A Higher Order Immersed Boundary Method for Viscous Compressible Flows

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**Science and Technology Corp.*

based on
C. Brehm, C. Hader, H.F. Fasel, AIAA-2014-2093

NASA Ames, AMS Seminar Series, 09/18/14

Outline

Motivation/Introduction
Motivation and introduction to immersed boundary methods.

Numerical Methods for Regular Grid Points
Space and Time Operators.

Immersed Boundary Method
All aspects of immersed boundary method for CNSE

Basic Concept
Introduces basic idea of immersed boundary method.

Extension to Compressible Navier-Stokes Equations
Presents additions to original IIM.

Imposition of Pressure BCs and Viscous Terms
Different BC extrapolations schemes and conservative/non-conservative IIM.

Error Convergence Study
Verifies formal order-of-accuracy with method of manufactured solution.

Stability Analysis
Demonstrates stability of immersed method.

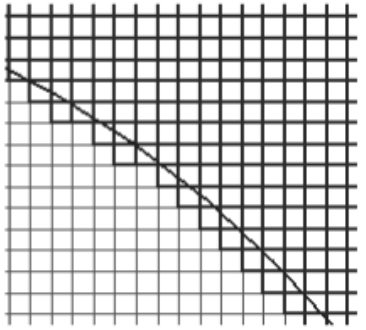
Computed Results
Flow around cylinder/sphere, porous wall, discrete roughness.

Motivation

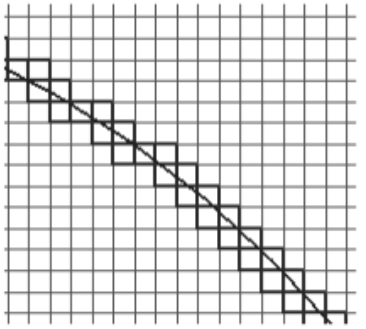
- Immersed Boundary Method -

- Pros
  - **Grid generation process is greatly simplified!**
  - Advantages for moving or deforming geometries: Fluid structure interaction, shape optimization, etc.
  - Cartesian mesh offers friendlier memory layout
- Cons
  - Usually lower order accurate (or robustness issues)
  - Boundary layer resolution for high Reynolds numbers
  - Less flexibility w.r.t. grid point distribution
- Main attributes of current method
  - Higher-order accurate
  - Sharp interface method
  - Optimal stencil coefficients w.r.t. numerical stability

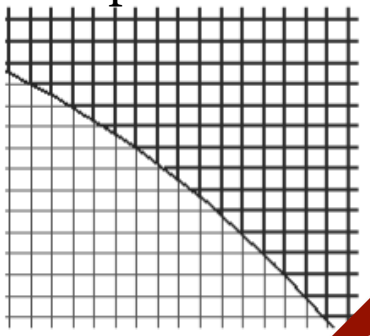
Stair-Step Approach



Forcing Approach



Sharp Interface

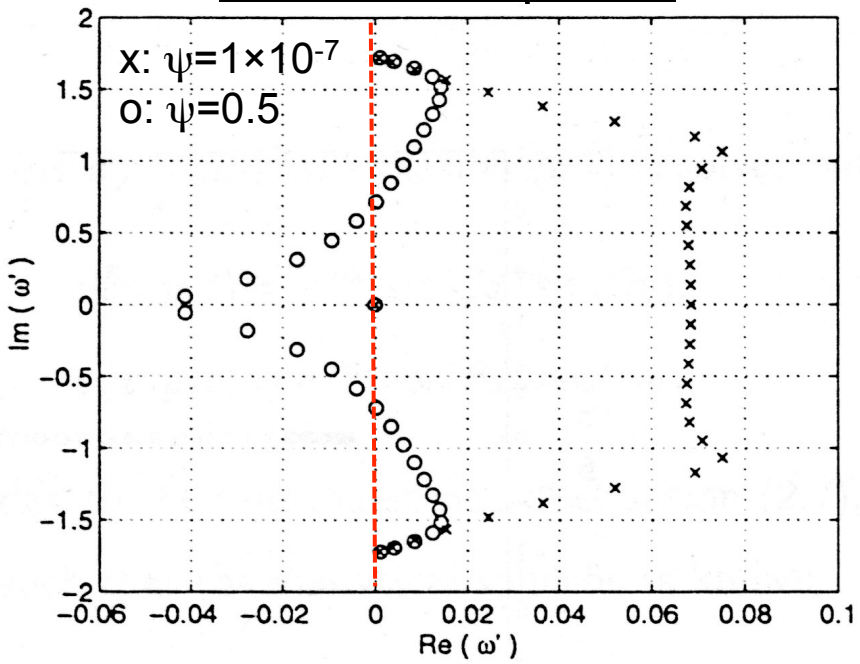


Motivation

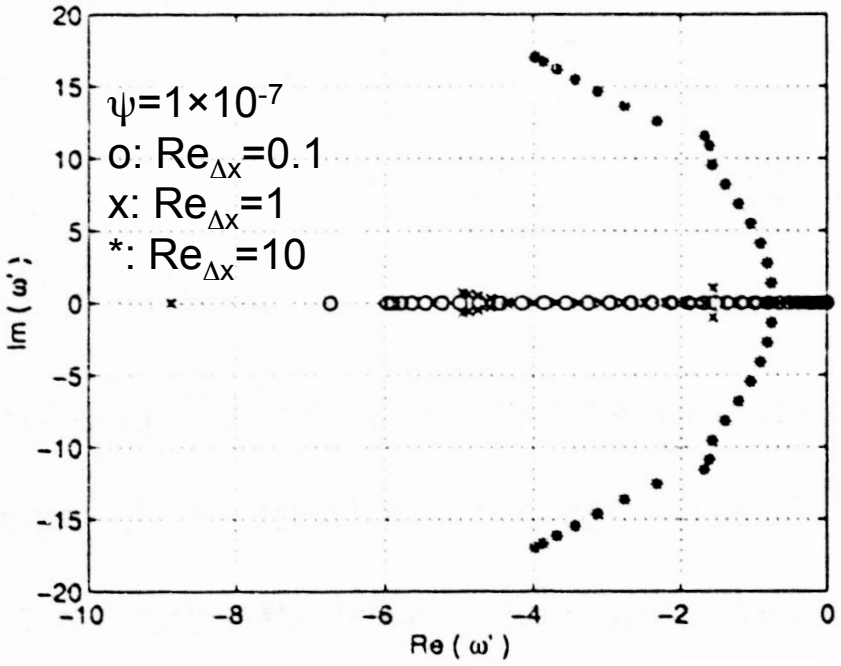
- Immersed Boundary Method -

Linnick's compact IB Method, $O(\Delta x^4)$, in streamfunction vorticity formulation:

Advection Equation



+Diffusion Equation



Definitions: $Re_{\Delta x} = c_x \Delta x/\mu = CFL/DFL$

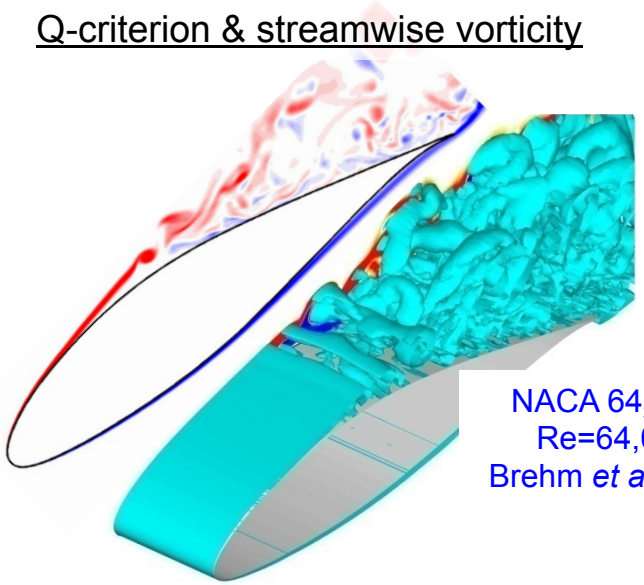
$CFL = c_x \Delta t/\Delta x$

$DFL = \mu \Delta t/\Delta x^2$

Linnick and Fasel (2004)

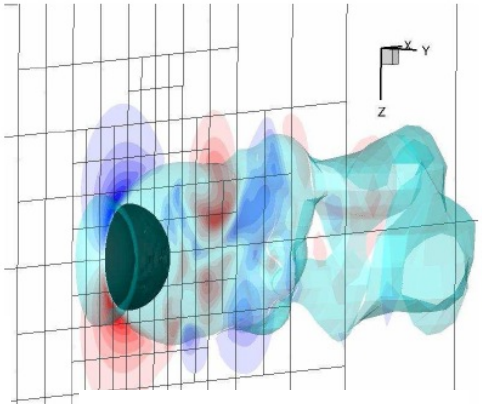
Incompressible Navier-Stokes

Q-criterion & streamwise vorticity



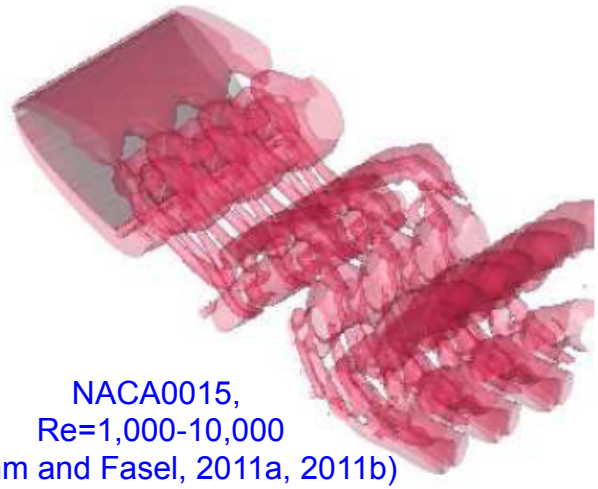
NACA 64_3-618,
Re=64,000,
Brehm *et al.*, 2008

Q-criterion & streamwise velocity



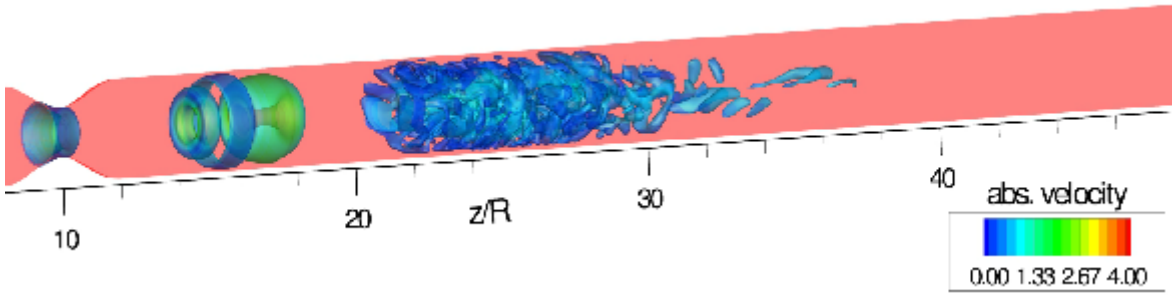
unsteady sphere flow,
Re=1,000

Q-criterion (Q=1)



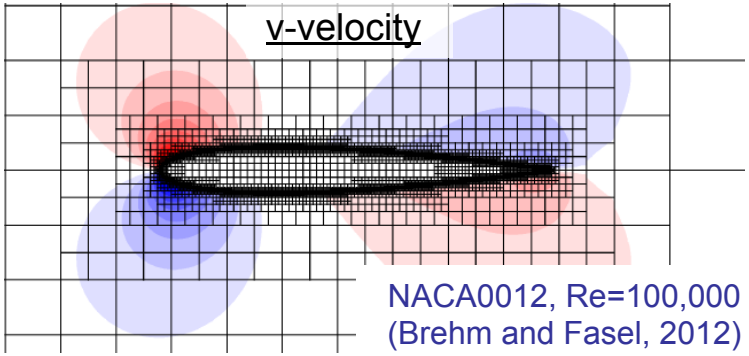
NACA0015,
Re=1,000-10,000
(Brehm and Fasel, 2011a, 2011b)

Q-criterion (Q=3)



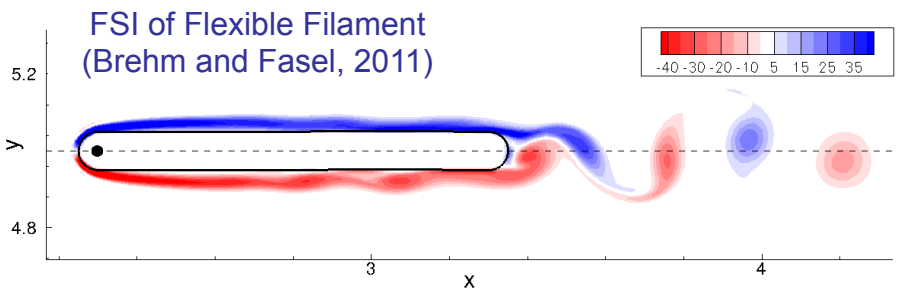
pulsatile stenotic flow, Re=500, S=75%, λ/R=8,Brehm *et al.*(2011, 2012)

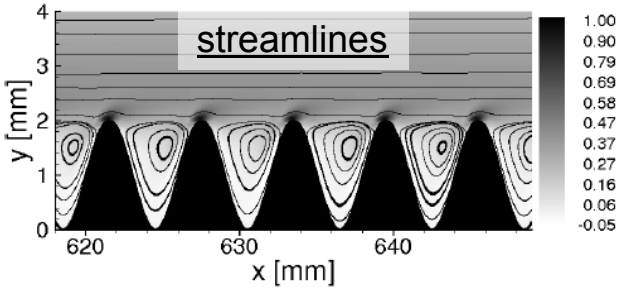
v-velocity



NACA0012, Re=100,000
(Brehm and Fasel, 2012)

FSI of Flexible Filament
(Brehm and Fasel, 2011)

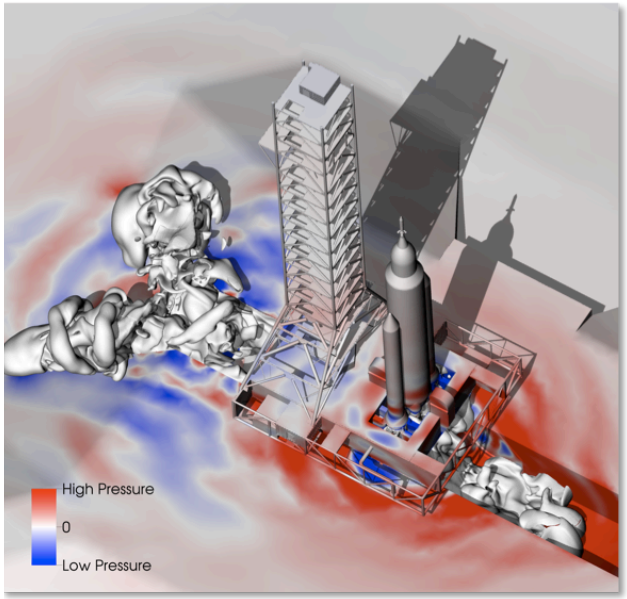




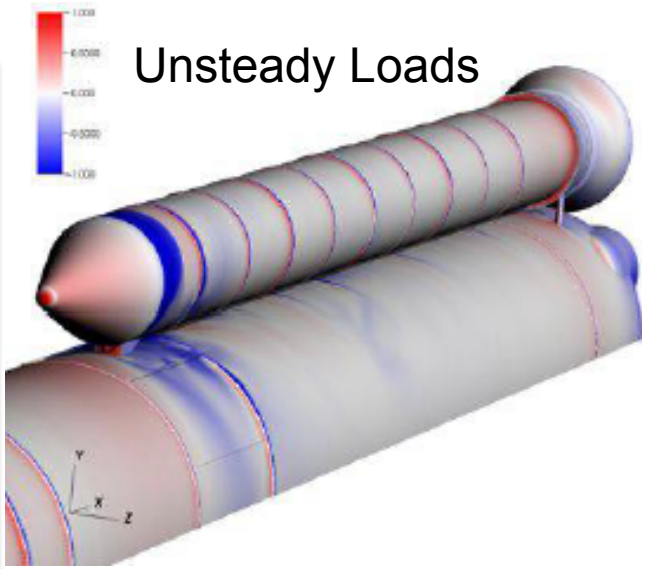
flow past distributed roughness,
Brehm *et al.* (2011),
experiments by Prof. Gaster, London, UK

Motivation – NASA Relevant IB Applications
Unsteady Flows

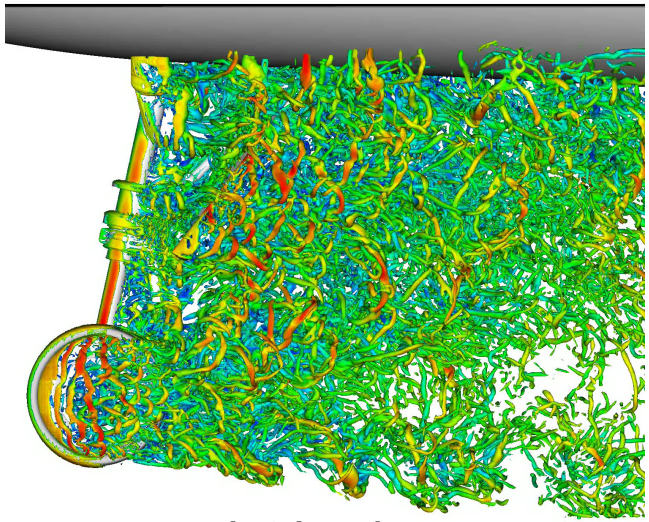
Ignition Over-Pressure Wave



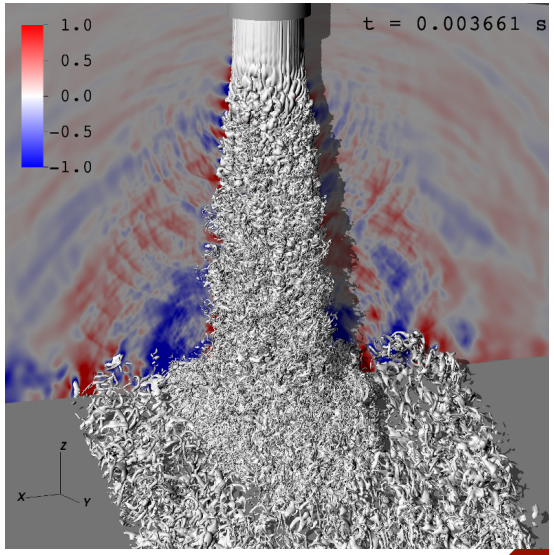
Unsteady Loads



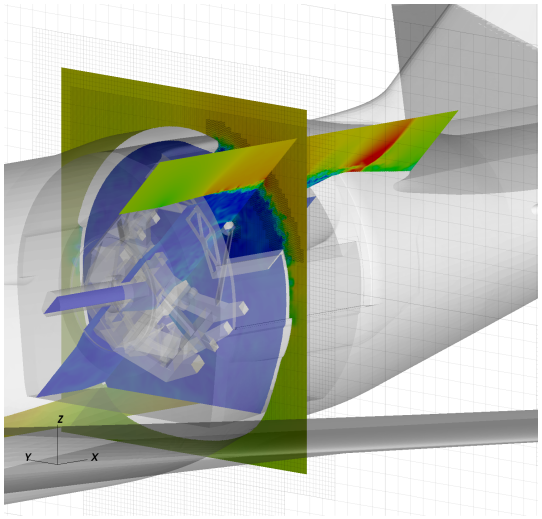
Landing Gear Noise (BANC III)



Jet Impingement



Open Cavity (SOFIA)



Flame Trench Pressure Environment



LAVA CFD solutions (LAVA developed at NASA Ames)

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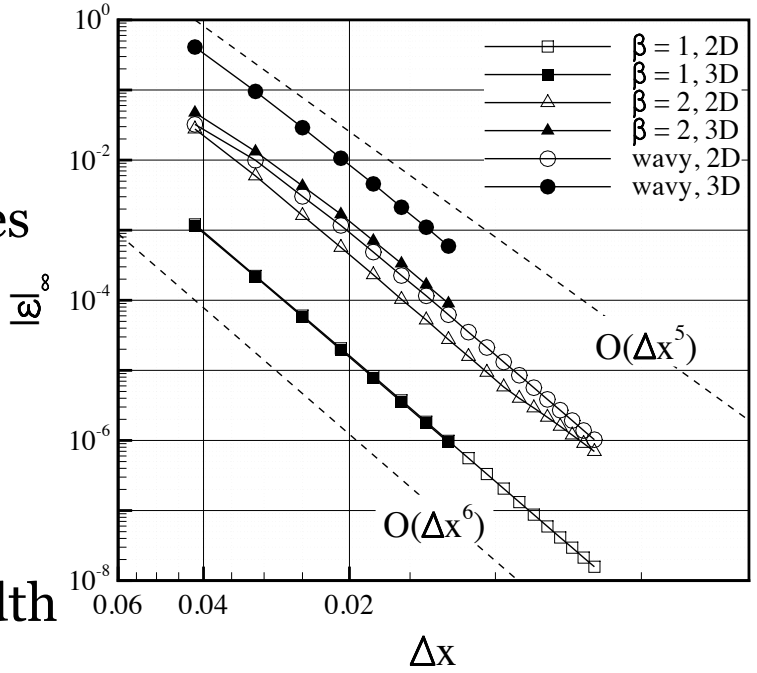
Interior Scheme

- Solving compressible Navier-Stokes equations
- Convective terms ~ O (Δx^{2n-1}), O(Δx^{2n})
  - Flux vector splitting
    (Lax Friedrichs, van Leer, AUSMPW+, etc.)
  - Various discretization options
    Here: node-centered upwind finite differences

$$\left.\frac{\partial \phi}{\partial x}\right|_i = \sum_{k=i-N_s}^{i+N_s} c_k \phi_k - \alpha \overline{\Delta x} \left.\frac{\partial^{2N_s-1}\phi}{\partial x^{2N_s-1}}\right|_i$$

c_k stencil coefficients, α upwind factor, Δx averaged grid spacing, and N_s stencil half width

Truncation Error (inviscid)
by method of manufactured solutions

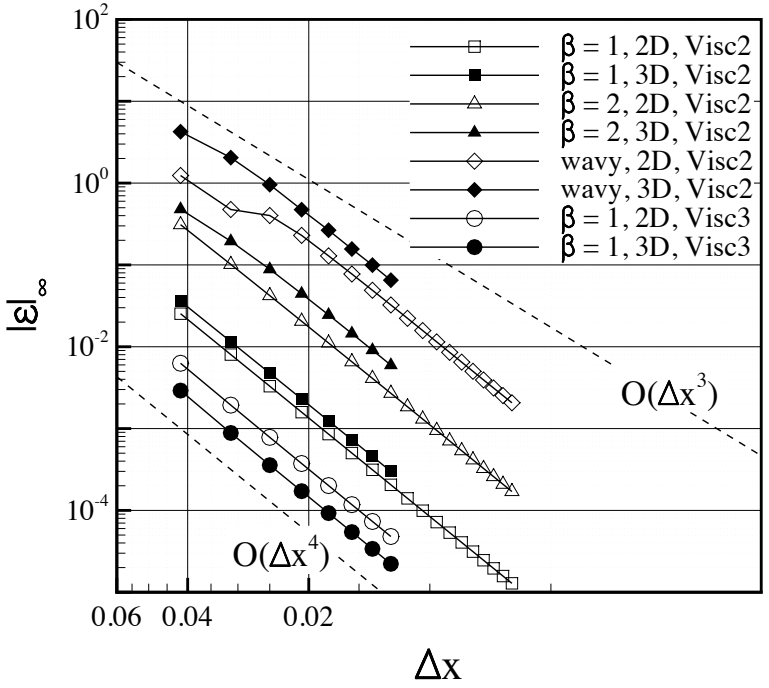


- For α=0: higher-order filtering (Visbal & Gaitonde, 1998)
- Higher-order shock capturing compact FD (AIAA-2014-1278)

Interior Scheme

- Runge-Kutta time-integrator (Shu-Osher form)
- Curvilinear coordinates
- Block-structured Cartesian with AMR
- Guard cell filling up to 4^{th}-order accurate
- MPI-parallel
- Viscous terms ~ $O(\Delta x^{2n})$, here $O(\Delta x^4)$

Truncation Error (viscous)
by method of manufactured solutions



- Conservative form (Visc2)

$$\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \left(\vec{\mathcal{F}} - \vec{\mathcal{F}}_v \right) = 0$$

- Non-conservative (Visc3)

$$\frac{\partial \left(\tau_{xy} \right)}{\partial y} = \frac{d\mu}{dT} \frac{\partial T}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x} \right)$$

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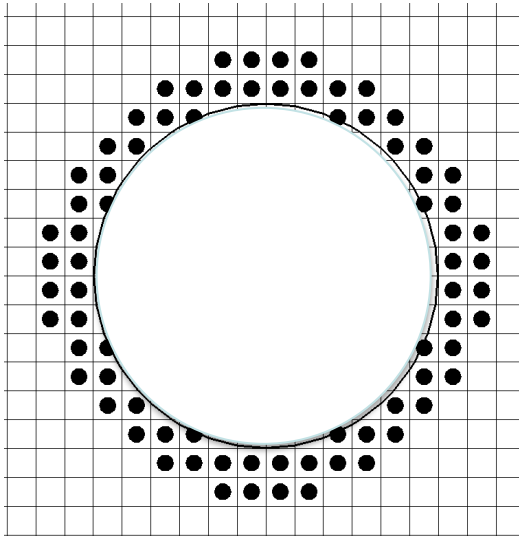
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Basic Concept of Stability Enhancement

- Initial observation:

"Stability of numerical scheme can be formulated as N-dimensional optimization problem"
(N=number of irregular grid points)



- Irregular grid point

- Enforce order-of-accuracy

$$\tau_i = \left(-c_x \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x}\left(\beta \frac{\partial \phi}{\partial x}\right) + f\right)_{x=x_i} - \left(\hat{\phi}_B \tilde{\alpha}_1 + \hat{\phi}_i \tilde{\alpha}_2 + \hat{\phi}_{i+1} \tilde{\alpha}_3 + \hat{\phi}_{i+2} \tilde{\alpha}_4 + \hat{\phi}_{i+3} \tilde{\alpha}_5 + C\right)$$

- Additional grid point is needed to introduce free parameter
- Objective function depends on the nature of the PDE, e.g., $\lambda_{r,max}$ or $\rho(A)$

$$\frac{\partial \phi}{\partial t} = -c_x \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x}\left(\beta \frac{\partial \phi}{\partial x}\right) + f \longrightarrow \underline{\mathbf{B}} \hat{\boldsymbol{\phi}}^{n+1} = \underline{\mathbf{A}} \hat{\boldsymbol{\phi}}^{n} + \hat{\mathbf{f}}$$

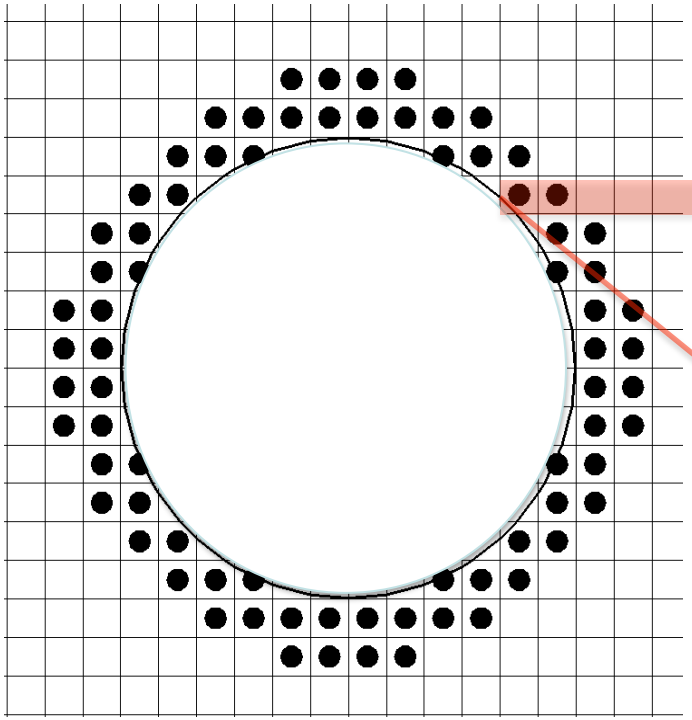
- Extract perturbation of irregular finite difference stencil (assume $\underline{\mathbf{B}}=\underline{\mathbf{I}}$)

$$\underline{\mathbf{A}} = \underline{\mathbf{E}}\underline{\mathbf{A}}_{id} \longrightarrow \rho(\underline{\mathbf{E}}) \leqslant 1$$

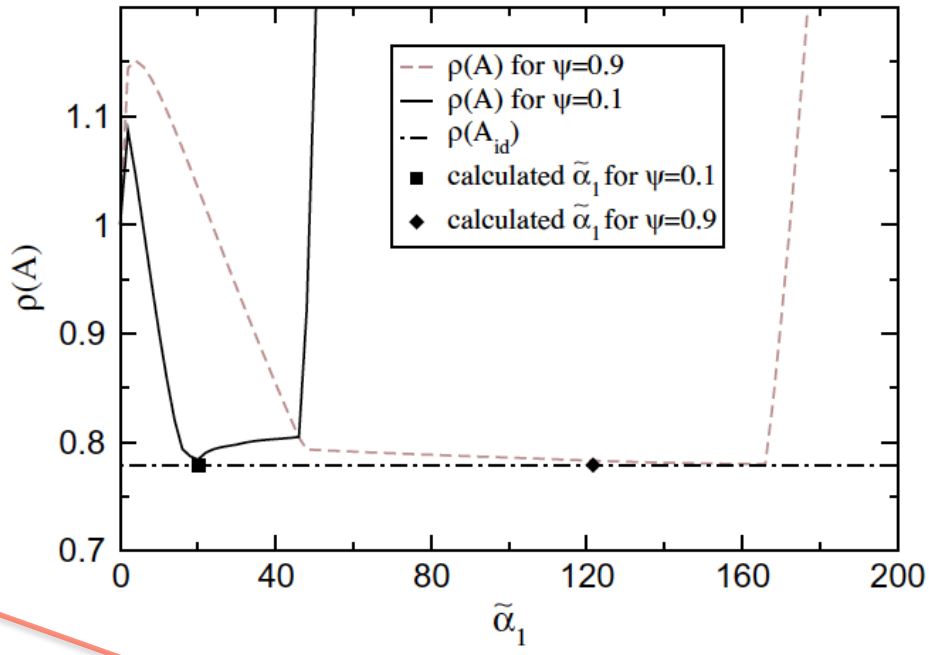
Brehm and Fasel (JCP, 2013)

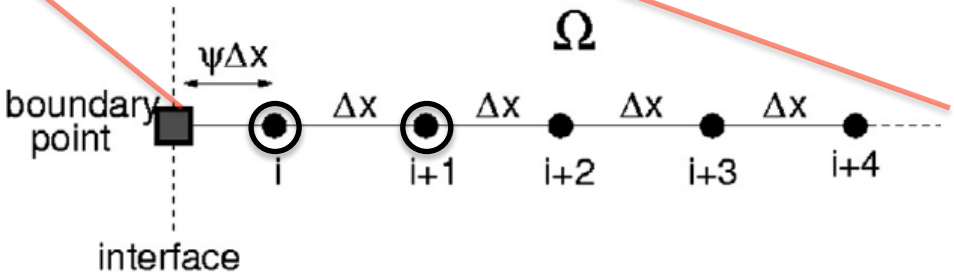
Basic Concept of Stability Enhancement

- Possible to solve N-dimensional optimization problem
- Apply localization of FD stencil
  - Turns N-dimensional problem into 1-D problem
  - More practical (e.g., FSI)
  - Localization has been demonstrated in Brehm and Fasel (2013)



optimization problem





Localization Assumption

- Analytical study for 1D advection-diffusion type equation

$$\underline{E}^T = \left(\underline{\mathrm{A}}_{id}^{-1}\right)^T \cdot \underline{A}^T$$

$$\underline{A}_{id}^T = \begin{bmatrix} 1-2\chi & \chi-\upsilon/2 & 0 & 0 & \cdots \\ \chi+\upsilon/2 & 1-2\chi & \chi-\upsilon/2 & 0 & \cdots \\ 0 & \chi+\upsilon/2 & 1-2\chi & \chi-\upsilon/2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

with $\chi=\mu\Delta t/\Delta x^2$ and $\nu=c_x\Delta t/\Delta x$

$$\underline{E} = \begin{bmatrix} a & b & c & d & e & \cdots & \cdots \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Recursion formula for (1,1)-coefficient of inverse of $\underline{\mathrm{A}}_{id}^T$

$$X_N = \frac{D_{N-1}}{D_N} = \frac{A}{B} \quad \text{with}$$

$$A = (1-2\chi) - (\chi^2 - \vartheta^2/4)X_{N-2}$$

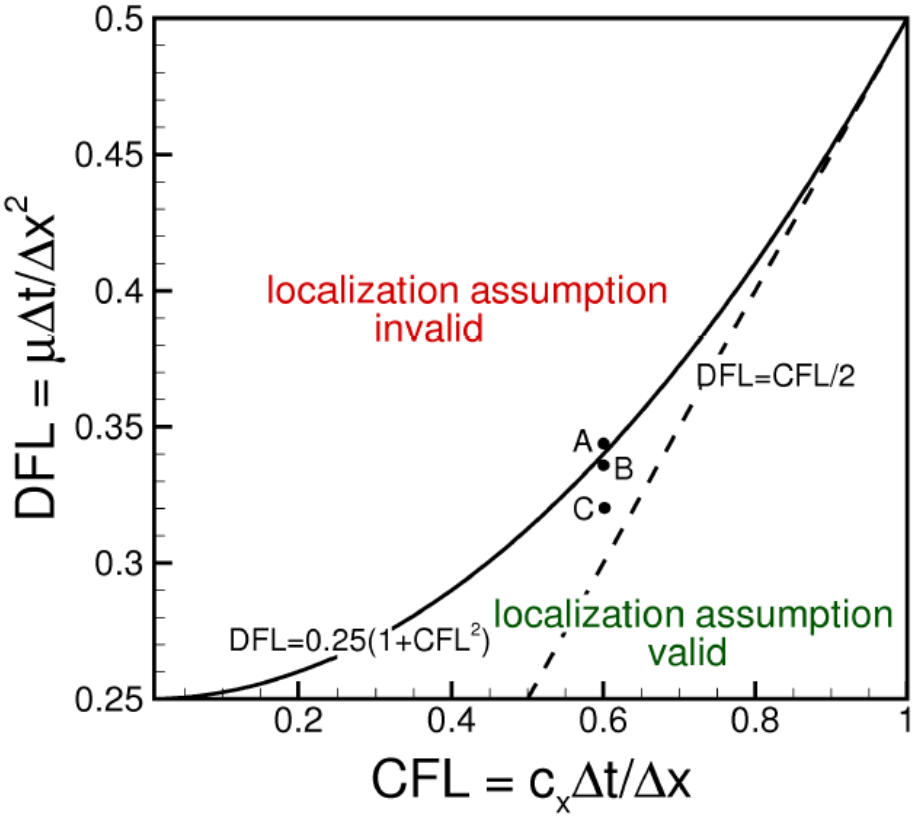
$$B = (1-4\chi+3\chi^2+\vartheta^2/4) - (\chi^2 - 2\chi^3 - \chi^2/4 + \chi\vartheta^2/2)X_{N-2}$$

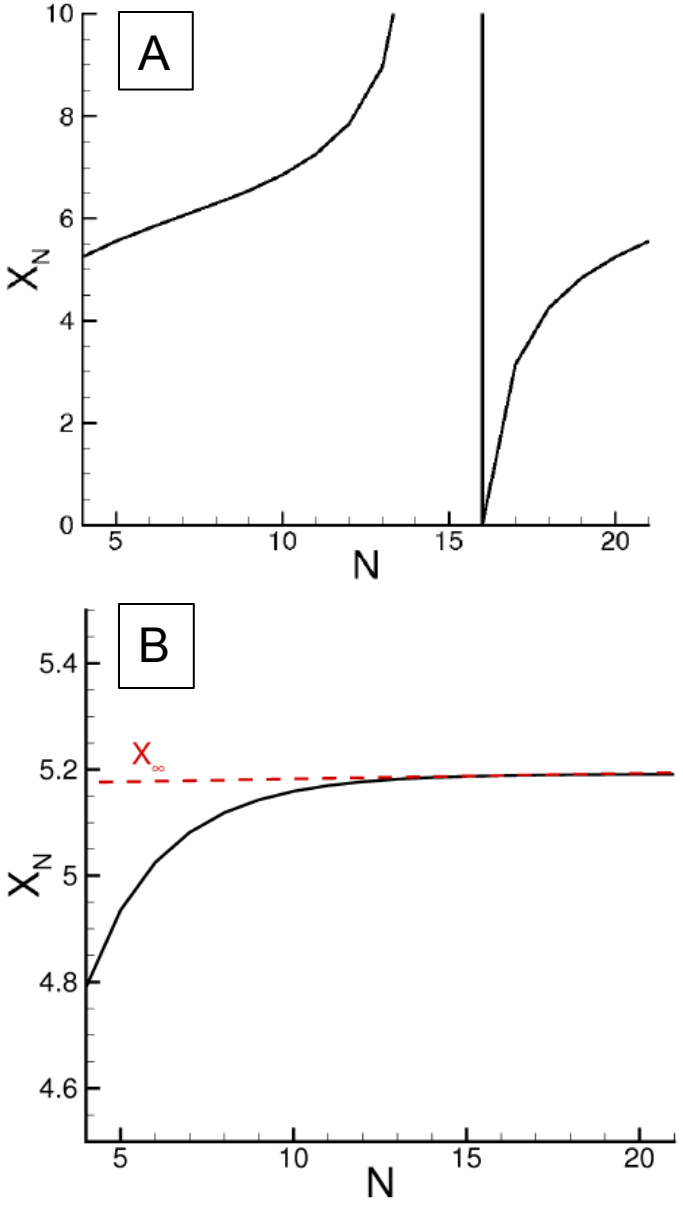
In the limit of

$$X_N \quad as \quad N \to \infty$$

$$X_{1,2} = \frac{1-2\vartheta \pm \sqrt{1-4\chi+\vartheta^2}}{2\chi^2 - \vartheta^2/2}$$

Localization Assumption



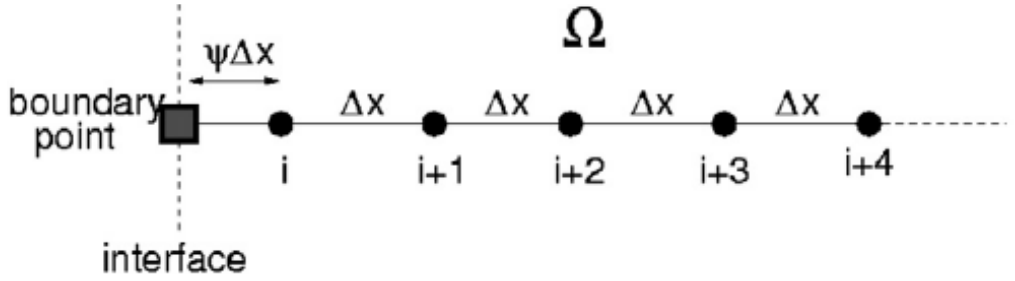


In the limit of $X_N \quad as \quad N \rightarrow \infty$

$$X_{1,2} = \frac{1 - 2\vartheta \pm \sqrt{1 - 4\chi + \vartheta^2}}{2\chi^2 - \vartheta^2 / 2}$$

Extension of IIM to Compressible NSE

- Basic ideas are discussed for 1D Euler-equation



$$\begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix}_t = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{pmatrix}_x$$

- Flux Vector Splitting (not excluding FDS)

$$\mathbf{W}_t = \mathbf{F}_x = \mathbf{F}_x^- + \mathbf{F}_x^+$$

- Linearization

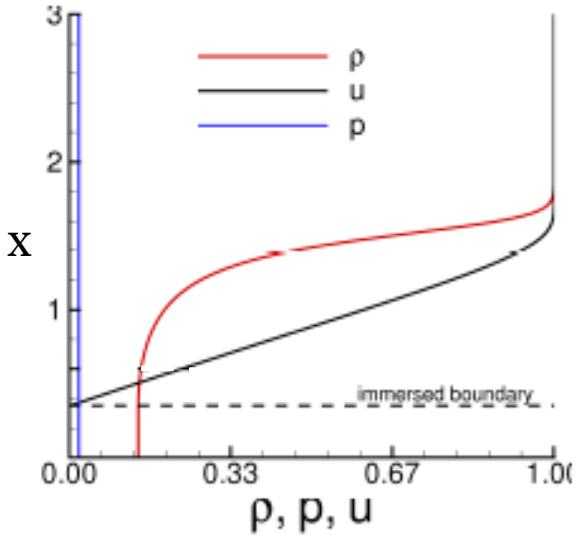
$$\mathbf{W}_t = \frac{\partial \mathbf{F}^+}{\partial \mathbf{W}} \cdot \frac{\partial \mathbf{W}}{\partial x} + \frac{\partial \mathbf{F}^-}{\partial \mathbf{W}} \cdot \frac{\partial \mathbf{W}}{\partial x}$$
$$\cong \frac{\partial \mathbf{F}^+}{\partial \mathbf{W}} \cdot (\underline{\mathbf{D}}_x^+ \cdot \mathbf{W}) + \frac{\partial \mathbf{F}^-}{\partial \mathbf{W}} \cdot (\underline{\mathbf{D}}_x^- \cdot \mathbf{W})$$

- Forward Difference, $\underline{\mathbf{D}}_x^+$
  - apply stencil optimization procedure
- Backward Difference, $\underline{\mathbf{D}}_x^-$
  - apply WLSQR stencil

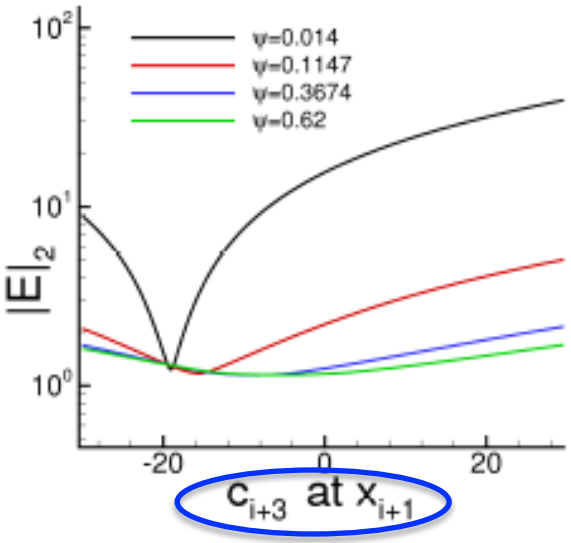
- Extensions of original approach:
  - strongly nonlinear equation
  - system of equation
  - different types of BCs
  - general 3D formulation

Local Optimization Problem

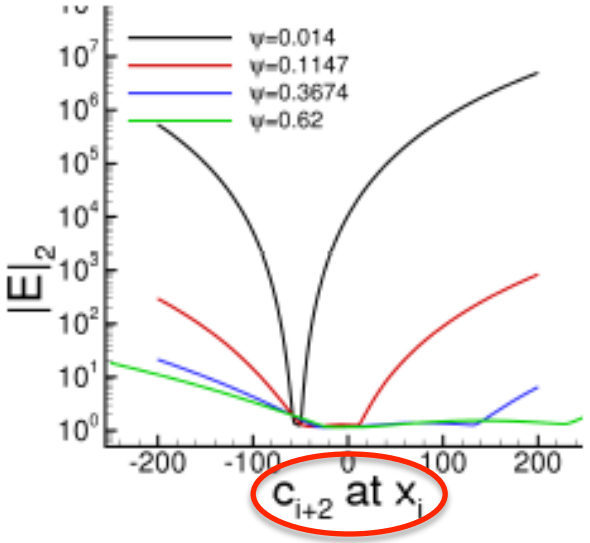
(ρ, u, p) profiles



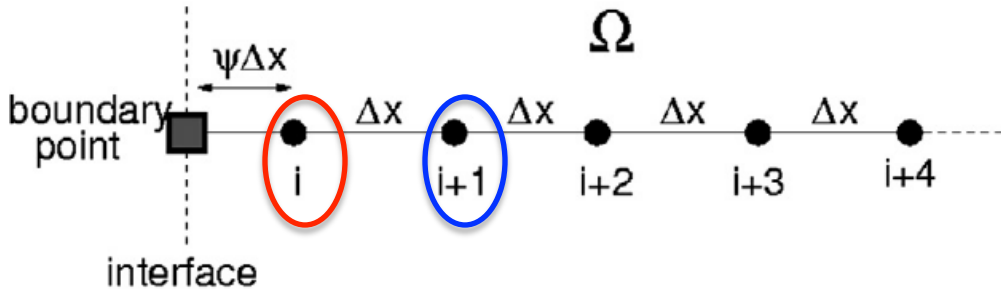
$|E|_2$ vs c_{i+3}



$|E|_2$ vs c_{i+2}

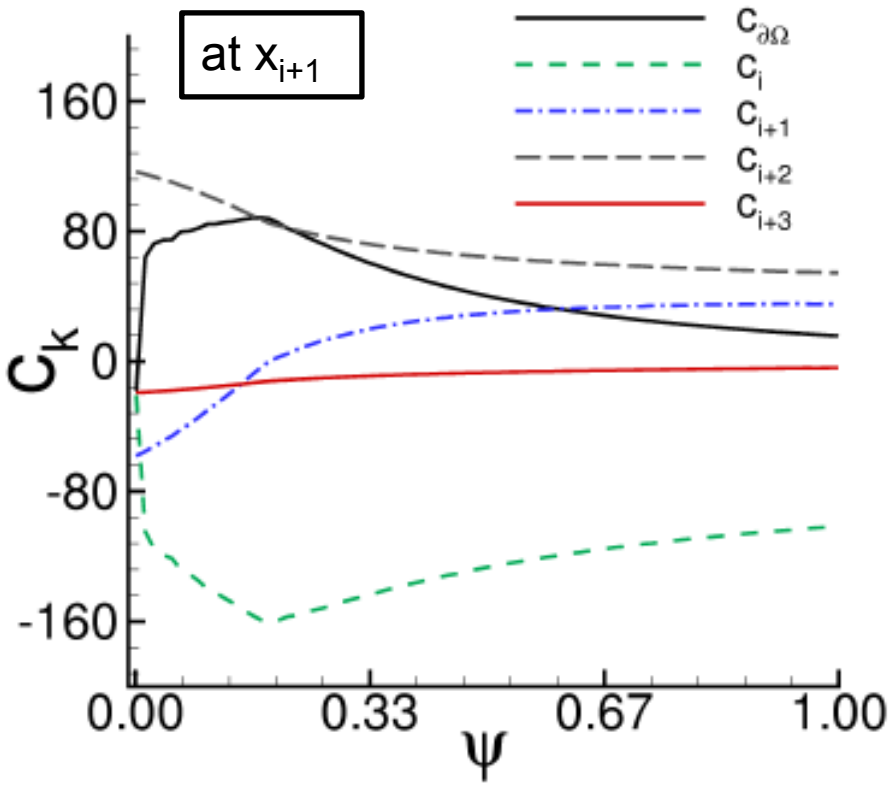


- Test problem conditions: M6 boundary layer flow
- Instead 1×2D we solve 2×1D
- For small ψ, $|E|_2$ distribution becomes more narrow
- Strong effect on c_{i+2} and c_{i+3}

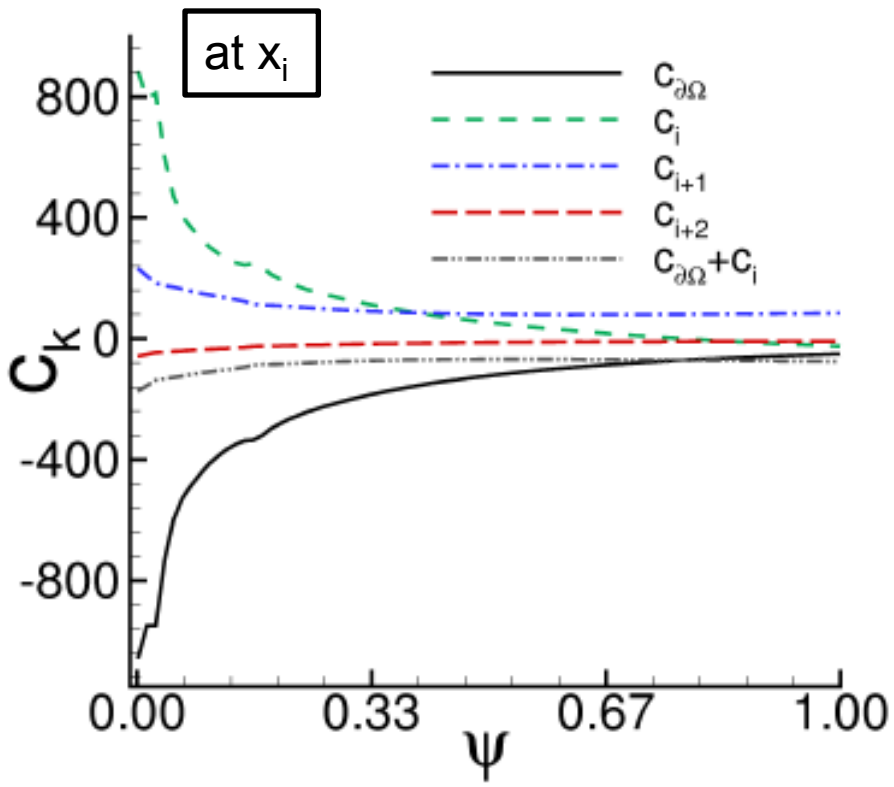


Optimal Stencil Coefficients

Stencil coefficient at x_{i+1}



Stencil coefficient at x_i



- coefficients well behaved for $\psi \rightarrow 0$ at x_{i+1}
- sum of $c_{\delta\Omega}+c_i$ well behaved for $\psi \rightarrow 0$ at x_i
- small adjustment for ψ close to 1

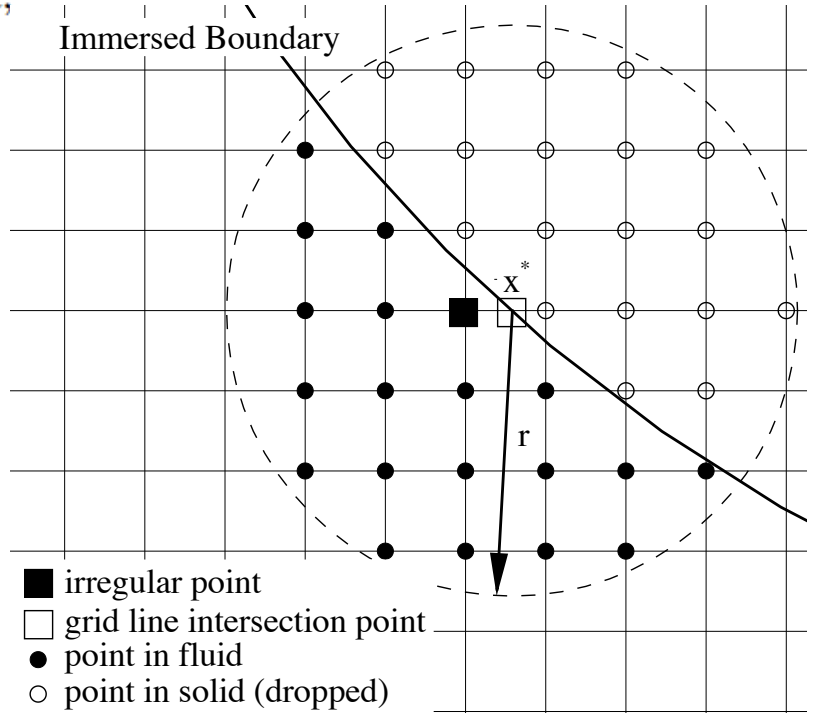
Imposition of Boundary Conditions

- Numerical scheme is very sensitive to extrapolation procedure of the BC
- Boundary conditions applied at grid line intersections
- Weighted least square method (WLSQR)

$$\Delta\phi^*_{i_k,j_k,k_k} = \Delta\phi^* - \Delta x^*_k\phi^*_x - \Delta y^*_k\phi^*_y - \Delta z^*_k\phi^*_z + \frac{(\Delta x^*_k)^2}{2!}\phi^*_{xx} - \frac{(\Delta y^*_k)^2}{2!}\phi^*_{yy} - \frac{(\Delta z^*_k)^2}{2!}\phi^*_{zz} +$$
$$\Delta x^*_k\Delta y^*_k\phi^*_{xy} - \Delta x^*_k\Delta z^*_k\phi^*_{xz} - \Delta y^*_k\Delta z^*_k\phi^*_{yz} - \text{ h.o.t},$$

$$w_k = \frac{1}{((\Delta x^*)^2 + (\Delta y^*)^2 + (\Delta z^*)^2)^{p/2}}, \text{ with } p/2 \geq 0$$

$$\left(\sum_{k=1}^{n_R+n_A} w_k(\Delta\phi^*_{i_k,j_k,k_k})^2\right) \rightarrow \min$$



- Points are selected from candidate point cloud close to immersed boundary
- Substitute Robin BC for pressure and temperature
- Rank based stencil selection step (WLSQR+)

Viscous Terms

- Conservative Approach

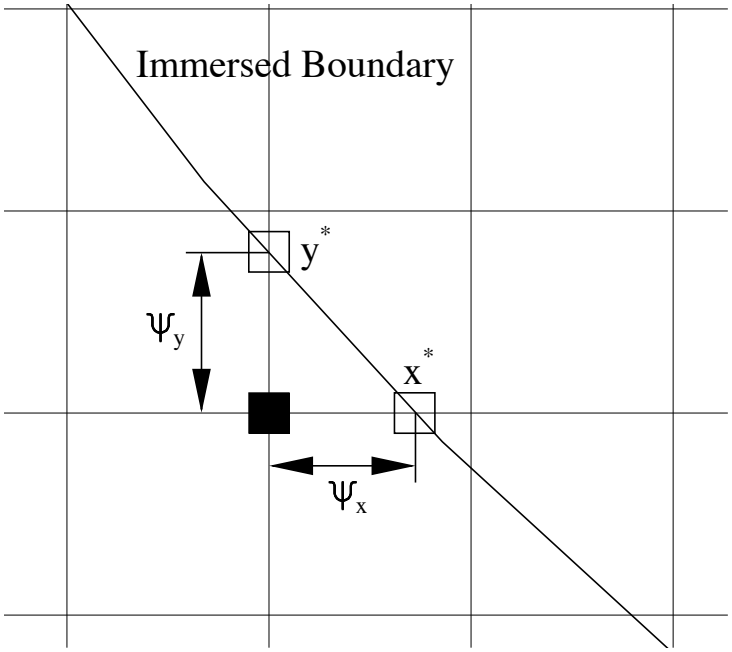
$$\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \left(\vec{\mathcal{F}} - \vec{\mathcal{F}}_v \right) = 0$$

  - Only first derivatives are applied

$$\left.\frac{\partial \phi}{\partial x}\right|_k = c_{\partial\Omega}\phi_{\partial\Omega} + \sum_{m=1}^{p_k+1} c_m \phi_{i+m-1}$$

  - Apply WLSQR+ to determine coefficients
  - Viscous flux boundary condition

BC for cross-derivatives



- Non-conservative approach

$$\frac{\partial (\tau_{xy})}{\partial y} = \frac{d\mu}{dT}\frac{\partial T}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right)$$

  - Apply WLSQR+ to determine coefficients of higher derivatives, e.g,

$$\left.\frac{\partial^2 \phi}{\partial x^2}\right|_k = c_{\partial\Omega}\phi_{\partial\Omega} + \sum_{m=1}^{p_k+2} c_m \phi_{i+m-1} \quad \text{and} \quad \left.\frac{\partial^2 \phi}{\partial x \partial y}\right|_{i,j,k} = c_{\partial\Omega}\phi_{\partial\Omega} + \sum_{m=1}^{n_R-1+n_A} c_m \phi_{i_m, j_m, k_m}$$

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Verifies formal order-of-accuracy with method of manufactured solution.

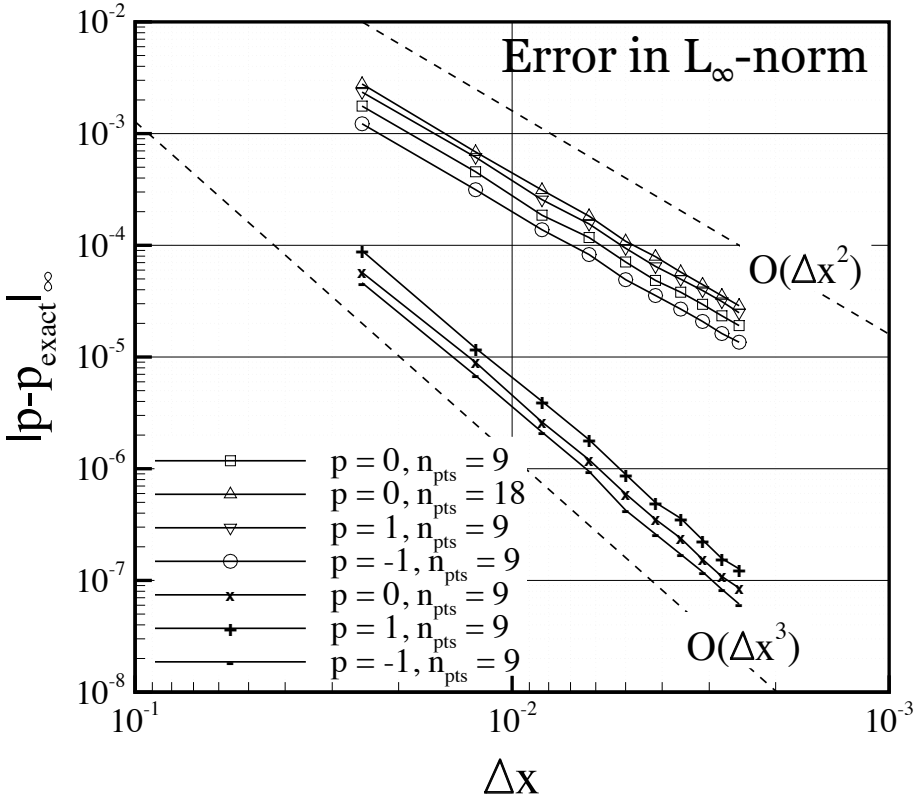
Stability Analysis
Demonstrates stability of immersed method.

Computed Results
Flow around cylinder/sphere, porous wall, discrete roughness.

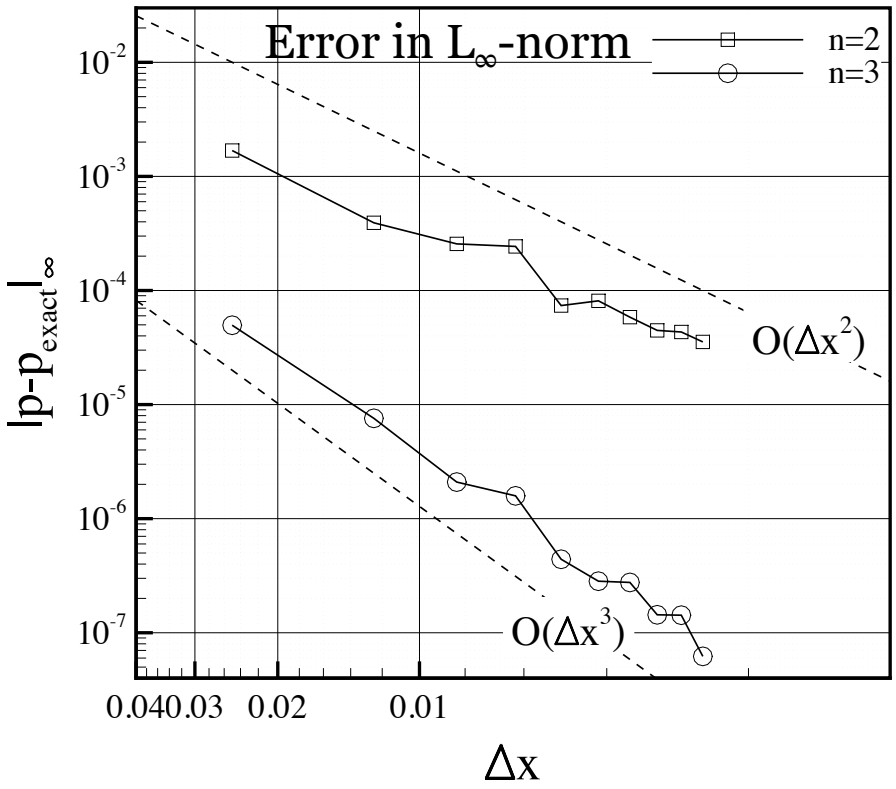
Truncation Error Study

- Boundary Condition Extrapolation

over-determined system



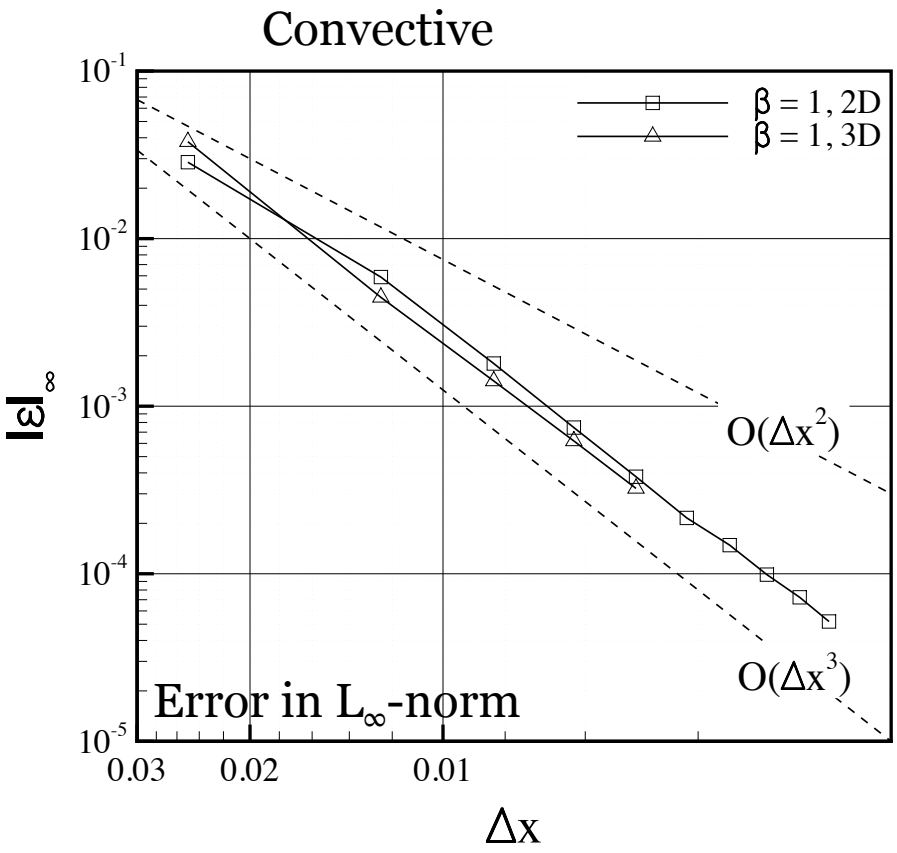
minimum number of grid points

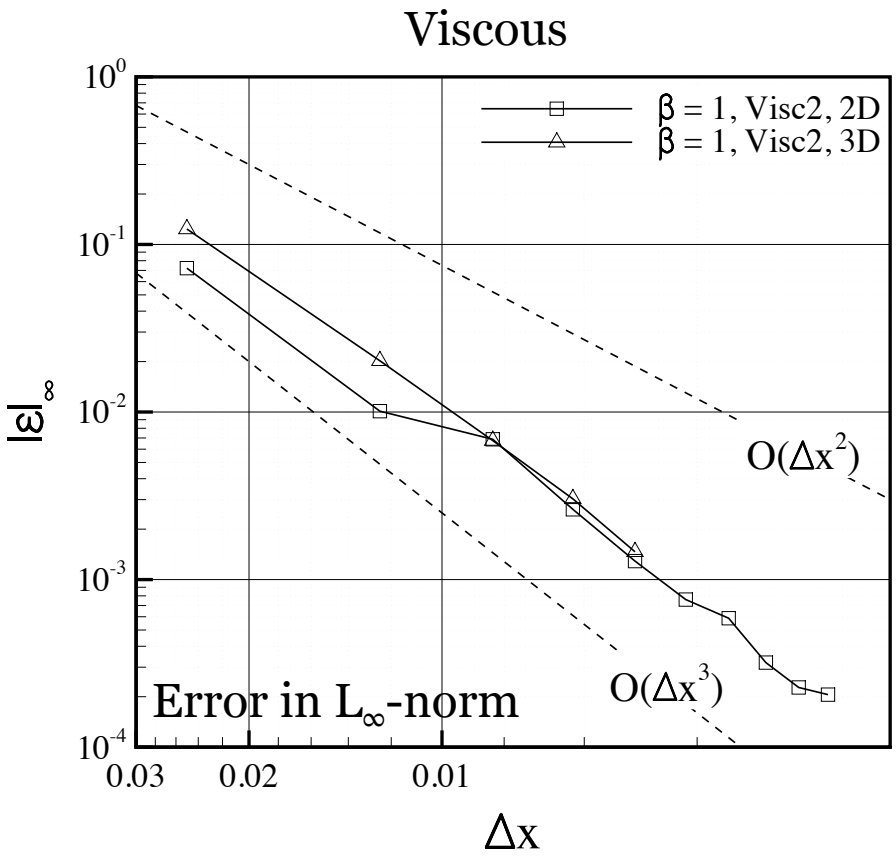


- Error increases for larger p
- Non-monotone behavior for minimum number of grid points (MNGP)
- Robustness issues for MNGP; use over-determined system (N_R+N_A)

Truncation Error Study

- Convective Terms





- Uses exact flux-BC at the wall
- Formal order-of-accuracy verified
- Monotone behavior for convective flux differentiation
- Non-conservative more accurate than conservative approach (not shown)

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Stability Analysis (1D Spatial)

- Lax-Richtmyer equivalence theorem:
  Ensuring convergence by consistency and Lax-stability in semi-discrete form
- Euler equations linearized about the mean flow field (here Mach 6 BL flow)
- Error equation is assumed to be homogeneous

$$\underline{\tilde{B}}\frac{d\varepsilon}{dt} = -\frac{1}{\Delta x}\underline{\tilde{A}}\varepsilon, \qquad \text{with BC:} \quad \varepsilon_0(t) = 0, \qquad \text{and IC:} \quad \varepsilon(t) = f(x_i)$$

- Considering ε-pseudo eigenvalue spectrum (non-orthogonal EVs)
- Generalized eigenvalue problem

$$\left(\underline{\tilde{B}}\lambda_\epsilon + \frac{1}{\Delta x}\underline{\tilde{A}} - \underline{AD}\right)\boldsymbol{v} = 0,$$

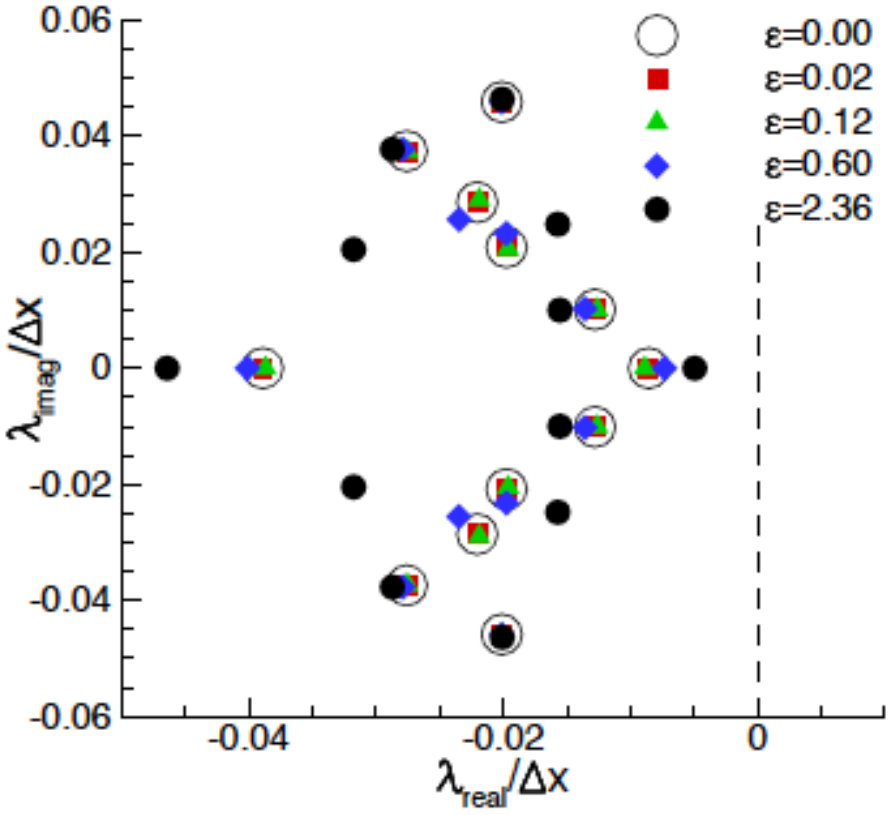
perturbation matrix with $\|\underline{D}\| = \epsilon.$

- Ensure algebraic stability of the full discretization for locally stable, one step methods

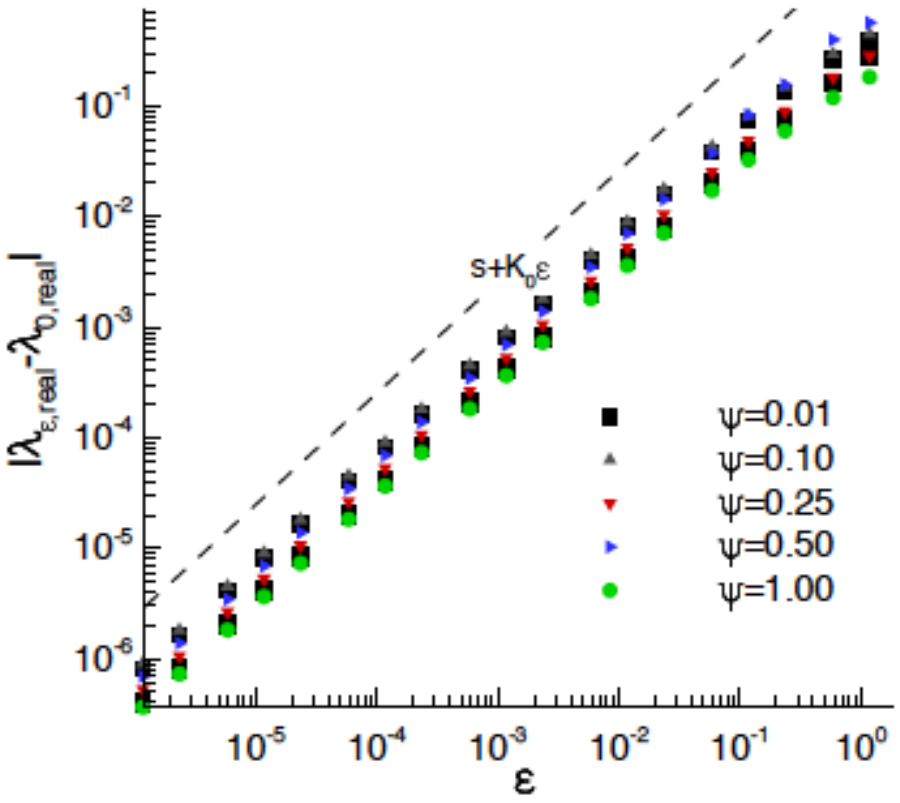
$$\lambda_{\epsilon,real} \leq s + K_0\epsilon \quad \forall\, \epsilon \geq 0$$

Stability Analysis (1D Spatial)

Close-Up of ε-Pseudo Eigenvalue Spectrum



Algebraic Stability



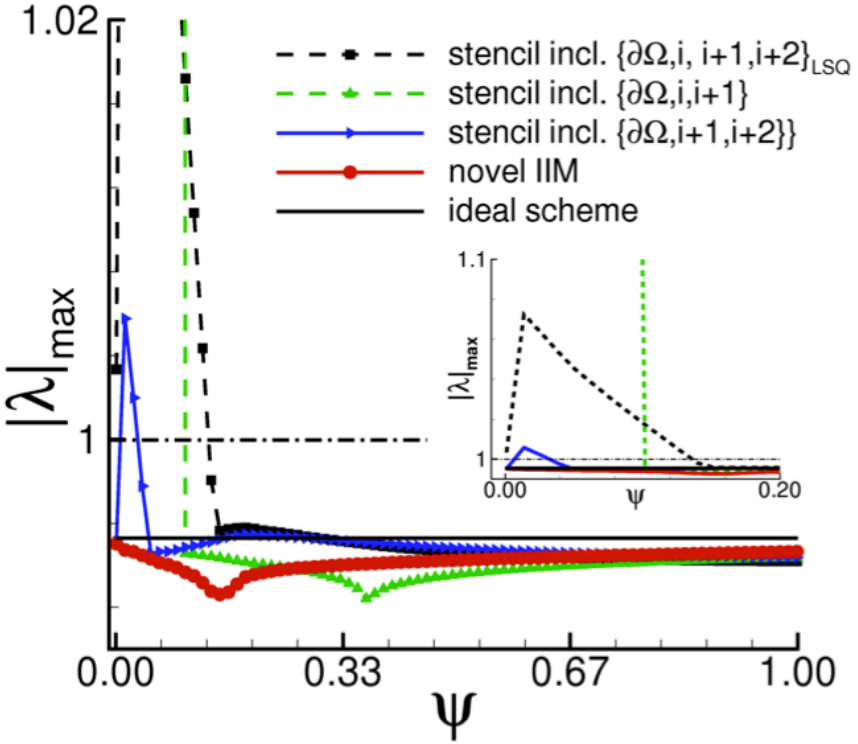
- Different realizations and grids considered but not shown here

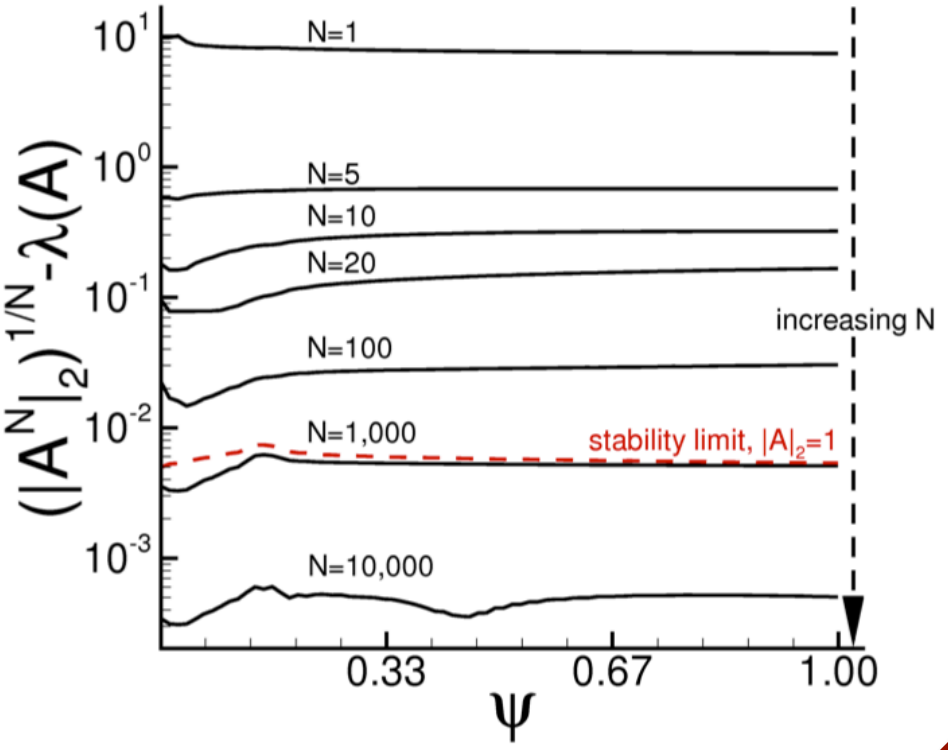
Stability Analysis (+temporal)

- 1D stability analysis of Euler equations (with RK4)
- None of the other schemes is stable in the entire range
- Convergence towards spectral radius by Gerschgorin formula

$$\rho(\underline{\mathbf{A}}) = \lim_{k\to\infty} \| (\underline{\mathbf{A}})^k \|^{1/k}, \forall\ k \in \mathbb{N}$$

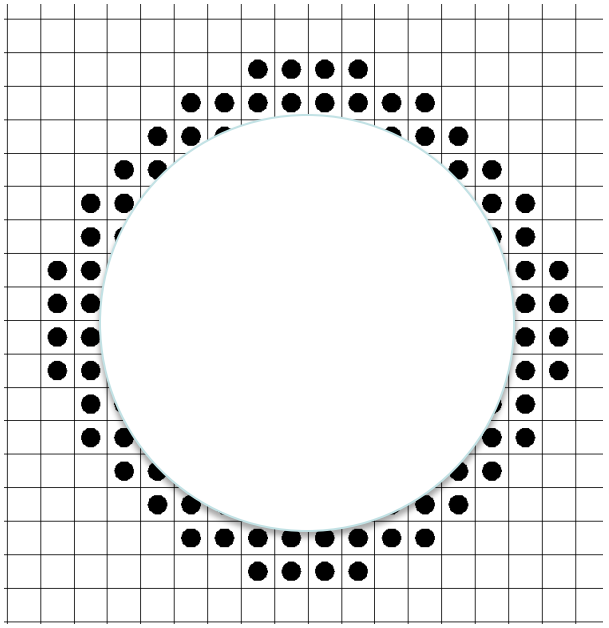
- Necessary condition $\rho(\underline{\mathbf{A}}) \leq 1$ and sufficient condition $|\underline{\mathbf{A}}| \leq 1 + C\Delta t$





2D Stability Analysis 2D

- Unit circle immersed into 4x4 domain
- Vary number of grid points
  N=20, 40, and 80
- Cylinder potential flow solution used as baseflow
- u_∞=34.72, p_∞=1 atm, T_∞=300, γ=1.4, R=287



- Irregular grid point

$$\rho(x,y) = \rho_\infty,$$

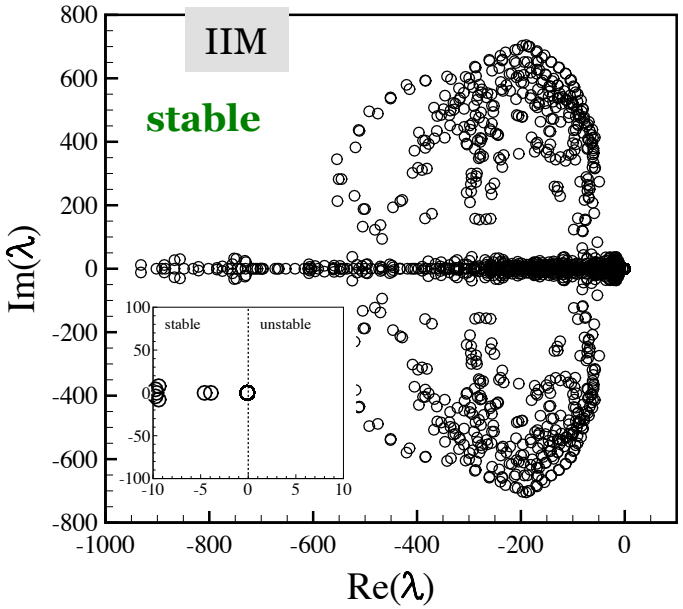
$$u(x,y) = \frac{U_\infty}{r^2}\left[(x-x_0)^2\left(1-\frac{R_0^2}{r^2}\right)+(y-y_0)^2\left(1+\frac{R_0^2}{r^2}\right)\right],$$

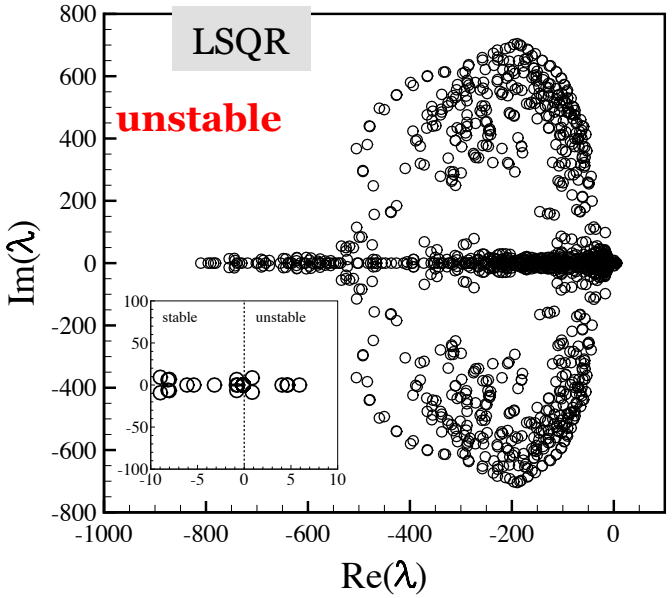
$$v(x,y) = \frac{U_\infty}{r^2}\left[(x-x_0)(y-y_0)\left(1-\frac{R_0^2}{r^2}\right)-(x-x_0)(y-y_0)\left(1+\frac{R_0^2}{r^2}\right)\right],$$

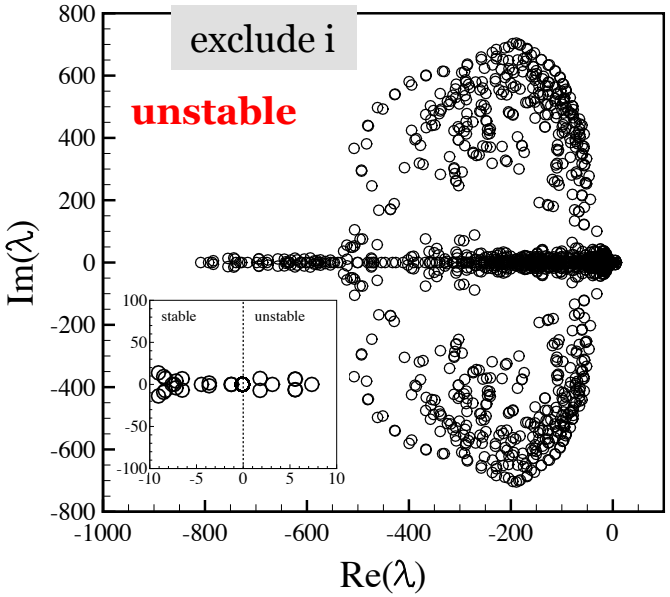
$$p(x,y) = \frac{1}{2}\rho_\infty\left(U_\infty^2-\left[u(x,y)^2+v(x,y)^2\right]\right), \text{ and}$$

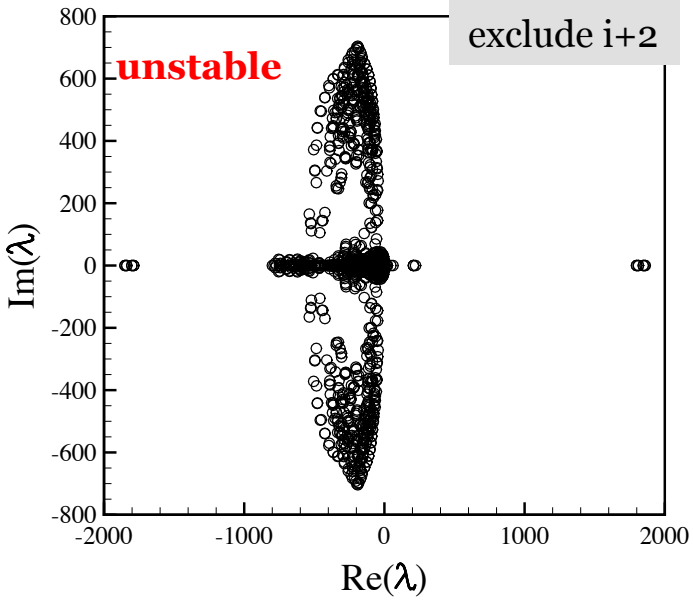
$$T(x,y) = \frac{p(x,y)}{R\rho(x,y)},$$

Stability Analysis 2D, Spatial Operator

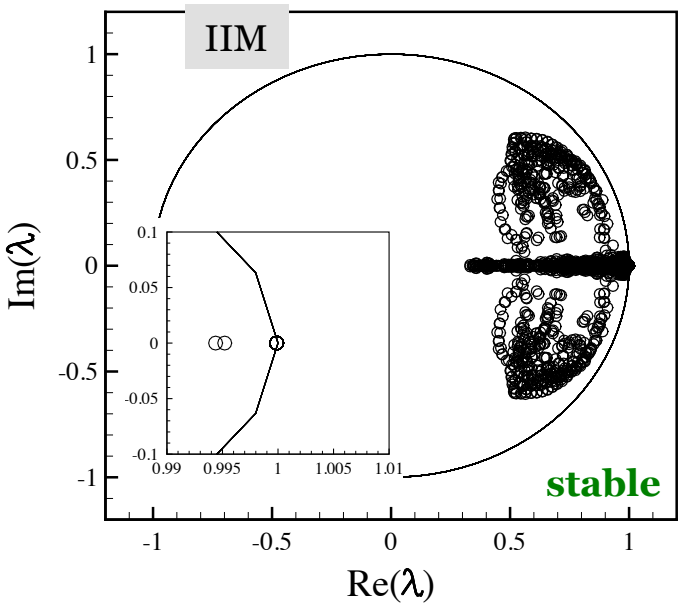


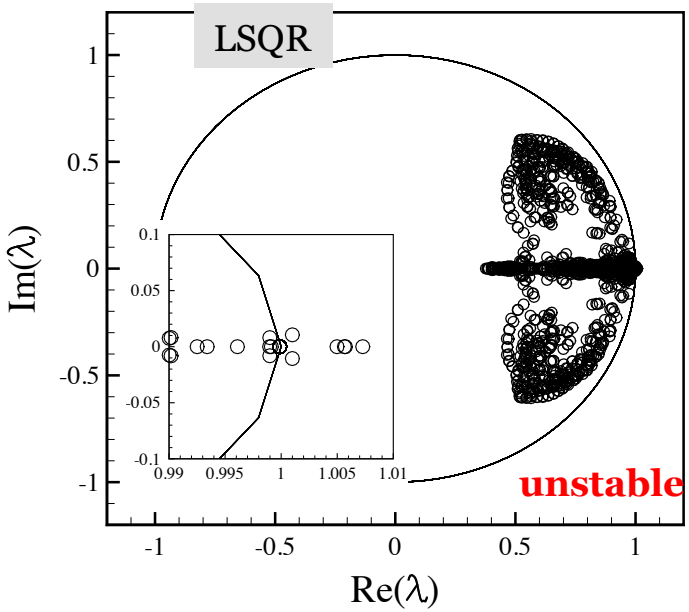


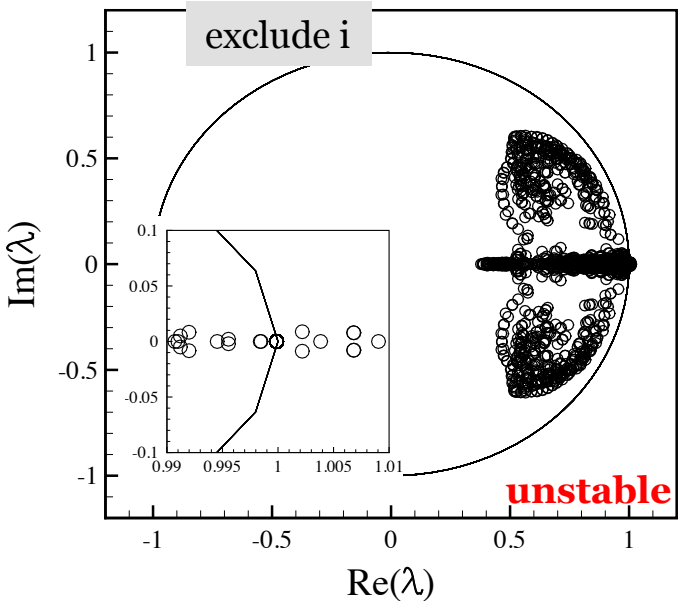


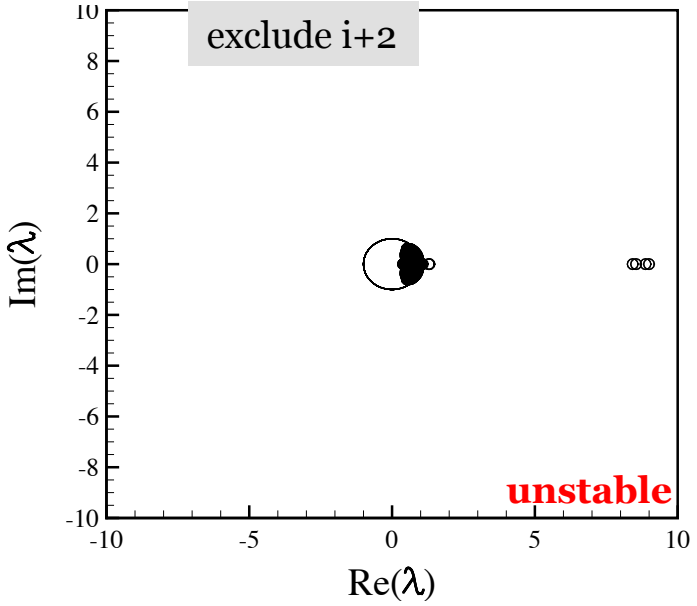


Stability Analysis 2D, Coupled Operator



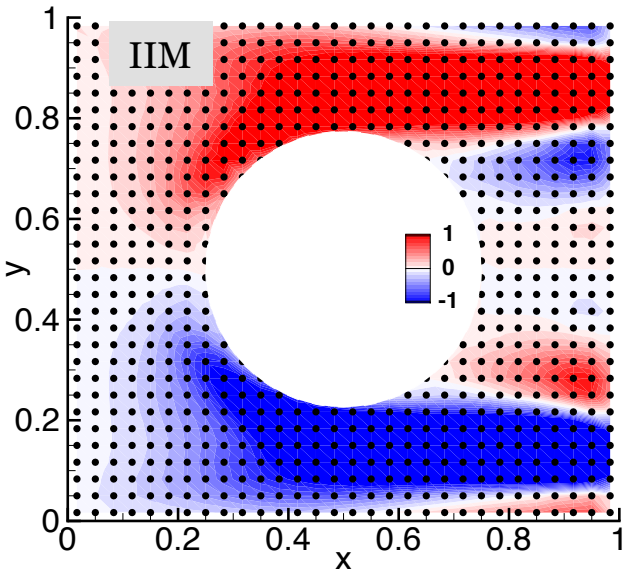


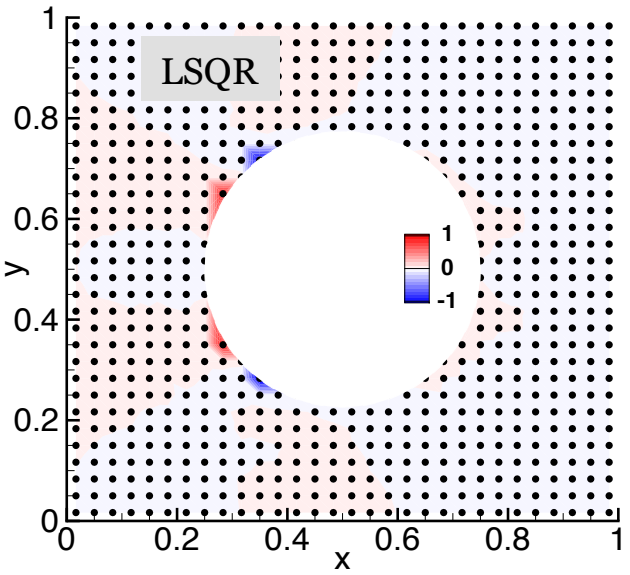


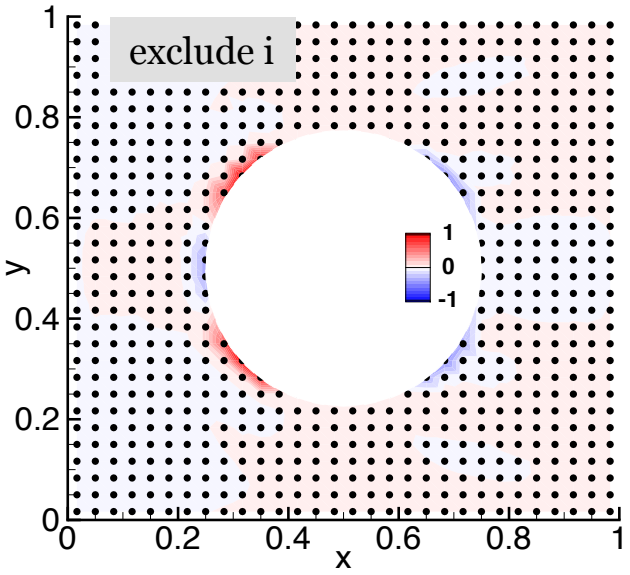


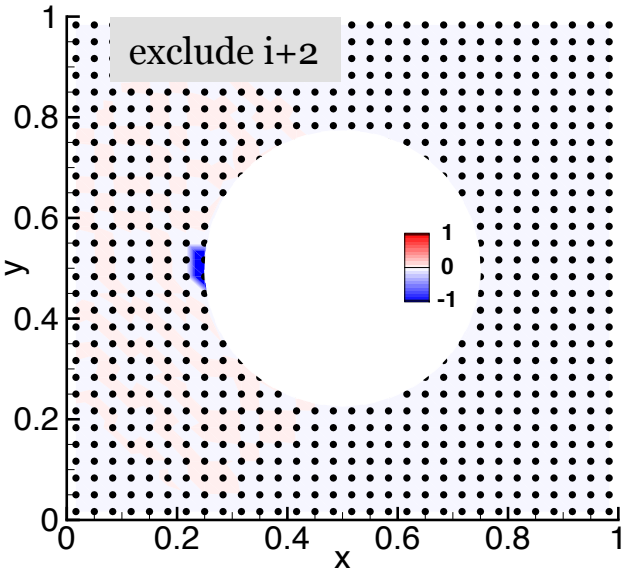
Stability Analysis 2D, Coupled Operator

x-momentum component of least decaying mode









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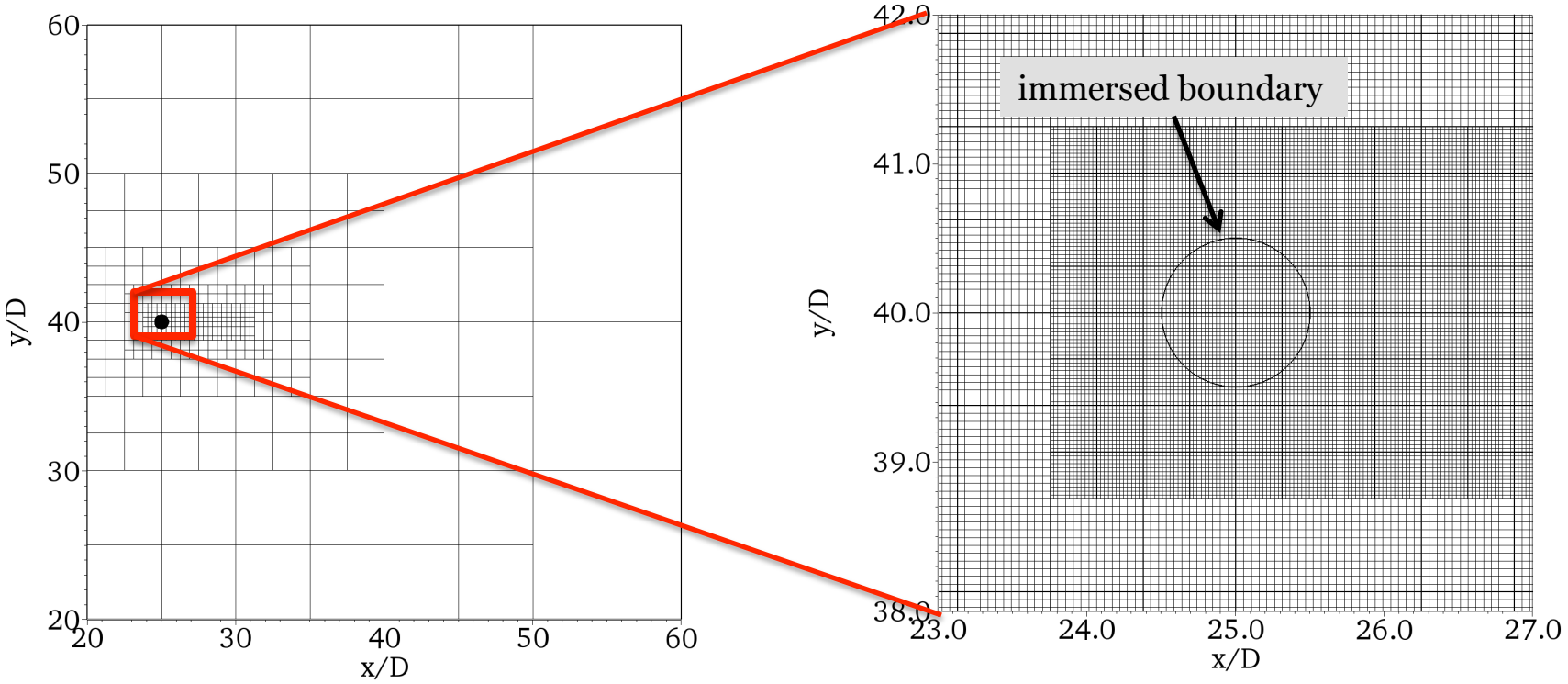
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Cylinder Flow

Flow parameters:
- U_∞ = 34.7 m/s
- p_∞ = 101327 Pa
- ρ_∞ = 1.1762 kg/m^3
- T_∞ = 300 K

- ❑ Domain size: length = 80D, height = 80D
- ❑ Cylinder location: x_o = 25D, y_o = 40D
- ❑ Resolution around cylinder: $\Delta x \approx D/60$
- ❑ Increased resolution in wake region to resolve shear layer instability



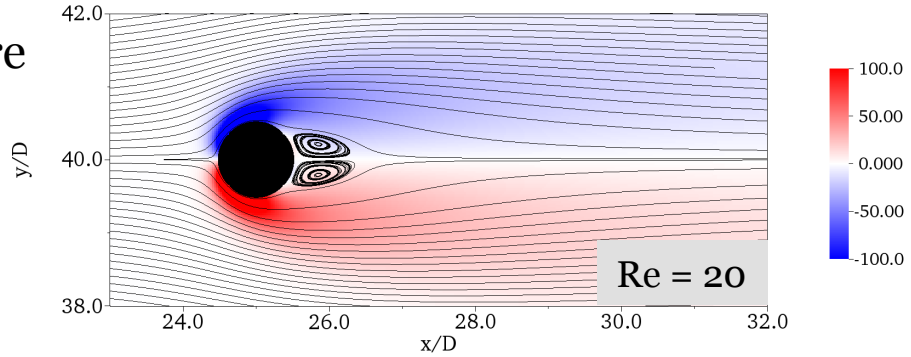
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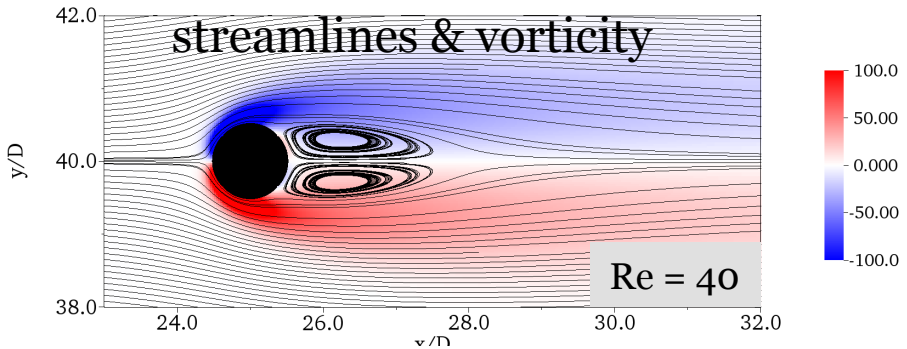
- Steady cases: Re = 20, 40
- Excellent match with results from literature

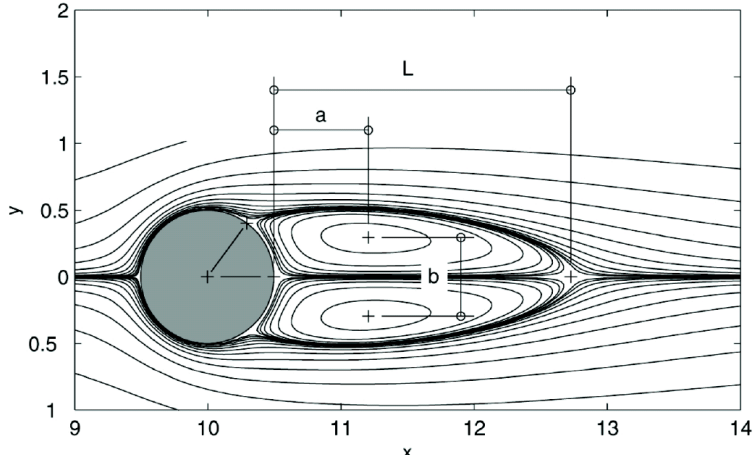
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- T_∞ = 300 K

	L	a	b	Θ	c_D
$Re_D = 20$:					
Fornberg[32]	0.91	—	—	45.7°	2.00
Dennis[33]	0.94	—	—	43.7°	2.05
Coutanceau[34]*	0.93	0.33	0.46	45.0°	—
Trittin[27]*	—	—	—	—	2.09
Linnick[7]	0.93	0.36	0.43	43.5°	2.06
Brehm[14]	0.94	0.36	0.43	43.5°	2.08
present study	**0.96**	**0.36**	**0.42**	**44°**	**2.02**
$Re_D = 40$:					
Fornberg[32]	2.24	—	—	55.6°	1.50
Dennis[33]	2.35	—	—	53.8°	1.52
Coutanceau[34]*	2.13	0.76	0.59	55.8°	—
Tritton[27]*	—	—	—	—	1.59
Linnick[7]	2.28	0.72	0.60	53.6°	1.54
Mittal[31]	—	—	—	—	1.53
Henderson[35]	—	—	—	—	1.54
Marella[36]	—	—	—	—	1.52
Brehm[14]	2.29	0.72	0.60	52.4°	1.57
present study	**2.26**	**0.72**	**0.58**	**52.9°**	**1.51**

(*) Results from Coutanceau[34] are extrapolated to $\lambda = 0$.







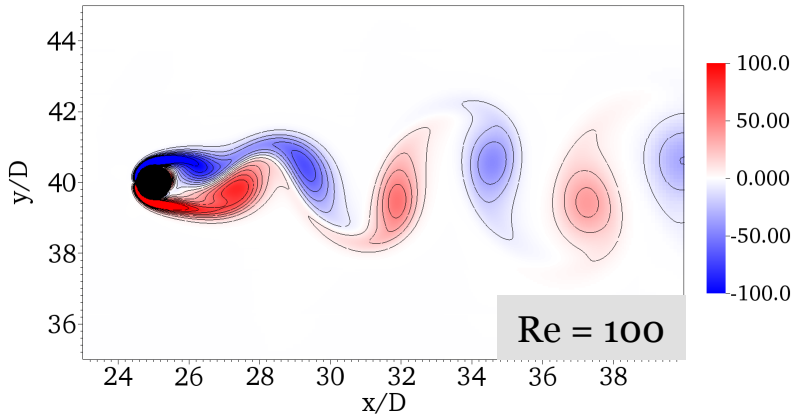
Cylinder Flow

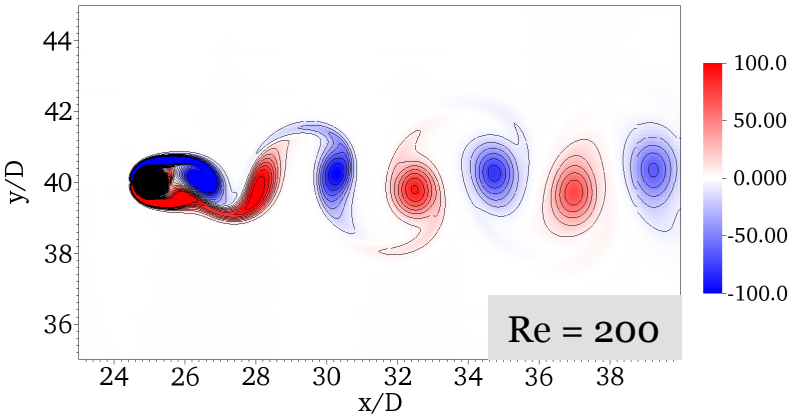
- Unsteady cases: Re = 100, 200
- Excellent match with results from literature

Flow parameters:
- U_∞ = 34.7 m/s
- p_∞ = 101327 Pa
- ρ_∞ = 1.1762 kg/m^3
- T_∞ = 300 K

	St	c_D	c_L
$Re_D = 100$:			
Berger[37]*	0.16–0.17	—	—
Liu[38]	0.165	1.35 ± 0.012	± 0.339
Linnick[7] ($\lambda = 0.056$)⊛	0.169	1.38 ± 0.010	± 0.337
Linnick[7] ($\lambda = 0.023$)⊛	0.166	1.34 ± 0.009	± 0.333
Mittal[31]	—	1.35	—
Henderson[35]	—	1.35	—
Marella[36]	—	1.36	—
Brehm[14]	0.170	1.38 ± 0.011	± 0.342
present study	**0.165**	**1.32 ± 0.01**	**± 0.32**
$Re_D = 200$:			
Berger[37]*	0.18–0.19	—	—
Belov[39]	0.193	1.19 ± 0.042	± 0.64
Rogers, Kwak*	0.185	1.23 ± 0.050	± 0.65
Miyake et al.*	0.196	1.34 ± 0.043	± 0.67
Liu[38]	0.192	1.31 ± 0.049	± 0.69
Linnick[7] ($\lambda = 0.056$)⊛	0.199	1.37 ± 0.046	± 0.70
Linnick[7] ($\lambda = 0.023$)⊛	0.197	1.34 ± 0.044	± 0.69
Brehm[14]	0.198	1.38 ± 0.044	± 0.70
present study	**0.192**	**1.3 ± 0.04**	**± 0.66**

vorticity



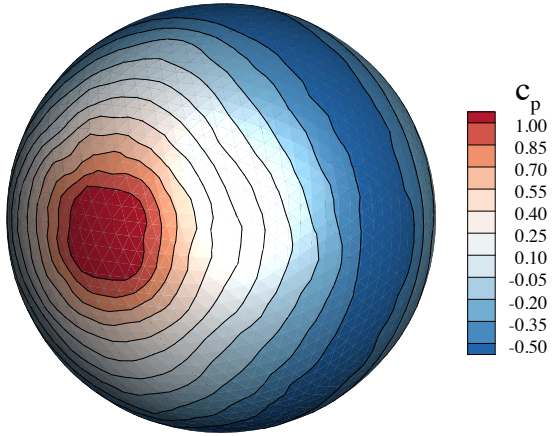


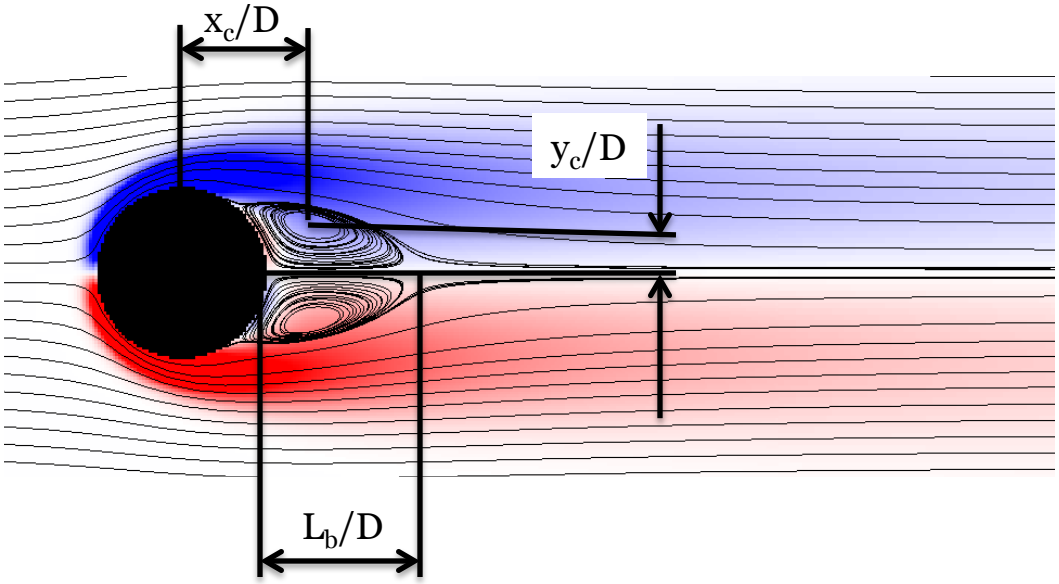
Flow Around Sphere

Flow parameters:
- U_∞ = 34.7 m/s
- p_∞ = 101327 Pa
- ρ_∞ = 1.1762 kg/m^3
- T_∞ = 300 K

- ❑ Steady case: Re = 100
- ❑ Domain size: length = 20D, height = 20D, width = 20D
- ❑ Resolution around cylinder: $\Delta x \approx D/60$
- ❑ Sphere location: x_0 = 5D, y_0 = 10D, z_0 = 10D
- ❑ Smooth pressure contours on immersed boundary
- ❑ Excellent match with results from literature

pressure contours



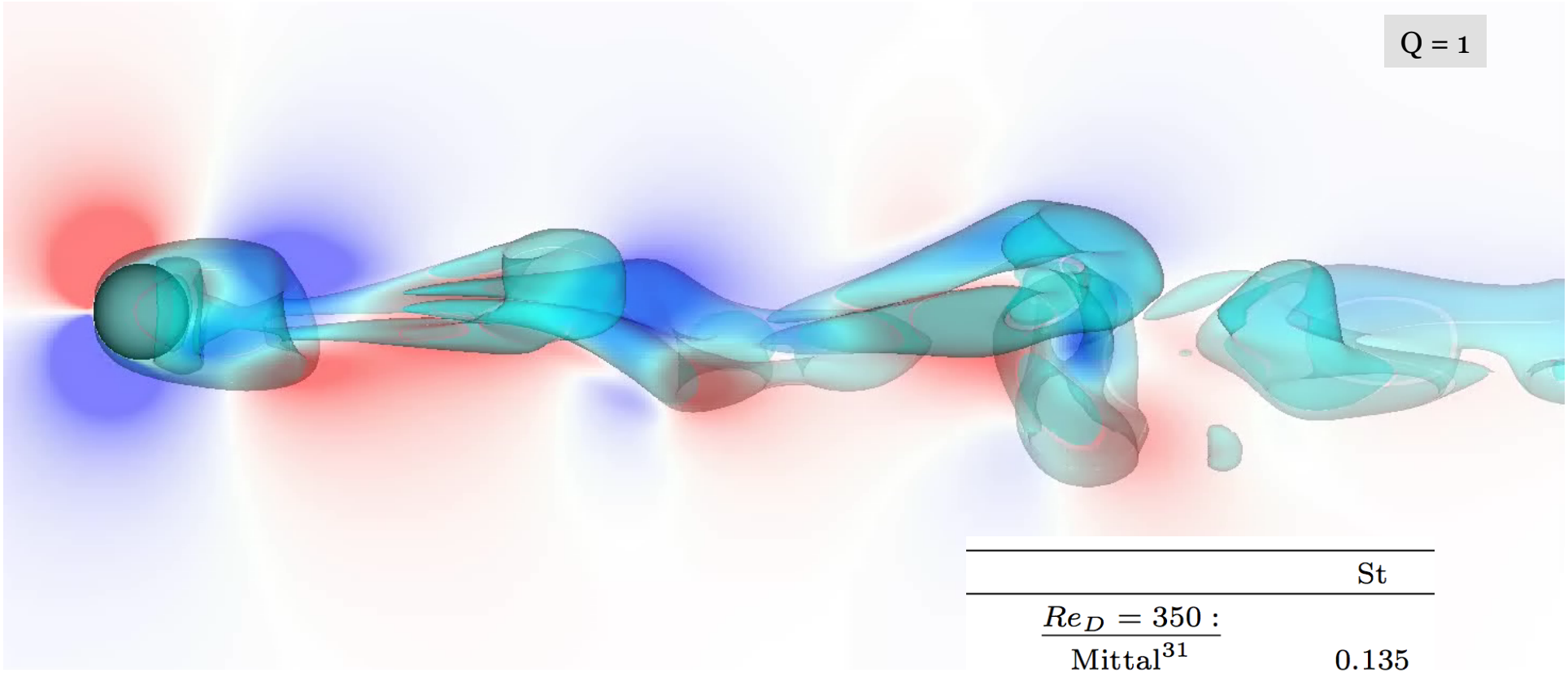


	x_c/D	y_c/D	L_b/D
$Re_D = 100$:			
Mittal[31]	0.742	0.278	0.84
Johnson and Patel[40]	0.75	0.29	0.88
Taneda[41]	0.745	0.28	0.8
Marella[36]	—	—	0.88
present study	**0.78**	**0.3**	**0.9**

Flow Around Sphere

Flow parameters:
- U_∞ = 34.7 m/s
- p_∞ = 101327 Pa
- ρ_∞ = 1.1762 kg/m^3
- T_∞ = 300 K

- ❑ Unsteady case: Re = 350
- ❑ Excellent match with results from literature

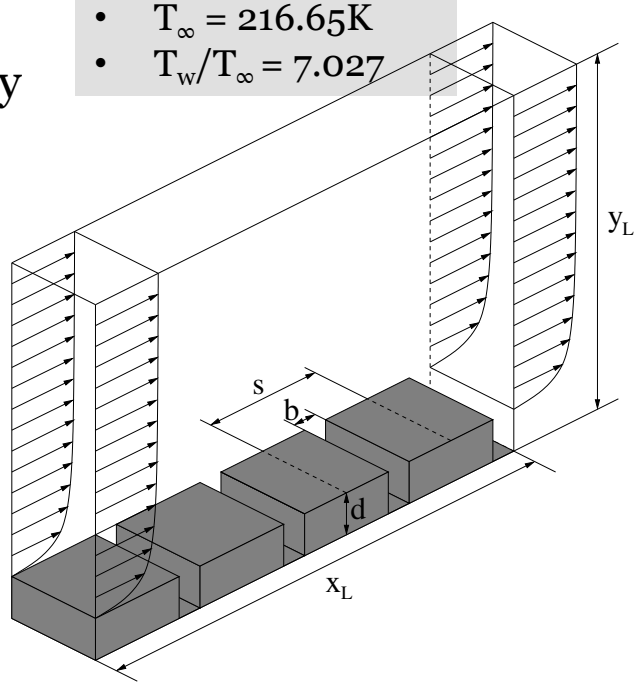


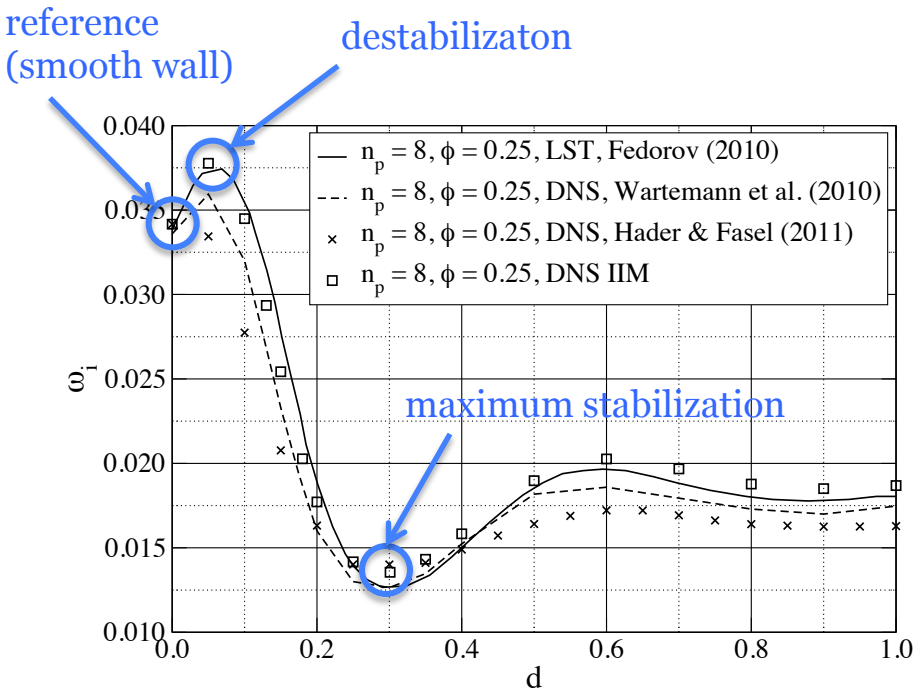
	St
$Re_D = 350$:	
Mittal[31]	0.135
Johnson and Patel[40]	0.137
Taneda[41]	—
Marella[36]	0.133
present study	**0.132**

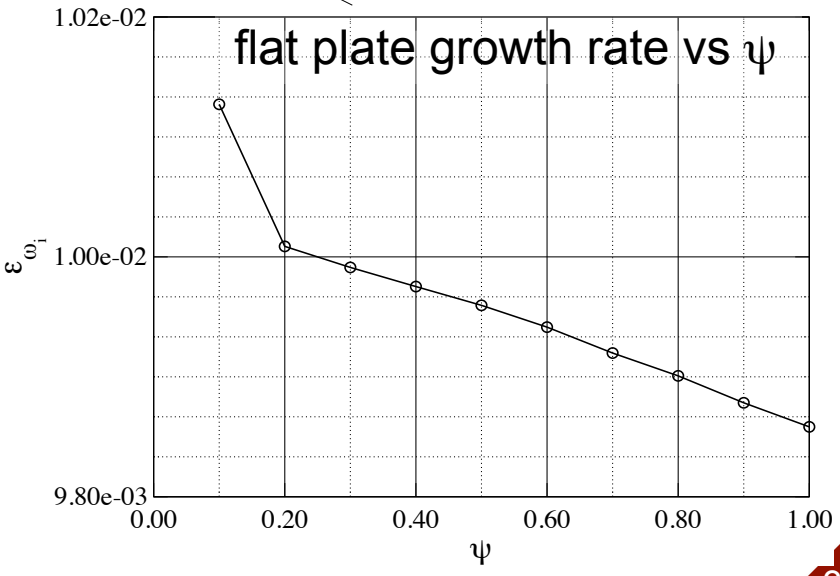
Hypersonic Transition

Flow parameters:
- M = 6
- Re_{δ^*} = 20000
- T_∞ = 216.65K
- T_w/T_∞ = 7.027

- Application of IIM: stabilization of boundary boundary layer disturbances through porous walls (TDNS)
- Physically resolving pores with IIM
- Good match between LST and linear DNS with IIM
- Temporal growth rate nearly independent of non-dimensional boundary distance (demonstrates robustness of method)



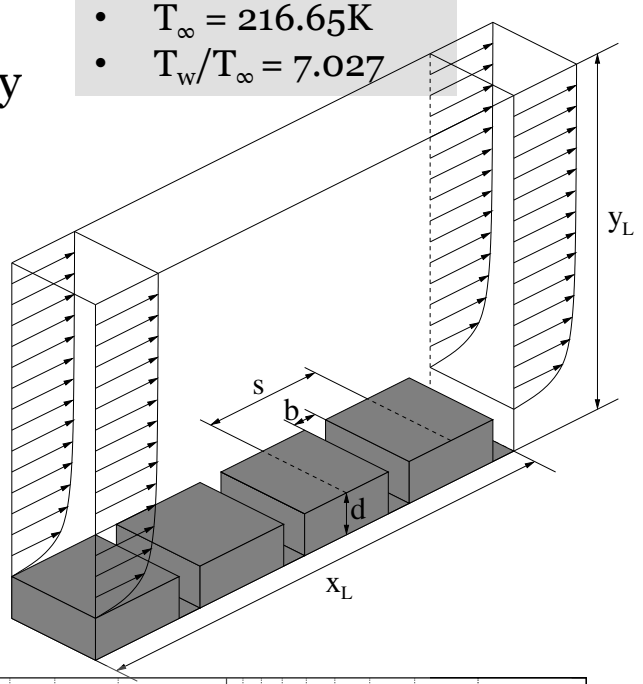


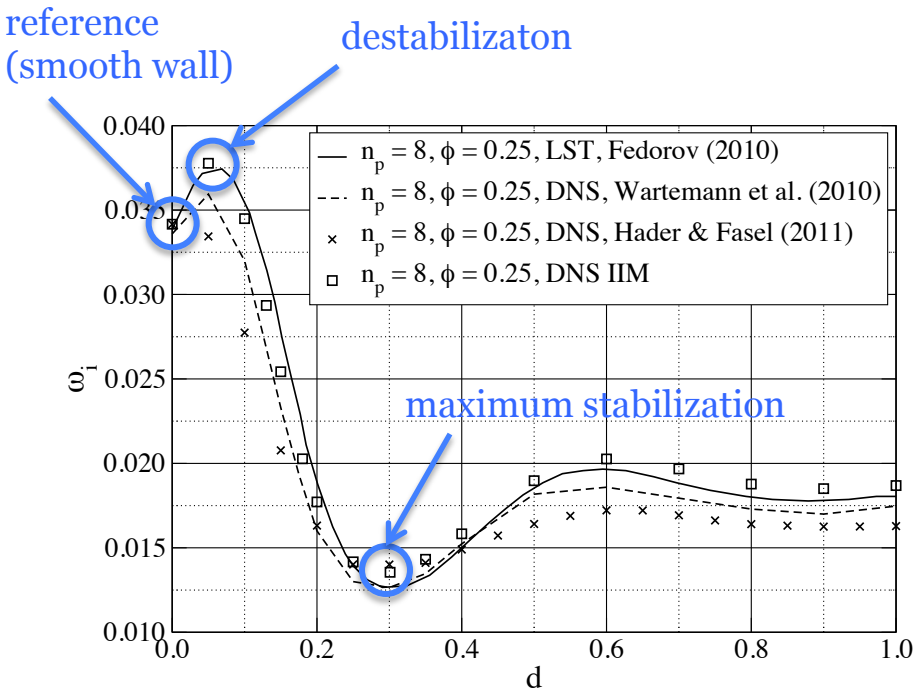


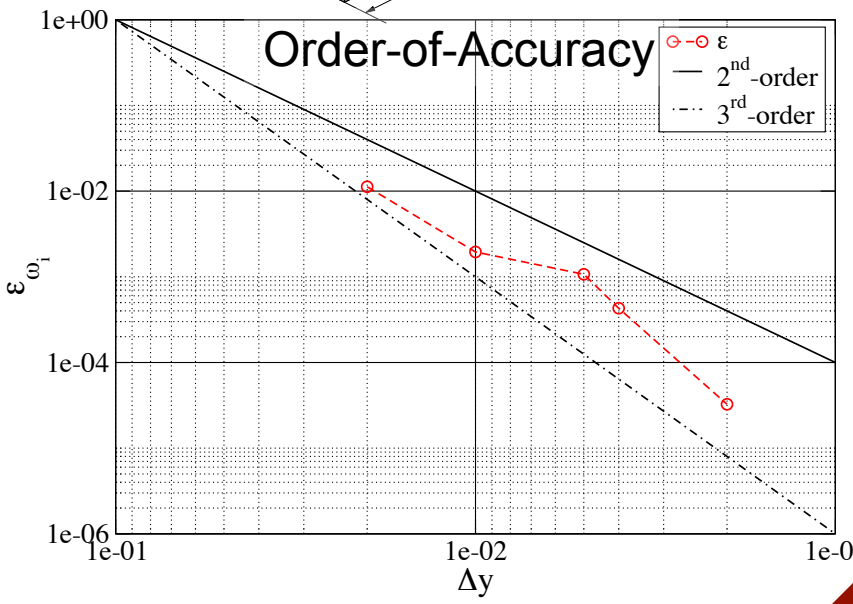
Hypersonic Transition

Flow parameters:
- M = 6
- Re_{δ^*} = 20000
- T_∞ = 216.65K
- T_w/T_∞ = 7.027

- Application of IIM: stabilization of boundary boundary layer disturbances through porous walls (TDNS)
- Physically resolving pores with IIM
- Good match between LST and linear DNS with IIM
- Temporal growth rate nearly independent of non-dimensional boundary distance (demonstrates robustness of method)

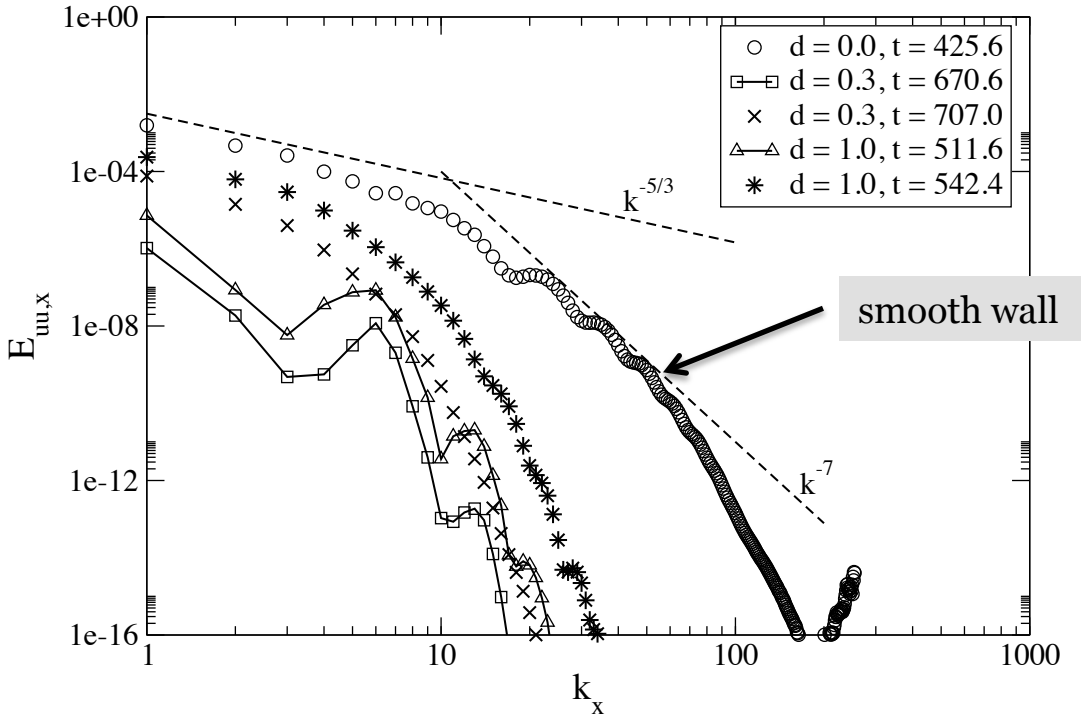






Hypersonic Transition

- Application of IIM in nonlinear regime
- Investigation of transition delay of porous walls
- Breakdown to small scales successfully delayed
- IIM successfully employed for linear and nonlinear high speed transition simulations



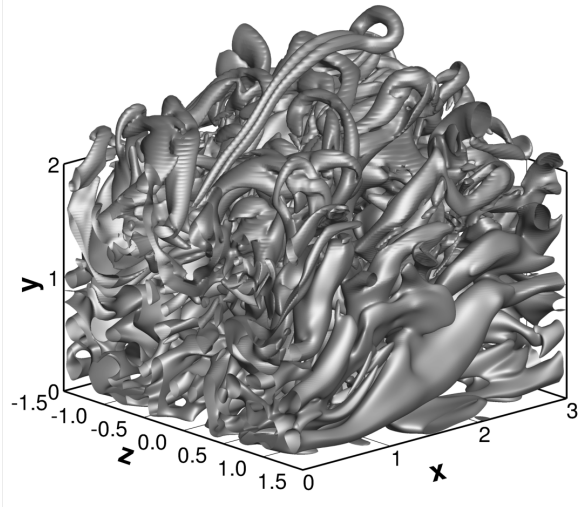
C. Hader, C. Brehm, H.F. Fasel

"Numerical Investigation of Transition Delay for Various Controlled Breakdown Scenarios in a Mach 6 Boundary Layer Using Porous Walls"

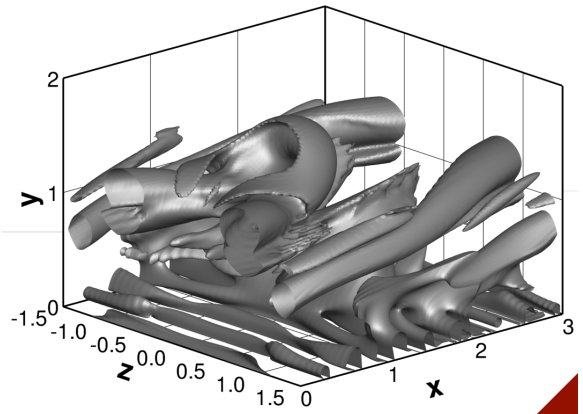
Flow parameters:

- M = 6
- Re_{δ^*} = 20000
- T_∞ = 216.65K
- T_w/T_∞ = 7.027

Smooth wall, t = 455

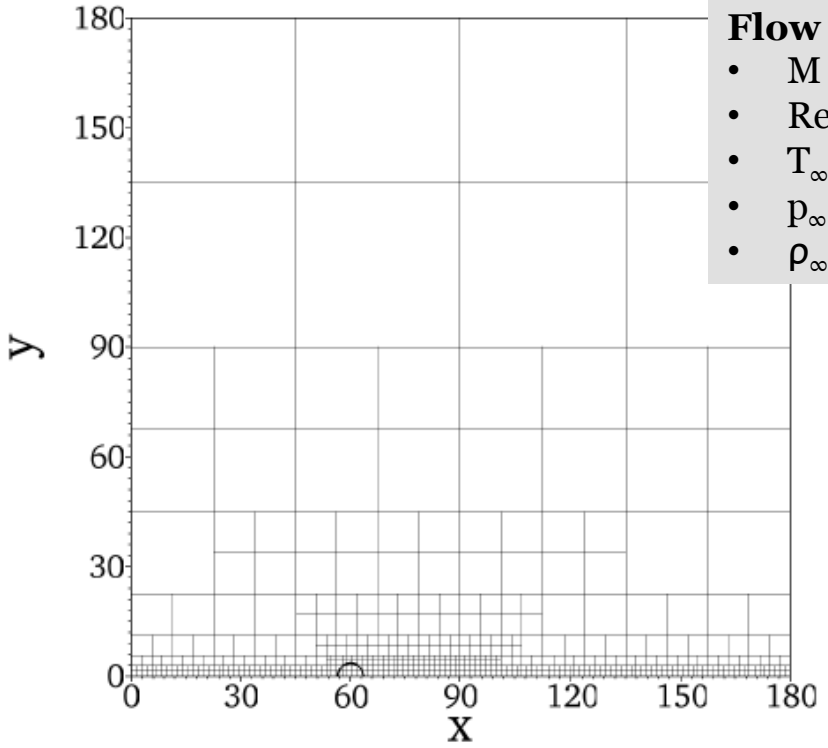


Porous wall, t = 543



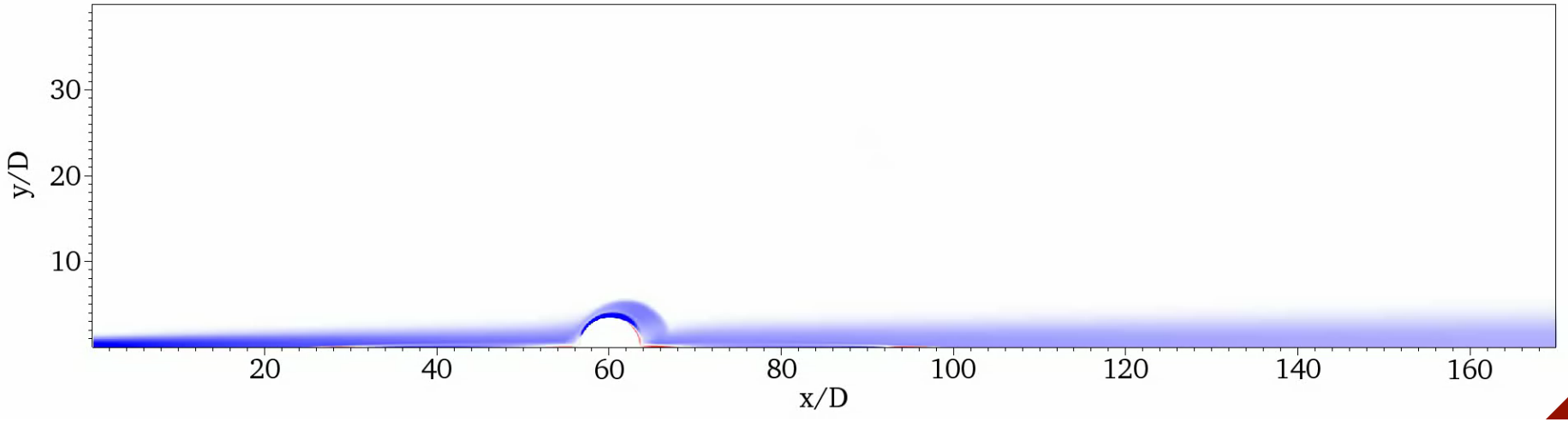
Flow Over Discrete Roughness

- 2D, $180\delta^* \times 180\delta^*$
- $n_{xb}=n_{yb}=20$
- $x_{in}=15\delta^*$
- Outflow buffer
- $y^+=1$
- Half circle, $R=\delta$
- Flow conditions as in Cherubini *et al.* (JFM, 2013)



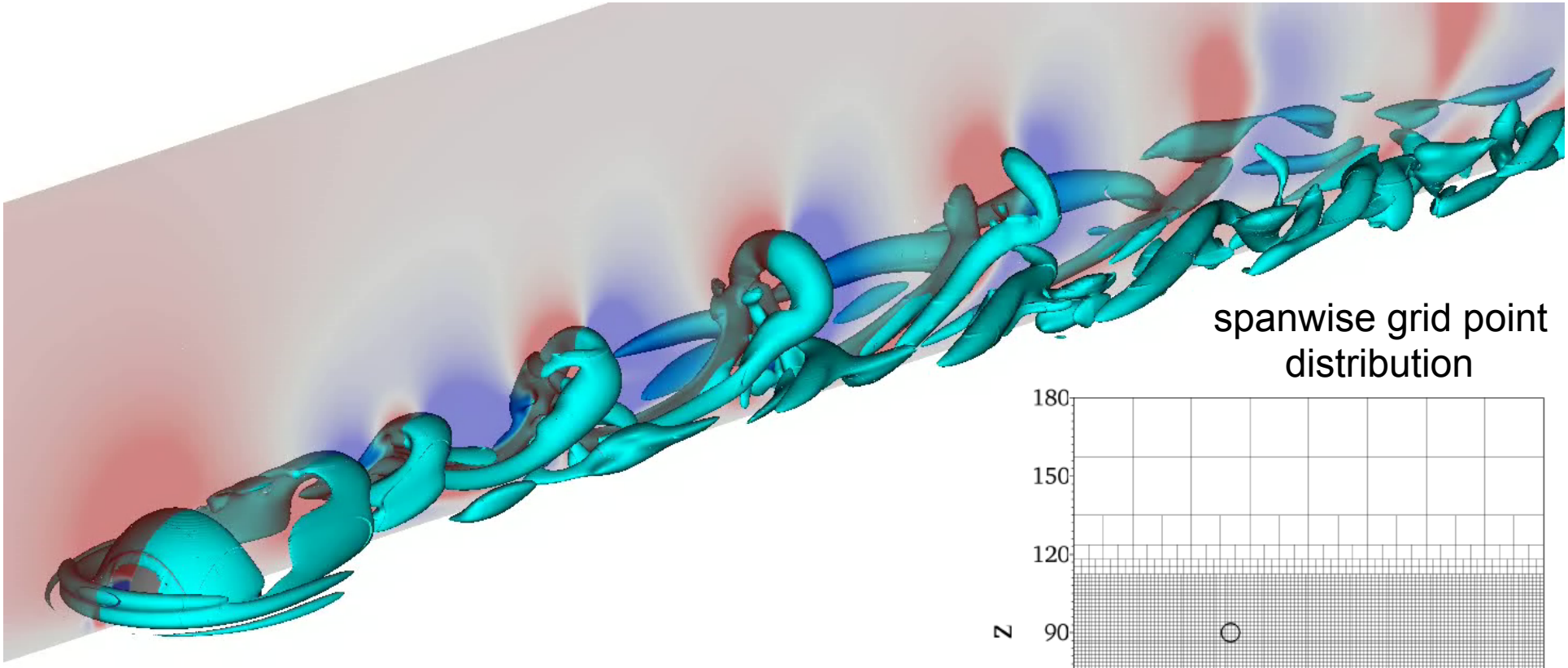
Flow Conditions:

- M = 0.1
- Re_{δ^*} = 164.57
- T_∞ = 300 K
- p_∞ = 101327 Pa
- ρ_∞ = 1.1762 kg/m^3



Flow Over Discrete Roughness

- 3D setup is the same as in 2D



spanwise grid point distribution

- Immersed boundary method is employed for biglobal stability analysis of discrete roughness element (continuation of Brehm and Fasel, 2012,2013)

Summary

- Higher-order immersed boundary method for compressible Navier-Stokes
  - Convective terms
  - Extrapolation of pressure and temperature
  - Viscous terms
- Optimization of irregular stencil coefficients with respect to numerical stability
- Superior stability characteristics
- Applied to various test problems

Outlook

- Improve method for viscous high Reynolds number flows
- Slip wall boundary conditions

Acknowledgements

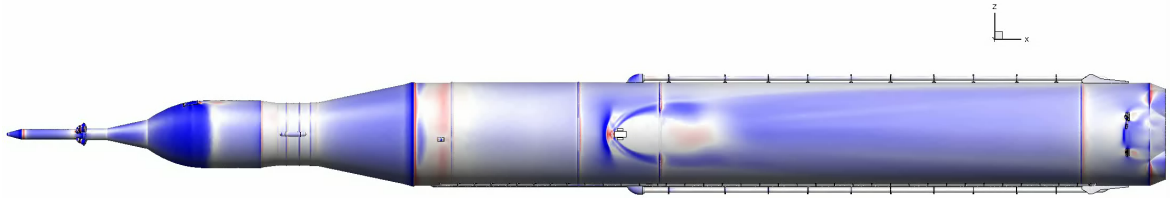
- Contributions from Christoph Hader and Prof. Fasel (University of Arizona)

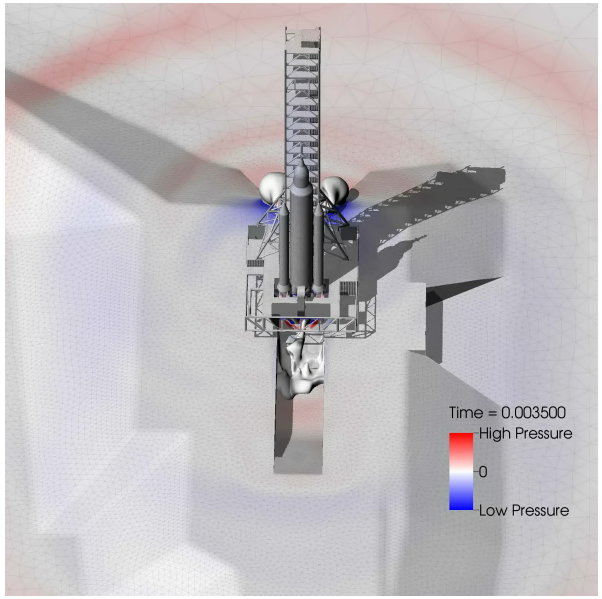
- Special thanks for contributions from LAVA group at Applied Modeling Simulations Branch, NASA Ames

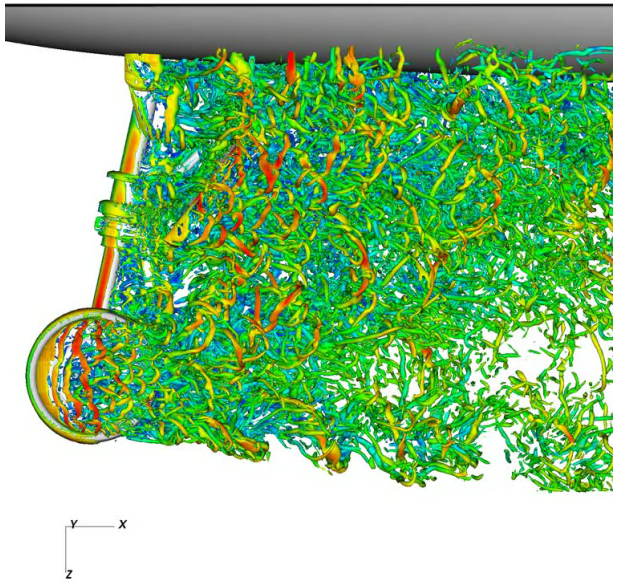
- Computer time provided by NASA Advanced Supercomputing (NAS) Division

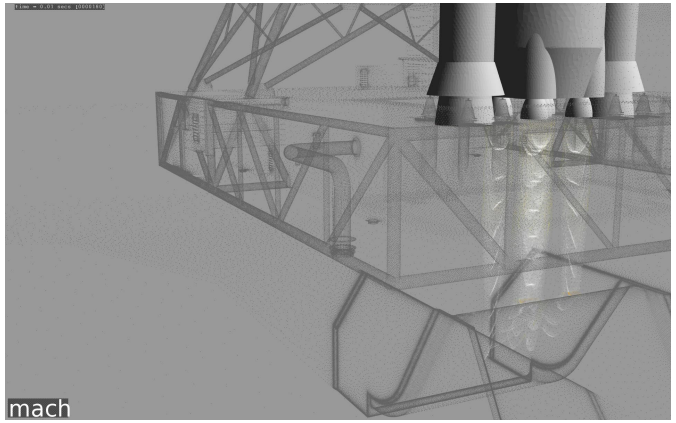
- Work was partially funded by Applied Modeling Simulations Branch ARC and Hypersonic Center for Laminar Turbulent Transition Research

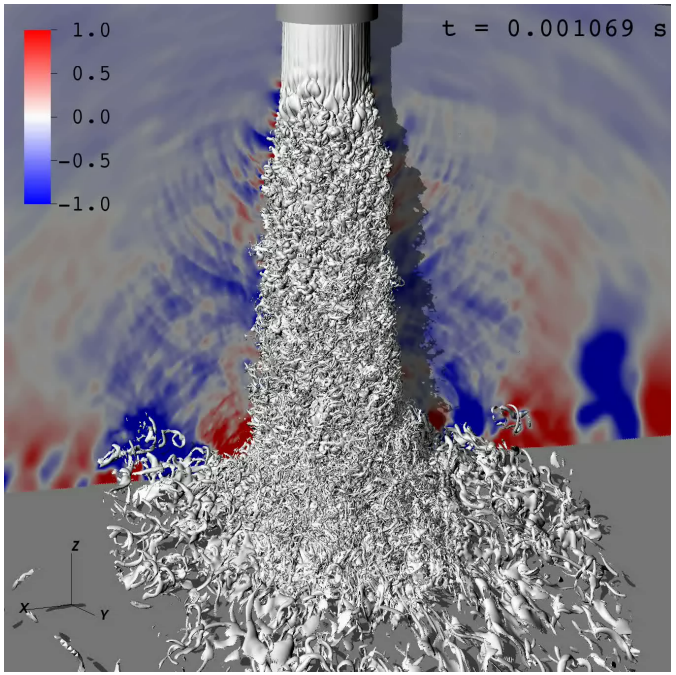
NASA Relevant IB Applications

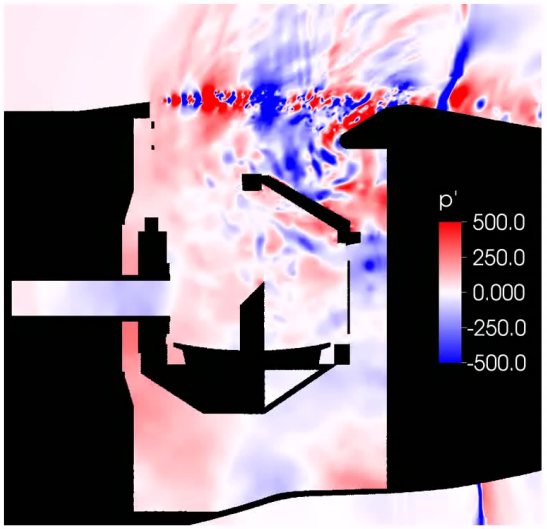


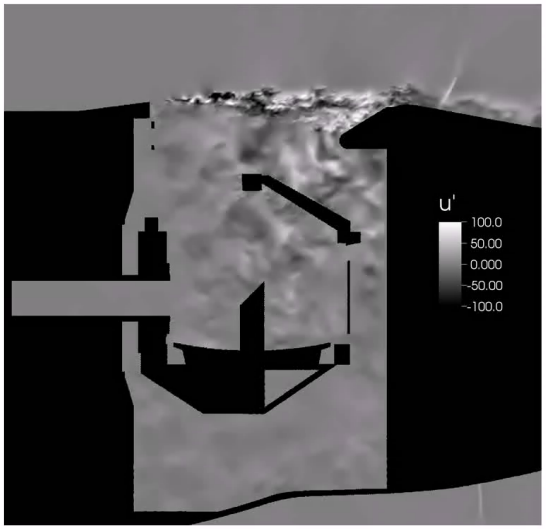












by LAVA (NASA Ames)