Problem Set 2 (*Instructor: T. Pulliam*)

Problem Set on Taylor Tables and Truncation Error

1. Using Taylor Tables find er_t for

   (a) $(\delta_x u)_j = (u_{j+1} - u_{j-1})/(2\Delta x)$

   (b) $(\delta_x u)_j = (-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2})/(12\Delta x)$

   (c) $\frac{1}{6}((\delta_x u)_{j+1} + 4(\delta_x u)_j + (\delta_x u)_{j-1}) = (u_{j+1} - u_{j-1})/(2\Delta x)$

2. The finite difference scheme for the 1st derivative is given by

$$(\delta_x u)_j = \frac{1}{\alpha \Delta x}(-u_{j+2} + \beta u_{j+1} - \beta u_{j-1} + u_{j-2})$$

   (a) Using Taylor Tables, find the values of α and β which result in a 4th order accurate method. *Hint: Multiply by $\alpha\Delta x$ to make the algebra easier*

   (b) Find er_t for the method.

   (c) If we set $\beta = 4$, find α and identify the order of acuracy.

3. Find, by means of a Taylor table, the values of a, b, c, and d that minimize the value of er_t in the expression

$$a\left(\frac{\partial u}{\partial x}\right)_{j-1} + \left(\frac{\partial u}{\partial x}\right)_j - \frac{1}{\Delta x}[bu_{j+1} + cu_j + du_{j-1}] = ? \tag{1}$$

   What is the resulting finite difference scheme and what is the value of er_t?

Problem Set on Modified Wave Numbers

4. Find the expression for the modified wave number (ik^*) in the following centered difference approximations to $(\delta_x u)_j$ in terms of Δx and k. This is done just as done in class where we let $u_j = e^{ikj\Delta x}$. (Cast the results in terms of $sin(k\Delta x)$ and $cos(k\Delta x)$).

   (a) $(\delta_x u)_j = (u_{j+1} - u_{j-1})/(2\Delta x)$

   (b) $(\delta_x u)_j = (-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2})/(12\Delta x)$

   (c) $\frac{1}{6}((\delta_x u)_{j+1} + 4(\delta_x u)_j + (\delta_x u)_{j-1}) = (u_{j+1} - u_{j-1})/(2\Delta x)$

5. Find the expression for the modified wave number in the following one sided difference approximations to $(\delta_x u)_j$ in terms of Δx and k. In this case there will be real and imaginary parts to the modified wave number. (Cast the results in terms of $sin(k\Delta x)$ and $cos(k\Delta x)$).

   (a) $(\delta_x u)_j = (u_j - u_{j-1})/\Delta x$

   (b) $(\delta_x u)_j = (3u_j - 4u_{j-1} + u_{j-2})/(2\Delta x)$

   (c) $2(\delta_x u)_j + (\delta_x u)_{j-1} = (u_{j+1} + 4u_j - 5u_{j-1})/(2\Delta x)$