Find the exact solution for the following first order generic ODE:

$$\frac{du}{dt} = \lambda u + a \tag{1}$$

The above equation (1) can be rewritten as follows:

$$\frac{du}{dt} - \lambda u - a = 0$$

Multiplying throughout by $e^{-\lambda t}$

$$e^{-\lambda t}\frac{du}{dt} - \lambda u e^{-\lambda t} - a e^{-\lambda t} = 0$$
$$\frac{d}{dt}(u e^{-\lambda t}) - a e^{-\lambda t} = 0$$

Integrating both sides over dt and using c_1 as a constant of integration

$$u(t)e^{-\lambda t} + \frac{a}{\lambda}e^{-\lambda t} = c_1$$
$$u(t) = c_1 e^{\lambda t} - \frac{a}{\lambda}\frac{e^{-\lambda t}}{e^{-\lambda t}}$$

$$\boxed{u(t) = c_1 e^{\lambda t} - \frac{a}{\lambda}}$$

At $t = 0$,

$$u(0) = c_1 - \frac{a}{\lambda}$$
$$c_1 = u(0) + \frac{a}{\lambda}$$

Hence, the exact solution to equation (1) is:

$$u(t) = \left(u(0) + \frac{a}{\lambda}\right)e^{\lambda t} - \frac{a}{\lambda} \tag{2}$$

$$\boxed{u(t) = u(0)e^{\lambda t} - \frac{a}{\lambda}(1 - e^{\lambda t})}$$