Problem Set on Modified Wave Numbers

1. Find the expression for the modified wave number (ik^*) in the following centered difference approximations to $(\delta_x u)_j$ in terms of Δx and k. This is done just as done in class where we let $u_j = e^{ikj\Delta x}$. (Cast the results in terms of $sin(k\Delta x)$ and $cos(k\Delta x)$).

   (a) $(\delta_x u)_j = (u_{j+1} - u_{j-1})/(2\Delta x)$

   (b) $(\delta_x u)_j = (-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2})/(12\Delta x)$

   (c) $\frac{1}{6}((\delta_x u)_{j+1} + 4(\delta_x u)_j + (\delta_x u)_{j-1}) = (u_{j+1} - u_{j-1})/(2\Delta x)$

2. Find the expression for the modified wave number in the following one sided difference approximations to $(\delta_x u)_j$ in terms of Δx and k. In this case there will be real and imaginary parts to the modified wave number. (Cast the results in terms of $sin(k\Delta x)$ and $cos(k\Delta x)$).

   (a) $(\delta_x u)_j = (u_j - u_{j-1})/\Delta x$

   (b) $(\delta_x u)_j = (3u_j - 4u_{j-1} + u_{j-2})/(2\Delta x)$

   (c) $2(\delta_x u)_j + (\delta_x u)_{j-1} = (u_{j+1} + 4u_j - 5u_{j-1})/(2\Delta x)$

3. For problems 1 and 2 plot the resulting expressions for the $(k^*\Delta x)$ against $k\Delta x$ for $k = 1, 2, ...(M-1)/2$ with $M = 51$ and $\Delta x = 2\pi/(M-1)$. (You can use Matlab or anything else you want.)

   (a) For problem 1 the results should all be pure imaginary, i.e. ik^*. Plot the imaginary part of $(ik^*\Delta x)$ against $k\Delta x$.

   (b) For problem 2 the results should all be complex. Plot the imaginary part (a real number) against $k\Delta x$. Then plot exponential of (- real part $(ik^*)\Delta x$) against $k\Delta x$.