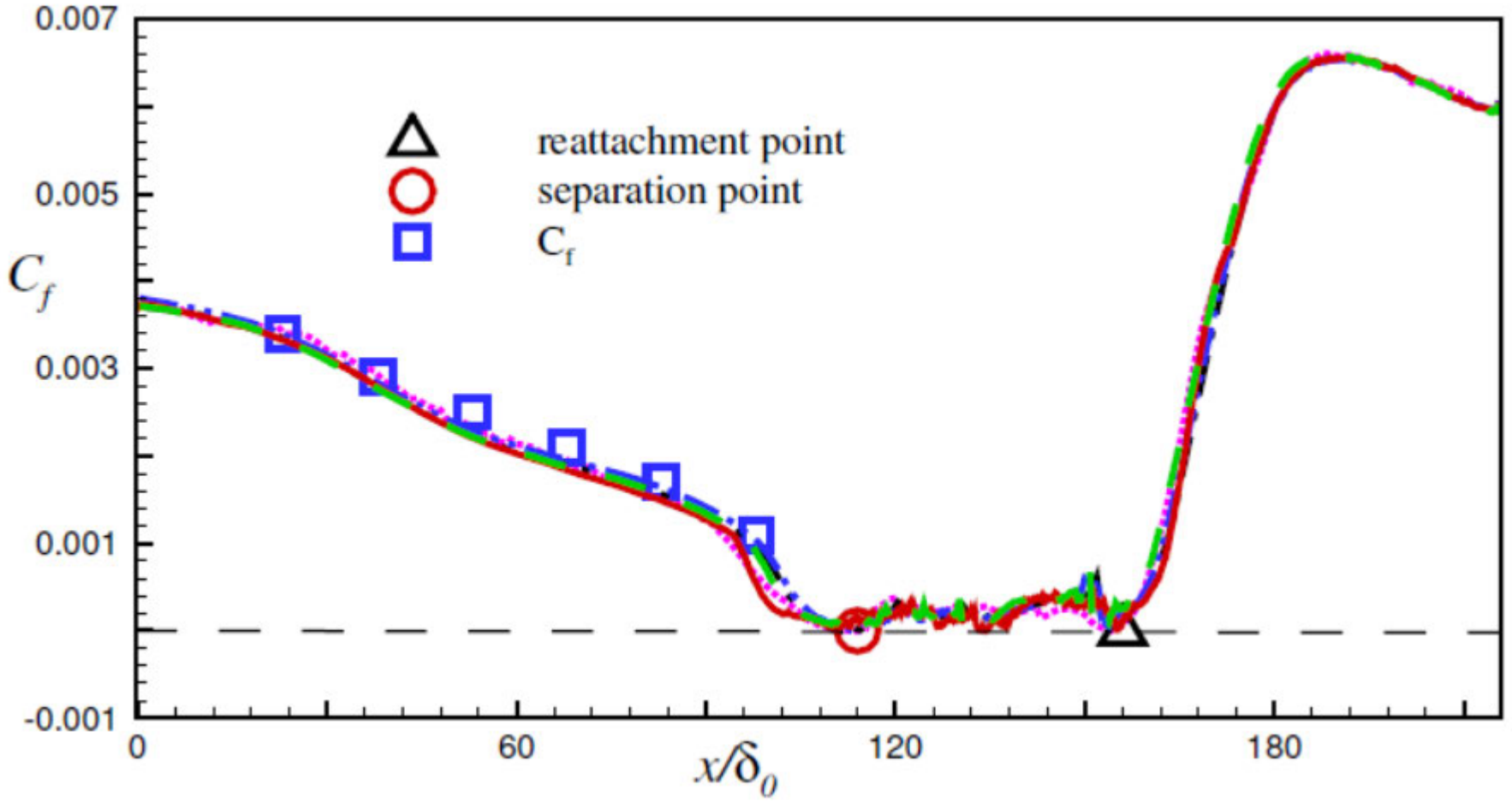
Case 1 : Wall Pressure Coefficient



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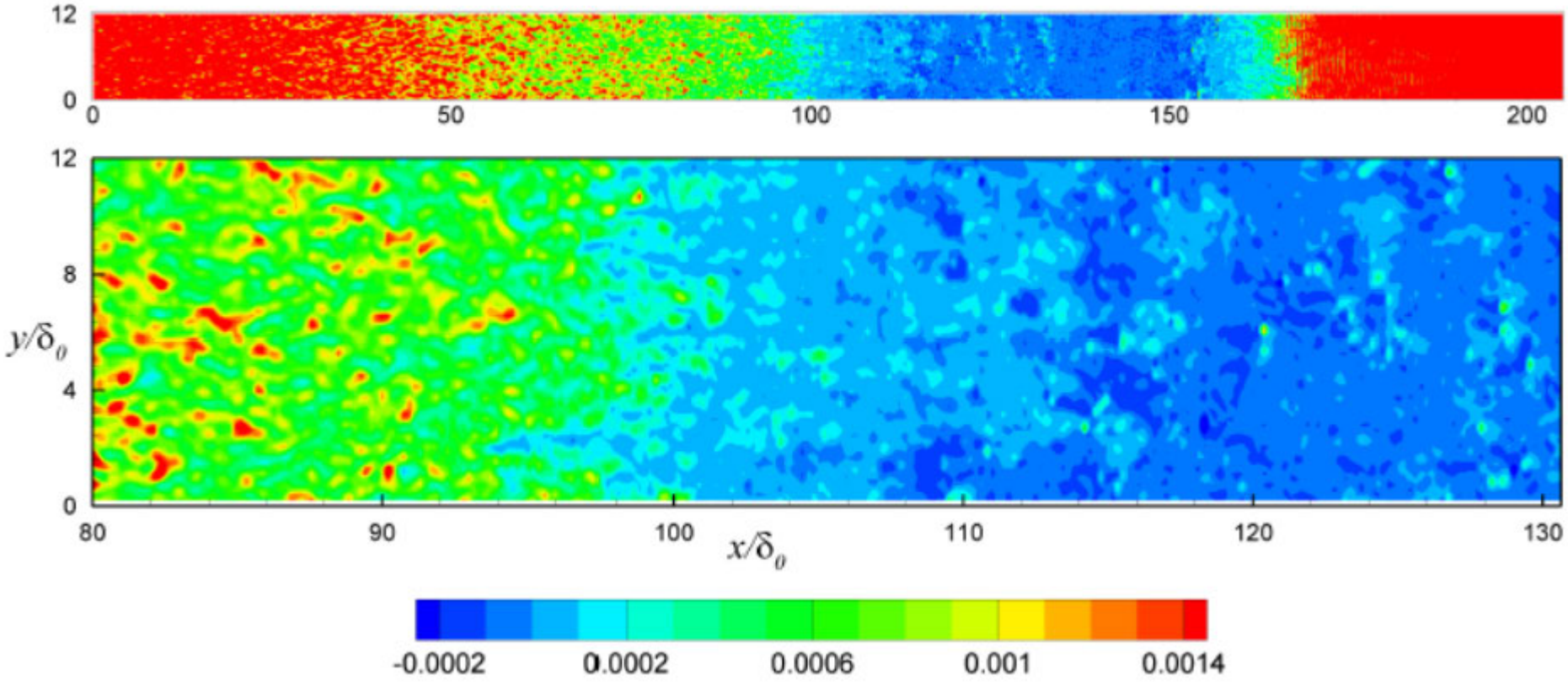
Case 1 : Wall Skin Friction Coefficient



Discovery through simulation

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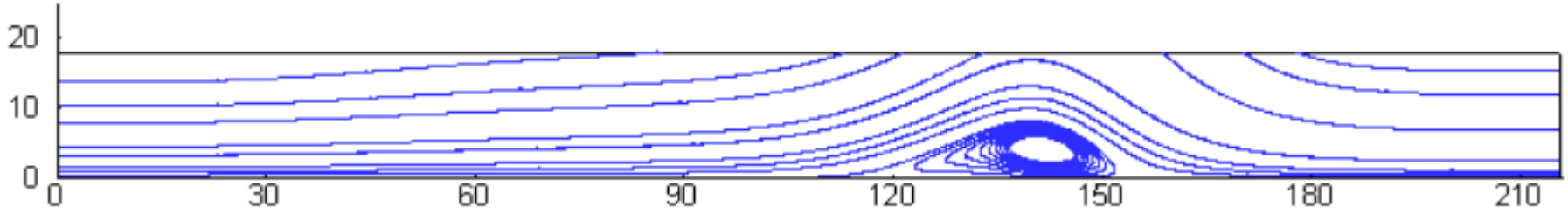
Case 1 : Surface Shear Stress

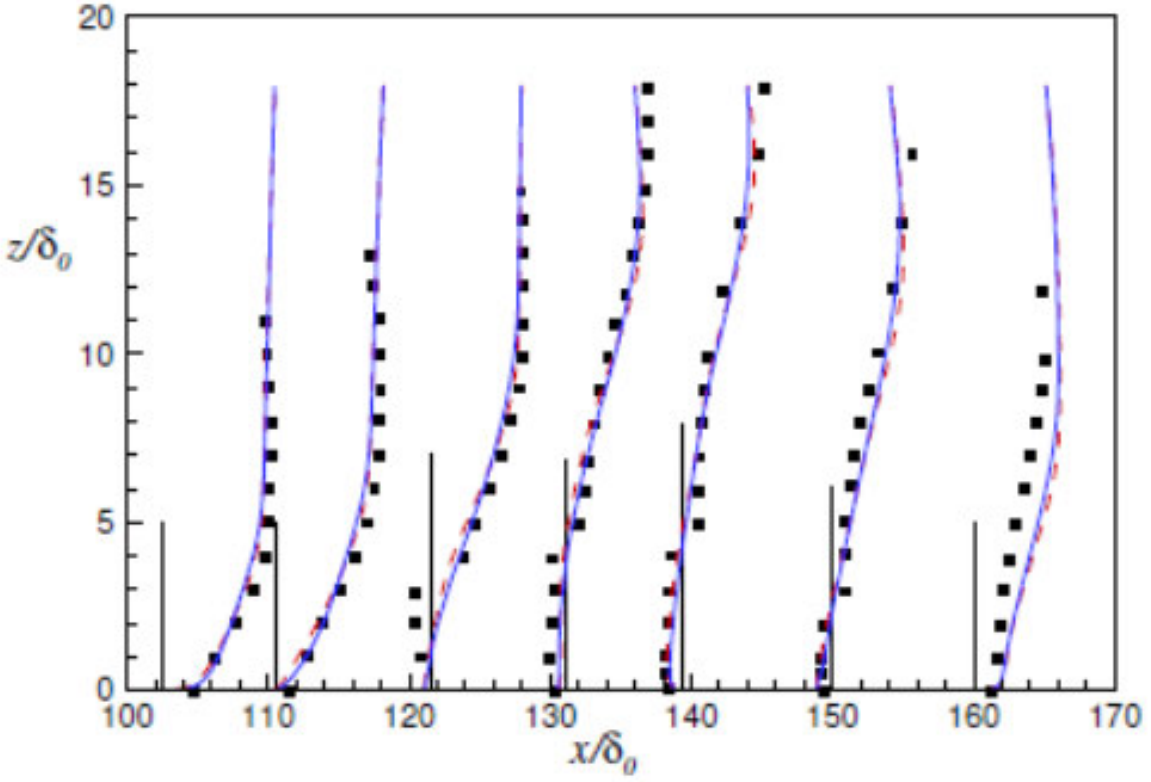


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Case 1 : Velocity Comparison With Experiment

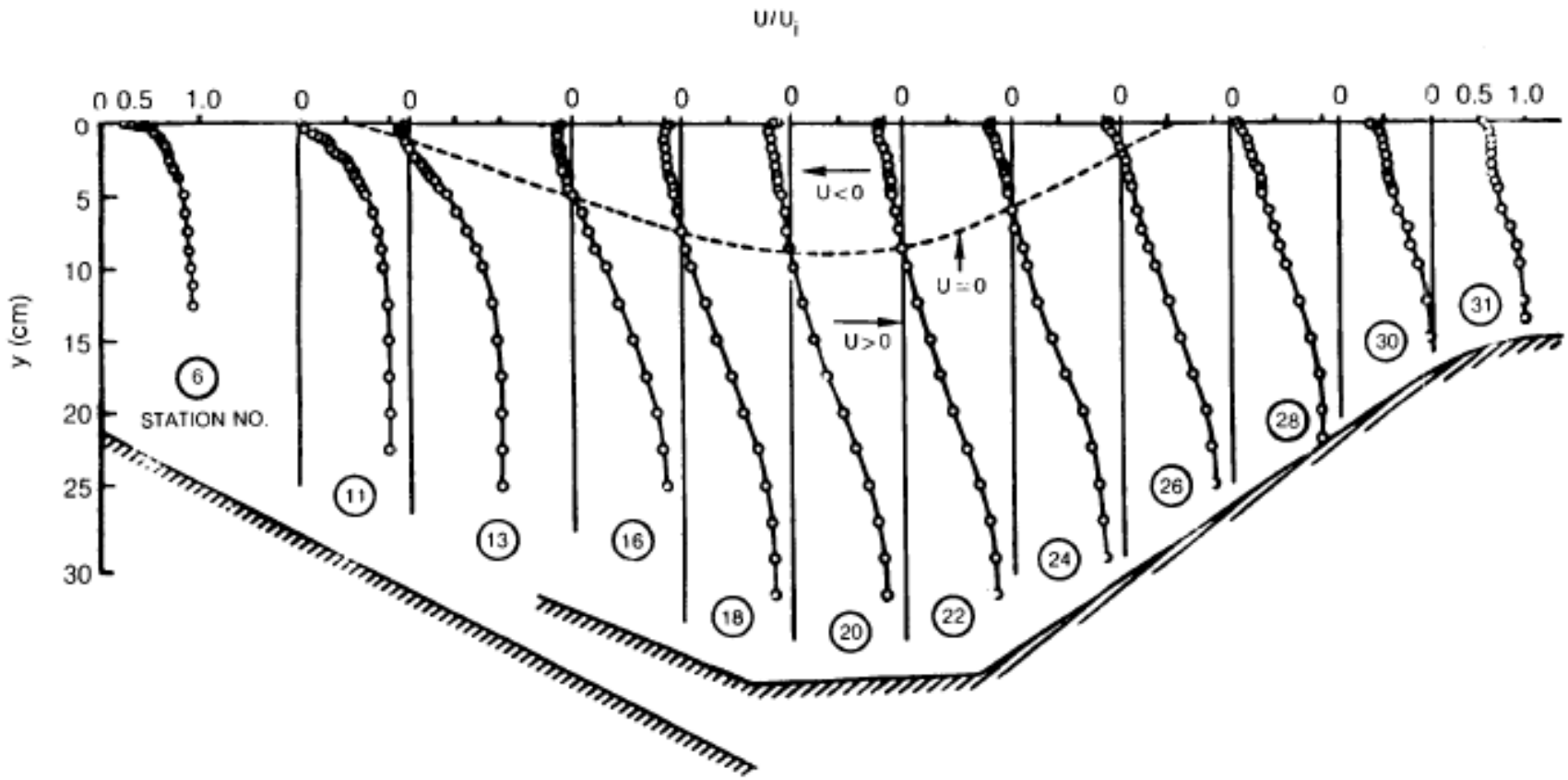




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Case 2 : Patrick(1987)

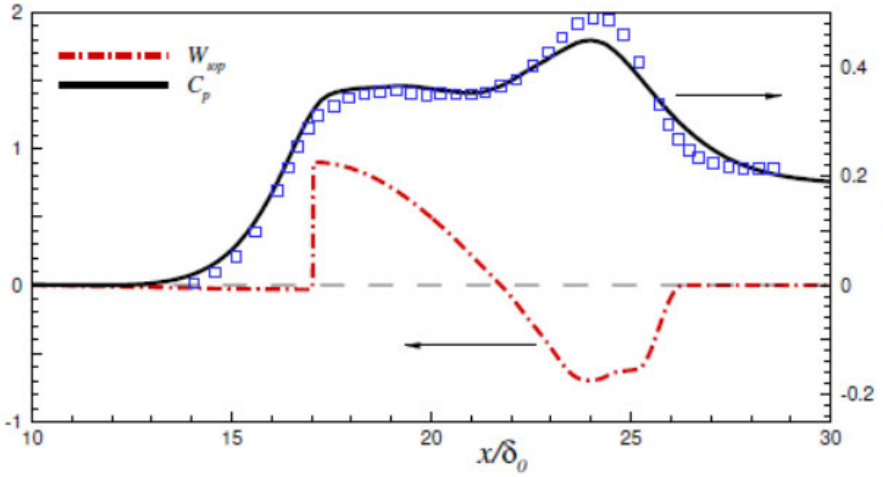


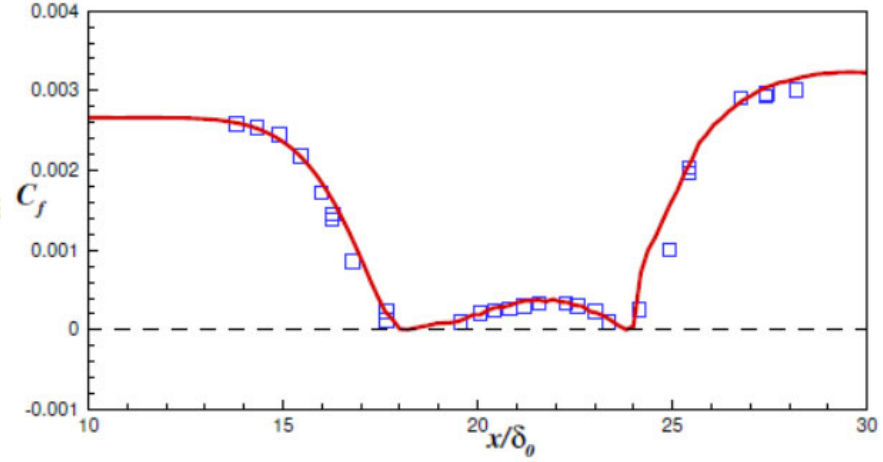
$Re_\theta = 11000$

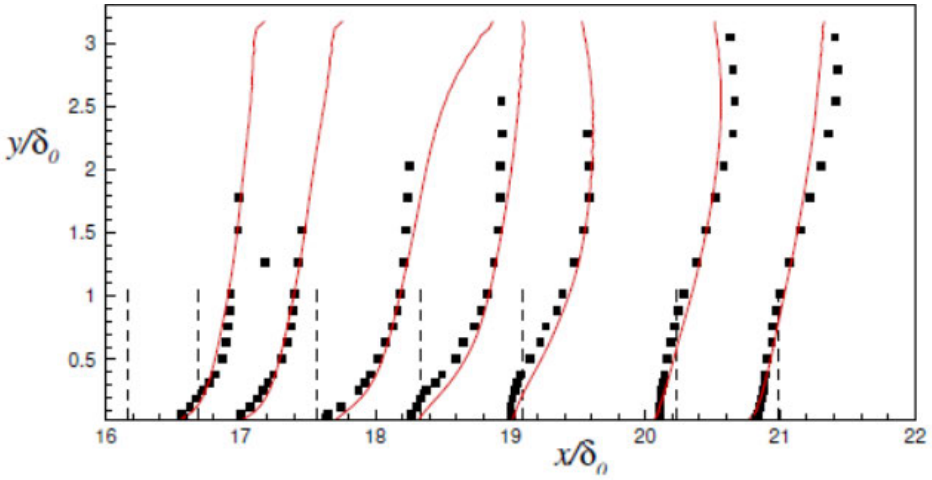
Discovery through simulation

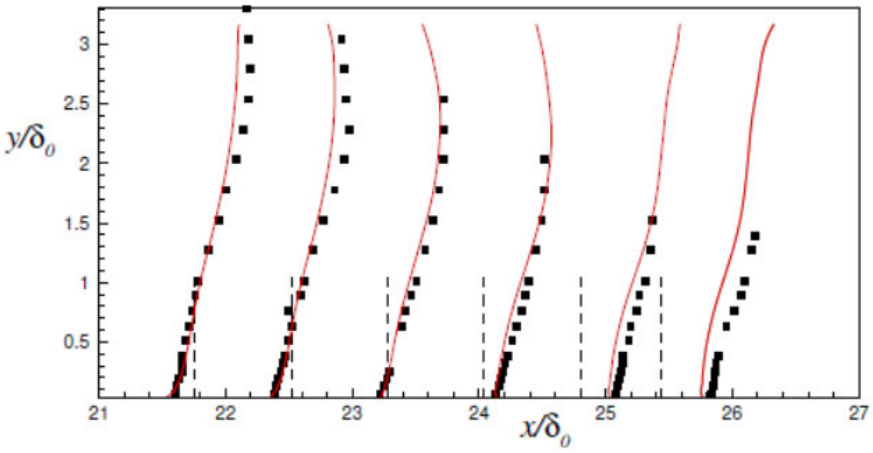
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Case 2 : Skin Friction Coefficient and Velocity









Domain: $35\delta_0 \times 12\delta_0 \times 3.2\delta_0$

Mesh: $384 \times 64 \times 112$

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Discovery through simulation

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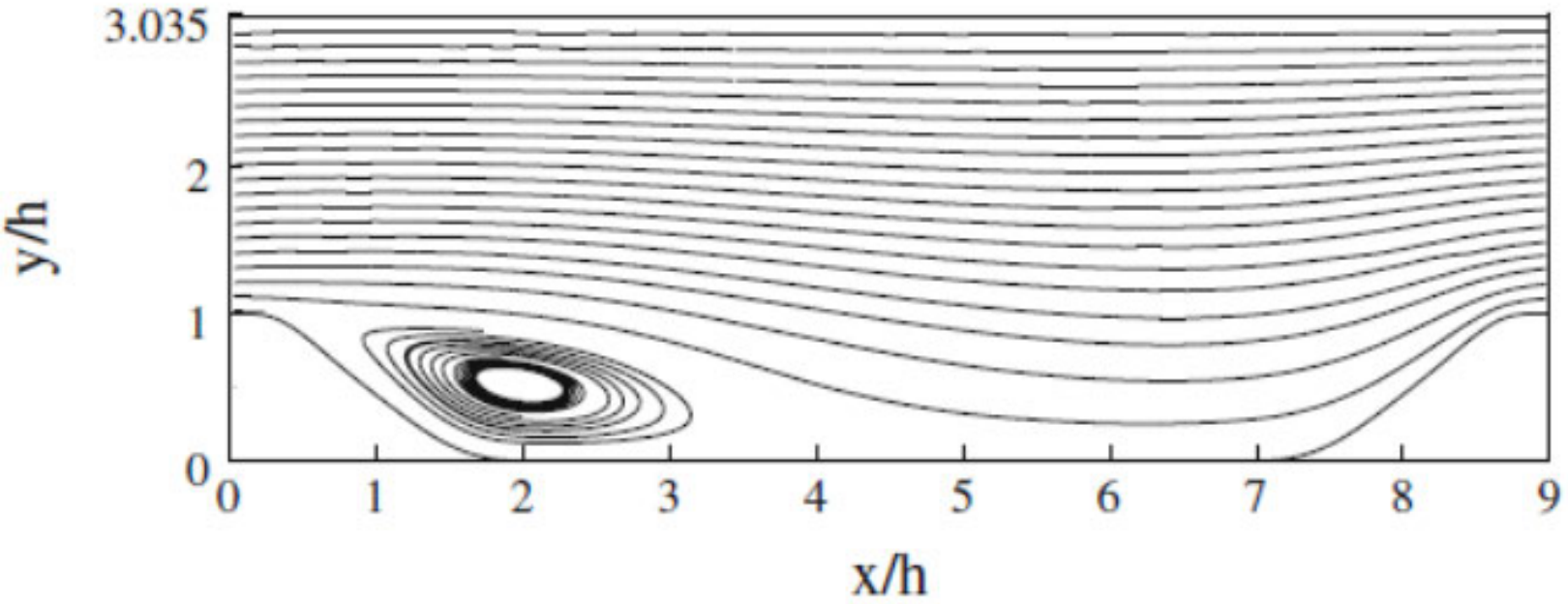
Conclusion II

- 2D wall model
  - *Allows back flow at the wall*
  - *Enables modeling of 2D surface wall shear stress field*
- Applied to two flat plate cases
  - *Perry & Fairlie JFM 1975*
  - *Patrick 1987*
  - *Good match of pressure and skin friction coefficients*

Application 3

Periodic Hill

Introduction to Periodic Hill



- *Experiments: Rapp & Manhart 2011*
- *Numerous LES and RANS simulations*

- *NASA Turbulence modeling resource: 2-D Periodic Hill*
- *ERCOFTAC UFR3-30*

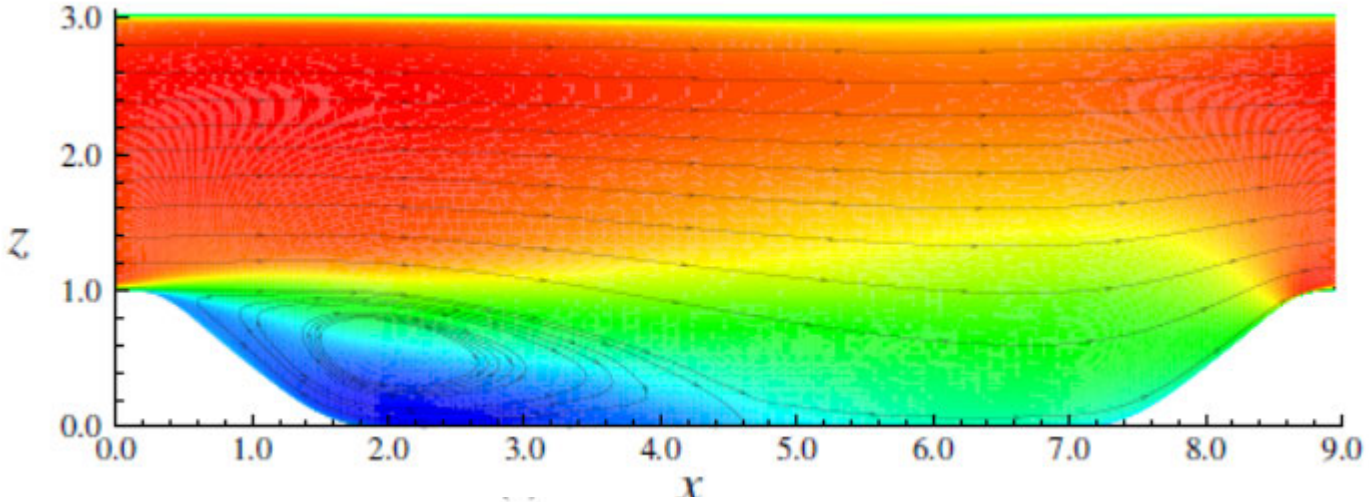
fps
Discovery through simulation

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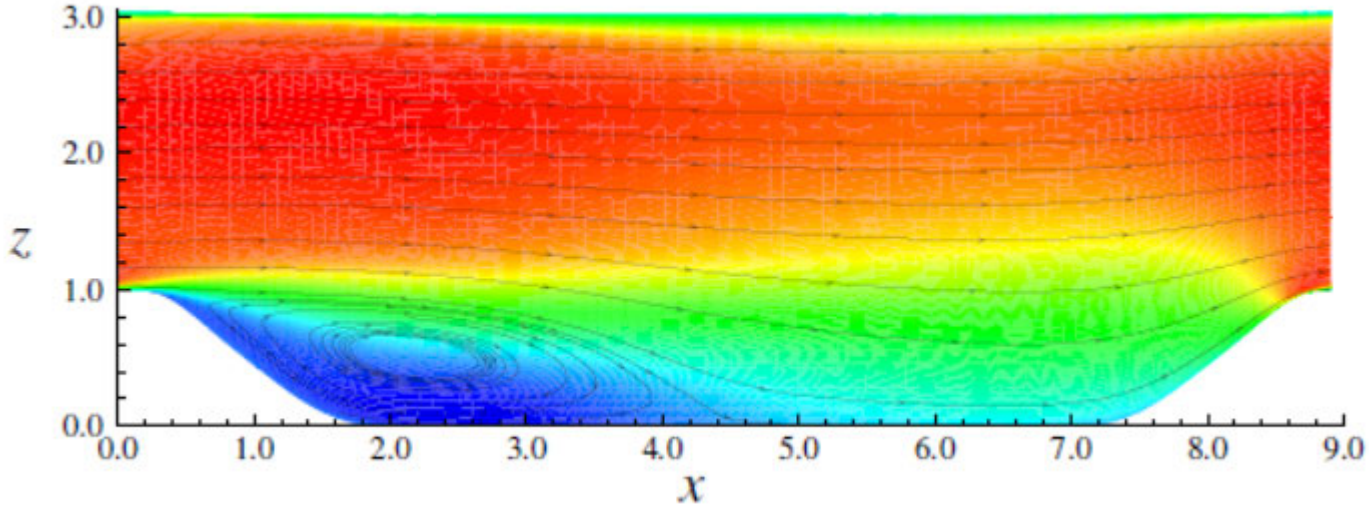
Hill case : Streamlines

- $Re_H = 1000$

- *Wall resolved LES*

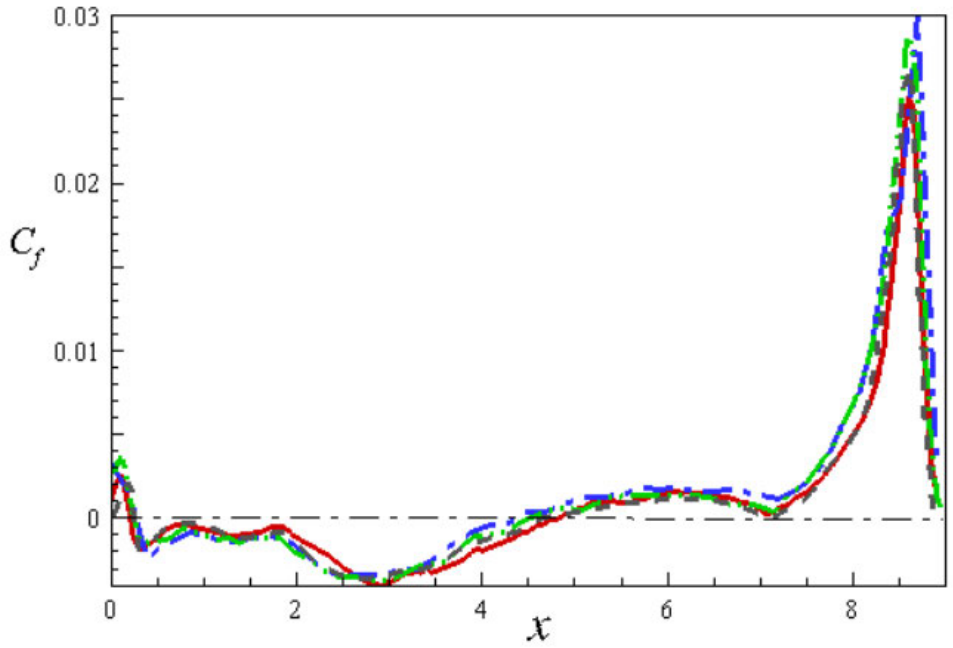


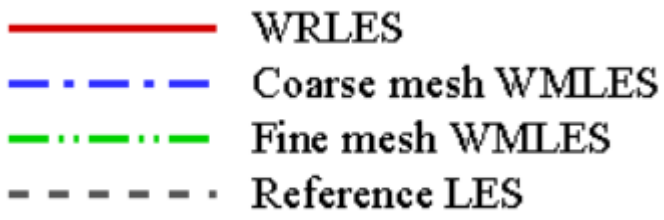
- *Wall modeled LES*



fps
Discovery through simulation

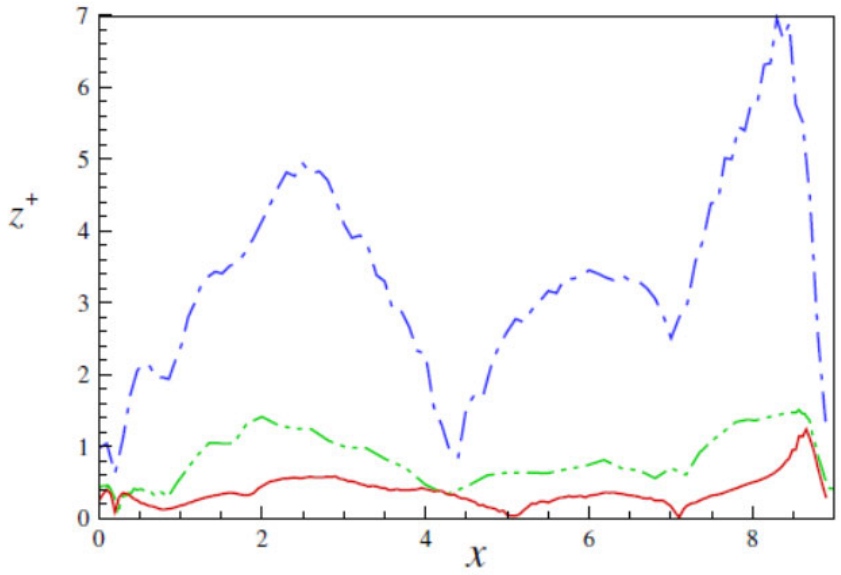
Hill case : Skin Friction Coefficient



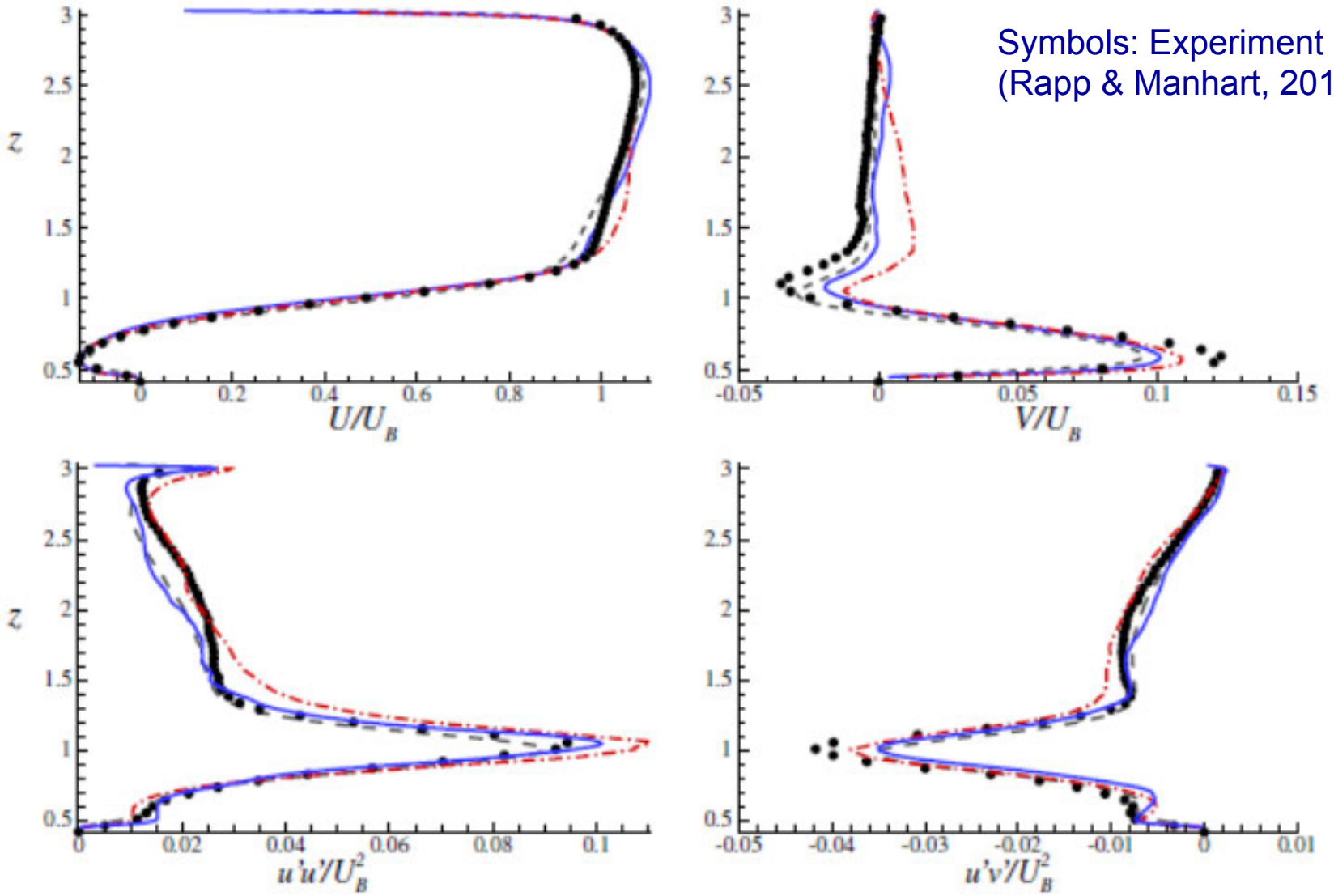


- *Frohlich et al, JFM, 2005*

Case	N_ξ	N_y	N_ζ	ζ^+	Type
H0	192	96	192	0.32	WR
H1	96	48	64	0.75	WM
H2	96	48	64	3.46	WM



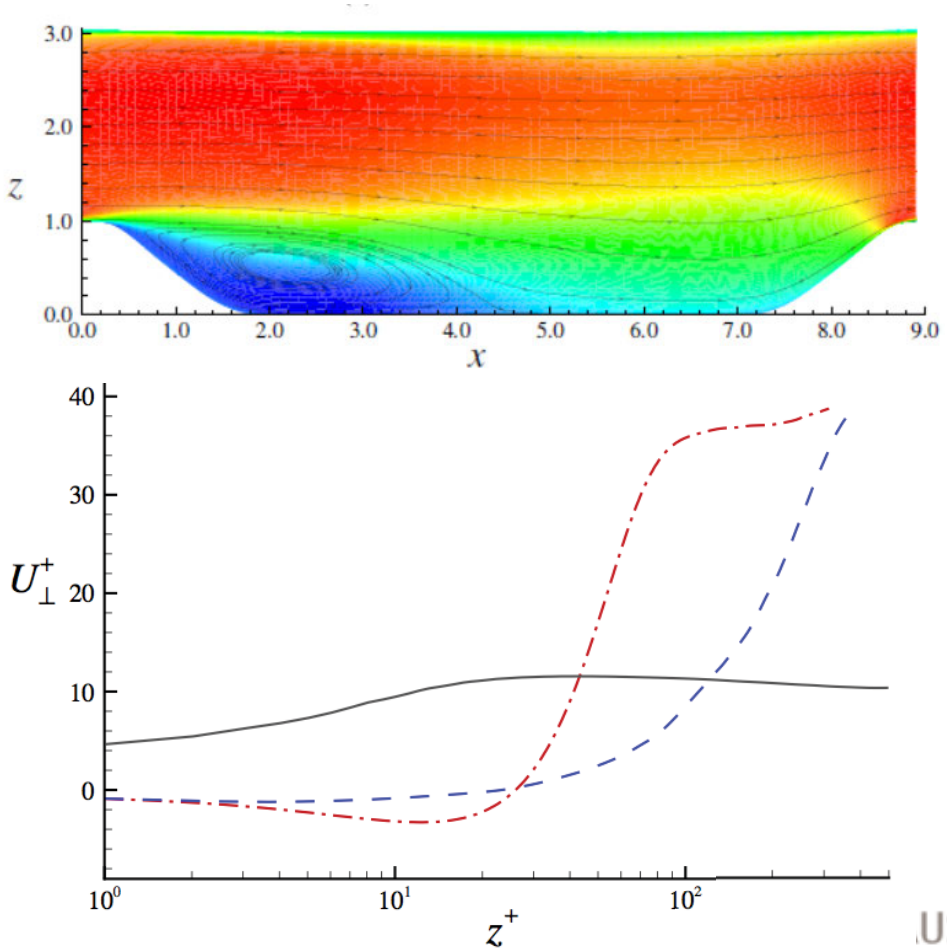
Hill case : Comparison at x=1.0



Symbols: Experiment
(Rapp & Manhart, 2011)

Conclusion III

- 2D wall model applied to curvilinear coordinates
  - *Some approximations had to be made in the ODE and slip velocity BC at the virtual wall*
- Applied to periodic hill case
  - *Re_H = 10595*



fps
Discovery through simulation

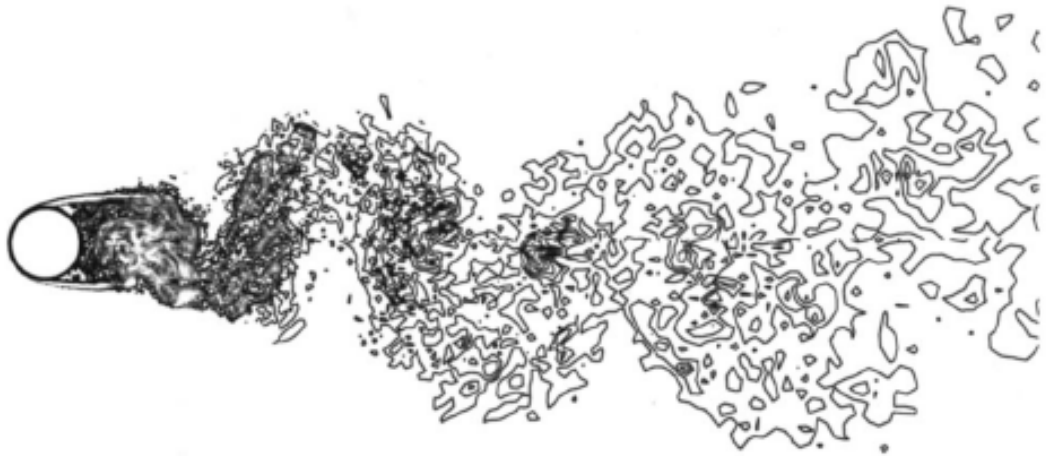
UST

Application 4

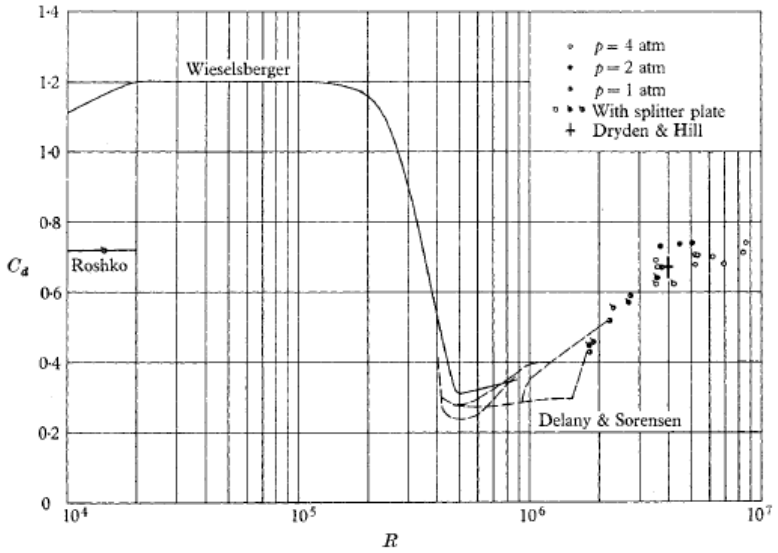
Flow over Cylinder

WORK IN PROGRESS

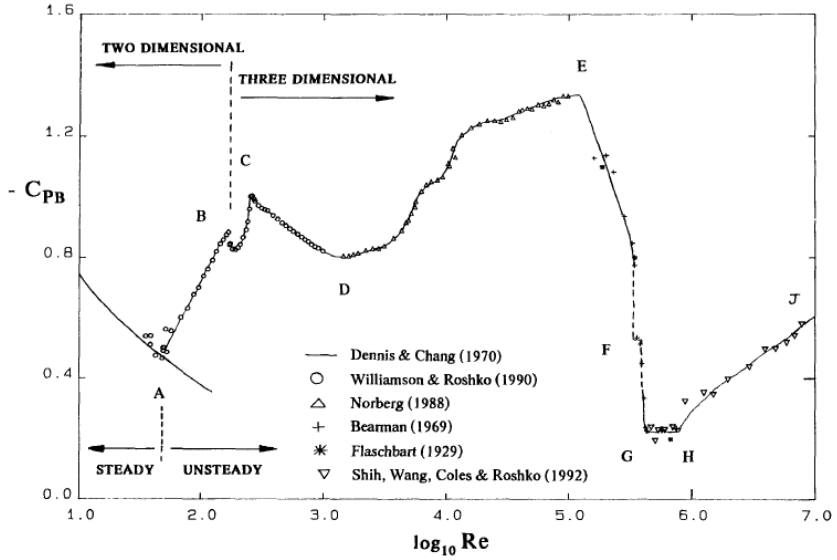
Cylinder Flow



- *Kravchenko & Moin, PoF, 2000*



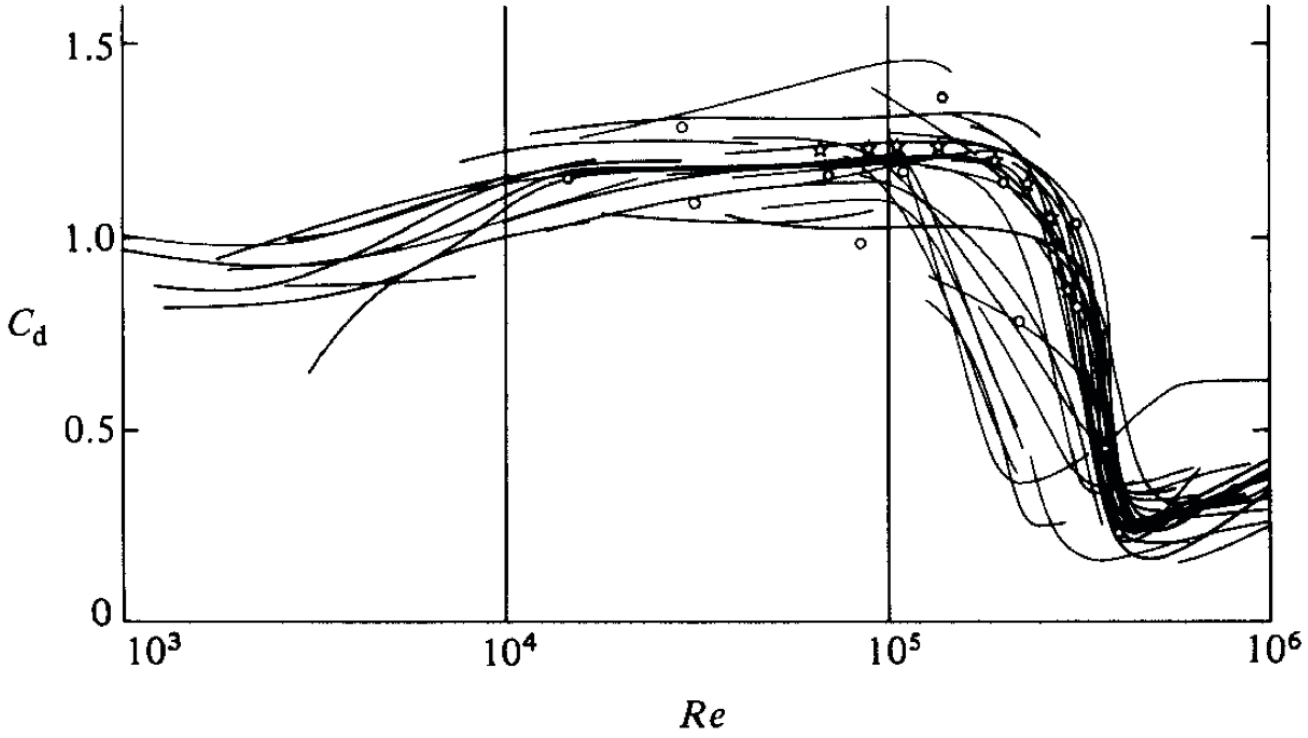
- *Roshko, JFM, 1961*



- *Roshko, JWEIA, 1993*

Drag coefficient

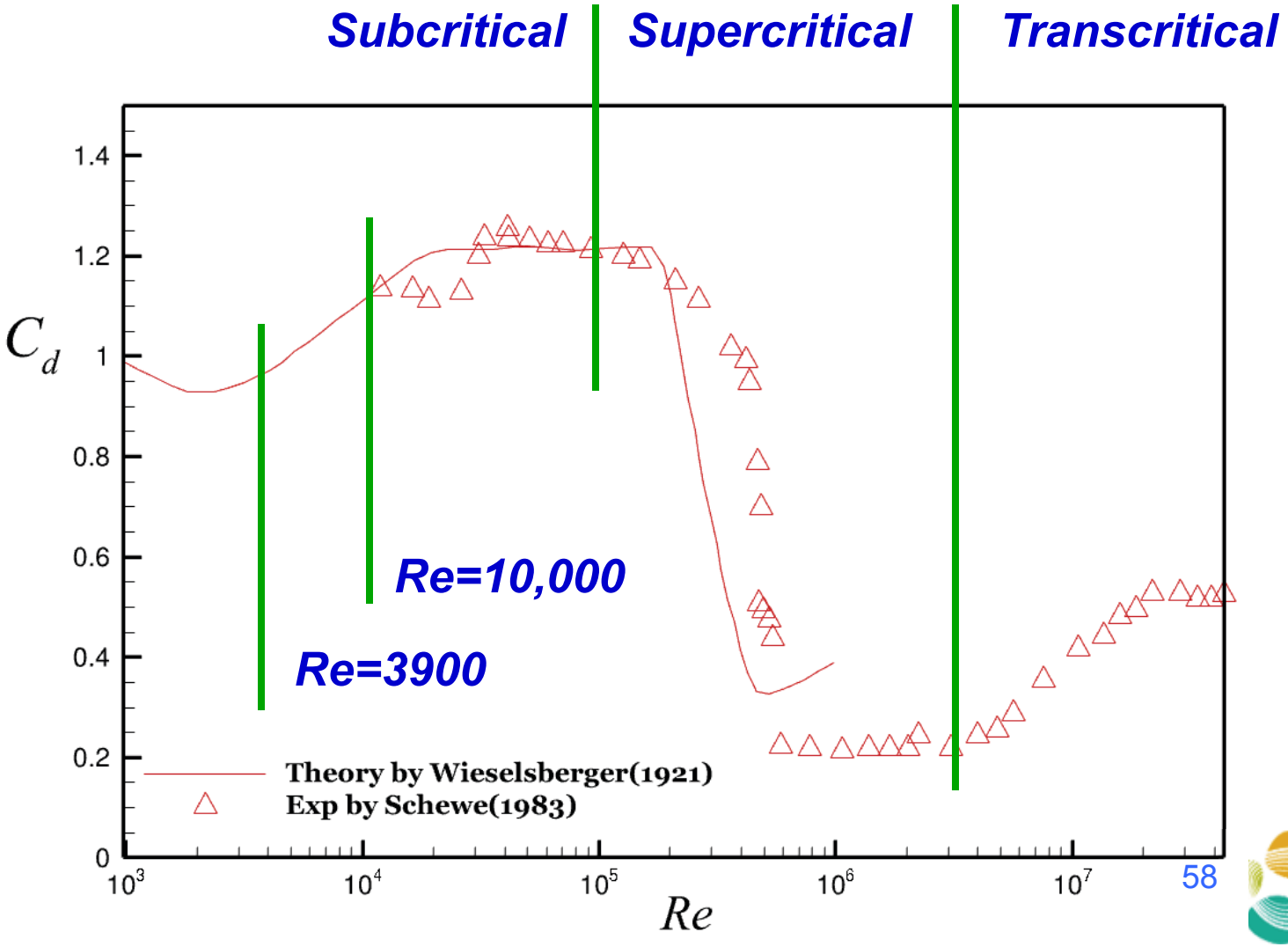
- Drag coefficient results from experiments do not agree with each other
- Figure from Cantwell & Coles (1983,JFM): exp. Data from literature



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Discovery through simulation

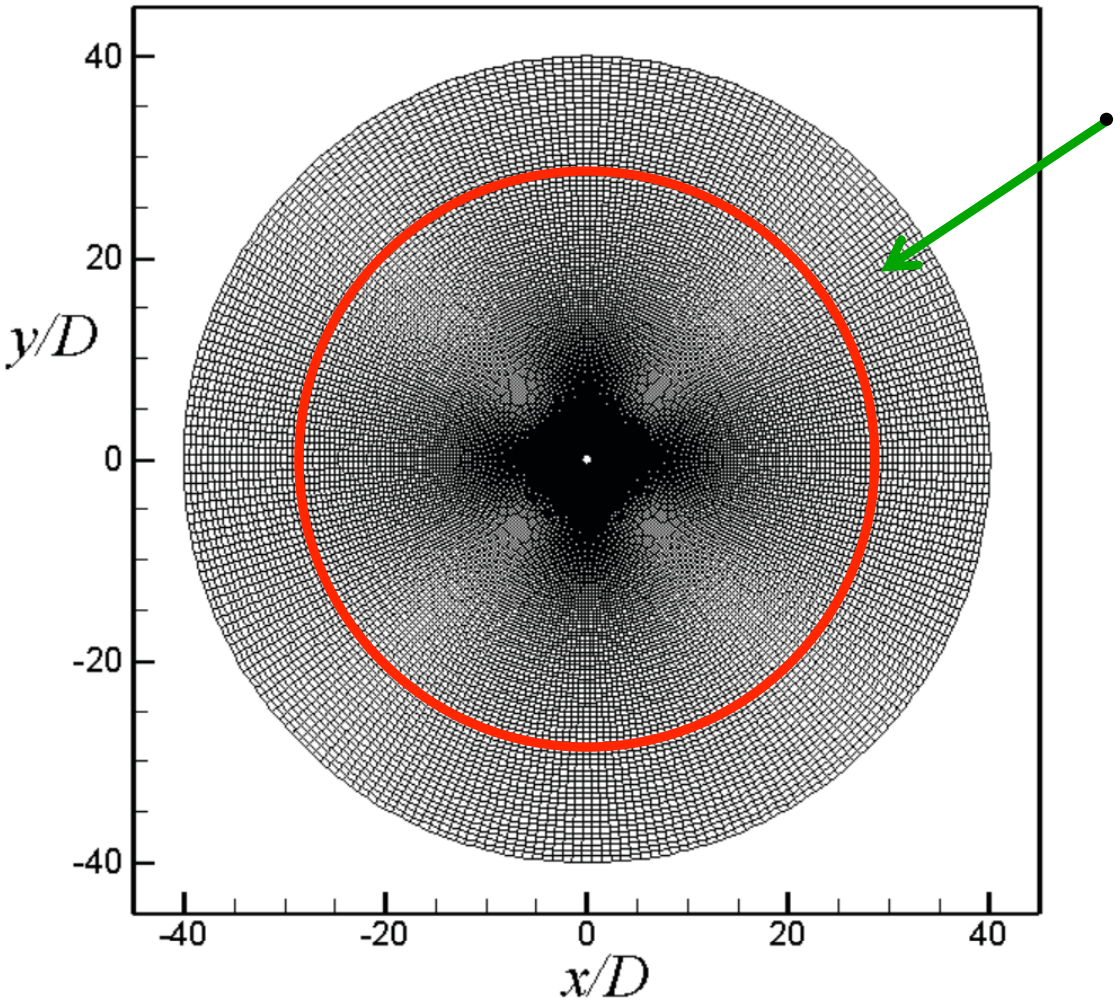
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Drag Coefficient



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Discovery through simulation
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Cylinder



Sponge region

- *Wall model*
  - *Cylindrical coordinate*
  - *2D version*

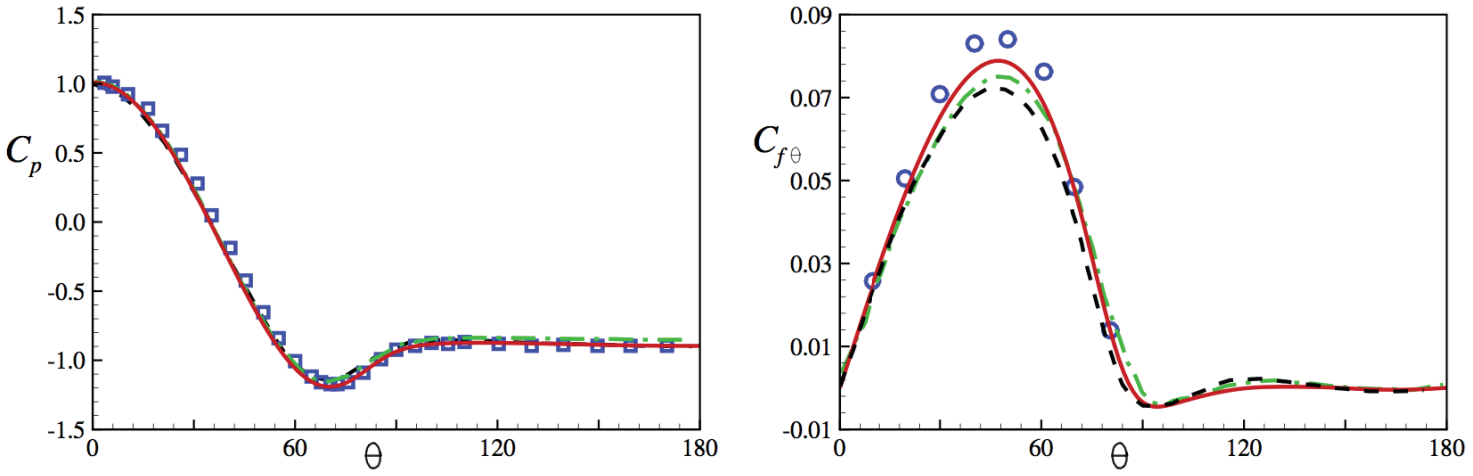
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Discovery through simulation

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Re=3900

- C_p : all simulations agree with Exp. Data
- $C_{f\theta}$: simulations agree with each other; a little off at peak value

$$C_{f\theta} = \overline{\left.\frac{du_\theta}{dr}\right|_{r=0}}$$

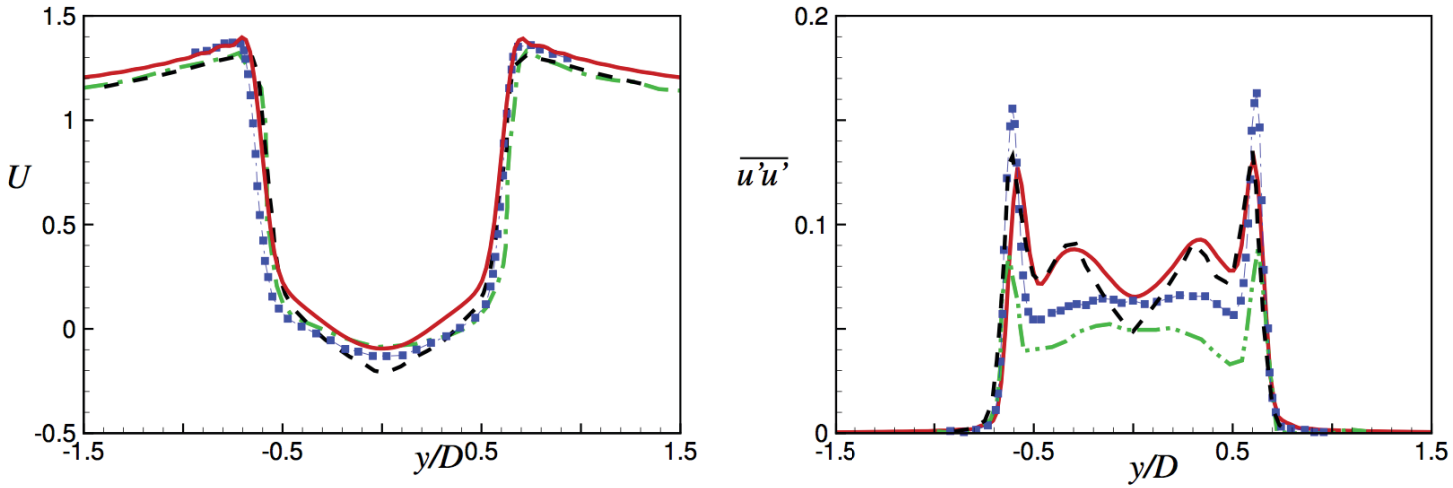


(a) pressure coefficient . (b) skin friction coefficient.

FIGURE 1. Comparison of pressure coefficient and skin friction coefficient. ——— , present LES; □ , experiments by Norberg (1993); ○ , experiments by Son & Harratty (1969); —·— , LES by Beaudan & Moin (1994) – – – – , DNS by Ma *et al.* (2000).

Re=3900

- U : all simulations agree with Exp. Data
- Present turbulent intensity agrees with LES (Moin 2000)



(a) mean streamwise velocity . (b) streamwise turbulent intensity.

FIGURE 2. Comparison of steamwise velocity and turbulent intensity. ■, experiments by Parnaudeau *et al.* (2008); ——— , present LES; – – – – , LES by Kravchenko & Moin (2000); —·— , DNS by Ma *et al.* (2000).

Subcritical region

- Different ways to evaluate C_f give slightly different results
- For Re=5000, results generally agree
  - *In capturing the peak of C_f, passively solving ODE gives best result*

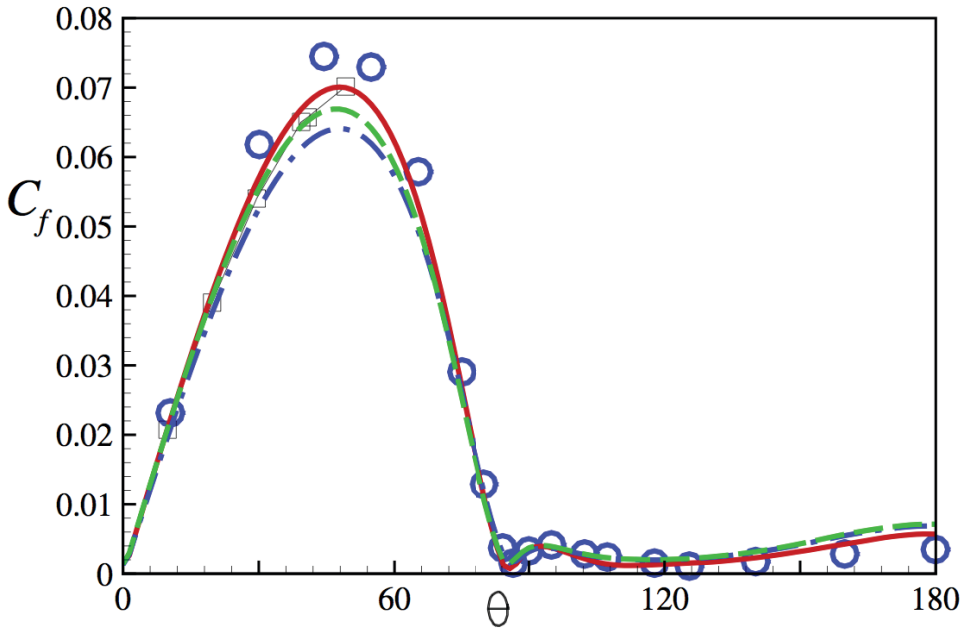
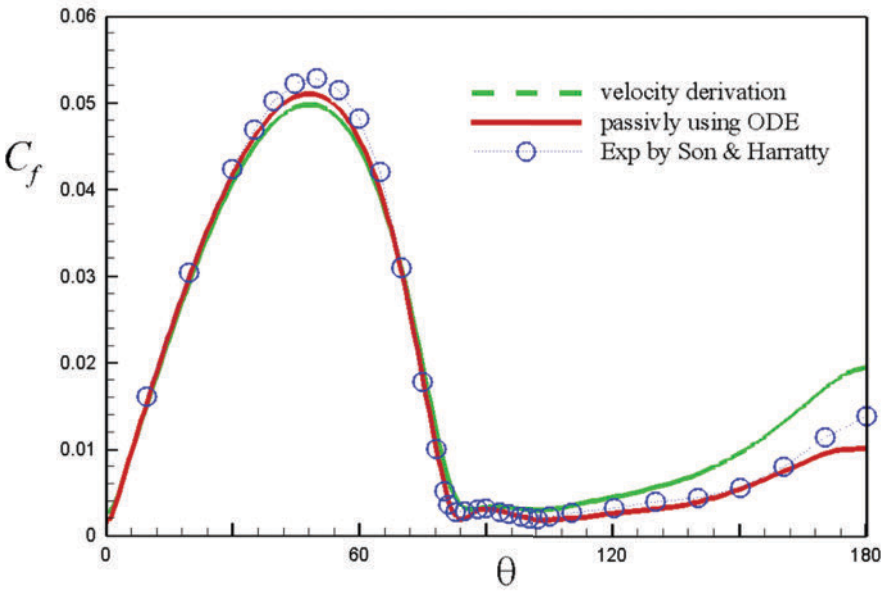
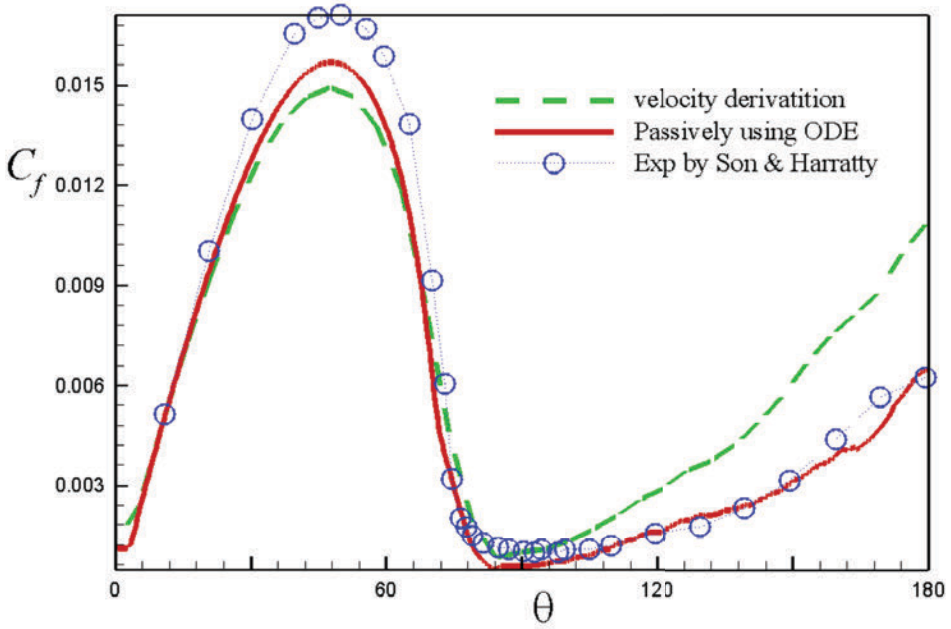


FIGURE 5. Comparison of steamwise velocity and turbulent intensity. ∘ , experiments by Parnaudeau *et al.* (2008); □ , theoretical estimate using method by Thom (1928) ——— , by solving ODE of η_0; - - - - , by 4-th order derivative of velocity field; —·— , by 2-ed order derivative of velocity field.

Subcritical region



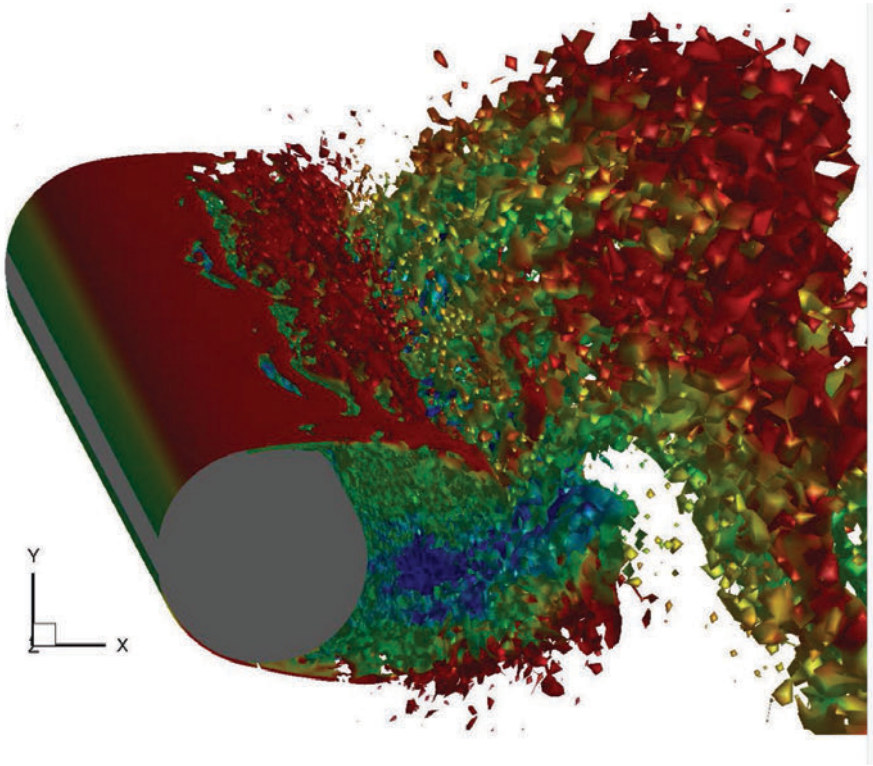
- $Re=10^4$



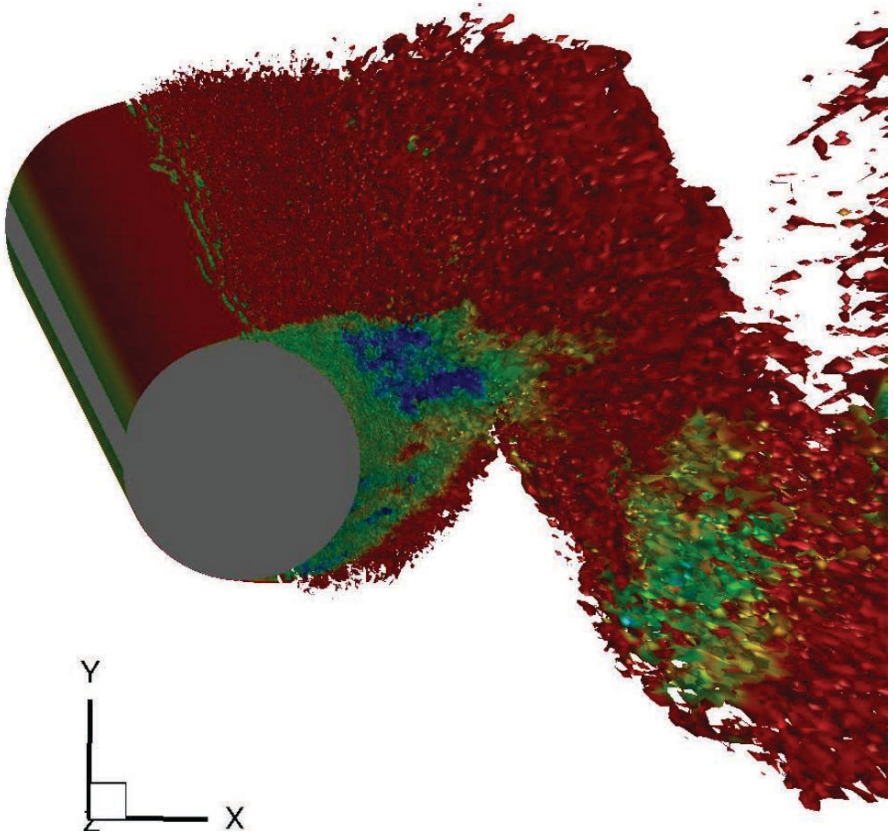
- $Re=10^5$

- *Passively using ODE to compute C_f always give best results*
- *Especially in separated region*

Visualization of Q isosurfaces

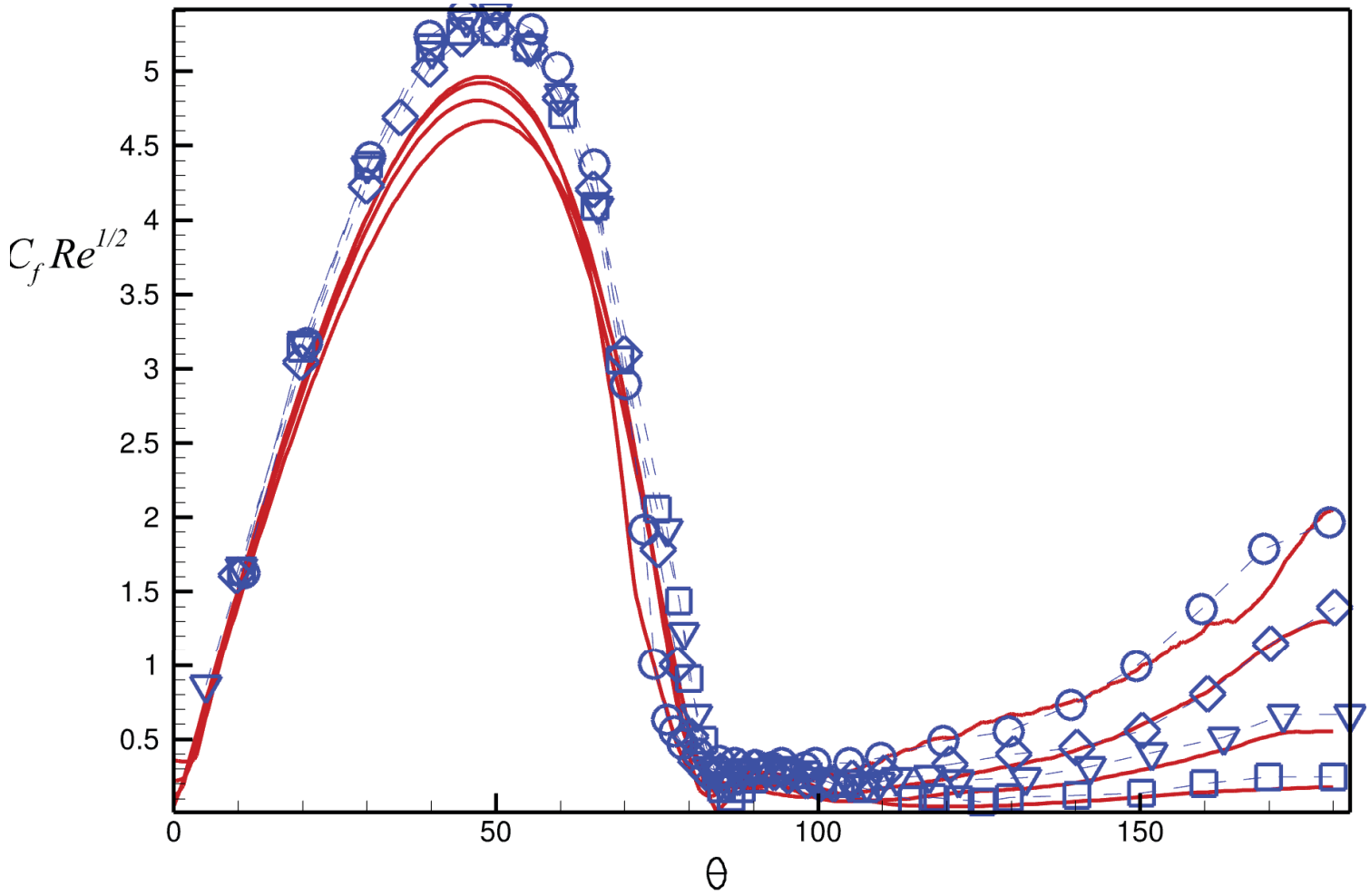


- $Re=10^4$



- $Re=10^5$

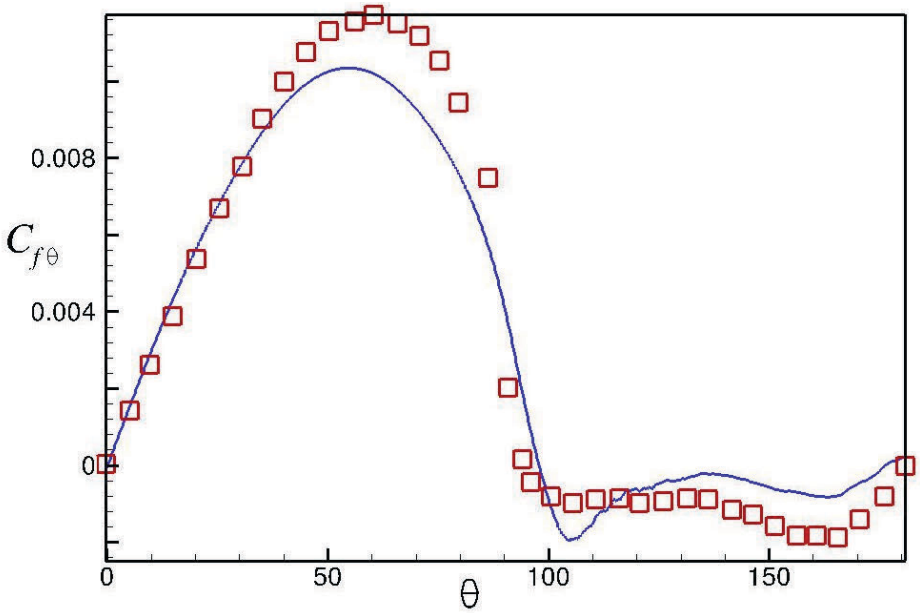
Scaled C_f



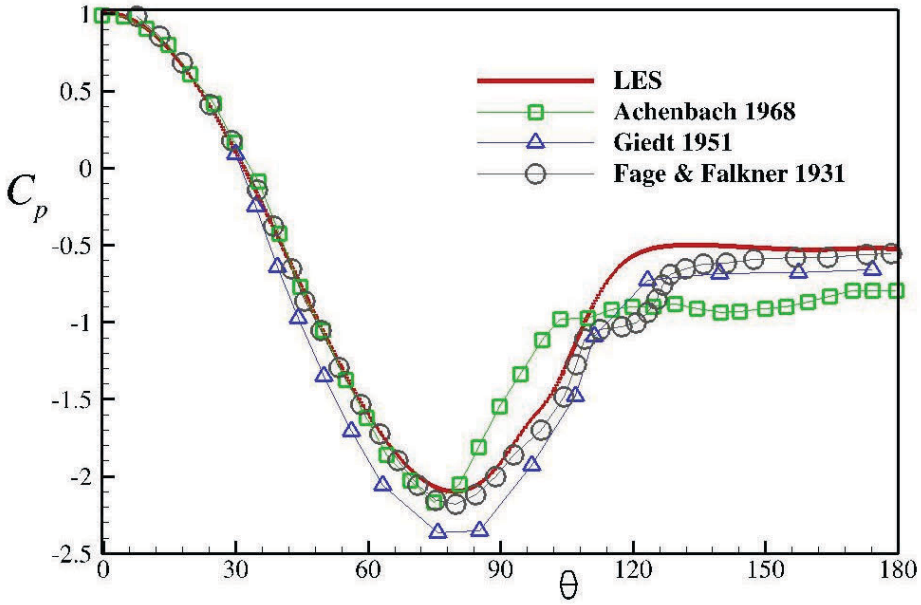
Symbols: Re=5K, 10K, 20K,100K

Re=2.6 x 10^5

- Mesh: 2048 x 512 x 192



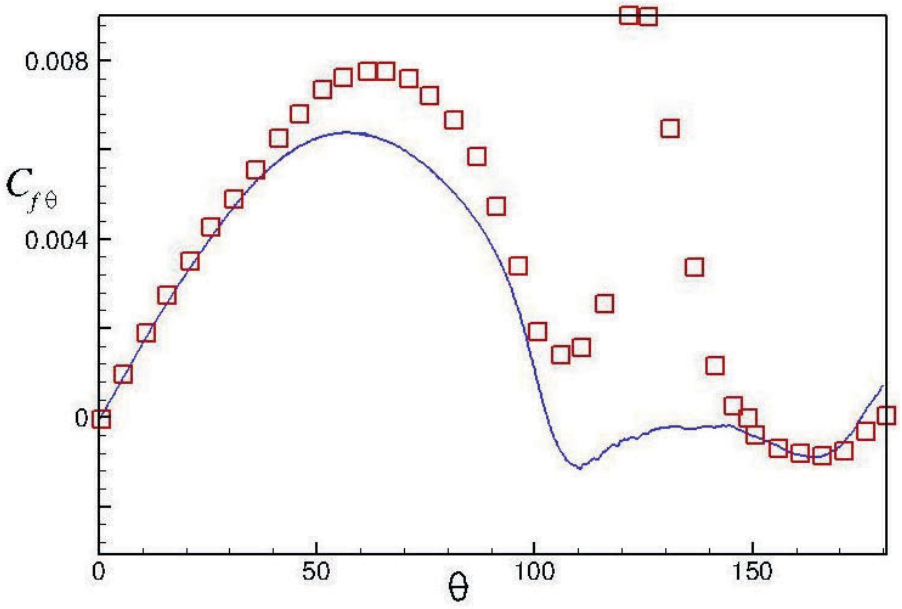
- *Skin friction coefficient*



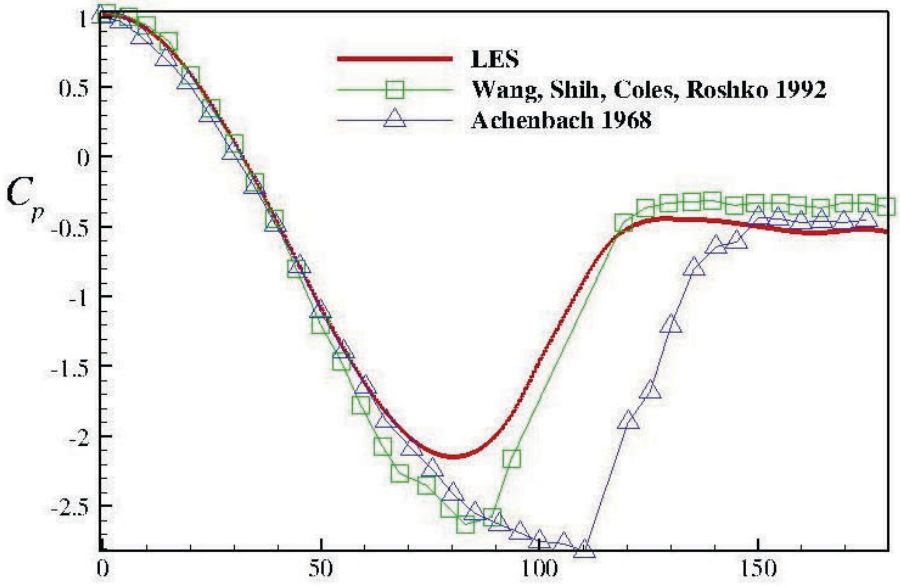
- *Pressure coefficient*

Re = 8.5 x10^5

- Mesh: 2048 x 512 x 192

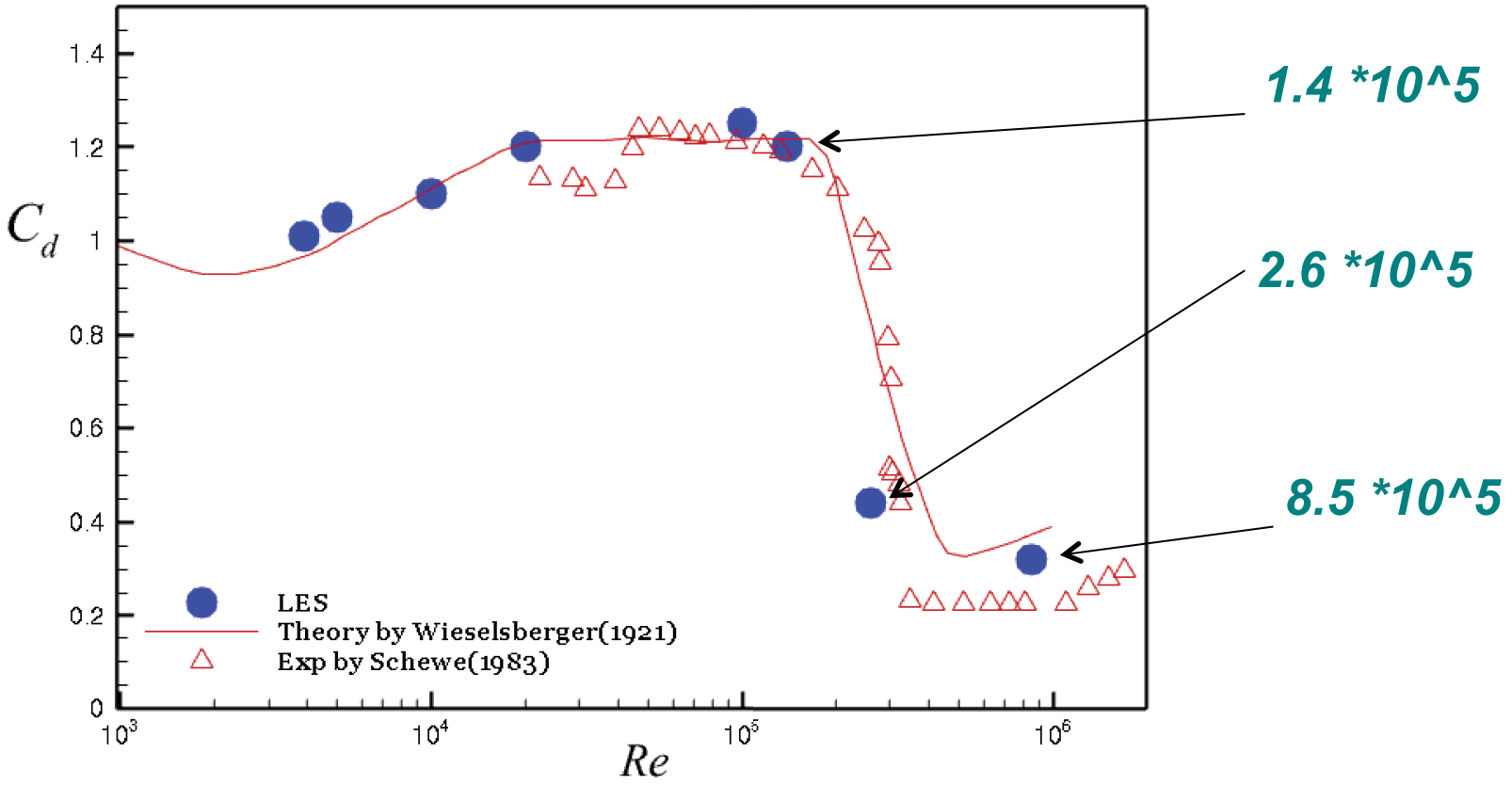


- *Skin friction coefficient*



- *Pressure coefficient*

Drag coefficient



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Discovery through simulation

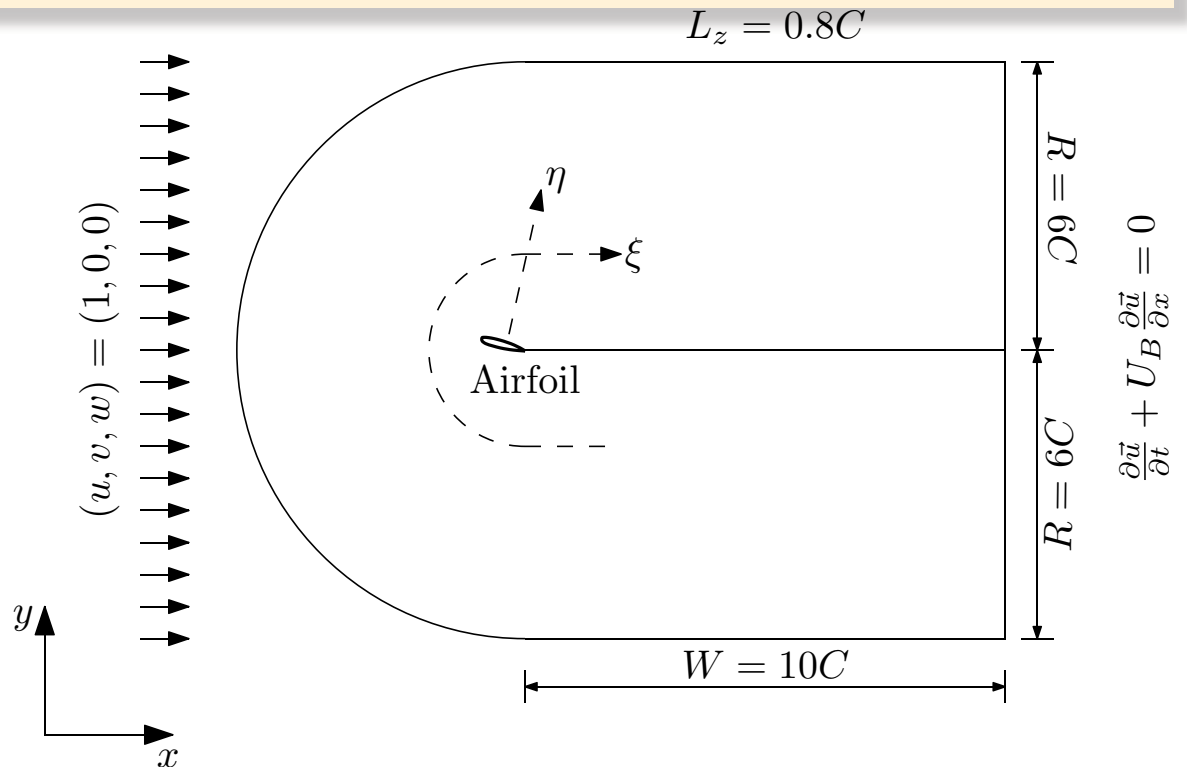
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Application 5

Flow over Airfoils

WORK IN PROGRESS

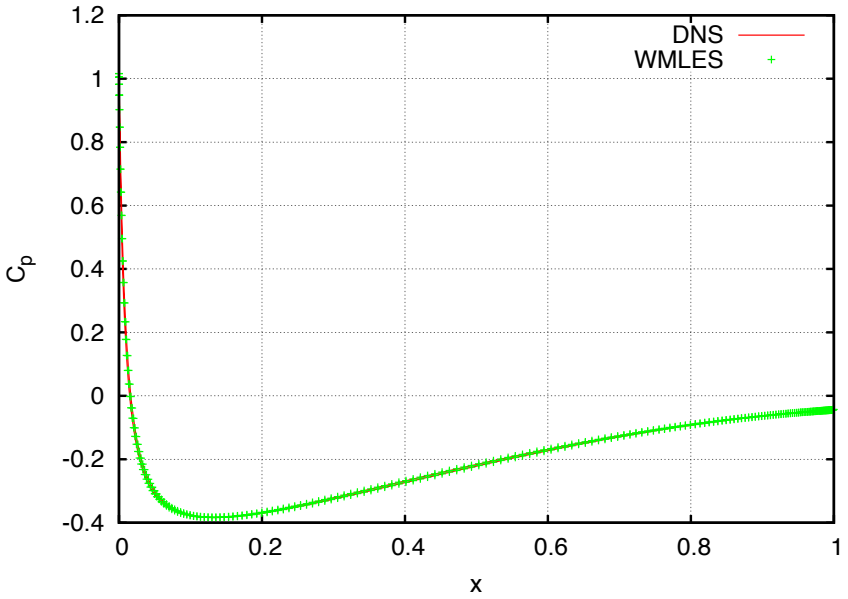
Numerical Setup

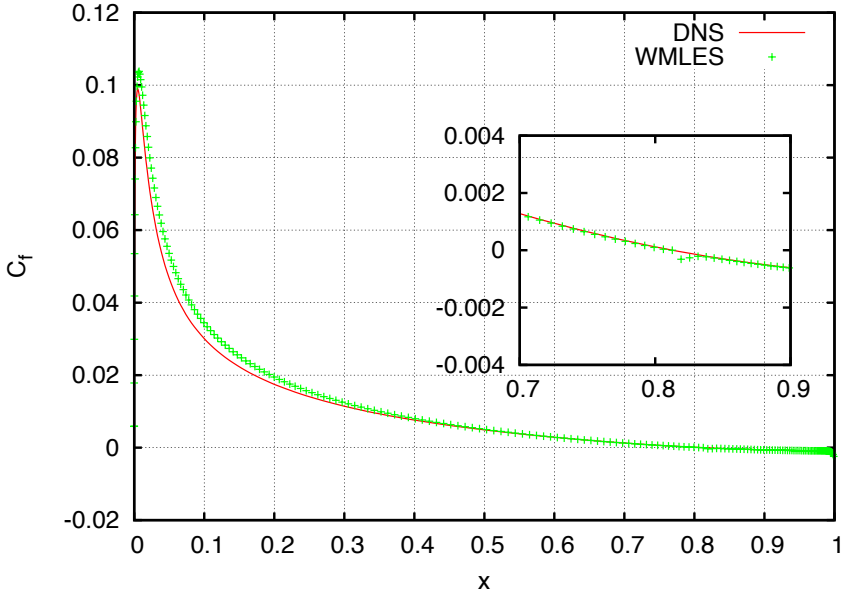


Airfoil	Simulation	*Re*	*AoA*	$N_x \times N_y \times N_z$	T_{end}
NACA0012	DNS	10^4	0-3	2048×256×128	$80C/U_0$
NACA0012	WMLES	10^4	0-3	1024×128×64	$80C/U_0$
NACA0018	WMLES	10^5	5	2048×256×128	$16C/U_0$

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Discovery through
UST

NACA0012, AOA=0



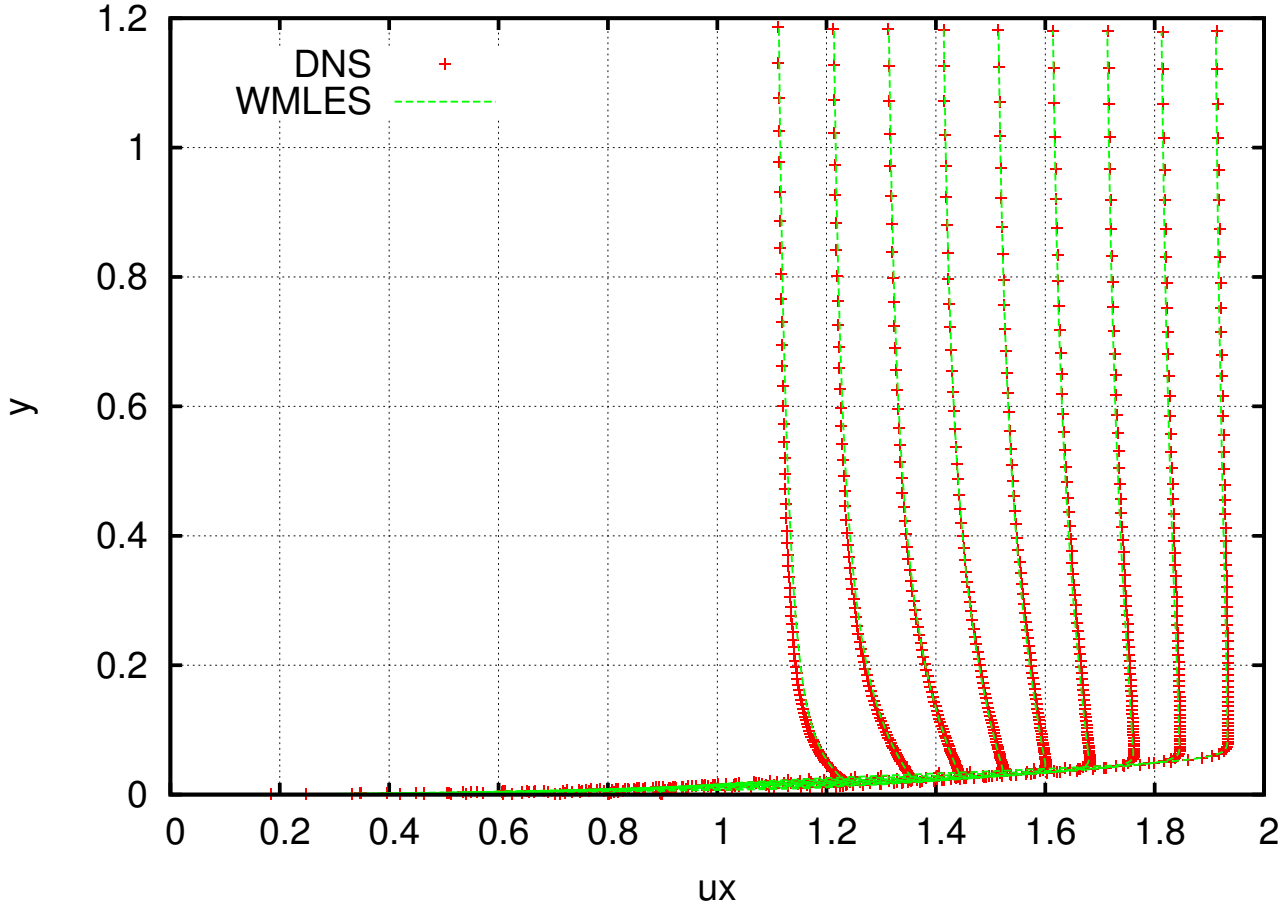


Spanwise- and time-averaged C_f and C_p

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Discovery through simulation

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NACA0012, AOA=0

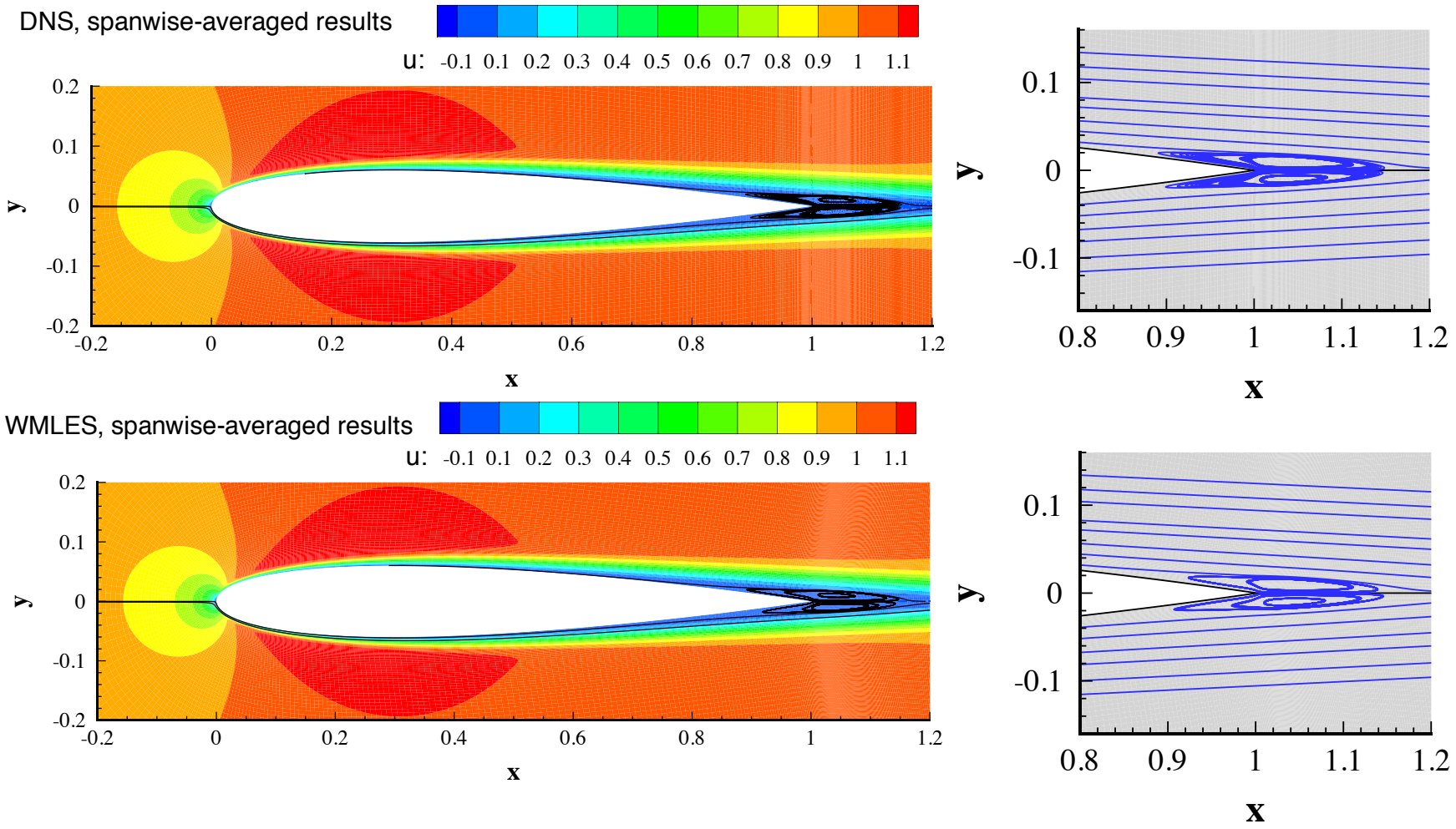


Spanwise- and time-averaged velocity distribution on the upper surface from x=0.1 to x=0.9

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NACA0012, AOA=0

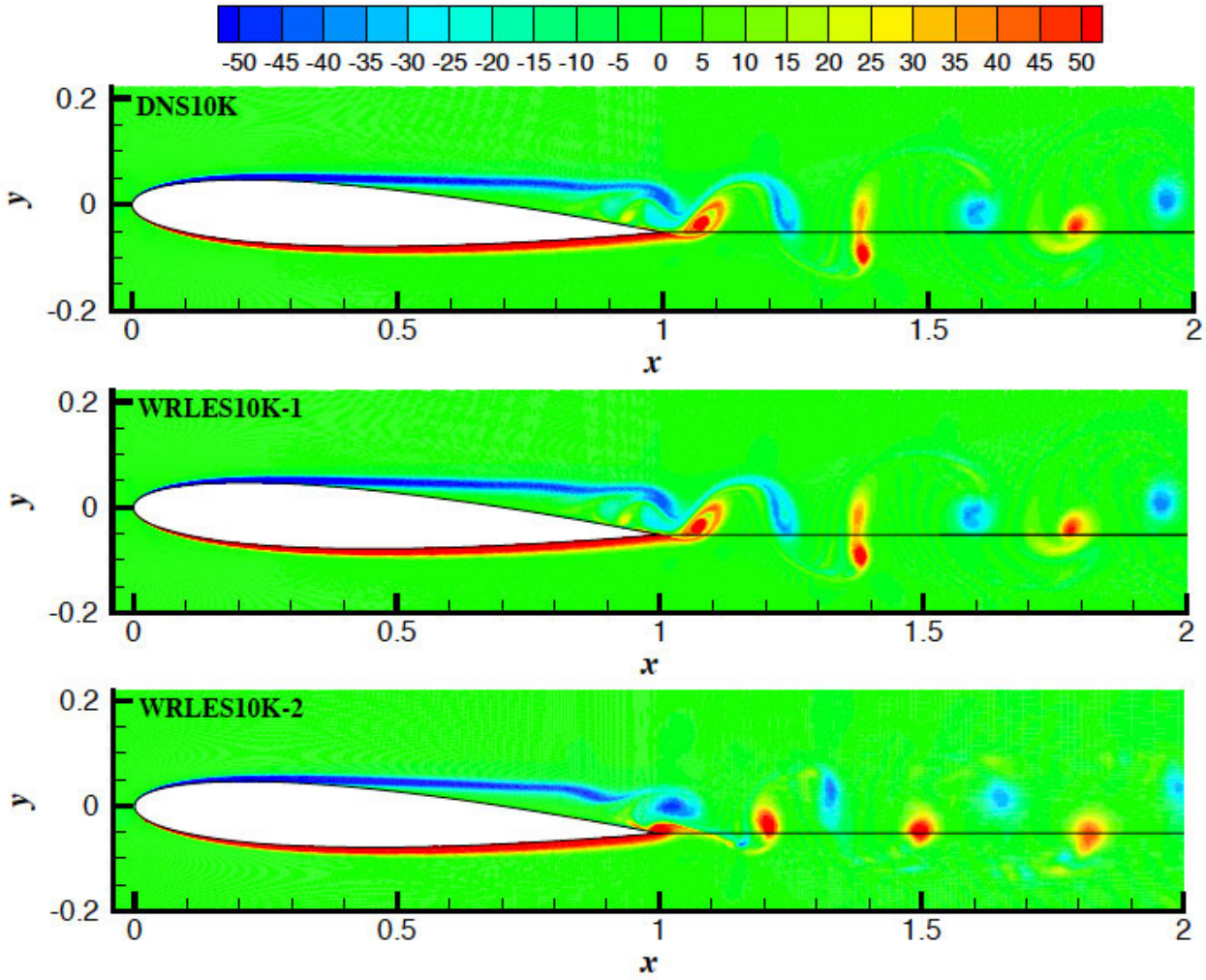


Spanwise- and time-averaged velocity contour and trailing edge separation

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NACA0012, AOA=3

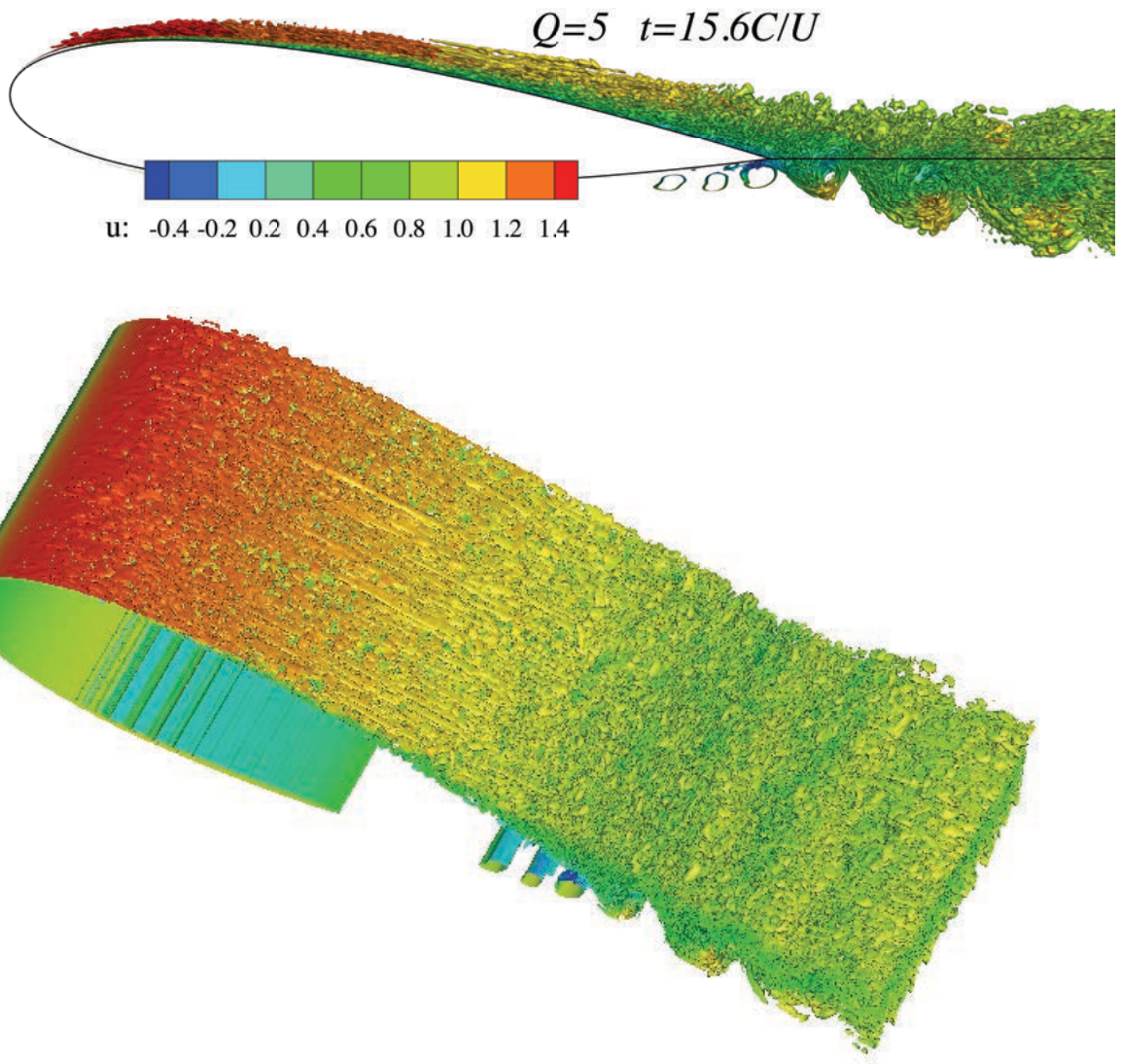


Spanwise- averaged vorticity

Discovery through simulation

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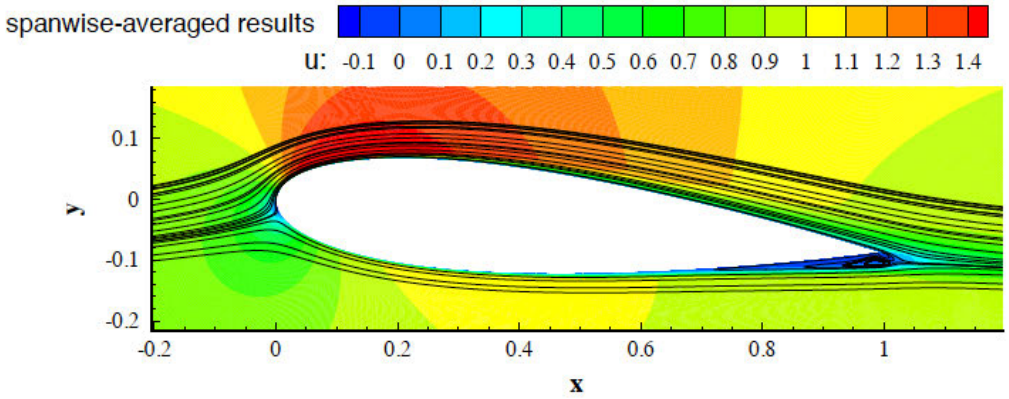
NACA0018, AOA=5



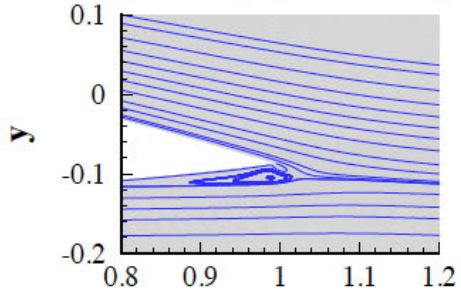
fps
Discovery through simulation

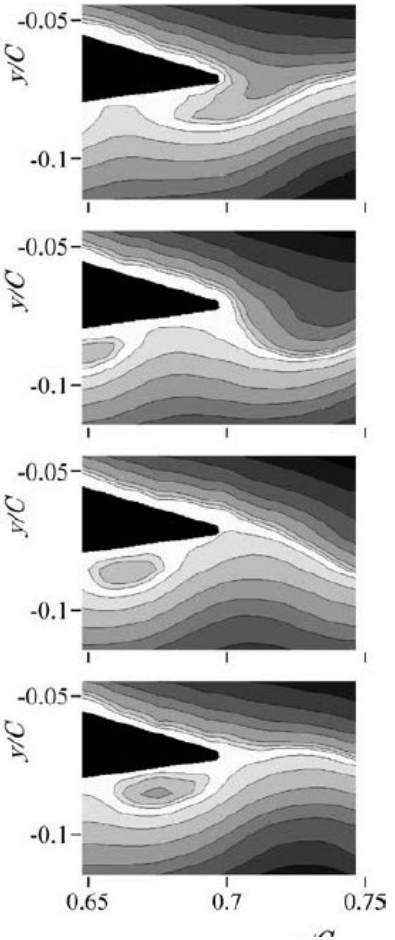
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NACA0018, AOA=5



The right figure is from Nakano *et al.* (2006), Experiments in Fluids. NACA0018, Re=1.6×10^5, AoA=6.









Discovery through simulation

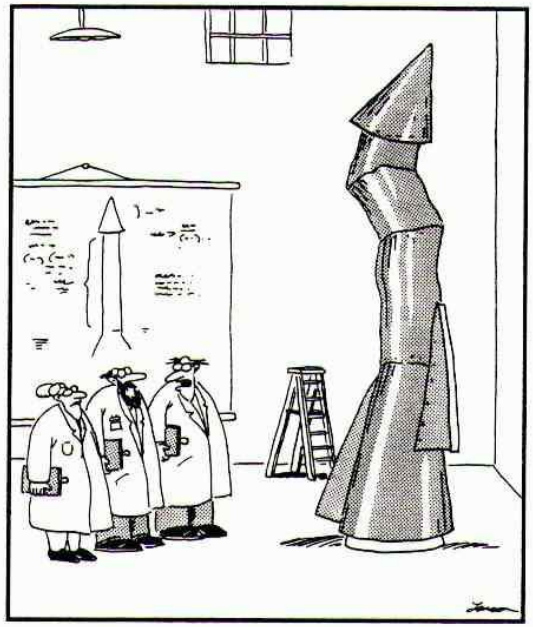
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Conclusion

- Wall-bounded turbulent flows may be computed with LES for the “outer” flow and a wall-model for the inner “universal” character
- Stretched spiral SGS model employed for outer LES
- Wall model ➔ ODE for shear stress and Dirichlet BC at a virtual wall
- Examples: flat plate TBL, separated/reattached TBL, periodic hill, flow over a cylinder with drag crisis, flow past an airfoil. Quantitative comparisons with DNS/other LES and experiments.

Thank you

- Acknowledgement
  - *KAUST Office of Competitive Research Funds under Award Number URF/1/1394-1*
  - *KAUST Supercomputing Laboratory for time on Shaheen I (IBM BGP) and Shaheen II (Cray XC-40) (10s of millions of cpu hours)*
- Questions?



"It's time we face reality, my friends. ... We're not exactly rocket scientists."

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Discovery through simulation

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Backup Slides

Power law by George & Castillo

- *Inner scaling:*

$$U^+ = \frac{u}{u_\tau} = C_i \left(\frac{z+a}{l^+} \right)^\gamma$$

- ***Empirical parameters:***

$$\gamma = \gamma_\infty + \frac{\alpha A}{(\ln Re_c)^{1+\alpha}}$$

$$\frac{C_o}{C_i} = \frac{C_{o\infty}}{C_{i\infty}} \exp\left[(1+\alpha)A/(\ln Re_c)^\alpha\right]$$

$$\frac{C_o}{C_{o\infty}} = 1 + c_1 \exp(c_2 Re_c),$$

- ***Outer scaling:***

$$\frac{u - U_\infty}{U_\infty} = C_o \left(\frac{z+a}{\delta_{99}} \right)^\gamma$$

- ***The friction law:***

$$\frac{U_\infty}{u_\tau} = \frac{C_i}{C_o} (Re_c)^\gamma$$

- ***With*** $Re_c = \dfrac{\delta_{99} u_\tau}{\nu}$

and $a^+, \gamma_\infty, A, \alpha, C_{o\infty}, C_{i\infty}, c_1, c_2$ ***constant***

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Discovery through simulation

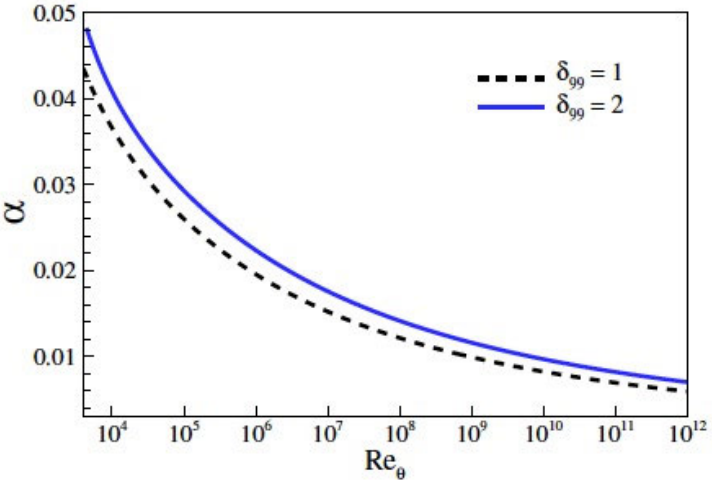
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Wall model using GC power law

- *New scaling relation*

$$\frac{\partial \langle u \rangle}{\partial t} = \frac{\partial \eta_w}{\partial t} \frac{\tilde{u}|_h}{2\eta_w} (1 + \alpha(Re)) \qquad \alpha = \tilde{u}|_h \mathscr{F}/2$$

$$\mathscr{F}(Re) = \left(\frac{c_1 c_2 Re}{e^{-c_2 Re} + c_1} + (1+\alpha) A \alpha \frac{\ln Re - \ln(z^+ + a^+)}{(\ln Re)^{2+\alpha}} \right) \left(\gamma_\infty + \frac{\alpha A}{(\ln Re)^{1+\alpha}} \right)^{-1}$$



- *New ODE for u_τ with power law*

$$\frac{\tilde{u}|_h}{2\eta_w}(1+\alpha)\frac{\partial \eta_w}{\partial t} + \frac{\partial \widetilde{uu}|_h}{\partial x} + \frac{\partial \widetilde{uv}|_h}{\partial y} = -\frac{1}{h}\widetilde{uw}|_h - \left.\frac{\partial \tilde{p}}{\partial x}\right|_h + \frac{\nu}{h}\left(\left.\frac{\partial \tilde{u}}{\partial z}\right|_h - \eta_w \right)$$

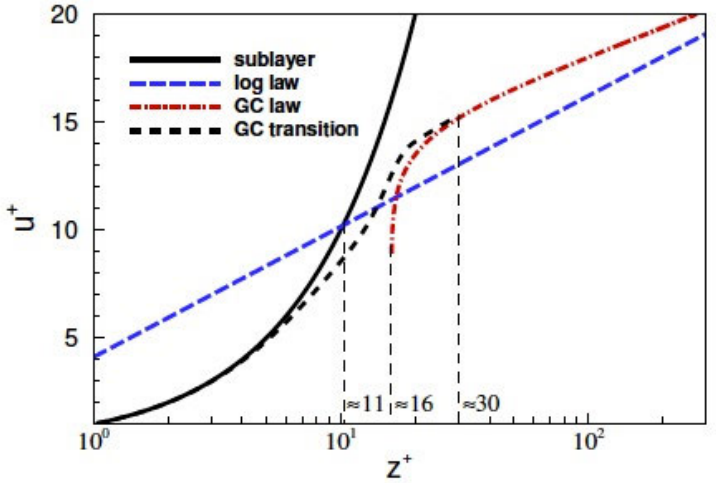
- *Velocity gradient relation*

$$\frac{d\tilde{u}^+}{dz^+} = A_1 \gamma (z^+ + a^+)^{(\gamma-1)}$$

$$A_1 = \frac{2(-T_{xz}/u_\tau)}{\gamma_{11} K^{1/2} z^{+\gamma}} \left(z^+ + a^+\right)^{-(\gamma-1)}$$

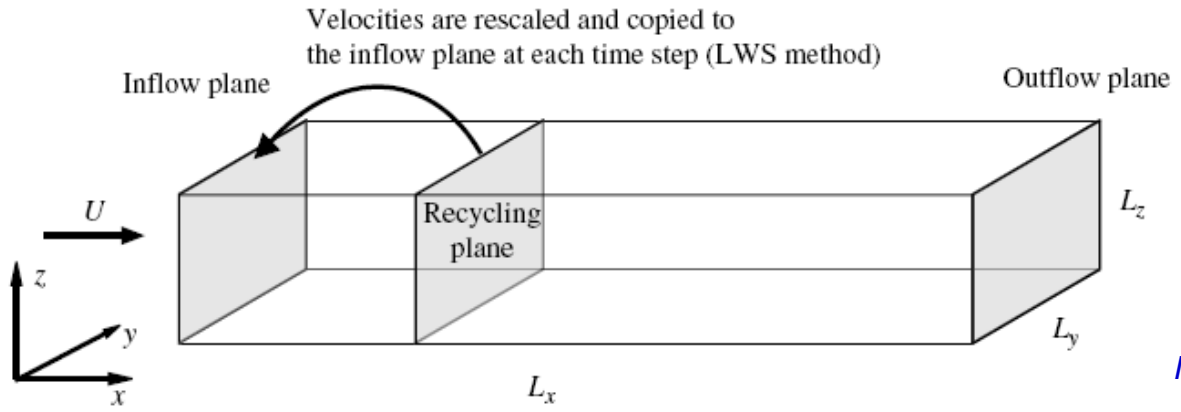
- *Slip velocity BC*

$$\tilde{u}^+|_{h_0} = A_1 (h_0^+ + a^+)^\gamma$$



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Discovery through simulation

Recycling method for inflow B.C.



Inoue & Pullin (2011)

- *Log law recycling:*

$$U_{in}^{inn} = \lambda U_{re}^{inn}, \quad U_{in}^{out} = \lambda U_{re}^{out} + (1-\lambda)U_{\infty},$$
$$W_{in}^{inn} = \lambda W_{re}^{inn}, \quad W_{in}^{out} = \lambda W_{re}^{out},$$
$$(u'_i)_{in}^{inn} = \lambda (u'_i)_{re}^{inn}, \quad (u'_i)_{in}^{out} = \lambda (u'_i)_{re}^{out}.$$

- *Parameters:*

$$\lambda = \frac{(u_\tau)_{in}}{(u_\tau)_{re}}$$

Lund et al. (1998)

- *GC law recycling:*

$$U_{in}^{inngc} = \lambda U_{re}^{inngc}, \quad U_{in}^{outgc} = \lambda_{o1} U_{re}^{outgc}$$
$$W_{in}^{inngc} = \lambda W_{re}^{inngc}, \quad U_{in}^{outgc} = \lambda_{o2} U_{re}^{outgc}$$
$$(u'_i)_{in}^{inngc} = \lambda (u'_i)_r e^{inner}, \quad (u')_{in}^{outgc} = \lambda_{o1} (u')_{re}^{outgc}$$
$$(v')_{in}^{outgc} = \lambda_{o2} (v')_{re}^{outgc} \quad (w')_{in}^{outgc} = \lambda_{o1} (w')_{re}^{outgc}.$$

- *Parameters:*

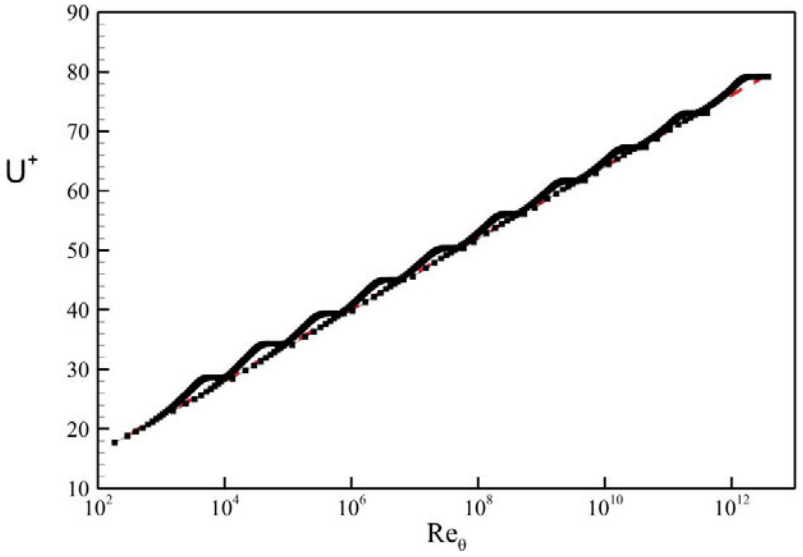
$$\lambda_{o1} = \frac{(U_\infty)_{in}}{(U_\infty)_{re}}, \quad \lambda_{o2} = \frac{(U_\infty d\delta/dx)_{in}}{(U_\infty d\delta/dx)_{re}}.$$

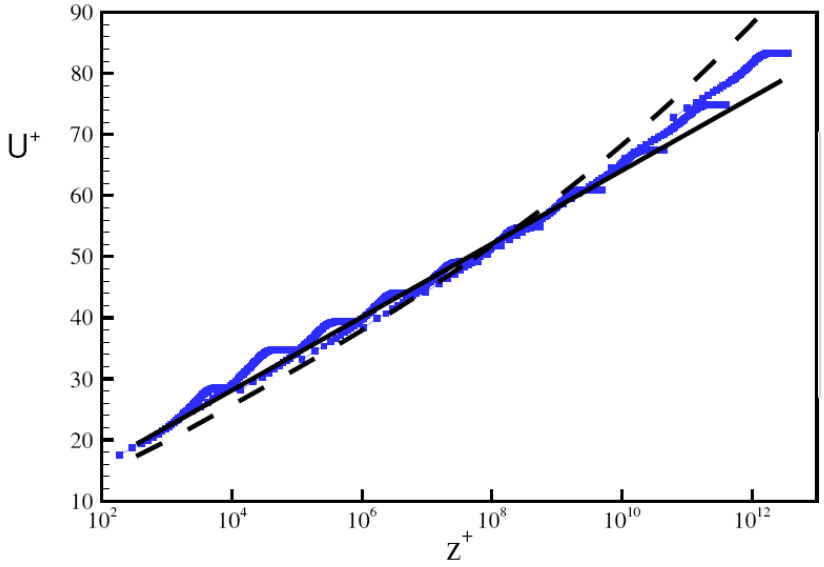
Bohr, Ph.D. thesis (2005)

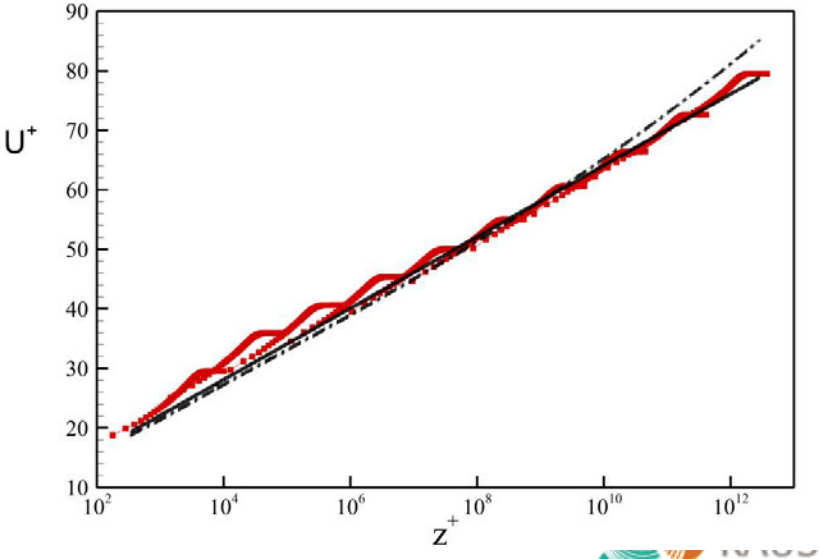
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Mean velocity

- *LES with log law*
  - *Self-consistent results*
- *LES with original GC law*
  - *Some deviation for medium Re case*
  - *Asymptotically back to log form*
- *LES with modified GC law*
  - *Mimic log law for medium Re cases*
  - *Bad performance for asymptotic cases*

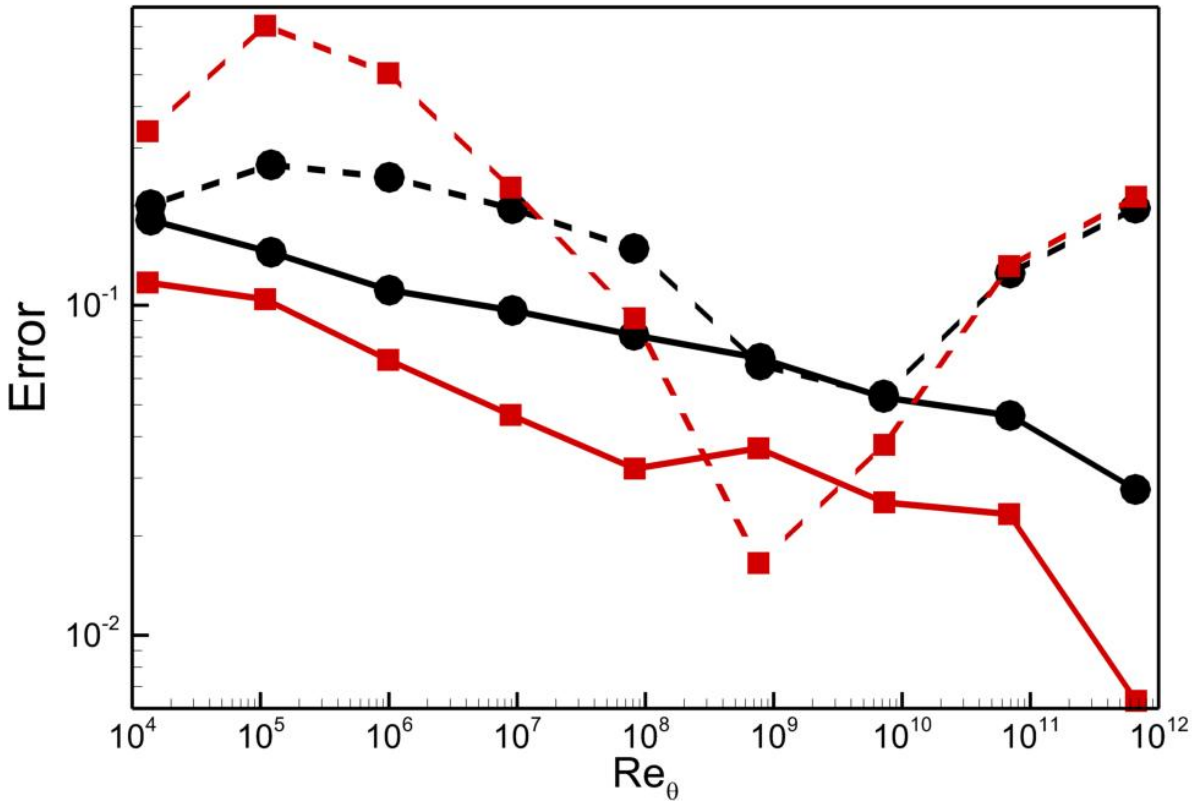






Log Law or Power Law?

- *Overlap region* $z < 0.15\delta_{99}$



Black: log law LES

Red: power law LES

dash: power law theory

Solid: log law theory

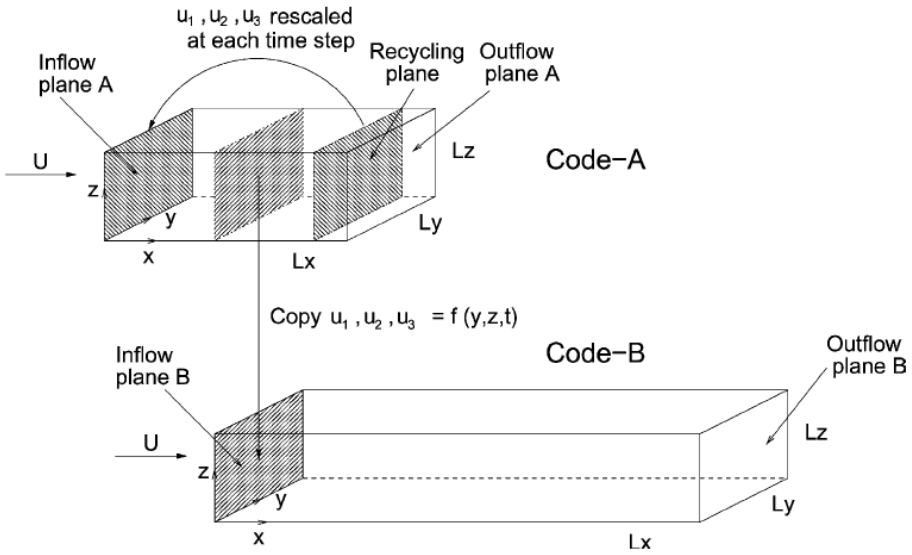
- *Errors between all LES results and log law decreases with Re*
- *Errors between LES and GC law have no asymptotic behavior*

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Numerical method

- *Numerical method*
  - *4-th order spectral/FD*
  - *Validation: ZPG, APGTBL*
- *AB code*
  - *A code: ZPG*
  - *B code: separation*

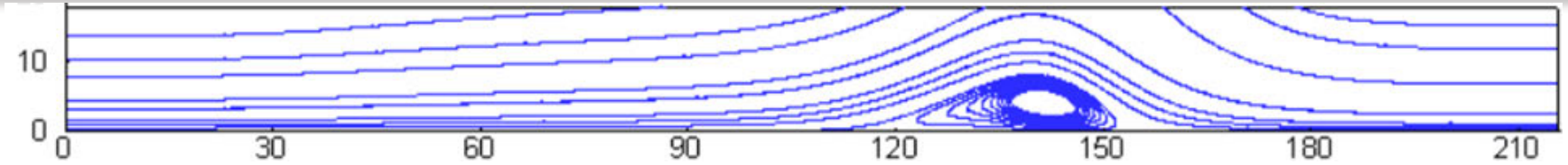


Figures from Ferrante & Elghobashi, 2004, JCP

code A			
Top	Bottom	Left	Right
$u = 1$	$u = u_w$	R	C
$\frac{\partial v}{\partial z} = 0$	$v = v_w$	R	C
$\frac{\partial v}{\partial z} = 0$	$w = w_w$	R	C
$p = 0$	$\frac{\partial p}{\partial z} = 0$	$\frac{\partial p}{\partial x} = 0$	$\frac{\partial p}{\partial x} = 0$

code B			
Top	Bottom	Left	Right
$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$	$u = u_w$	D	C
$\frac{\partial v}{\partial z} = 0$	$v = v_w$	D	C
$w = f(x)$	$w = w_w$	D	C
$\frac{\partial p}{\partial x} = 0$	$\frac{\partial p}{\partial x} = 0$	$\frac{\partial p}{\partial x} = 0$	$\frac{\partial p}{\partial x} = 0$

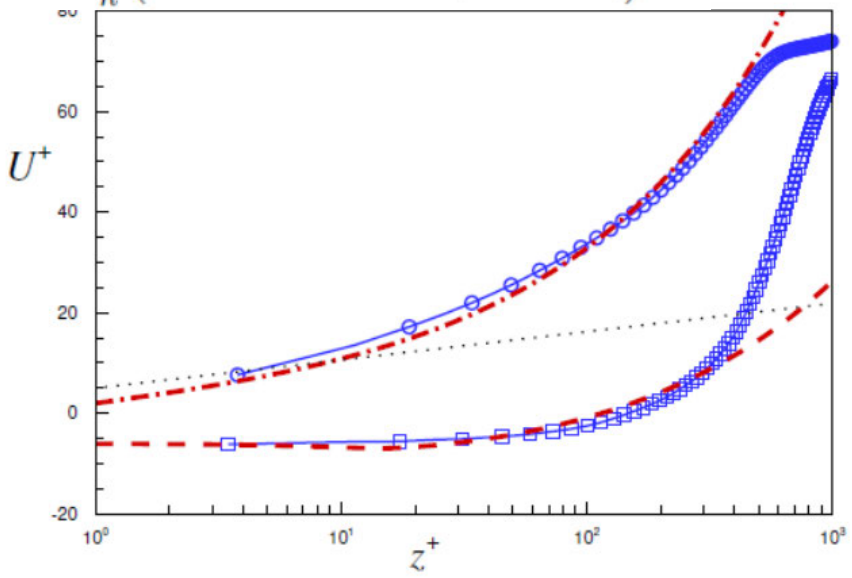
Case 1 : velocity comparison with theory



- ***Skote & Henningson (2000)***

$$U^+ = \frac{1}{\kappa}\left(\ln z^+ - 2\ln\frac{\sqrt{1+\lambda z^+}+1}{2} + 2(\sqrt{1+\lambda z^+}-1)\right) + B$$

$$U^+ = \frac{1}{\kappa}\left(2\sqrt{\lambda z^+ - 1} - 2\arctan(\sqrt{\lambda z^+ - 1})\right) + C$$

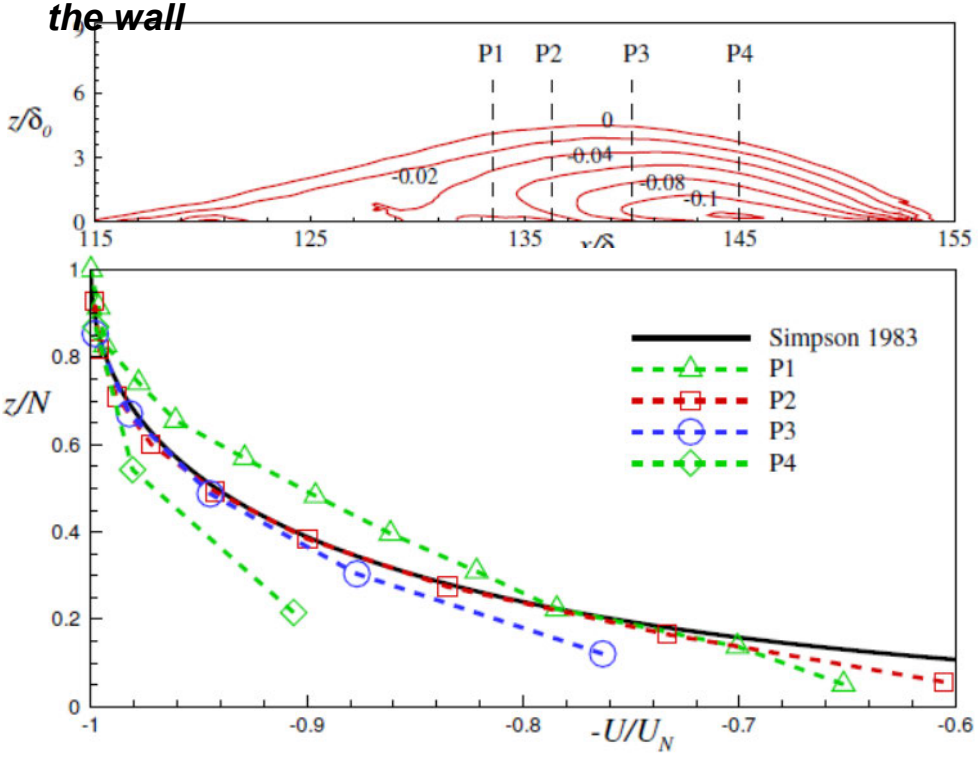


$$U^+ = \frac{u}{u_\tau}, \quad \lambda = \left(\frac{u_p}{u_\tau}\right)^3, \quad u_p = \left(\nu \left.\frac{dp}{dx}\right|_{z=0}\right)^{1/3}$$

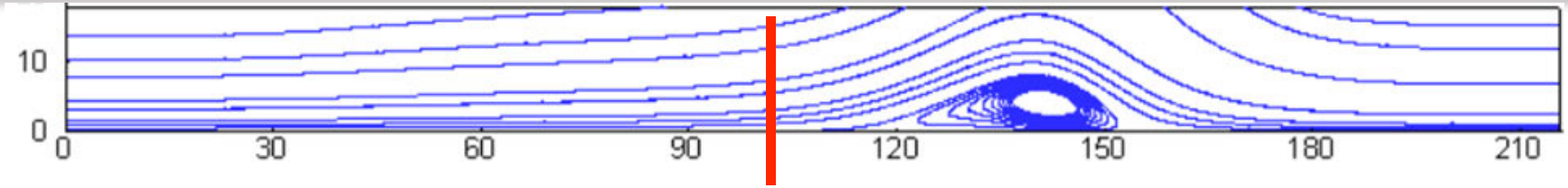
- ***Simpson (1983)***

$$\frac{U}{|U_N|} = A\left(\frac{y}{N} - \ln\left|\frac{y}{N}\right| - 1\right) - 1,$$

U_N = max back-flow velocity N = its distance from the wall



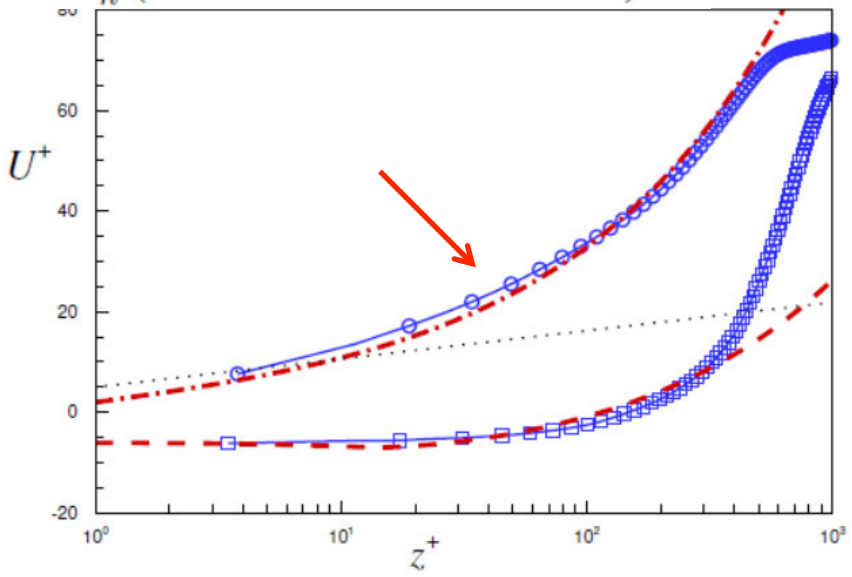
Case 1 : velocity comparison with theory



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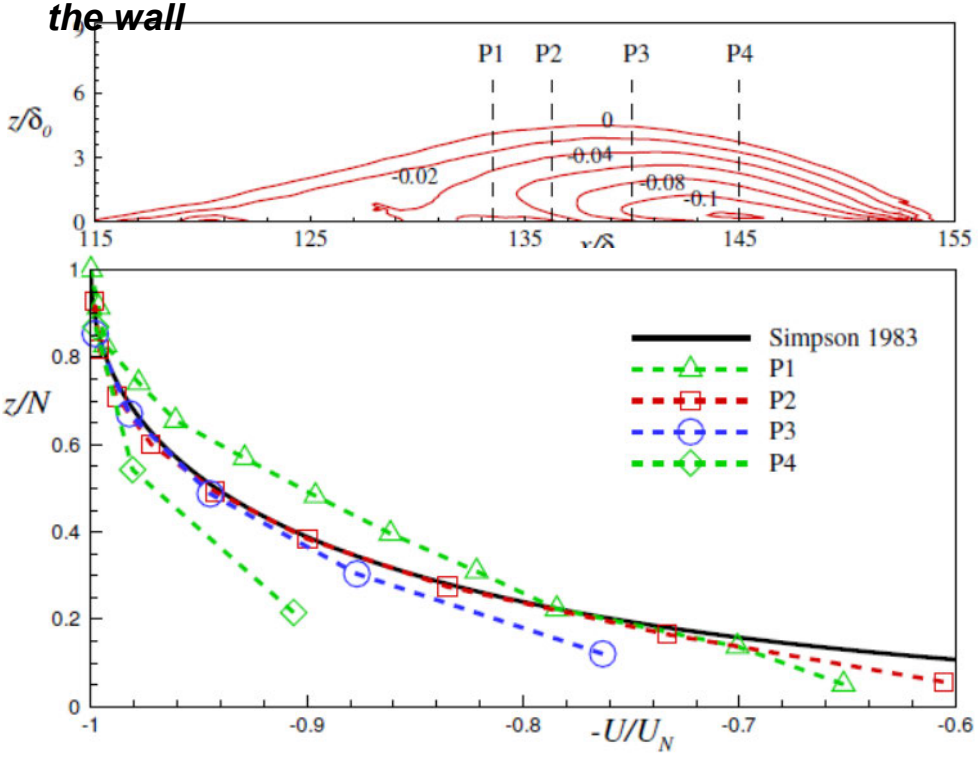


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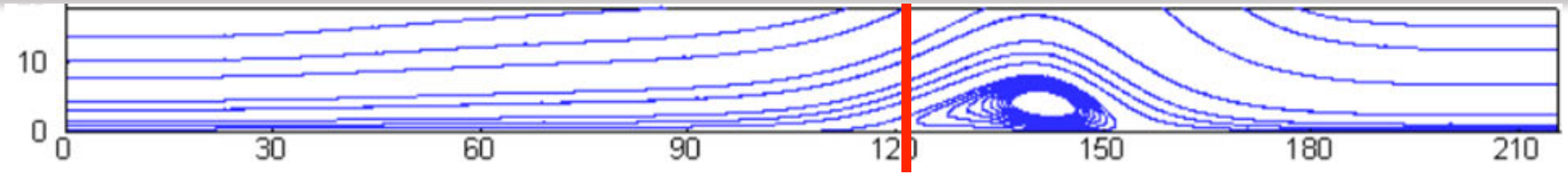
- ***Simpson (1983)***

$$\frac{U}{|U_N|} = A\left(\frac{y}{N} - \ln\left|\frac{y}{N}\right| - 1\right) - 1,$$

U_N = max back-flow velocity N = its distance from the wall



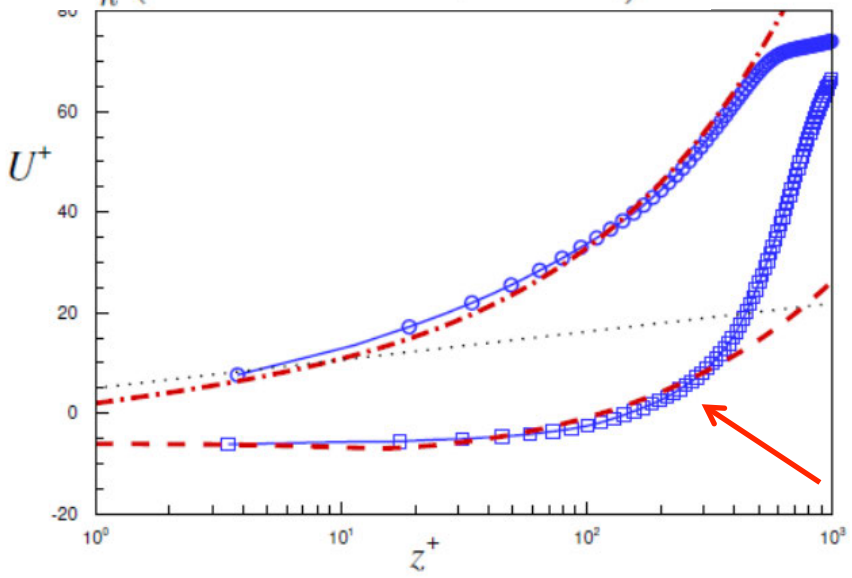
Case 1 : velocity comparison with theory



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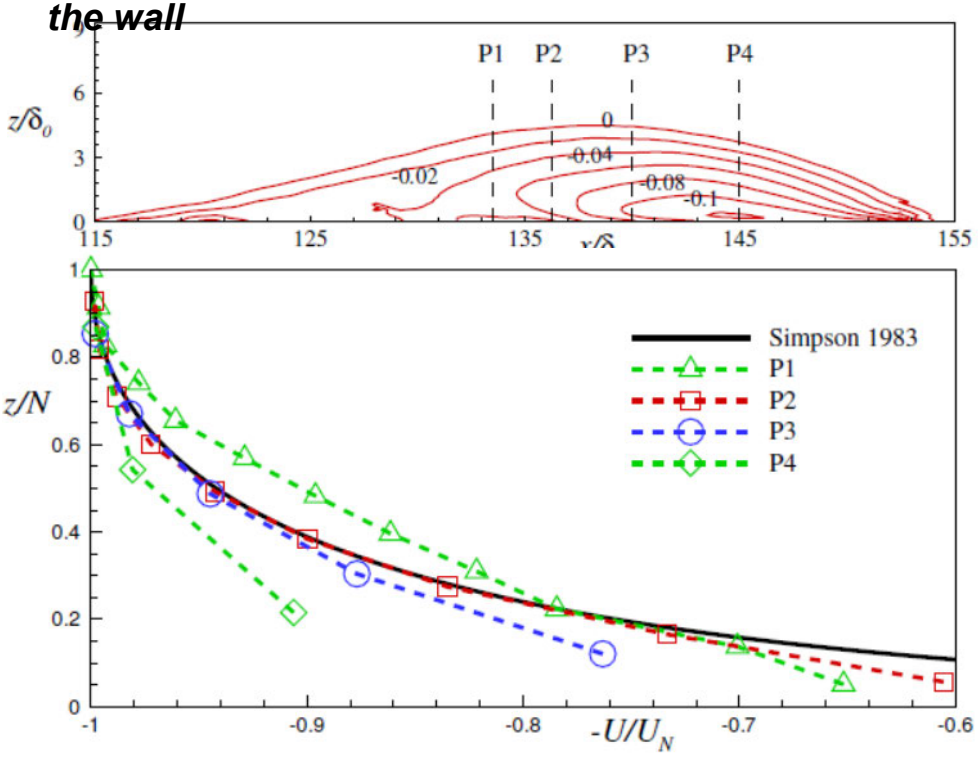


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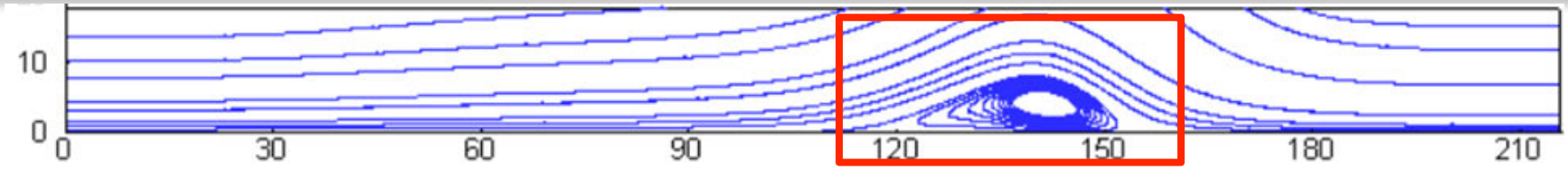
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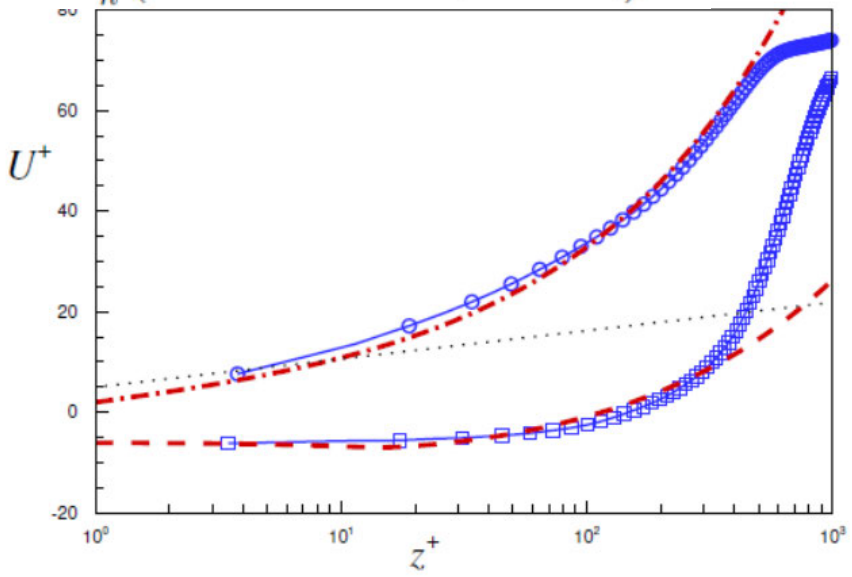
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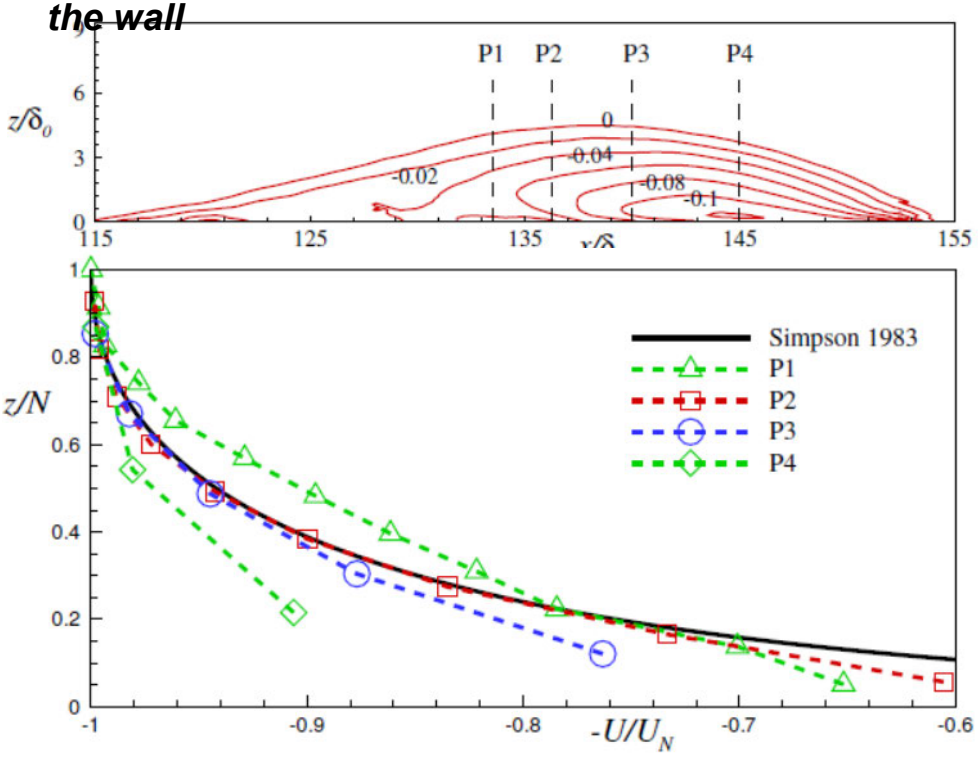


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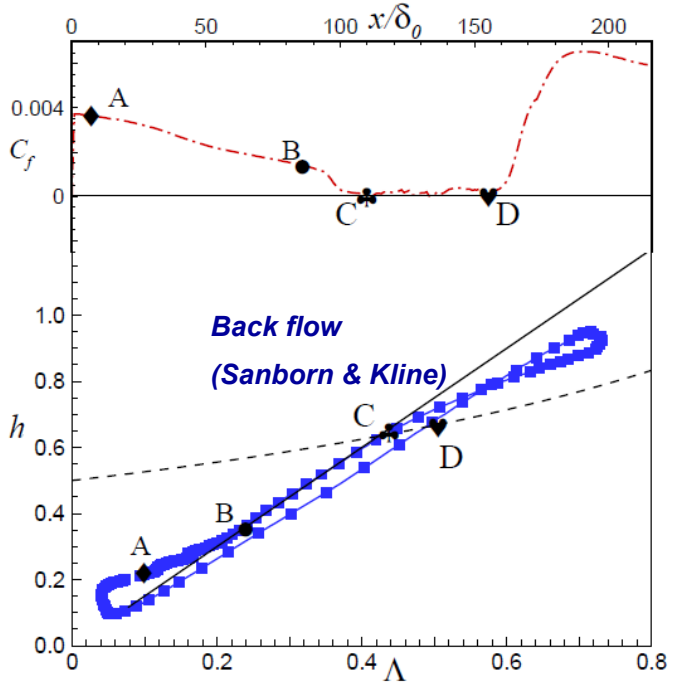
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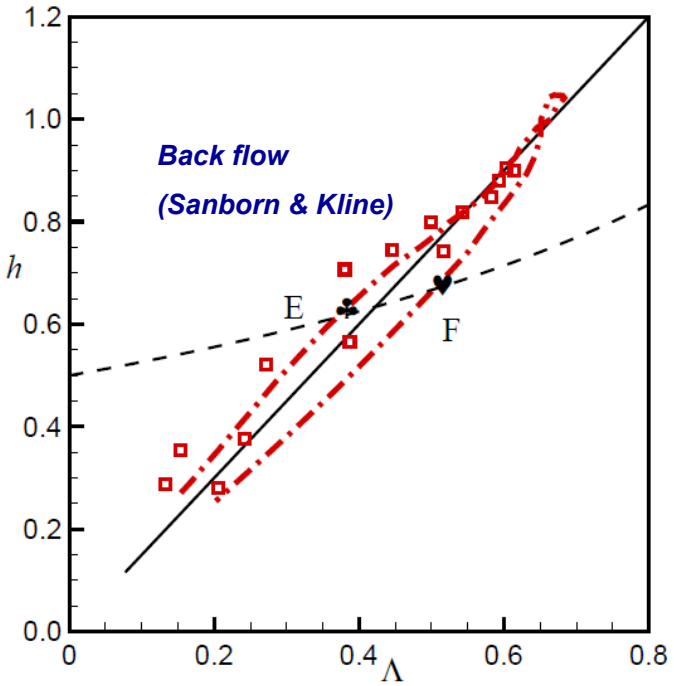
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More on Boundary Layer Thickness



(a): $h - \Lambda$ plot for case by Perry & Fairlie. —·— for C_f; horizontal line for $C_f = 0$;



(b): $h - \Lambda$ plot for case by Patrick. □ , experimental data; —·— , LES result.

$$h = (H - 1)/H$$

$$H \equiv \delta^*/\theta$$

$$\Lambda \equiv \delta^*/\delta_{99}$$

——— $h = 1.5\Lambda$: ***(Simpson 1983, Perry & Schofield, 1973)***

- - - - - - $h = 1/(2 - \Lambda)$: ***(Sanborn & Kline, 1961)***

fps
Discovery through simulation

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